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Crazy Representations and Selfie Numbers

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Abstract

This summary brings author's work on numbers divided in two parts. First part is on "*Crazy Representations of Natural Numbers*" and second is on "*Selfie Numbers*". The Selfie numbers may also be considered as generalized or wild narcissistic numbers, where natural numbers are represented by their own digits with certain operations.

The work is summarized as:

- 1 Numbers in terms of 1 to 9 and 9 to 1 [1];
- 2 Single Digit Representations [3];
- 3 Single Letter Representations [4, 8];
- 4 Palindromic and Number Patterns [9];
- 5 Fibonacci Sequences and Extensions [9];
- 6 Running Expressions [5];
- 7 Representations in Order of Digits Appearing in Numbers and Reverse [6];
- 8 Representations in Increasing and Decreasing Orders of Consecutive Digits [2];
- 9 Representations in Increasing and Decreasing Orders of Non Consecutive Digits [7].

PART I: CRAZY REPRESENTATIONS

In this part, natural numbers are represented in four different ways. The idea of Fibonacci sequence and its extension are also discussed.

1 Numbers in terms of 1 to 9 and 9 to 1

In 2014, a work is done writing natural numbers in increasing and decreasing orders of 1 to 9 and 9 to 1. See examples below:

$$\begin{aligned}
 999 &:= 12 \times 3 \times (4 + 5) + (67 + 8) \times 9 = 9 + 8 + 7 + 654 + 321. \\
 2535 &:= 1 + 2345 + (6 + 7 + 8) \times 9 = 9 + 87 \times (6 + 5 \times 4 + 3) + 2 + 1. \\
 2607 &:= 123 \times 4 \times 5 + 6 + (7 + 8) \times 9 = 987 + 6 \times 54 \times (3 + 2) \times 1 \\
 11807 &:= 1 \times 234 \times (5 + 6 \times 7) + 89 = -9 + 8 + 7 \times (6 + 5) \times (4 \times 3)^2 \times 1.
 \end{aligned}$$

For full work, refer to link below [1] (Jan., 2014):

<http://arxiv.org/abs/1302.1479>.

This work brings the numbers from 0 to 11111 using only basic operations, i.e., *addition*, *subtraction*, *multiplication*, *division* and *potentiation*. The only number not available with these operation is 10958, and can be written as

$$10958 = 12 \times 3 + \sqrt{4} + 5! \times (67 + 8 \times \sqrt{9}).$$

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2 Single Digit Representations

In the previous work all the nine digits are used to write natural numbers. Here the work is done writing numbers for each digit separately. See examples below:

$$\begin{aligned}
 717 &= (1 + 1)^{11} - 11^{(1+1+1)} & 995 &= (11 - 1)^{(1+1+1)} - (11 - 1)/(1 + 1) \\
 &= 22^2 + 222 + 22/2 & &= 22 + 2 \times (22^2 + 2) + 2/2 \\
 &= 3^{(3+3)} - 3 - 3 \times 3 & &= 3 \times 333 - 3 - 3/3 \\
 &= 4 \times (4 \times 44 + 4) - 4 + 4/4 & &= 4 \times (4^4 - 4 - 4) + 4 - 4/4 \\
 &= (55 \times (55 + 5 + 5) + 5 + 5)/5 & &= 5 \times (5 + 5) \times (5 \times 5 - 5) - 5 \\
 &= (6 \times 6/(6 + 6))^6 - 6 - 6 & &= 666 + 6 \times 66 - 66 - 6/6 \\
 &= 777 - 7 \times 7 - 77/7 & &= (7 + 7) \times (77 - 7) + 7 + 7 + 7/7 \\
 &= 8 \times 88 + (88 + 8 + 8)/8 & &= 888 + 88 + 8 + 88/8 \\
 &= 9 \times 9 \times 9 - (99 + 9)/9. & &= 999 - (9 + 9 + 9 + 9)/9.
 \end{aligned}$$

For full work, refer to link [3] (Feb. 2015):

<http://arxiv.org/abs/1502.03501>.

3 Single Letter Representations

In the previous work, it is observed that some numbers can be written in symmetric form, such as

$$\begin{aligned}
 5 &= \frac{11 - 1}{1 + 1} = \frac{22 - 2}{2 + 2} = \frac{33 - 3}{3 + 3} = \frac{44 - 4}{4 + 4} = \frac{55 - 5}{5 + 5} = \frac{66 - 6}{6 + 6} = \frac{77 - 7}{7 + 7} = \frac{88 - 8}{8 + 8} = \frac{99 - 9}{9 + 9}; \\
 6 &= \frac{11 + 1}{1 + 1} = \frac{22 + 2}{2 + 2} = \frac{33 + 3}{3 + 3} = \frac{44 + 4}{4 + 4} = \frac{55 + 5}{5 + 5} = \frac{66 + 6}{6 + 6} = \frac{77 + 7}{7 + 7} = \frac{88 + 8}{8 + 8} = \frac{99 + 9}{9 + 9};
 \end{aligned}$$

etc.

Motivated by this idea, instead working for each digit separately, we can work with a *single letter "a"* to write natural numbers. See examples below:

$$\begin{aligned}
 5 &:= (aa - a)/(a + a); \\
 6 &:= (aa + a)/(a + a); \\
 561 &:= (aaaa + aa)/(a + a); \\
 925 &:= (aaaaa - aa)/(aa + a); \\
 1089 &:= (aaaa - aa - aa)/a; \\
 4477 &:= (aaa/(a + a + a) \times aa \times aa)/(a \times a),
 \end{aligned}$$

where $a \in \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$, and $aa = 10^2 \times a + a$, etc.

For full work, refer to links below [4, 8](April, June, 2015):

<http://rgmia.org/papers/v18/v18a40.pdf>. or <http://rgmia.org/papers/v18/v18a73.pdf>

The first link is up to 3000 numbers, while second link extend it to 5000 numbers.

4 Palindromic and Number Patterns

The idea of single digit representations is applied to palindromic and number patterns. The study is also extended to prime patterns, doubly symmetric patterns, etc. See some examples below:

$$\begin{aligned}
 11 &:= (a \times aa)/(a \times a). \\
 121 &:= (aa \times aa)/(a \times a). \\
 12321 &:= (aaa \times aaa)/(a \times a). \\
 1234321 &:= (aaaa \times aaaa)/(a \times a). \\
 123454321 &:= (aaaaa \times aaaaa)/(a \times a). \\
 12345654321 &:= (aaaaaa \times aaaaaa)/(a \times a). \\
 1234567654321 &:= (aaaaaaa \times aaaaaaa)/(a \times a). \\
 123456787654321 &:= (aaaaaaaa \times aaaaaaaa)/(a \times a). \\
 12345678987654321 &:= (aaaaaaaaa \times aaaaaaaaa)/(a \times a).
 \end{aligned}$$

$$\begin{aligned}
 1156 &:= 34^2 = ((aa + aa + aa + a)/a)^{(a+a)/a}. \\
 111556 &:= 334^2 = ((aaa + aaa + aaa + a)/a)^{(a+a)/a}. \\
 11115556 &:= 3334^2 = ((aaaa + aaaa + aaaa + a)/a)^{(a+a)/a}. \\
 1111155556 &:= 33334^2 = ((aaaaa + aaaaa + aaaaa + a)/a)^{(a+a)/a}. \\
 111111555556 &:= 333334^2 = ((aaaaaa + aaaaaa + aaaaaa + a)/a)^{(a+a)/a}. \\
 11111115555556 &:= 3333334^2 = ((aaaaaaa + aaaaaaa + aaaaaaa + a)/a)^{(a+a)/a}. \\
 1111111155555556 &:= 33333334^2 = ((aaaaaaaa + aaaaaaaa + aaaaaaaa + a)/a)^{(a+a)/a}.
 \end{aligned}$$

$$\begin{aligned}
 99 = 98 + 1 & & :=(aaa - aa - a)/(a \times a). \\
 999 = 987 + 12 & & :=(aaaa - aaa - a)/(a \times a). \\
 9999 = 9876 + 123 & & :=(aaaaa - aaaa - a)/(a \times a). \\
 99999 = 98765 + 1234 & & :=(aaaaaa - aaaaa - a)/(a \times a). \\
 999999 = 987654 + 12345 & & :=(aaaaaaa - aaaaaa - a)/(a \times a). \\
 9999999 = 9876543 + 123456 & & :=(aaaaaaaa - aaaaaaa - a)/(a \times a). \\
 99999999 = 98765432 + 1234567 & & :=(aaaaaaaaa - aaaaaaaa - a)/(a \times a). \\
 999999999 = 987654321 + 12345678 & & :=(aaaaaaaaaa - aaaaaaaaa - a)/(a \times a). \\
 9999999999 = 9876543210 + 123456789 & & :=(aaaaaaaaaaa - aaaaaaaaaa - a)/(a \times a).
 \end{aligned}$$

For full work, refer to link below [9](July, 2015):

<http://rgmia.org/papers/v18/v18a99.pdf>.

5 Fibonacci Sequence and Extensions

Fibonacci sequence and its extensions are well known in the literature. Its extensions or variations are: *Lucas sequences*, *Tribonacci sequence*, *Tetranacci sequence*, etc. The following pattern represent the denominators in

each case. The numerator is always 1. Here below are patterns to get initial values of each sequence:

Fibonacci sequence	Lucas Sequence	Tribonacci Sequence	Tetranacci sequence
$\frac{1}{89}$	$\frac{19}{89}$	$\frac{1}{889}$	$\frac{1}{8889}$
$\frac{1}{9899}$	$\frac{199}{9899}$	$\frac{1}{989899}$	$\frac{1}{98989899}$
$\frac{1}{998999}$	$\frac{1999}{998999}$	$\frac{1}{998998999}$	$\frac{1}{998998998999}$
$\frac{1}{99989999}$	$\frac{19999}{99989999}$	$\frac{1}{999899989999}$	$\frac{1}{9998999899989999}$
$\frac{1}{9999899999}$	$\frac{199999}{9999899999}$	$\frac{1}{999989999899999}$	$\frac{1}{99998999989999899999}$

Denominators satisfies very interesting symmetrical property:

Fibonacci sequence	Tribonacci sequence	Tetranacci sequence
$89 = 10^2 - 11$	$889 = 10^3 - 111$	$8889 = 10^4 - 1111$
$9899 = 10^4 - 101$	$989899 = 10^6 - 10101$	$98989899 = 10^8 - 1010101$
$998999 = 10^6 - 1001$	$998998999 = 10^9 - 1001001$	$998998998999 = 10^{12} - 1001001001$
$99989999 = 10^8 - 10001$	$999899989999 = 10^{12} - 100010001$	$9998999899989999 = 10^{16} - 1000100010001.$

In case of Lucas sequence the difference is only in the numerator. In case of Fibonacci sequence is 1 while in case of Lucas sequence it is a pattern as 19, 199, etc. For more details refer to [9](July, 2015):

<http://rgmia.org/papers/v18/v18a99.pdf>

6 Running Expressions

It is well know that one can write, $12 = 3 \times 4$, $56 = 7 \times 8$. Here 9 remains alone. The aim of this work is to see how we can write expressions using 9 digits in a sequence in increasing and decreasing ways. The decreasing case 9 to 0 is also considered. The expressions are separated either by single or by double equality signs. See example below:

$$1 + 23 + 45 + 6! = 789$$

$$1234 = -5 + 6! + 7 + 8^{\sqrt{9}}$$

$$987 = 6! + 5! + (4 + 3) \times 21.$$

$$24 := 1 + 23 = 4 + 5!/6 = 7 + 8 + 9$$

$$:= 9 + 8 + 7 = (6 - 5) \times 4! = 3 + 21.$$

$$120 := 1 \times (2 + 3)! = 4 + 5!/6 + 7 + 89$$

$$:= 98 + 7 + 6 + 5 + 4 = (3 + 2)! \times 1$$

$$:= \sqrt{9} + 87 + 6 \times 5 = \sqrt{4} \times 3 \times 2 \times 10.$$

For full work, refer to the link below [5] (March, 2015):

<http://rgmia.org/papers/v18/v18a27.pdf>.

PART II: SELFIE NUMBERS

Numbers represented by their own digits by certain operations are understood as "Selfie Number". These numbers are divided in two categories. These two categories are again divided in two each, i.e., one in order of digits appearing in the numbers and their reverse, and the second is in increasing and decreasing order of digits. In increasing and decreasing order of digits, again two categories are worked, one for consecutive digits and another for non consecutive.

7 Representations in Order of Digits Appearing in Numbers and Reverse

Here the numbers are written in terms of same digit as appearing in the numbers and their reverse order. See examples below:

7.1 Same Order of Digits

$$\begin{aligned} 936 &= (\sqrt{9})!^3 + 6!; \\ 1296 &= \sqrt{(1+2)!^9/6}; \\ 2896 &= 2 \times (8 + (\sqrt{9})!! + 6!); \\ 12969 &= 1 \times 2 \times 9 \times 6! + 9. \end{aligned}$$

7.2 Reverse Order

$$\begin{aligned} 936 &= 6! + (3!)^{\sqrt{9}}; \\ 1296 &= 6^{(\sqrt{9}+2-1)}; \\ 2896 &= (6! + (\sqrt{9})!! + 8) \times 2; \\ 20167 &= 7 + (6 + 1 + 0)!/2. \end{aligned}$$

Some Interesting symmetrical results are also found, for example:

$14400 := (1+4)!^{\sqrt{4}} + 0 + 0 = 0 + (0!+4)!^{\sqrt{4}} \times 1$	$64800 = 6!^{\sqrt{4}}/8 + 0 + 0.$
$14401 := (1+4)!^{\sqrt{4}} + 0 + 1 = 1 + (0!+4)!^{\sqrt{4}} \times 1$	$64801 = 6!^{\sqrt{4}}/8 + 0 + 1.$
$14402 := (1+4)!^{\sqrt{4}} + 0 + 2 = 2 + (0!+4)!^{\sqrt{4}} \times 1$	$64802 = 6!^{\sqrt{4}}/8 + 0 + 2.$
$14403 := (1+4)!^{\sqrt{4}} + 0 + 3 = 3 + (0!+4)!^{\sqrt{4}} \times 1$	$64803 = 6!^{\sqrt{4}}/8 + 0 + 3.$
$14404 := (1+4)!^{\sqrt{4}} + 0 + 4 = 4 + (0!+4)!^{\sqrt{4}} \times 1$	$64804 = 6!^{\sqrt{4}}/8 + 0 + 4.$
$14405 := (1+4)!^{\sqrt{4}} + 0 + 5 = 5 + (0!+4)!^{\sqrt{4}} \times 1$	$64805 = 6!^{\sqrt{4}}/8 + 0 + 5.$
$14406 := (1+4)!^{\sqrt{4}} + 0 + 6 = 6 + (0!+4)!^{\sqrt{4}} \times 1$	$64806 = 6!^{\sqrt{4}}/8 + 0 + 6.$
$14407 := (1+4)!^{\sqrt{4}} + 0 + 7 = 7 + (0!+4)!^{\sqrt{4}} \times 1$	$64807 = 6!^{\sqrt{4}}/8 + 0 + 7.$
$14408 := (1+4)!^{\sqrt{4}} + 0 + 8 = 8 + (0!+4)!^{\sqrt{4}} \times 1$	$64808 = 6!^{\sqrt{4}}/8 + 0 + 8.$
$14409 := (1+4)!^{\sqrt{4}} + 0 + 9 = 9 + (0!+4)!^{\sqrt{4}} \times 1$	$64809 = 6!^{\sqrt{4}}/8 + 0 + 9.$

In the increasing case, above two sequences can be extended up to 14499 and 64899 respectively.

$$14410 := (1 + 4)!^{\sqrt{4}} + 10.$$

$$64810 := 6!^{\sqrt{4}}/8 + 10.$$

$$14411 := (1 + 4)!^{\sqrt{4}} + 11.$$

$$64811 := 6!^{\sqrt{4}}/8 + 11.$$

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$$14498 := (1 + 4)!^{\sqrt{4}} + 98.$$

$$64898 := 6!^{\sqrt{4}}/8 + 98.$$

$$14499 := (1 + 4)!^{\sqrt{4}} + 99.$$

$$64899 := 6!^{\sqrt{4}}/8 + 99.$$

These are only two numbers, where we maximum number of sequential representations for five digits. Still, we have the following two symmetrical patterns:

$36 := 3! \times 6.$	$5^2 := 25 = 5^2.$
$360 := 3! \times 60.$	$5^3 := 125 = 5^{2+1}.$
$3600 := 3! \times 600.$	$5^4 := 625 = 5^{-2+6}.$
$36000 := 3! \times 6000.$	$5^5 := 3125 = 5^{2+1 \times 3}.$
$360000 := 3! \times 60000.$	$5^6 := 15625 = 5^{(2 \times 6 - 5 - 1)}.$
$3600000 := 3! \times 600000.$	$5^7 := 78125 = 5^{2 + \sqrt{18+7}}.$
$36000000 := 3! \times 6000000.$	$5^8 := 390625 = 5^{2+6+0 \times 9 \times 3}.$

For full details, refer to link [6] (April, 2015):

<http://rgmia.org/papers/v18/v18a32.pdf>

Above study is done for representations of numbers in digit's order and their reverse. The work is done only up to five digits. For higher order, the quantity of numbers is too high to write in a paper, for example, for 6 digits, there are approx. 60000 numbers.

Following two sections are working with numbers where we don't have repetition of digits. Representations are considered for consecutive and non consecutive digits in increasing and decreasing orders for each case.

8 Representations in Increasing and Decreasing Orders of Consecutive Digits

Here we shall represent numbers in consecutive order of digits. The results are obtained in two ways, one in increasing order and another in decreasing order. See some examples below:

8.1 Increasing Order of digits

$$456 = 4 \times (5! - 6);$$

$$573846 = -3!! - \sqrt{4} + (5! - 6) \times 7! + 8;$$

$$3654127 = 123 + 4 + (5 + 6!) \times 7!;$$

$$165479328 = (12 + 3!! \times 456) \times 7 \times 8 \times 9.$$

8.2 Decreasing Order of Digits

$$\begin{aligned} 3456 &= 6/5 \times 4 \times 3!!; \\ 40312 &= (4!/3)! + 2 - 10; \\ 13287456 &= (8 + 76) \times (54^3 + (2 + 1)!!); \\ 178459362 &= \sqrt{\sqrt{9^8}} \times (765 \times 4 \times 3!! + 2 \times 1). \end{aligned}$$

For full details, refer to link [2] (Nov., 2014):

<http://rgmia.org/papers/v17/v17a140.pdf>.

Following order in both sides, we have only few numbers:

$$\begin{aligned} 456 &= (-6 + 5!) \times 4. \\ 3456 &= 6/5 \times 4 \times 3!!; \\ 34567 &= 7 + 6! \times (5 + 43). \end{aligned}$$

Some interesting "twin selfies" are found. See examples below:

$$\begin{aligned} 645879 &= 4^5 \times (6 + 7!)/8 - 9. & 256734 &= (2 \times 3 + 45) \times (-6 + 7!). \\ 645897 &= 4^5 \times (6 + 7!)/8 + 9. & 257346 &= (2 \times 3 + 45) \times (+6 + 7!). \\ 64385279 &= 23 \times \sqrt{4} \times 5 \times 6^7 + 8 - 9. & 51862734 &= (8! + (7 + 6)^5 - 4) \times 3! \times 21. \\ 64385297 &= 23 \times \sqrt{4} \times 5 \times 6^7 + 8 + 9. & 51863742 &= (8! + (7 + 6)^5 + 4) \times 3! \times 21. \end{aligned}$$

Here we worked with numbers with representations in increasing and decreasing orders in consecutive way. In the decreasing case ending with zero, the work is done only up to seven digits. For further order, 8, 9 and 10 digits ending in zero, shall be done elsewhere.

9 Representations in Increasing and Decreasing Orders of Non Consecutive Digits

Previous section worked with consecutive representations. Here the work is for non-consecutive digits but the representation are in increasing and decreasing orders. See some examples below:

9.1 Increasing Order of Digits

$$\begin{aligned} 936 &= 3!! + 6^{\sqrt{9}}; \\ 1296 &= (1 + 2)! \times 6^{\sqrt{9}}; \\ 8397 &= -3 - 7! + 8!/\sqrt{9}; \\ 241965 &= (1 + (2 \times 4)! + 5) \times 6 + 9. \end{aligned}$$

9.2 Decreasing Order of Digits

$$\begin{aligned}
 936 &= (\sqrt{9})!! + 6^3; \\
 1296 &= ((\sqrt{9})! \times 6)^2 \times 1; \\
 20148 &= (8! - 4)/2 - 10; \\
 435609 &= 9 + (6! - 5!/\sqrt{4})^{(3-0!)}.
 \end{aligned}$$

As in section 6, here also, some interesting symmetries are found. Here below are some examples,

$15637 = -1 + 3! + 5^6 + 7 = 7 + 6 + 5^{3!} - 1.$	$790 = (\sqrt{9})!! + 70.$
$15638 = -1 + 3! + 5^6 + 8 = 8 + 6 + 5^{3!} - 1.$	$791 = (\sqrt{9})!! + 71.$
$15639 = -1 + 3! + 5^6 + 9 = 9 + 6 + 5^{3!} - 1.$	$792 = (\sqrt{9})!! + 72.$
$30245 = 5 + (4 + 3)! \times (2 + 0)!.$	$793 = (\sqrt{9})!! + 73.$
$30246 = 6 + (4 + 3)! \times (2 + 0)!.$	$794 = (\sqrt{9})!! + 74.$
$30247 = 7 + (4 + 3)! \times (2 + 0)!.$	$795 = (\sqrt{9})!! + 75.$
$30248 = 8 + (4 + 3)! \times (2 + 0)!.$	$796 = (\sqrt{9})!! + 76.$
$30249 = 9 + (4 + 3)! \times (2 + 0)!.$	

We observe that the numbers 936 and 1296 are in both ways (increasing and decreasing) appearing in Sections 5 and 7. Here below are some more examples written in four different ways:

$$\begin{aligned}
 35278 &= 3! + (5 + 2) \times 7! - 8 = 8! - 7! - \sqrt{25} + 3 = -2 + (3 + 5)! \times 7/8 = 8! \times 7/(5 + 3) - 2. \\
 97632 &= -(\sqrt{9})!! + 7! + 6^{3!} \times 2 = 2 \times 3!^6 + 7! - (\sqrt{9})!! = 2 \times 3!^6 + 7! - (\sqrt{9})!! = -(\sqrt{9})!! + 7! + 6^{3!} \times 2. \\
 23694 &= (-2 + (\sqrt{36})!) \times (9 + 4!) = (4! + 9) \times ((6 - 3)!! - 2) = (-2 + 3!!) \times (4 \times 6 + 9) = (9 + 6 \times 4) \times (3!! - 2).
 \end{aligned}$$

For full details, refer to the link [7](May, 2015):

<http://rgmia.org/papers/v18/v18a70.pdf>.

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