

Multi-Digits Magic Squares

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Abstract. *In this work, we shall construct magic squares written in multi-digits formats. In some situations, different digits are considered according to order of magic squares. Magic squares related to combination with repetitions are also considered. In each case study is extended to palindromic magic squares.*

1. Introduction

Magic squares are generally constructed using sequential or continued numbers such as $1, 2, \dots, n^2$. Here in this work we shall write magic squares using *multi-digits in a symmetrical form or equally distributed*. By *symmetrical form or equally distributed* we understand that for each magic square, the number of digits in each cell is of same order, for example, 3-digits, 112, 121, 133, etc; 4-digits, 1441, 2331, 1115, etc.; 5-digits, 25532, 12221, 23334, etc. This we have considered in three forms:

- (i) **Combinations without Repetitions** - These are understood as *magic squares* formed by *combinations of digits without repetitions*, where in each cell there is no repetition, such as 1243, 1423, 1234, etc. Number of digits considered in each cell are of same order as of magic square. For simplicity, we shall write as *different digits magic squares*.

In case of order 3, we know that there are only $3!=6$ different digits having the three numbers 1, 2 and 3, such as, 123, 132, 213, 231, 312 and 321. For further orders, we have much more possibilities, i.e., for order 4: $4!=24$; for order 5: $5!=120$; for order 6: $6!=720$, etc. We have given *different digits magic squares* for the orders 4 to 10.

- (ii) **Combinations with repetitions** - These kind of magic square are considered writing combinations with repetitions, such as, 112, 121, 122, etc. See the table below:

Possibilities	Combinations	Examples	Magic square
$3^3=27$	1,2,3 - 3 by 3	123, 113, 133, etc	-
$4^3=64=8^2$	1,2,3,4 - 3 by 3	123, 124, 113, etc.	Order 8
$3^4=81=9^2$	1,2,3 - 4 by 4	1123, 1223, 1233, etc	Order 9
$4^4=256=16^2$	1,2,3,4 - 4 by 4	1234, 1123, 1114, etc	Order 16
$5^4=625=25^2$	1,2,3,4,5 - 4 by 4	12345, 11234, 11145, etc	Order 25
....

Table 1

- (iii) **Palindromic** - Here we construct palindromic magic squares. If we want to write *3-digits palindromes* using only 1 and 2 we have exactly four (2^2) palindromes, i.e., 111, 121, 212 and 222. The same is true for 4-digits. The table below give exact quantity in each case:

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3-digits palindromes using numbers	Total Palindromes
1, 2	$4 = 2^2$
1, 2, 3	$9 = 3^2$
1, 2, 3, 4	$16 = 4^2$
1, 2, 3, 4, 5	$25 = 5^2$
1, 2, 3, 4, 5, 6	$36 = 6^2$
1, 2, 3, 4, 5, 6, 7	$49 = 7^2$
1, 2, 3, 4, 5, 6, 7, 8	$64 = 8^2$
1, 2, 3, 4, 5, 6, 7, 8, 9	$81 = 9^2$
0, 1, 2, 3, 4, 5, 6, 7, 8, 9	90

Table 2

This idea is to construct *palindromic magic squares* in each case. Table 2 relates only 3-digits palindromes. See the table below giving exact values with higher order palindromes resulting in interesting magic squares.

Possibilities	Total Palindromes
7-digits (1,2)	$16=4^2$
5-digits (1,2,3,4)	$64=8^2$
7-digits (1,2,3)	$81=9^2$
7-digits (1,2,3,4)	$256=16^2$
7-digits (1,2,3,4,5)	$625=25^2$
9-digits (1,2,3,4)	$1024=32^2$
7-digits (1,2,3,4,5,6)	$1296=36^2$
7-digits (1,2,3,4,5,6,7)	$2401=49^2$
7-digits (1,2,3,4,5,6,7,8)	$4096=64^2$
7-digits (1,2,3,4,5,6,7,8,9)	$6561=81^2$
....

Table 3

Magic squares obtained under above three situations are considered *“multi-digits magic squares”*. In this work, we shall construct magic squares of orders 3-10, 16 and 25 using all three possibilities given in section 1, except the orders 16 and 25, where we have obtained results for the situations given in (ii) and (iii).

2. Magic Squares Order 3

For order 3, we have only $3!=6$ different digits having the three numbers 1, 2 and 3 given as: 123, 132, 213, 231, 312 and 321. We observe that in 6 digits adding 111, 222 and 333, still we don't have magic square. On the other side if we allow repetitions, we have $3^3 = 27$, given by

111	112	113	121	122	123	131	132	133
211	212	213	221	222	223	231	232	233
311	312	313	321	322	323	331	332	333

Table 4

Each row of Table 4 gives us a magic square of order 3 given in result below.

Result 1. Magic squares of order 3 based on values of each row of above table 4 are given by

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			366				666				966
121	133	112	366	221	233	212	666	321	333	312	966
113	122	131	366	213	222	231	666	313	322	331	966
132	111	123	366	232	211	223	666	332	311	323	966
366	366	366	366	666	666	666	666	966	966	966	966

We have exactly 9 palindromes of 3-digits using numbers 1, 2 and 3, resulting in a magic square. See result below.

Result 2. 3-digits palindromic magic square of order 3 with numbers 1, 2 and 3 is given by

			666
212	333	121	666
131	222	313	666
323	111	232	666
666	666	666	666

3. Magic Squares Order 4

This section deals with magic squares of order 4 in two different forms. One on *different digits* and second on *palindromic numbers* with 3 and 4-digits.

3.1. Different digits

We have $4! = 24$ combinations of different digits having only four numbers, 1, 2, 3 and 4. These combinations are as follows:

1234	1243	1324	1342	1412	1421	2134	2143	2314	2341	2413	2431
3124	3142	3214	3241	3412	3421	4123	4132	4213	4231	4312	4321

Out of 24, below is a magic square of order 4 using 16 different values

Result 3. Magic square of order 4 with *different digits* [2] considering 4 by 4 out of 1,2,3 and 4:

				11110
1234	4132	4312	1432	11110
2413	2341	2143	4213	11110
3142	3214	3412	1342	11110
4321	1423	1243	4123	11110
11110	11110	11110	11110	11110

Result 4. Magic square of order 4 with *different digits* considering 3 by 3 out of 1,2,3 and 4:

				1110					1110
123	413	431	143	1110	234	132	312	432	1110
241	234	214	421	1110	413	341	143	213	1110
314	321	341	134	1110	142	214	412	342	1110
432	142	124	412	1110	321	423	243	123	1110
1110	1110	1110	1110	1110	1110	1110	1110	1110	1110

3.2. Palindromic

We can write 3 and 4-digits pan diagonal palindromic magic square of order 4 using four digits. See the result below.

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Result 5. 3 and 4-digits palindromic pan diagonal magic squares of order 4 with numbers 1, 2, 3 and 4 are respectively given by

	1110	1110	1110	1110	
	232	343	111	424	1110
1110	121	414	242	333	1110
1110	444	131	323	212	1110
1110	313	222	434	141	1110
	1110	1110	1110	1110	1110

	11110	11110	11110	11110	
	2332	3443	1111	4224	11110
11110	1221	4114	2442	3333	11110
11110	4444	1331	3223	2112	11110
11110	3113	2222	4334	1441	11110
	11110	11110	11110	11110	11110

Analysing 7-digits palindromes, we can make exactly 16 palindromes having the numbers 1 and 2. This gives us a pan diagonal magic square of order 4.

Result 6. 7-digits palindromic pan diagonal magic squares of order 4 just with numbers 1 and 2 is given by

	6666666	6666666	6666666	6666666	
	1221221	2122212	1111111	2212122	6666666
6666666	1112111	2211122	1222221	2121212	6666666
6666666	2222222	1121211	2112112	1211121	6666666
6666666	2111112	1212121	2221222	1122211	6666666
	6666666	6666666	6666666	6666666	6666666

4. Magic Squares Order 5

This section deals with magic squares of order 5 in two different forms. One on different digits and second on palindromic numbers with 3 and 5-digits.

4.1. Different digits

We have total $5! = 120$ combinations of different digits having only 5 numbers, 1, 2, 3, 4 and 5. Below is a magic square of order 5 using 25 of 120.

Result 7. Different digits magic square of order 5 using 5 different digits in each cell [3] is given by

					166665	
	12345	12354	34512	53142	54312	166665
	54321	45321	21453	13425	32145	166665
	12435	43215	34251	35412	41352	166665
	35421	12534	52134	51234	15342	166665
	52143	53241	24315	13452	23514	166665
	166665	166665	166665	166665	166665	166665

There are much more possibilities of writing above type of magic square [3], only one is given to justify its existence.

Result 8. Magic square of order 5 with different digits considering 4 by 4 with 1, 2, 3, 4 and 5:

					16665	
	2345	2354	4512	3142	4312	16665
	4321	5321	1453	3425	2145	16665
	2435	3215	4251	5412	1352	16665
	5421	2534	2134	1234	5342	16665
	2143	3241	4315	3452	3514	16665
	16665	16665	16665	16665	16665	16665

					16665	
	1234	1235	3451	5314	5431	16665
	5432	4532	2145	1342	3214	16665
	1243	4321	3425	3541	4135	16665
	3542	1253	5213	5123	1534	16665
	5214	5324	2431	1345	2351	16665
	16665	16665	16665	16665	16665	16665

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4.2. Palindromic

We can write 3 and 5-digits *pan diagonal palindromic magic square* of order 5 using five digits 1, 2, 3, 4 and 5.

Result 9. 3 and 5-digits *palindromic pan diagonal magic squares* of order 5 with 5 numbers 1, 2, 3, 4 and 5 are respectively given by

	1665	1665	1665	1665	1665	
	111	222	333	444	555	1665
1665	434	545	151	212	323	1665
1665	252	313	424	535	141	1665
1665	525	131	242	353	414	1665
1665	343	454	515	121	232	1665
1665	1665	1665	1665	1665	1665	

	166665	166665	166665	166665	166665	166665
	11311	22322	33333	44344	55355	166665
166665	43334	54345	15351	21312	32323	166665
166665	25352	31313	42324	53335	14341	166665
166665	52325	13331	24342	35353	41314	166665
166665	34343	45354	51315	12321	23332	166665
166665	166665	166665	166665	166665	166665	166665

In [10], we have given a 5-digits *pan diagonal bimagic square* of order 25, where each block of order 5 is also a magic square. In subsection 2.10, we have given *pan diagonal palindromic magic square* of order 25 with 7-digits *palindromes* using only five numbers 1, 2, 3, 4 and 5.

5. Magic Squares Order 6

This section deals with magic squares of order 6 in two different forms. One on *different digits* and second on *palindromic numbers* with 3 and 6-digits.

5.1. Different digits

We have total $6! = 720$ combinations of *different digits* having six numbers, 1, 2, 3, 4, 5 and 6. Based on these combinations, here below is a magic square of order 6 [1] with combinations of six different digits in each cell.

Result 10. *Magic squares* of order 6 with 6 different digits [1] in each cell using the numbers 1 to 6 are given by

						2333331
532614	245163	361245	653421	124356	416532	2333331
326145	451632	612453	534216	243561	165324	2333331
261453	516324	124536	342165	435612	653241	2333331
614532	163245	245361	421653	356124	532416	2333331
145326	632451	453612	216534	561243	324165	2333331
453261	324516	536124	165342	612435	241653	2333331
2333331	2333331	2333331	2333331	2333331	2333331	2333331

						2333331
124536	612453	361245	536124	453612	245361	2333331
516324	451632	245163	324516	632451	163245	2333331
261453	326145	532614	453261	145326	614532	2333331
653241	165324	416532	241653	324165	532416	2333331
435612	243561	124356	612435	561243	356124	2333331
342165	534216	653421	165342	216534	421653	2333331
2333331	2333331	2333331	2333331	2333331	2333331	2333331

Below are example of 5-digits *magic square of order 6* choosing out of six numbers, i.e., 1, 2, 3, 4, 5 and 6.

Result 11. *Magic squares* of order 6 with *different digits* considering 5 by 5 with 1, 2, 3, 4, 5 and 6 are given by

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3546271	1234657	1374526	5312476	4376152	7642513	7624513	3111108
2341567	4653712	6431275	5327416	7365214	3265471	1726453	3111108
4256317	6517234	3461752	6745321	7143562	1752346	1234576	3111108
7354612	2653471	5137264	3465217	3715246	6423751	2361547	3111108
7654213	5143276	4723651	1274365	3416725	1245637	7653241	3111108
4723561	3654127	7345126	3254671	3561742	5314267	3257614	3111108
1234567	7254631	2637514	5731642	1532467	5467123	7253164	3111108
3111108	3111108	3111108	3111108	3111108	3111108	3111108	3111108

and

3546271	1234657	1374526	5312476	4376152	7642513	7624513	3111108
2341567	4653712	6431275	5327416	7365214	3265471	1726453	3111108
4256317	7523164	3461752	4756321	7143562	2735416	1234576	3111108
7354612	2653471	5137264	3465217	3715246	6423751	2361547	3111108
7654213	4157326	4723651	1352476	3416725	2153476	7653241	3111108
4723561	3654127	7345126	3254671	3561742	5314267	3257614	3111108
1234567	7234651	2637514	7642531	1532467	3576214	7253164	3111108
3111108	3111108	3111108	3111108	3111108	3111108	3111108	3111108

There are much more possibilities [4], but we have written only 2 to show its existence.

Result 14. In the above result remove 1st, 2nd and 3rd initial digit of each member, we get respectively 6, 5 and 4-digits magic squares of *different digits* using the numbers 1 to 7:

546271	234657	374526	312476	376152	642513	624513	3111108
341567	653712	431275	327416	365214	265471	726453	3111108
256317	517234	461752	745321	143562	752346	234576	3111108
354612	653471	137264	465217	715246	423751	361547	3111108
654213	143276	723651	274365	416725	245637	653241	3111108
723561	654127	345126	254671	561742	314267	257614	3111108
234567	254631	637514	731642	532467	467123	253164	3111108
3111108	3111108	3111108	3111108	3111108	3111108	3111108	3111108

46271	34657	74526	12476	76152	42513	24513	311108	6271	4657	4526	2476	6152	2513	4513	31108
41567	53712	31275	27416	65214	65471	26453	311108	1567	3712	1275	7416	5214	5471	6453	31108
56317	17234	61752	45321	43562	52346	34576	311108	6317	7234	1752	5321	3562	2346	4576	31108
54612	53471	37264	65217	15246	23751	61547	311108	4612	3471	7264	5217	5246	3751	1547	31108
54213	43276	23651	74365	16725	45637	53241	311108	4213	3276	3651	4365	6725	5637	3241	31108
23561	54127	45126	54671	61742	14267	57614	311108	3561	4127	5126	4671	1742	4267	7614	31108
34567	54631	37514	31642	32467	67123	53164	311108	3567	4631	7514	1642	2467	7123	3164	30108
311108	311108	311108	311108	311108	311108	311108	311108	30108	31108	31108	31108	31108	31108	31108	31108

Still, we can have magic squares of order 7 with 7 numbers 1 to 7 considering 3 by 3. We shall deal it in another work.

6.2. Palindromic

Similar to other cases here also we have *pan diagonal palindromic magic squares* of order 7 with 3 and 7-digits, having numbers 1, 2, 3, 4, 5, 6 and 7.

Result 15. 3 and 7-digits *pan diagonal palindromic magic square* of order 7 with numbers 1, 2, 3, 4, 5, 6 and 7 are given by

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856341	412785	781634	345278	638527	274163	523416	167852	3999996
278563	634127	163452	527816	416385	852741	341678	785234	3999996
345678	781234	412385	856741	167452	523816	274563	638127	3999996
527416	163852	634527	278163	785634	341278	852341	416785	3999996
381274	745638	456781	812345	123856	567412	238167	674523	3999996
563812	127456	678123	234567	741238	385674	816745	452381	3999996
812745	456381	745238	381674	674123	238567	567812	123456	3999996
234167	678523	127856	563412	452781	816345	385274	741638	3999996

56341	12785	81634	45278	38527	74163	23416	67852	399996	6341	2785	1634	5278	8527	4163	3416	7852	39996
78563	34127	63452	27816	16385	52741	41678	85234	399996	8563	4127	3452	7816	6385	2741	1678	5234	39996
45678	81234	12385	56741	67452	23816	74563	38127	399996	5678	1234	2385	6741	7452	3816	4563	8127	39996
27416	63852	34527	78163	85634	41278	52341	16785	399996	7416	3852	4527	8163	5634	1278	2341	6785	39996
81274	45638	56781	12345	23856	67412	38167	74523	399996	1274	5638	6781	2345	3856	7412	8167	4523	39996
63812	27456	78123	34567	41238	85674	16745	52381	399996	3812	7456	8123	4567	1238	5674	6745	2381	39996
12745	56381	45238	81674	74123	38567	67812	23456	399996	2745	6381	5238	1674	4123	8567	7812	3456	39996
34167	78523	27856	63412	52781	16345	85274	41638	399996	4167	8523	7856	3412	2781	6345	5274	1638	39996

7.2. Palindromic

Result 18. 3 and 8-digits pan diagonal palindromic bimagic squares of order 8 using numbers 1 to 8 are given by

282	616	545	151	434	868	777	323	3996
424	878	767	333	252	646	515	181	3996
111	585	656	242	363	737	828	474	3996
373	727	838	464	141	555	686	212	3996
565	131	222	676	717	383	454	848	3996
747	353	484	818	575	121	232	666	3996
636	262	171	525	888	414	343	757	3996
858	444	313	787	626	272	161	535	3996

and

28822882	61166116	54455445	15511551	43344334	86688668	77777777	32233223	39999996
42244224	87788778	76677667	33333333	25522552	64466446	51155115	18811881	39999996
11111111	58855885	65566556	24422442	36633663	73377337	82288228	47744774	39999996
37733773	72272272	83388338	46644664	14411441	55555555	68866886	21122112	39999996
56655665	13311331	22222222	67766776	71177117	38833883	45544554	84488448	39999996
74477447	35533553	48844884	81188118	57755775	12211221	23322332	66666666	39999996
63366336	26622662	17711771	52255225	88888888	41144114	34433443	75577557	39999996
85588558	44444444	31133113	78877887	62266226	27722772	16611661	53355335	39999996

Both the above pan diagonal magic square are bimagic with bimagic sums $Sb_{8 \times 8} := 2428644$ and $Sb_{8 \times 8} := 24260075687432244$ respectively. In both the example, each block of order 2x4 or 4x2 has the same sum as of magic square, but each block of order 4 is not a magic square. Both examples are compact of order 4. Below are examples of pan diagonal palindromic magic squares of order 8 where each block of order 4 is a magic square.

Result 19. Pan diagonal palindromic magic squares of order 8 with numbers 1 to 8 are given by

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		3996	3996	3996	3996	3996	3996	3996	3996	
	3996	111	858	444	585	212	757	343	686	3996
3996		484	545	151	818	383	646	252	717	3996
3996		555	414	888	141	656	313	787	242	3996
3996		848	181	515	454	747	282	616	353	3996
3996		121	868	434	575	222	767	333	676	3996
3996		474	535	161	828	373	636	262	727	3996
3996		565	424	878	131	666	323	777	232	3996
3996		838	171	525	464	737	272	626	363	3996
		3996	3996	3996	3996	3996	3996	3996	3996	

and

		39999996	39999996	39999996	39999996	39999996	39999996	39999996	39999996	39999996	
	39999996	11111111	85588558	44444444	58855885	21122112	75577557	34433443	68866886	39999996	
39999996		48844884	54455445	15511551	81188118	38833883	64466446	25522552	71177117	39999996	
39999996		55555555	41144114	88888888	14411441	65566556	31133113	78877887	24422442	39999996	
39999996		84488448	18811881	51155115	45544554	74477447	28822882	61166116	35533553	39999996	
39999996		12211221	86688668	43344334	57755775	22222222	76677667	33333333	67766776	39999996	
39999996		47744774	53355335	16611661	82288228	37733773	63366336	26622662	72277227	39999996	
39999996		56655665	42244224	87788778	13311331	66666666	32233223	77777777	23322332	39999996	
39999996		83388338	17711771	52255225	46644664	73377337	27722772	62266226	36633663	39999996	
		39999996	39999996	39999996	39999996	39999996	39999996	39999996	39999996	39999996	

Above magic squares are *not bimagic* but each block of order 4 is a magic square with same magic sum. Also, each block of order 2 has the same sum as of magic square of order 4. Moreover, both are *compact* of order 4.

Considering *5-digits palindromes*, we have exactly 64 palindromes using the four numbers 1, 2, 3 and 4. It brings following result.

Result 20. *5-digits pan diagonal palindromic magic squares* of order 8 only with four numbers 1, 2, 3 and 4 are given by

		222220	222220	222220	222220	222220	222220	222220	222220	
	222220	14441	33133	31413	12121	23332	44244	42324	21212	222220
222220		23232	44344	42224	21312	14141	33433	31113	12421	222220
222220		11111	32423	34143	13431	22222	41314	43234	24342	222220
222220		22322	41214	43334	24242	11411	32123	34443	13131	222220
222220		32223	11311	13231	34343	41114	22422	24142	43434	222220
222220		41414	22122	24442	43134	32323	11211	13331	34243	222220
222220		33333	14241	12321	31213	44444	23132	21412	42124	222220
222220		44144	23432	21112	42424	33233	14341	12221	31313	222220
		222220	222220	222220	222220	222220	222220	222220	222220	222220

and

		222220	222220	222220	222220	222220	222220	222220	222220	
	222220	11111	44144	23432	32423	13131	42124	21412	34443	222220
222220		24442	31413	12121	43134	22422	33433	14141	41114	222220
222220		32123	23132	44444	11411	34143	21112	42424	13431	222220
222220		43434	12421	31113	24142	41414	14441	33133	22122	222220
222220		11211	44244	23332	32323	13231	42224	21312	34343	222220
222220		24342	31313	12221	43234	22322	33333	14241	41214	222220
222220		32223	23232	44344	11311	34243	21212	42324	13331	222220
222220		43334	12321	31213	24242	41314	14341	33233	22222	222220
		222220	222220	222220	222220	222220	222220	222220	222220	222220

First example is bimagic with bimagic sum $Sb_{8 \times 8} := 7183377060$, where each block of order 2×4 or 4×2 are of same sum. The second one is *not bimagic* but each block of order 4 is a magic square with sum $S_{4 \times 4} := 111110$. Moreover, both are compact of order 4.

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7.3. Combinations with Repetitions

Making combinations of four numbers 1,2,3 and 4 with repetitions considering 3 by 3, we have exactly $64 = 8^2$ possibilities. This gives us following result.

Result 21. *Pan diagonal magic squares* of order 8 with 1, 2, 3 and 4 considering 3 by 3 are given by

		2220	2220	2220	2220	2220	2220	2220	2220											
	2220	144	331	314	121	233	442	423	212	2220										
	2220	232	443	422	213	141	334	311	124	2220	2220	111	441	234	324	131	421	214	344	
	2220	111	324	341	134	222	413	432	243	2220	2220	244	314	121	431	224	334	141	411	
	2220	223	412	433	242	114	321	344	131	2220	2220	321	231	444	114	341	211	424	134	
	2220	322	113	132	343	411	224	241	434	2220	2220	434	124	311	241	414	144	331	221	
	2220	414	221	244	431	323	112	133	342	2220	2220	112	442	233	323	132	422	213	343	
	2220	333	142	123	312	444	231	214	421	2220	2220	243	313	122	432	223	333	142	412	
	2220	441	234	211	424	332	143	122	313	2220	2220	322	232	443	113	342	212	423	133	
	2220									2220	2220	433	123	312	242	413	143	332	222	
		2220	2220	2220	2220	2220	2220	2220	2220			2220	2220	2220	2220	2220	2220	2220	2220	2220

The difference between above two examples is that the first one is bimagic ($Sb_{8 \times 8} := 717060$) with each block of order 2×4 or 4×2 having the same sum. The second example is not bimagic, but each block of order 4 is a magic square of order 4. Moreover, both are compact of order 4.

8. Magic Squares Order 9

This section deals with magic squares of order 9 in three different forms. One on *different digits*, second on *palindromic numbers* with 3, 7 and 9-digits, and third on *combinations with repetitions* using only the numbers 1, 2 and 3.

8.1. Different digits

We have total $9! = 362880$ combinations of different digits having numbers 1 to 9. Based on these combinations, below are magic squares formed by different combinations of above nine numbers in each cell.

Result 22. *Different digits bimagic square* of order 9 using 9 numbers 1 to 9 in each cell is given by

									4999999995
123456789	297531864	345678912	486729153	561894237	618942375	759183426	834267591	972315648	4999999995
459783126	534867291	672915348	723156489	897231564	945378612	186429753	261594837	318642975	4999999995
786129453	861294537	918342675	159483726	234567891	372615948	423756189	597831264	645978312	4999999995
378612945	156489723	231564897	642975318	429753186	594837261	915348672	783126459	867291534	4999999995
615948372	483726159	567891234	978312645	756189423	831264597	342675918	129453786	294537861	4999999995
942375618	729153486	894237561	315648972	183426759	267591834	678912345	456789123	531864297	4999999995
264597831	312645978	189423756	537861294	675918342	453786129	891234567	948372615	726159483	4999999995
591834267	648972315	426759183	864297531	912345678	789123456	237561894	375618942	153486729	4999999995
837261594	975318642	753186429	291534867	348672915	126459783	564897231	612945378	489723156	4999999995
4999999995	4999999995	4999999995	4999999995	4999999995	4999999995	4999999995	4999999995	4999999995	4999999995

Above magic square is *bimagic* with bimagic sum $Sb_{9 \times 9} := 3376740371623259625$. Even though above magic square has the property that each block of order 3 has the same as of magic square,

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but it is neither compact nor pan diagonal. Reorganizing the values of above example, we can write a compact and pan diagonal magic square of order 9. In this case, it is not bimagic.

Result 23. Different digits pan diagonal magic square of order 9 is given by

	499999995	499999995	499999995	499999995	499999995	499999995	499999995	499999995	499999995	499999995
499999995	183456729	918345672	291834567	729183456	672918345	567291834	456729183	345672918	834567291	499999995
499999995	615978342	261597834	426159783	342615978	834261597	783426159	978342615	597834261	159783426	499999995
499999995	759123486	675912348	867591234	486759123	348675912	234867591	123486759	912348675	591234867	499999995
499999995	291564837	729156483	372915648	837291564	483729156	648372915	564837291	156483729	915648372	499999995
499999995	534897261	153489726	615348972	261534897	726153489	972615348	897261534	489726153	348972615	499999995
499999995	867231594	486723159	948672315	594867231	159486723	315948672	231594867	723159486	672315948	499999995
499999995	372645918	837264591	183726459	918372645	591837264	459183726	645918372	264591837	726459183	499999995
499999995	426789153	342678915	534267891	153426789	915342678	891534267	789153426	678915342	267891534	499999995
499999995	948312675	594831267	759483126	675948312	267594831	126759483	312675948	831267594	483126759	499999995
499999995	499999995	499999995	499999995	499999995	499999995	499999995	499999995	499999995	499999995	499999995

The above magic square is pan diagonal but not bimagic. Also it is compact of order 3.

Result 24. In the example given in Result 23, remove 1st, 2nd, 3rd and 4th members in the beginning of each member, we get respectively 8, 7, 6 and 5-digits magic squares of different digits using the numbers 1 to 9:

	499999995	499999995	499999995	499999995	499999995	499999995	499999995	499999995	499999995	499999995
499999995	83456729	18345672	91834567	29183456	72918345	67291834	56729183	45672918	34567291	499999995
499999995	15978342	61597834	26159783	42615978	34261597	83426159	78342615	97834261	59783426	499999995
499999995	59123486	75912348	67591234	86759123	48675912	34867591	23486759	12348675	91234867	499999995
499999995	91564837	29156483	72915648	37291564	83729156	48372915	64837291	56483729	15648372	499999995
499999995	34897261	53489726	15348972	61534897	26153489	72615348	97261534	89726153	48972615	499999995
499999995	67231594	86723159	48672315	94867231	59486723	15948672	31594867	23159486	72315948	499999995
499999995	72645918	37264591	83726459	18372645	91837264	59183726	45918372	64591837	26459183	499999995
499999995	26789153	42678915	34267891	53426789	15342678	91534267	89153426	78915342	67891534	499999995
499999995	48312675	94831267	59483126	75948312	67594831	26759483	12675948	31267594	83126759	499999995
499999995	499999995	499999995	499999995	499999995	499999995	499999995	499999995	499999995	499999995	499999995

	499999995	499999995	499999995	499999995	499999995	499999995	499999995	499999995	499999995	499999995
499999995	3456729	8345672	1834567	9183456	2918345	7291834	6729183	5672918	4567291	499999995
499999995	5978342	1597834	6159783	2615978	4261597	3426159	8342615	7834261	9783426	499999995
499999995	9123486	75912348	67591234	86759123	48675912	34867591	23486759	12348675	91234867	499999995
499999995	1564837	9156483	2915648	7291564	3729156	8372915	4837291	6483729	5648372	499999995
499999995	4897261	3489726	5348972	1534897	6153489	2615348	7261534	9726153	8972615	499999995
499999995	7231594	6723159	8672315	4867231	9486723	5948672	1594867	3159486	2315948	499999995
499999995	2645918	7264591	3726459	8372645	1837264	9183726	5918372	4591837	6459183	499999995
499999995	6789153	2678915	4267891	3426789	5342678	1534267	9153426	8915342	7891534	499999995
499999995	8312675	4831267	9483126	7594831	6759483	2675948	1267594	3126759	8312675	499999995
499999995	499999995	499999995	499999995	499999995	499999995	499999995	499999995	499999995	499999995	499999995

	499999995	499999995	499999995	499999995	499999995	499999995	499999995	499999995	499999995	499999995
499999995	456729	345672	834567	183456	918345	291834	729183	672918	567291	499999995
499999995	978342	597834	159783	615978	261597	426159	342615	834261	783426	499999995
499999995	123486	912348	591234	759123	675912	486759	348675	234867	123486	499999995
499999995	564837	156483	915648	291564	729156	372915	837291	483729	648372	499999995
499999995	897261	489726	348972	534897	153489	615348	261534	726153	972615	499999995
499999995	231594	723159	672315	867231	486723	948672	594867	159486	315948	499999995
499999995	645918	264591	726459	372645	837264	183726	918372	591837	459183	499999995
499999995	789153	267891	426789	342678	534267	153426	915342	891534	789153	499999995
499999995	312675	831267	483126	948312	759483	675948	267594	126759	312675	499999995
499999995	499999995	499999995	499999995	499999995	499999995	499999995	499999995	499999995	499999995	499999995

and

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	499995	499995	499995	499995	499995	499995	499995	499995	499995	499995
	56729	45672	34567	83456	18345	91834	29183	72918	67291	499995
499995	78342	97834	59783	15978	61597	26159	42615	34261	83426	499995
499995	23486	12348	91234	59123	75912	67591	86759	48675	34867	499995
499995	64837	56483	15648	91564	29156	72915	37291	83729	48372	499995
499995	97261	89726	48972	34897	53489	15348	61534	26153	72615	499995
499995	31594	23159	72315	67231	86723	48672	94867	59486	15948	499995
499995	45918	64591	26459	72645	37264	83726	18372	91837	59183	499995
499995	89153	78915	67891	26789	42678	34267	53426	15342	91534	499995
499995	12675	31267	83126	48312	94831	59483	75948	67594	26759	499995
	499995	499995	499995	499995	499995	499995	499995	499995	499995	499995

All the above magic squares are *compact* of order 3. Similarly we can write four more magic squares with 8, 7, 6 and 5-digits using the Result 22 with *different digits* from 1 to 9. In each case the magic squares obtained are *bimagic*.

8.2. Palindromic

Let us write *palindromic magic squares* of order 9 with 3 and 9-digits *palindromes*.

Result 25. 3-digits palindromic bimagic and pan diagonal magic squares of order 9 are respectively given by

										4995
111	292	353	484	545	636	767	828	979	4995	4995
464	525	676	717	898	959	181	242	333	4995	4995
787	848	939	161	222	373	414	595	656	4995	4995
393	151	212	646	434	585	929	777	868	4995	4995
626	474	565	999	757	818	343	131	282	4995	4995
949	737	888	323	171	262	696	454	515	4995	4995
252	313	191	535	686	444	878	969	727	4995	4995
575	666	424	858	919	797	232	383	141	4995	4995
838	989	747	272	363	121	555	616	494	4995	4995
4995	4995	4995	4995	4995	4995	4995	4995	4995	4995	4995

											4995
111	242	373	434	565	696	727	858	989	4995	4995	4995
424	555	686	717	848	979	131	262	393	4995	4995	4995
737	868	999	121	252	383	414	545	676	4995	4995	4995
343	171	212	666	494	535	959	787	828	4995	4995	4995
656	484	525	949	777	818	363	191	232	4995	4995	4995
969	797	838	353	181	222	646	474	515	4995	4995	4995
272	313	141	595	636	464	888	929	757	4995	4995	4995
585	626	454	878	919	747	292	333	161	4995	4995	4995
898	939	767	282	323	151	575	616	444	4995	4995	4995
4995	4995	4995	4995	4995	4995	4995	4995	4995	4995	4995	4995

First magic square is *bimagic* with bimagic sum $Sb_{9 \times 9} := 3390285$ but not *pan diagonal*. The second is *pan diagonal* but not *bimagic*. In both cases the sum of 9 members of each block of order 3 has the same sum as of magic square. Moreover, the second magic square is *compact* of order 3.

Result 26. 9-digits palindromic bimagic and pan diagonal magic squares of order 9 are respectively given by

											499999995
111111111	222999222	333555333	444888444	555444555	666333666	777666777	888222888	999777999	499999995	499999995	499999995
444666444	555222555	666777666	777111777	888999888	999555999	111888111	222444222	333333333	499999995	499999995	499999995
777888777	888444888	999333999	111666111	222222222	333777333	444111444	555999555	666555666	499999995	499999995	499999995
333999333	111555111	222111222	666444666	444333444	555888555	999222999	777777777	888666888	499999995	499999995	499999995
666222666	444777444	555666555	999999999	777555777	888111888	333444333	111333111	222888222	499999995	499999995	499999995
999444999	777333777	888888888	333222333	111777111	222666222	666999666	444555444	555111555	499999995	499999995	499999995
222555222	333111333	111999111	555333555	666888666	444444444	888777888	999666999	777222777	499999995	499999995	499999995
555777555	666666666	444222444	888555888	999111999	777999777	222333222	333888333	111444111	499999995	499999995	499999995
888333888	999888999	777444777	222777222	333666333	111222111	555555555	666111666	444999444	499999995	499999995	499999995
499999995	499999995	499999995	499999995	499999995	499999995	499999995	499999995	499999995	499999995	499999995	499999995

and

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	499999995	499999995	499999995	499999995	499999995	499999995	499999995	499999995	499999995	499999995
	111111111	222444222	333777333	444333444	555666555	666999666	777222777	888555888	999888999	499999995
499999995	444222444	555555555	666888666	777111777	888444888	999777999	111333111	222666222	333999333	499999995
499999995	777333777	888666888	999999999	111222111	222555222	333888333	444111444	555444555	666777666	499999995
499999995	333444333	111777111	222111222	666666666	444999444	555333555	999555999	777888777	888222888	499999995
499999995	666555666	444888444	555222555	999444999	777777777	888111888	333666333	111999111	222333222	499999995
499999995	999666999	777999777	888333888	333555333	111888111	222222222	666444666	444777444	555111555	499999995
499999995	222777222	333111333	111444111	555999555	666333666	444666444	888888888	999222999	777555777	499999995
499999995	555888555	666222666	444555444	888777888	999111999	777444777	222999222	333333333	111666111	499999995
499999995	888999888	999333999	777666777	222888222	333222333	111555111	555777555	666111666	444444444	499999995
	499999995	499999995	499999995	499999995	499999995	499999995	499999995	499999995	499999995	499999995

First magic square is *bimagic* with bimagic sum $Sb_{9,9} := 3517039990002961485$ but not *pan diagonal*. The second is *pan diagonal* but not *bimagic*. In both cases the sum of 9 members of each block of order 3 has same sum as of magic square. Moreover, the second magic square is *compact* of order 3.

According to Table 3, there are exactly 81 numbers of 7-digits *palindormes* with numbers 1, 2 and 3. Based on this, below are two *palindromic magic square* of order 9.

Result 27. 7-digits *pan diagonal palindromic magic squares* of order 9 only with numbers 1, 2, and 3 are respectively given by

										19999998
	1111111	1233321	1322231	2132312	2221222	2313132	3123213	3212123	3331333	19999998
	2123212	2212122	2331332	3111113	3233323	3322233	1132311	1221221	1313131	19999998
	3132313	3221223	3313133	1123211	1212121	1331331	2111112	2233322	2322232	19999998
	1333331	1122211	1211121	2321232	2113112	2232322	3312133	3131313	3223223	19999998
	2312132	2131312	2223222	3333333	3122213	3211123	1321231	1113111	1232321	19999998
	3321233	3113113	3232323	1312131	1131311	1223221	2333332	2122212	2211122	19999998
	1222221	1311131	1133311	2213122	2332332	2121212	3231323	3323233	3112113	19999998
	2231322	2323232	2112112	3222223	3311133	3133313	1213121	1332331	1121211	19999998
	3213123	3332333	3121213	1231321	1323231	1112111	2222222	2311132	2133312	19999998
19999998	19999998	19999998	19999998	19999998	19999998	19999998	19999998	19999998	19999998	19999998

and

											19999998
	1111111	1221221	1331331	2113112	2223222	2333332	3112113	3222223	3332333	19999998	19999998
19999998	2112112	2222222	2332332	3111113	3221223	3331333	1113111	1223221	1333331	19999998	19999998
19999998	3113113	3223223	3333333	1112111	1222221	1332331	2111112	2221222	2331332	19999998	19999998
19999998	1321231	1131311	1211121	2323232	2133312	2213122	3322233	3132313	3212123	19999998	19999998
19999998	2322232	2132312	2212122	3321233	3131313	3211123	1323231	1133311	1213121	19999998	19999998
19999998	3323233	3133313	3213123	1322231	1132311	1212121	2321232	2131312	2211122	19999998	19999998
19999998	1231321	1311131	1121211	2233322	2313132	2123212	3232323	3312133	3122213	19999998	19999998
19999998	2232322	2312132	2122212	3231323	3311133	3121213	1233321	1313131	1123211	19999998	19999998
19999998	3233323	3313133	3123213	1232321	1312131	1122211	2231322	2311132	2121212	19999998	19999998
	19999998	19999998	19999998	19999998	19999998	19999998	19999998	19999998	19999998	19999998	19999998

First magic square is *bimagic* with bimagic sum $Sb_{9,9} := 50505077616162$. In both examples, each block of order 3 are of same sum as of magic square. The second one is not *bimagic* but *pan diagonal*. Moreover, second magic square is also *compact* of order 3.

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										499999995
437195628	053719462	894621053	789532104	628053719	371964280	210476895	105387946	946208537	562840371	499999995
946280537	105378946	280537194	671943280	567804321	432159678	328065719	719426805	894612053	053791462	499999995
895621043	719462805	328056719	462805371	953710462	046298537	537149628	671934280	280573194	104387956	499999995
780532194	678943210	537194628	046289537	194628053	805317946	953701462	462850371	321065789	219476805	499999995
371946280	562804371	956710432	805371946	219467805	780523194	194682053	043298567	437159628	628035719	499999995
014378956	820537194	761943280	357194628	406289537	593701462	642850371	238065719	179426805	985612043	499999995
503719462	984621053	179462805	238056719	342195678	657840321	761934280	820573194	015387946	496208537	499999995
652804371	496280537	015378946	120467895	731946280	268035719	879523104	984612053	503791462	347159628	499999995
268053719	347195628	403289567	914628053	870532194	129476805	085317946	596701432	652840371	731964280	499999995
129467805	231056789	642805371	593710462	085371946	914682053	406298537	357149628	768934210	870523194	499999995
499999995	499999995	499999995	499999995	499999995	499999995	499999995	499999995	499999995	499999995	499999995

										499999995
37195628	53719462	94621053	89532104	28053719	71964280	10476895	05387946	46208537	62840371	499999995
46280537	05378946	80537194	71943280	67804321	32159678	28065719	19426805	94612053	53791462	499999995
95621043	19462805	28056719	62805371	53710462	46298537	37149628	71934280	80573194	04387956	499999995
80532194	78943210	37194628	46289537	94628053	05317946	53701462	62850371	21065789	19476805	499999995
71946280	62804371	56710432	05371946	19467805	80523194	94682053	43298567	37159628	28035719	499999995
14378956	20537194	61943280	57194628	06289537	93701462	42850371	38065719	79426805	85612043	499999995
03719462	84621053	79462805	38056719	42195678	57840321	61934280	20573194	15387946	96208537	499999995
52804371	96280537	15378946	20467895	31946280	68035719	79523104	84612053	03791462	47159628	499999995
68053719	47195628	03289567	14628053	70532194	29476805	85317946	96701432	52840371	31964280	499999995
29467805	31056789	42805371	93710462	85371946	14682053	06298537	57149628	68934210	70523194	499999995
499999995	499999995	499999995	499999995	499999995	499999995	499999995	499999995	499999995	499999995	499999995

										499999995
4371956	0537194	8946210	7895321	6280537	3719642	2104768	1053879	9462085	5628403	499999995
9462805	1053789	2805371	6719432	5678043	4321596	3280657	7194268	8946120	0537914	499999995
8956210	7194628	3280567	4628053	9537104	0462985	5371496	6719342	2805731	1043879	499999995
7805321	6789432	5371946	0462895	1946280	8053179	9537014	4628503	3210657	2194768	499999995
3719462	5628043	9567104	8053719	2194678	7805231	1946820	0432985	4371596	6280357	499999995
0143789	8205371	7619432	3571946	4062895	5937014	6428503	2380657	1794268	9856120	499999995
5037194	9846210	1794628	2380567	3421956	6578403	7619342	8205731	0153879	4962085	499999995
6528043	4962805	0153789	1204678	7319462	2680357	8795231	9846120	5037914	3471596	499999995
2680537	3471956	4032895	9146280	8705321	1294768	0853179	5967014	6528403	7319642	499999995
1294678	2310567	6428053	5937104	0853719	9146820	4062985	3571496	7689342	8705231	499999995
499999995	499999995	499999995	499999995	499999995	499999995	499999995	499999995	499999995	499999995	499999995

										499999995
437195	053719	894621	789532	628053	371964	210476	105387	946208	562840	499999995
946280	105378	280537	671943	567804	432159	328065	719426	894612	053791	499999995
895621	719462	328056	462805	953710	046298	537149	671934	280573	104387	499999995
780532	678943	537194	046289	194628	805317	953701	462850	321065	219476	499999995
371946	562804	956710	805371	219467	780523	194682	043298	437159	628035	499999995
014378	820537	761943	357194	406289	593701	642850	238065	179426	985612	499999995
503719	984621	179462	238056	342195	657840	761934	820573	015387	496208	499999995
652804	496280	015378	120467	731946	268035	879523	984612	503791	347159	499999995
268053	347195	403289	914628	870532	129476	085317	596701	652840	731964	499999995
129467	231056	642805	593710	085371	914682	406298	357149	768934	870523	499999995
499999995	499999995	499999995	499999995	499999995	499999995	499999995	499999995	499999995	499999995	499999995

and

										499999995
43719	05371	89462	78953	62805	37196	21047	10538	94620	56284	499999995
94628	10537	28053	67194	56780	43215	32806	71942	89461	05379	499999995
89562	71946	32805	46280	95371	04629	53714	67193	28057	10438	499999995
78053	67894	53719	04628	19462	80531	95370	46285	32106	21947	499999995
37194	56280	95671	80537	21946	78052	19468	04329	43715	62803	499999995
01437	82053	76194	35719	40628	59370	64285	23806	17942	98561	499999995
50371	98462	17946	23805	34219	65784	76193	82057	01538	49620	499999995
65280	49628	01537	12046	73194	26803	87952	98461	50379	34715	499999995
26805	34719	40328	91462	87053	12947	08531	59670	65284	73196	499999995
12946	23105	64280	59371	08537	91468	40629	35714	76893	87052	499999995
499999995	499999995	499999995	499999995	499999995	499999995	499999995	499999995	499999995	499999995	499999995

Still, there are magic squares of order 10 with ten numbers 0 to 9, considering 3 by 3 and 4 by 4. This shall be studied elsewhere.

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9.2. Palindromic

Example 31. 3-digits palindromic magic square of order 10 is given by

000	484	151	919	878	232	747	363	595	626	4995
353	111	676	828	292	989	404	535	040	767	4995
616	575	222	494	959	060	383	808	737	141	4995
191	868	545	333	707	424	979	010	656	282	4995
939	202	080	565	444	717	858	696	121	373	4995
848	636	303	787	161	555	090	929	272	414	4995
474	020	797	252	515	343	666	181	909	838	4995
262	949	434	101	686	898	525	777	313	050	4995
727	393	969	646	030	171	212	454	888	505	4995
585	757	818	070	323	606	131	242	464	999	4995
4995	4995	4995	4995	4995	4995	4995	4995	4995	4995	4995

Since we have only 90 palindromes of 3-digits, other 10 numbers are used as 000, 010, 020, ..., 090 to complete the magic square.

Example 32. 10-digits palindromic magic square of order 10 having all the 10 digits is given by

4990000994	4996336994	4998448994	4991221994	4999889994	4997117994	4993553994	4994994994	4992772994	4995665994	49950004940
4994224994	4991111994	4993663994	4990880994	4995335994	4998008994	4999779994	4992552994	4997997994	4996446994	49950004940
4996996994	4995559994	4992222994	4995005994	4998668994	4994774994	4997447994	4991331994	4990110994	4993883994	49950004940
4999449994	4998778994	4990990994	4993333994	4992002994	4991661994	4994114994	4995885994	4996556994	4997227994	49950004940
4993773994	4997007994	4991551994	4998998994	4994444994	4996886994	4990330994	4999669994	4995225994	4992112994	49950004940
4991881994	4992992994	4996116994	4997667994	4990770994	4995555994	4998228994	4993443994	4999339994	4994004994	49950004940
4997337994	4994884994	4995775994	4992442994	4991991994	4999229994	4996666994	4998118994	4993003994	4990550994	49950004940
4995115994	4990660994	4999009994	4994554994	4996226994	4993993994	4992882994	4997777994	4991441994	4998338994	49950004940
4992662994	4993223994	4994334994	4999119994	4997557994	4990440994	4995995994	4996006994	4998888994	4991771994	49950004940
4998558994	4995445994	4997887994	4996776994	4993113994	4992332994	4991001994	4990220994	4994664994	4999999994	49950004940
49950004940	49950004940	49950004940	49950004940	49950004940	49950004940	49950004940	49950004940	49950004940	49950004940	49950004940

We have written only one example just to have an idea, but there are much more possibilities of writing 10-digits palindromic magic square of order 10 as there 90000 palindromes of 10-digits.

10. Magic Squares Order 16

This section deals with magic squares of order 16 in two different forms. One on 7-digits palindromes and second on combinations with repetitions using only the numbers 1, 2, 3 and 4. For 5-digits palindromic magic squares of order 16 refer to Taneja [9].

10.1. Palindromic

In [9], author wrote bimagic and pan diagonal magic squares of order 16 with 5-digits palindromes. From, Table 3, we observe that there are exactly 256 7-digits palindromes with the numbers 1, 2, 3 and 4. Based on this we have two different types of palindromic magic squares of order 16. One is bimagic and another is pan diagonal. In both the cases, the each block of order 4 is a magic square. See the results below.

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Result 33. 7-digits palindromic bimagic square of order 16 having the numbers 1, 2, 3 and 4 is given by

1111111	3232323	4343434	2424242	1223221	3144413	4431344	2312132	1334331	3413143	4122214	2241422	1442441	3321233	4214124	2133312
4324234	2443442	1132311	3211123	4412144	2331332	1244421	3123213	4141414	2222222	1313131	3434343	4233324	2114112	1421241	3342433
2432342	4311134	3224223	1143411	2344432	4423244	3112113	1231321	2213122	4134314	3441443	1322231	2121212	4242424	3333333	1414141
3243423	1124211	2411142	4332334	3131313	1212121	2323232	4444444	3422243	1341431	2234322	4113114	3314133	1433341	2142412	4221224
1342431	3421243	4114114	2233322	1434341	3313133	4222224	2141412	1123211	3244423	4331334	2412142	1211121	3132313	4443444	2324232
4133314	2214122	1321231	3442443	4241424	2122212	1413141	3334333	4312134	2431342	1144411	3223223	4424244	2343432	1232321	3111113
2221222	4142414	3433343	1314131	2113112	4234324	3341433	1422241	2444442	4323234	3212123	1131311	2332332	4411144	3124213	1243421
3414143	1333331	2242422	4121214	3322233	1441441	2134312	4213124	3231323	1112111	2423242	4344434	3143413	1224221	2311132	4432344
1423241	3344433	4231324	2112112	1311131	3432343	4143414	2224222	1242421	3121213	4414144	2333332	1134311	3213123	4322234	2441442
4212124	2131312	1444441	3323233	4124214	2243422	1332331	3411143	4433344	2314132	1221221	3142413	4341434	2422242	1113111	3234323
2144412	4223224	3312133	1431341	2232322	4111114	3424243	1343431	2321232	4442444	3133313	1214121	2413142	4334334	3241423	1122211
3331333	1412141	2123212	4244424	3443443	1324231	2211122	4132314	3114113	1233321	2342432	4421244	3222223	1141411	2434342	4313134
1234321	3113113	4422244	2341432	1142411	3221223	4314134	2433342	1411141	3332333	4243424	2124212	1323231	3444443	4131314	2212122
4441444	2322232	1213121	3134313	4333334	2414142	1121211	3242423	4224224	2143412	1432341	3311133	4112114	2231322	1344431	3423243
2313132	4434344	3141413	1222221	2421242	4342434	3233323	1114111	2132312	4211124	3324233	1443441	2244422	4123214	3412143	1331331
3122213	1241421	2334332	4413144	3214123	1133311	2442442	4321234	3343433	1424241	2111112	4232324	3431343	1312131	2223222	4144414

The above bimagic square has the property that each block of order 4 is also a magic square. The magic sums are $S_{16 \times 16} = 4444440$, $Sb_{16 \times 16} = 143658905634120$ and $S_{4 \times 4} = 1111110$. Sb means bimagic sum. Above magic square is not *pan diagonal*.

Below is an example of *pan diagonal palindromic magic square* of order 16.

Result 34. 7-digits *pan diagonal palindromic magic square* of order 16 having the numbers 1, 2, 3 and 4 is given by

2121212	3213123	1344431	4432344	2422242	3314133	1243421	4131314	2223222	3111113	1442441	4334334	2324232	3412143	1141411	4233324
1332331	4444444	2113112	3221223	1231321	4143414	2414142	3322233	1434341	4342434	2211122	3123213	1133311	4241424	2312132	3424243
4413144	1321231	3232323	2144412	4114114	1222221	3331333	2443442	4311134	1423241	3134313	2242422	4212124	1124211	3433343	2341432
3244423	2132312	4421244	1313131	3343433	2431342	4122214	1214121	3142413	2234322	4323234	1411141	3441443	2333332	4224224	1112111
2224222	3112113	1441441	4333334	2323232	3411143	1142411	4234324	2122212	3214123	1343431	4431344	2421242	3313133	1244421	4132314
1433341	4341434	2212122	3124213	1134311	4242424	2311132	3423243	1331331	4443444	2114112	3222223	1232321	4144414	2413142	3321233
4312134	1424241	3133313	2241422	4211124	1123211	3434343	2342432	4414144	1322231	3231323	2143412	4113114	1221221	3332333	2444442
3141413	2233322	4324234	1412141	3442443	2334332	4223224	1111111	3243423	2131312	4422244	1314131	3344433	2432342	4121214	1213121
2322232	3414143	1143411	4231324	2221222	3113113	1444441	4332334	2424242	3312133	1241421	4133314	2123212	3211123	1342431	4434344
1131311	4243424	2314132	3422443	1432341	4344434	2213122	3121213	1233321	4141414	2412142	3324233	1334331	4442444	2111112	3223223
4214124	1122211	3431343	2343432	4313134	1421241	3132313	2244422	4112114	1224221	3333333	2441442	4411144	1323231	3234323	2142412
3443443	2331332	4222224	1114111	3144413	2232322	4321234	1413141	3341433	2433342	4124214	1212121	3242423	2134312	4423244	1311131
2423242	3311133	1242421	4134314	2124212	3212123	1341431	4433344	2321232	3413143	1144411	4232324	2222222	3114113	1443441	4331334
1234321	4142414	2411142	3323233	1333331	4441444	2112112	3224223	1132311	4244424	2313132	3421243	1431341	4343434	2214122	3122213
4111114	1223221	3334333	2442442	4412144	1324231	3233323	2141412	4213124	1121211	3432343	2344432	4314134	1422241	3131313	2243422
3342433	2434342	4123214	1211121	3241423	2133312	4424244	1312131	3444443	2332332	4221224	1113111	3143413	2231322	4322234	1414141

The above *bimagic square* has the property that each block of order 4 is also a magic square. The magic sums are $S_{16 \times 16} = 4444440$ and $S_{4 \times 4} = 1111110$. Above magic square is *pan diagonal* but not *bimagic*.

10.2. Combinations with Repetitions

By allowing repetitions in numbers 1, 2, 3 and 4, we have exactly $4^4 = 256 = 16^2$ possibilities. These values gives us both *bimagic* and *pan diagonal magic squares* [9] given in the following results.

Result 35. *Bimagic square* of order 16 formed by digits 1, 2, 3 and 4 is given by

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1111	3232	4343	2424	1223	3144	4431	2312	1334	3413	4122	2241	1442	3321	4214	2133	44440
4324	2443	1132	3211	4412	2331	1244	3123	4141	2222	1313	3434	4233	2114	1421	3342	44440
2432	4311	3224	1143	2344	4423	3112	1231	2213	4134	3441	1322	2121	4242	3333	1414	44440
3243	1124	2411	4332	3131	1212	2323	4444	3422	1341	2234	4113	3314	1433	2142	4221	44440
1342	3421	4114	2233	1434	3313	4222	2141	1123	3244	4331	2412	1211	3132	4443	2324	44440
4133	2214	1321	3442	4241	2122	1413	3334	4312	2431	1144	3223	4424	2343	1232	3111	44440
2221	4142	3433	1314	2113	4234	3341	1422	2444	4323	3212	1131	2332	4411	3124	1243	44440
3414	1333	2242	4121	3322	1441	2134	4213	3231	1112	2423	4344	3143	1224	2311	4432	44440
1423	3344	4231	2112	1311	3432	4143	2224	1242	3121	4414	2333	1134	3213	4322	2441	44440
4212	2131	1444	3323	4124	2243	1332	3411	4433	2314	1221	3142	4341	2422	1113	3234	44440
2144	4223	3312	1431	2232	4111	3424	1343	2321	4442	3133	1214	2413	4334	3241	1122	44440
3331	1412	2123	4244	3443	1324	2211	4132	3114	1233	2342	4421	3222	1141	2434	4313	44440
1234	3113	4422	2341	1142	3221	4314	2433	1411	3332	4243	2124	1323	3444	4131	2212	44440
4441	2322	1213	3134	4333	2414	1121	3242	4224	2143	1432	3311	4112	2231	1344	3423	44440
2313	4434	3141	1222	2421	4342	3233	1114	2132	4211	3324	1443	2244	4123	3412	1331	44440
3122	1241	2334	4413	3214	1133	2442	4321	3343	1424	2111	4232	3431	1312	2223	4144	44440
44440	44440	44440	44440	44440	44440	44440	44440	44440	44440	44440	44440	44440	44440	44440	44440	44440

Its bimagic sum is $Sb_{16 \times 16} := 143634120$. Each block of order 4 is a magic square with sum $S_{4 \times 4} := 11110$.

Result 36. *Pan diagonal magic square* of order 16 formed by digits 1, 2, 3 and 4 with each block of order 4 also a *pan diagonal magic square* with sum $S_{4 \times 4} := 11110$ is given by

2333	3242	1213	4322	2234	3341	1314	4221	2431	3144	1111	4424	2132	3443	1412	4123	44440
1222	4313	2342	3233	1321	4214	2241	3334	1124	4411	2444	3131	1423	4112	2143	3432	44440
4342	1233	3222	2313	4241	1334	3321	2214	4444	1131	3124	2411	4143	1432	3423	2112	44440
3213	2322	4333	1242	3314	2221	4234	1341	3111	2424	4431	1144	3412	2123	4132	1443	44440
2432	3143	1112	4423	2131	3444	1411	4124	2334	3241	1214	4321	2233	3342	1313	4222	44440
1123	4412	2443	3132	1424	4111	2144	3431	1221	4314	2341	3234	1322	4213	2242	3333	44440
4443	1132	3123	2412	4144	1431	3424	2111	4341	1234	3221	2314	4242	1333	3322	2213	44440
3112	2423	4432	1143	3411	2124	4131	1444	3214	2321	4334	1241	3313	2222	4233	1342	44440
2134	3441	1414	4121	2433	3142	1113	4422	2232	3343	1312	4223	2331	3244	1211	4324	44440
1421	4114	2141	3434	1122	4413	2442	3133	1323	4212	2243	3332	1224	4311	2344	3231	44440
4141	1434	3421	2114	4442	1133	3122	2413	4243	1332	3323	2212	4344	1231	3224	2311	44440
3414	2121	4134	1441	3113	2422	4433	1142	3312	2223	4232	1343	3211	2324	4331	1244	44440
2231	3344	1311	4224	2332	3243	1212	4323	2133	3442	1413	4122	2434	3141	1114	4421	44440
1324	4211	2244	3331	1223	4312	2343	3232	1422	4113	2142	3433	1121	4414	2441	3134	44440
4244	1331	3324	2211	4343	1232	3223	2312	4142	1433	3422	2113	4441	1134	3121	2414	44440
3311	2224	4231	1344	3212	2323	4332	1243	3413	2122	4133	1442	3114	2421	4434	1141	44440
44440	44440	44440	44440	44440	44440	44440	44440	44440	44440	44440	44440	44440	44440	44440	44440	44440

The magic squares appearing in Results 35 and 36 can be applied to genetic tables [6,8] representing 1, 2, 3 and 4 by letters C, A, T and G respectively.

11. Magic Squares Order 25

This section deals with *magic squares* of order 25 in two different forms. One on *7-digits palindromes* and second on *combinations with repetitions* using only the numbers 1, 2, 3 and 4. For *5-digits palindromic magic square* of order 25 refer to Taneja [10].

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Combining Parts 1 and 2 as given above, we get *pan diagonal bimagic squares* of order 25 with magic sums $S_{25 \times 25} := 83333325$ and $Sb_{25 \times 25} := 32828307272725$ with each block of order 5 also a *pan diagonal magic squares* with sum $S_{5 \times 5} := 16666665$.

11.2. Combinations with Repetitions

Allowing repetitions in numbers 1, 2, 3 4 and 5 writing in 4-digits forms, we have exactly $5^4 = 625 = 25^2$ possibilities with repetitions. These values brings a *pan diagonal bimagic square* of order 25 given in result below.

Result 38. *Pan diagonal bimagic square* of order 25 formed by digits 1, 2, 3, 4 and 5 written 4-digits forms is given by

1111	2244	3322	4455	5533	4343	5421	1554	2132	3215	2525	3153	4231	5314	1442	5252	1335	2413	3541	4124	3434	4512	5145	1223	2351
4422	5555	1133	2211	3344	2154	3232	4315	5443	1521	5331	1414	2542	3125	4253	3513	4141	5224	1352	2435	1245	2323	3451	4534	5112
2233	3311	4444	5522	1155	5415	1543	2121	3254	4332	3142	4225	5353	1431	2514	1324	2452	3535	4113	5241	4551	5134	1212	2345	3423
5544	1122	2255	3333	4411	3221	4354	5432	1515	2143	1453	2531	3114	4242	5325	4135	5213	1341	2424	3552	2312	3445	4523	5151	1234
3355	4433	5511	1144	2222	1532	2115	3243	4321	5454	4214	5342	1425	2553	3131	2441	3524	4152	5235	1313	5123	1251	2334	3412	4545
5225	1353	2431	3514	4142	3452	4535	5113	1241	2324	1134	2212	3345	4423	5551	4311	5444	1522	2155	3233	2543	3121	4254	5332	1415
3531	4114	5242	1325	2453	1213	2341	3424	4552	5135	4445	5523	1151	2234	3312	2122	3255	4333	5411	1544	5354	1432	2515	3143	4221
1342	2425	3553	4131	5214	4524	5152	1235	2313	3441	2251	3334	4412	5545	1123	5433	1511	2144	3222	4355	3115	4243	5321	1454	2532
4153	5231	1314	2442	3525	2335	3413	4541	5124	1252	5512	1145	2223	3351	4434	3244	4322	5455	1533	2111	1421	2554	3132	4215	5343
2414	3542	4125	5253	1331	5141	1224	2352	3435	4513	3323	4451	5534	1112	2245	1555	2133	3211	4344	5422	4232	5315	1443	2521	3154
4334	5412	1545	2123	3251	2511	3144	4222	5355	1433	5243	1321	2454	3532	4115	3425	4553	5131	1214	2342	1152	2235	3313	4441	5524
2145	3223	4351	5434	1512	5322	1455	2533	3111	4244	3554	4132	5215	1343	2421	1231	2314	3442	4525	5153	4413	5541	1124	2252	3335
5451	1534	2112	3245	4323	3133	4211	5344	1422	2555	1315	2443	3521	4154	5232	4542	5125	1253	2331	3414	2224	3352	4435	5513	1141
3212	4345	5423	1551	2134	1444	2522	3155	4233	5311	4121	5254	1332	2415	3543	2353	3431	4514	5142	1225	5535	1113	2241	3324	4452
1523	2151	3234	4312	5445	4255	5333	1411	2544	3122	2432	3515	4143	5221	1354	5114	1242	2325	3453	4531	3341	4424	5552	1135	2213
3443	4521	5154	1232	2315	1125	2253	3331	4414	5542	4352	5435	1513	2141	3224	2534	3112	4245	5323	1451	5211	1344	2422	3555	4133
1254	2332	3415	4543	5121	4431	5514	1142	2225	3353	2113	3241	4324	5452	1535	5345	1423	2551	3134	4212	3522	4155	5233	1311	2444
4515	5143	1221	2354	3432	2242	3325	4453	5531	1114	5424	1552	2135	3213	4341	3151	4234	5312	1445	2523	1333	2411	3544	4122	5255
2321	3454	4532	5115	1243	5553	1131	2214	3342	4425	3235	4313	5441	1524	2152	1412	2545	3123	4251	5334	4144	5222	1355	2433	3511
5132	1215	2343	3421	4554	3314	4442	5525	1153	2231	1541	2124	3252	4335	5413	4223	5351	1434	2512	3145	2455	3533	4111	5244	1322
2552	3135	4213	5341	1424	5234	1312	2445	3523	4151	3411	4544	5122	1255	2333	1143	2221	3354	4432	5515	4325	5453	1531	2114	3242
5313	1441	2524	3152	4235	3545	4123	5251	1334	2412	1222	2355	3433	4511	5144	4454	5532	1115	2243	3321	2131	3214	4342	5425	1553
3124	4252	5335	1413	2541	1351	2434	3512	4145	5223	4533	5111	1244	2322	3455	2215	3343	4421	5554	1132	5442	1525	2153	3231	4314
1435	2513	3141	4224	5352	4112	5245	1323	2451	3534	2344	3422	4555	5133	1211	5521	1154	2232	3315	4443	3253	4331	5414	1542	2125
4241	5324	1452	2535	3113	2423	3551	4134	5212	1345	5155	1233	2311	3444	4522	3332	4415	5543	1121	2254	1514	2142	3225	4353	5431

Magic sums are $S_{25 \times 25} := 83325$ and $Sb_{25 \times 25} := 328227275$ with each block of order 5 also a *pan diagonal magic squares* with sum $S_{5 \times 5} := 16665$.

Final Comments

We have worked with three kinds of multi-digits magic squares. Two of them are connected with *combinations of numbers with and without repetitions*. The third one is on *palindromic numbers*. In case of *combinations without repetitions*, may magic squares are obtained considering order of digits as order of magic squares and in some cases less orders also give interesting magic squares. For order 8, 9, 16 and 25 we have situations of exact digits in *combinations with repetitions* as well as in *palindromic numbers*. See the table below:

Orders	Combinations with repetitions	Palindromic numbers
8	1,2,3,4 - 3 by 3 => $4^3=64=8^2$ (Result 17)	5-digits (1,2,3,4) => $64=8^2$ (Result 16)
9	1,2,3 - 4 by 4 => $3^4=81=9^2$ (Result 23)	7-digits (1,2,3) => $81=9^2$ (Result 22)
16	1,2,3,4 - 4 by 4 => $4^4=256=16^2$ (Results 29 and 30)	7-digits (1,2,3,4) => $256=16^2$ (Results 27 and 28)
25	1,2,3,4,5 - 4 by 4 => $5^4=625=25^2$ (Result 32)	7-digits (1,2,3,4,5) => $625=25^2$ (Result 31)

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Still we have exactly 16 palindromes of 7-digits with 1 and 2 forming a *pan diagonal magic square* of order 4 (Result 5). According Table 2, we have, $n^4 = (n^2)^2$. This means, we have *combinations with repetitions* of n numbers considering 4 by 4 give perfect squares to bring higher order magic squares. In Table 3, we have *palindromic* possibilities for higher order magic squares. Studies on both these aspects shall be dealt elsewhere. For more studied on magic squares refer [6]-[10].

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