**Schur Convexity and Schur-Geometrically Concavity of Seiffert’s Mean**

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**Abstract.** The Schur-concavity and Schur-geometrically convexity of the Seiffert’s mean with two positive numbers $a, b$ in $\mathbb{R}^2_{++}$ are discussed. Besides, some new inequalities are obtained.

**Keywords:** Seiffert’s mean, Schur-convexity, Schur-geometrically concavity, inequality

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§1 Introduction

Seiffert’s mean $^1, p.43$ of two positive numbers $a$ and $b$ is defined as follows

$$P = P(a, b) = \begin{cases} \frac{a - b}{4\arctan\frac{a/b - \pi}{a}} & a \neq b \\ a & a = b \end{cases}$$

In recent years, some further generalizations and applications about Seiffert’s mean have been obtained in [2-5] and the references therein.

In this paper, the Schur-concavity and Schur-geometrically convexity of the Seiffert’s mean with two positive numbers $a, b$ in $\mathbb{R}^2_{++} := (0, +\infty) \times (0, +\infty)$ are discussed. Besides, some new inequalities are obtained.

§2 Main Results

**Theorem 1.** $P(a, b)$ is Schur-concave with $(a, b)$ in $\mathbb{R}^2_{++}$.

**Theorem 2.** $P(a, b)$ is Schur-geometrically convex with $(a, b)$ in $\mathbb{R}^2_{++}$.

§3 Applications

**Theorem 3.** For $(a, b) \in \mathbb{R}^2_{++}$, with $a \leq b$, we have

$$G(a, b) \leq P\left(\frac{\frac{1}{2} \frac{1}{3} \frac{1}{2}}{a^2 b^2}, \frac{1}{2} \frac{1}{3} \frac{1}{2} \right) \leq P(a, b) \leq P\left(\frac{3a + b}{4}, \frac{a + 3b}{4}\right) \leq A(a, b),$$

where $G(a, b)$ and $A(a, b)$ is the arithmetic-mean and the geometry respectively.

**Theorem 4.** Let $0 < a < b$, $c \geq 0$. Then
\[ (a + b + 2c) \left( \arctan \frac{a + c}{\sqrt{b + c}} \right) - (a + b) \left( \arctan \frac{a}{\sqrt{b}} \right) \geq \frac{c\pi}{2}. \]

REFERENCES