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Representations of Palindromic, Prime and Number Patterns

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Abstract

This work brings representations of palindromic and number patterns in terms of single letter "a". Some examples of prime number patterns are also considered. Different classifications of palindromic patterns are considered, such as, palindromic decompositions, double symmetric patterns, number pattern decompositions, etc. Numbers patterns with power are also studied.

1 Introduction

Let us consider

$$f^n(10) = 10^n + 10^{n-1} + ... + 10^2 + 10 + 10^0,$$

For $a \in \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$, we can write

$$af^n(10) = \underbrace{aaa...a}_{(n+1)-times},$$

In particular,

$$\begin{aligned}
aa &= f^1(10) = a10 + a & \Longrightarrow 11 &:= \frac{aa}{a}.\\
aaa &= f^2(10) = a10^2 + a10 + a & \Longrightarrow 111 &:= \frac{aaa}{a}.\\
aaaa &= f^3(10) = a10^3 + a10^2 + a10 + a & \Longrightarrow 1111 &:= \frac{aaaa}{a}.\\
aaaaa &= f^4(10) = a10^4 + a10^3 + a10^2 + a10 + a & \Longrightarrow 11111 &:= \frac{aaaaa}{a}.\\
&......
\end{aligned}$$

In [11, 13] author wrote natural numbers in terms of single letter "a". See some examples below:

$$\begin{aligned}
5 &:= \frac{aa-a}{a+a}.\\
56 &:= \frac{aaa+a}{a+a}.\\
582 &:= \frac{aaaaa+aaaa}{aa+aa-a}.\\
1233 &:= \frac{aaaa+aaa+aa}{a}.\\
4950 &:= \frac{aaaaa-aaaa-aaa+aa}{a+a}.
\end{aligned}$$

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The work on representations of natural numbers using *single letter* is first of its kind [8, 13]. Author also worked with representation of numbers using *single digit* for each value from 1 to 9 separately. Representation of numbers using all the digits from 1 to 9 in increasing and decreasing ways is done by author [6]. Comments to this work can be seen at [1, 5]. Different studies on numbers, such as, *selfie numbers, running expressions*, etc. refer to author's work [7, 9, 10, 11, 12].

Our aim is to write palindromic, prime and number patterns in terms of single letter "a". For studies on patterns refer to [2, 3, 4].

2 Palindromic and Numbers Patterns

Before proceeding further, here below are some general ways of writing palindromic numbers. Odd and even orders are considered separately.

2.1 General Form of Palindromic Numbers

We shall divide palindromic numbers in two parts, *odd and even orders*.

2.1.1 Odd Order Palindromes

Odd order palindromes are those where the number of digits are odd, i.e, aba, $abcba$, etc. See the general form below:

$$\begin{aligned}
aba &:= (10^2 + 10^0) \times a + 10^1 \times b. \\
abcba &:= (10^4 + 10^0) \times a + (10^3 + 10^1) \times b + 10^2 \times c. \\
abcdcba &:= (10^6 + 10^0) \times a + (10^5 + 10^1) \times b + (10^4 + 10^2) \times c + 10^3 \times d. \\
abcdedcba &:= (10^8 + 10^0) \times a + (10^7 + 10^1) \times b + (10^6 + 10^2) \times c + (10^5 + 10^3) \times d + 10^4 \times e.
\end{aligned}$$

equivalently,

$$\begin{aligned}
aba &:= 101 \times a + 10 \times b. \\
abcba &:= 10001 \times a + 1010 \times b + 100 \times c. \\
abcdcba &:= 1000001 \times a + 100010 \times b + 10100 \times c + 1000 \times d. \\
abcdedcba &:= 100000001 \times a + 10000010 \times b + 1000100 \times c + 101000 \times d + 10000 \times e.
\end{aligned}$$

2.1.2 Even Order Palindromes

Even order palindromes are those where the number of digits are even, i.e, $abba$, $abccba$, etc. See the general form below:

$$\begin{aligned}
abba &:= (10^3 + 10^0) \times a + (10^2 + 10^1) \times b. \\
abccba &:= (10^5 + 10^0) \times a + (10^4 + 10^1) \times b + (10^3 + 10^2) \times c. \\
abcddcba &:= (10^7 + 10^0) \times a + (10^6 + 10^1) \times b + (10^5 + 10^2) \times c + (10^4 + 10^3) \times d. \\
abcdeedcba &:= (10^9 + 10^0) \times a + (10^8 + 10^1) \times b + (10^7 + 10^2) \times c + (10^6 + 10^3) \times d + (10^5 + 10^4) \times e.
\end{aligned}$$

equivalently,

$$
\begin{aligned}
abba &:= 1001 \times a + 110 \times b. \\
abccba &:= 100001 \times a + 10010 \times b + 1100 \times c. \\
abcddcba &:= 10000001 \times a + 1000010 \times b + 100100 \times c + 11000 \times d. \\
abcdeedcba &:= 1000000001 \times a + 100000010 \times b + 10000100 \times c + 1001000 \times d + 110000 \times e.
\end{aligned}
$$

Combining both orders, we have

$$
\begin{aligned}
aba &:= (10^2 + 10^0) \times a + 10^1 \times b. \\
abba &:= (10^3 + 10^0) \times a + (10^2 + 10^1) \times b. \\
abcba &:= (10^4 + 10^0) \times a + (10^3 + 10^1) \times b + 10^2 \times c. \\
abccba &:= (10^5 + 10^0) \times a + (10^4 + 10^1) \times b + (10^3 + 10^2) \times c. \\
abcdcba &:= (10^6 + 10^0) \times a + (10^5 + 10^1) \times b + (10^4 + 10^2) \times c + 10^3 \times d. \\
abcddcba &:= (10^7 + 10^0) \times a + (10^6 + 10^1) \times b + (10^5 + 10^2) \times c + (10^4 + 10^3) \times d. \\
abcdedcba &:= (10^8 + 10^0) \times a + (10^7 + 10^1) \times b + (10^6 + 10^2) \times c + (10^5 + 10^3) \times d + 10^4 \times e. \\
abcdeedcba &:= (10^9 + 10^0) \times a + (10^8 + 10^1) \times b + (10^7 + 10^2) \times c + (10^6 + 10^3) \times d + (10^5 + 10^4) \times e. \\
&\ldots\ldots
\end{aligned}
$$

It is understood that *even order palindromes* are different from *even numbers*. *Even numbers* are multiple of 2, while *even order palindromes* has *even number of digits*, for examples, 99, 1221, 997766, etc. The same is with *odd order palindromes*. We observed that the general way of writing both orders is different.

2.1.3 General Form of Particular Palindromes

If we consider some a simplified form of palindromes, for example, 121, 1221, etc. In this case we can write a simplified general form. Here the intermediate values are equal except extremes. The general form is given by

$$
\begin{aligned}
aba &:= (9 \times a + b) \times 11 + 2 \times a - b. \\
abba &:= (9 \times a + b) \times 111 + 2 \times a - b. \\
abbba &:= (9 \times a + b) \times 1111 + 2 \times a - b. \\
abbbba &:= (9 \times a + b) \times 11111 + 2 \times a - b. \\
abbbbba &:= (9 \times a + b) \times 111111 + 2 \times a - b. \\
abbbbbba &:= (9 \times a + b) \times 1111111 + 2 \times a - b. \\
abbbbbbba &:= (9 \times a + b) \times 11111111 + 2 \times a - b. \\
&\ldots\ldots
\end{aligned}
$$

where $a \in \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$, and $b \in \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9\}$.

Before proceeding further, let us clarify, the difference between *"Palindromic patterns"* and *"Number patterns"*.

2.2 Palindromic Patterns

When there is symmetry in representing palindromes, we call as *"palindromic patterns"*. See the examples below:

393	1111	11211	6112116
3993	2222	22322	6223226
39993	3333	33433	6334336
399993	4444	44544	6445446
....	....	....	....

2.3 Number Patterns

When there is symmetry in representing numbers similar as palindromic patterns, we call *"number patterns"*. See examples below:

399	1156	123	$23^2 = 549$
3999	111556	1234	$233^2 = 54289$
39999	11115556	12345	$2333^2 = 5442889$
399999	1111155556	123456	$23333^2 = 544428889$
....	....	....	....

3 Representations of Palindromic and Number Patterns

Below are examples of *palindromic* and *number patterns* in terms of single letter "*a*". These examples are divided in subsections. All the examples are followed by their respective decompositions. The following patterns are considered:

(i) Number Patterns with Palindromic Decompositions;
(ii) Palindromic Patterns with Number Pattern Decompositions;
(iii) Palindromic Patterns with Palindromic Decompositions;
(iv) Number Patterns with Power;
(v) Repeated Digits Patterns;
(vi) Doubly Symmetric Patterns;
(vii) Number Patterns with Number Pattern Decompositions.

3.1 Number Patterns with Palindromic Decompositions

Here below are examples of palindromic patterns with palindromic decompositions.

Example 1.

$$
\begin{aligned}
11 &= 11 && :=(a \times aa)/(a \times a).\\
121 &= 11 \times 11 && :=(aa \times aa)/(a \times a).\\
12321 &= 111 \times 111 && :=(aaa \times aaa)/(a \times a).\\
1234321 &= 1111 \times 1111 && :=(aaaa \times aaaa)/(a \times a).\\
123454321 &= 11111 \times 11111 && :=(aaaaa \times aaaaa)/(a \times a).\\
12345654321 &= 111111 \times 111111 && :=(aaaaaa \times aaaaaa)/(a \times a).\\
1234567654321 &= 1111111 \times 1111111 && :=(aaaaaaa \times aaaaaaa)/(a \times a).\\
123456787654321 &= 11111111 \times 11111111 && :=(aaaaaaaa \times aaaaaaaa)/(a \times a).\\
12345678987654321 &= 111111111 \times 111111111 && :=(aaaaaaaaa \times aaaaaaaaa)/(a \times a).
\end{aligned}
$$

Example 2.

$$
\begin{aligned}
11 &= 1 \times 11 && :=(a \times aa)/(a \times a).\\
1221 &= 11 \times 111 && :=(aa \times aaa)/(a \times a).\\
123321 &= 111 \times 1111 && :=(aaa \times aaaa)/(a \times a).\\
12344321 &= 1111 \times 11111 && :=(aaaa \times aaaaa)/(a \times a).\\
1234554321 &= 11111 \times 111111 && :=(aaaaa \times aaaaaa)/(a \times a).\\
123456654321 &= 111111 \times 1111111 && :=(aaaaaa \times aaaaaaa)/(a \times a).\\
12345677654321 &= 1111111 \times 11111111 && :=(aaaaaaa \times aaaaaaaa)/(a \times a).\\
1234567887654321 &= 11111111 \times 111111111 && :=(aaaaaaaa \times aaaaaaaaa)/(a \times a).\\
123456789987654321 &= 111111111 \times 1111111111 && :=(aaaaaaaaa \times aaaaaaaaaa)/(a \times a).
\end{aligned}
$$

Example 3. *The following example is represented in two different forms. One with product decomposition and another with potentiation.*

$$
\begin{aligned}
1089 &= 11 \times 99 && :=aa \times (aaa - aa - a)/(a \times a).\\
110889 &= 111 \times 999 && :=aaa \times (aaaa - aaa - a)/(a \times a).\\
11108889 &= 1111 \times 9999 && :=aaaa \times (aaaaa - aaaa - a)/(a \times a).\\
1111088889 &= 11111 \times 99999 && :=aaaaa \times (aaaaaa - aaaaa - a)/(a \times a).\\
111110888889 &= 111111 \times 999999 && :=aaaaaa \times (aaaaaaa - aaaaaa - a)/(a \times a).\\
11111108888889 &= 1111111 \times 9999999 && :=aaaaaaa \times (aaaaaaaa - aaaaaaa - a)/(a \times a).\\
1111111088888889 &= 11111111 \times 99999999 && :=aaaaaaaa \times (aaaaaaaaa - aaaaaaaa - a)/(a \times a).
\end{aligned}
$$

The above decomposition can also we written as $11 \times 99 = 33^2$, $111 \times 999 = 333^2$, etc. We can rewrite the above representation using as potentiation:

$$
\begin{aligned}
1089 &= 11 \times 99 = 33^2 && :=((aa + aa + aa)/a)^{(a+a)/a}.\\
110889 &= 111 \times 999 = 333^2 && :=((aaa + aaa + aaa)/a)^{(a+a)/a}.\\
11108889 &= 1111 \times 9999 = 3333^2 && :=((aaaa + aaaa + aaaa)/a)^{(a+a)/a}.\\
1111088889 &= 11111 \times 99999 = 33333^2 && :=((aaaaa + aaaaa + aaaaa)/a)^{(a+a)/a};.\\
111110888889 &= 111111 \times 999999 = 333333^2 && :=((aaaaaa + aaaaaa + aaaaaa)/a)^{(a+a)/a};.\\
11111108888889 &= 1111111 \times 9999999 = 3333333^2 && :=((aaaaaaa + aaaaaaa + aaaaaaa)/a)^{(a+a)/a}\\
1111111088888889 &= 11111111 \times 99999999 = 33333333^2 && :=((aaaaaaaa + aaaaaaaa + aaaaaaaa)/a)^{(a+a)/a}.
\end{aligned}
$$

More examples on potentiation are given in section 3.4.

Example 4.

$$
\begin{aligned}
7623 = 11 \times 9 \times 77 \quad &:= aa \times (aaa - aa - a) \times (aa - a - a - a - a)/(a \times a \times a).\\
776223 = 111 \times 9 \times 777 \quad &:= aaa \times (aaaa - aaa - a) \times (aa - a - a - a - a)/(a \times a \times a).\\
77762223 = 1111 \times 9 \times 7777 \quad &:= aaaa \times (aaaaa - aaaa - a) \times (aa - a - a - a - a)/(a \times a \times a).\\
7777622223 = 11111 \times 9 \times 77777 \quad &:= aaaaa \times (aaaaaa - aaaaa - a) \times (aa - a - a - a - a)/(a \times a \times a).\\
777776222223 = 111111 \times 9 \times 777777 \quad &:= aaaaaa \times (aaaaaaa - aaaaaa - a) \times (aa - a - a - a - a)/(a \times a \times a).\\
77777762222223 = 1111111 \times 9 \times 7777777 \quad &:= aaaaaaa \times (aaaaaaaa - aaaaaaa - a) \times (aa - a - a - a - a)/(a \times a \times a).\\
7777777622222223 = 11111111 \times 9 \times 77777777 \quad &:= aaaaaaaa \times (aaaaaaaaa - aaaaaaaa - a) \times (aa - a - a - a - a)/(a \times a \times a).
\end{aligned}
$$

3.2 Palindromic Patterns with Number Pattern Decompositions

Example 5.

$$
\begin{aligned}
1 = 0 \times 9 + 1 \quad &:= a/a.\\
11 = 1 \times 9 + 2 \quad &:= aa/a.\\
111 = 12 \times 9 + 3 \quad &:= aaa/a.\\
1111 = 123 \times 9 + 4 \quad &:= aaaa/a.\\
11111 = 1234 \times 9 + 5 \quad &:= aaaaa/a.\\
111111 = 12345 \times 9 + 6 \quad &:= aaaaaa/a.\\
1111111 = 123456 \times 9 + 7 \quad &:= aaaaaaa/a.\\
11111111 = 1234567 \times 9 + 8 \quad &:= aaaaaaaa/a.\\
111111111 = 12345678 \times 9 + 9 \quad &:= aaaaaaaaa/a.\\
1111111111 = 123456789 \times 9 + 10 \quad &:= aaaaaaaaaa/a.
\end{aligned}
$$

Example 6.

$$
\begin{aligned}
88 = 9 \times 9 + 7 \quad &:= (aa - a - a - a) \times aa/(a \times a).\\
888 = 98 \times 9 + 6 \quad &:= (aa - a - a - a) \times aaa/(a \times a).\\
8888 = 987 \times 9 + 5 \quad &:= (aa - a - a - a) \times aaaa/(a \times a).\\
88888 = 9876 \times 9 + 4 \quad &:= (aa - a - a - a) \times aaaaa/(a \times a).\\
888888 = 98765 \times 9 + 3 \quad &:= (aa - a - a - a) \times aaaaaa/(a \times a).\\
8888888 = 987654 \times 9 + 2 \quad &:= (aa - a - a - a) \times aaaaaaa/(a \times a).\\
88888888 = 9876543 \times 9 + 1 \quad &:= (aa - a - a - a) \times aaaaaaaa/(a \times a).\\
888888888 = 98765432 \times 9 + 0 \quad &:= (aa - a - a - a) \times aaaaaaaaa/(a \times a).\\
8888888888 = 987654321 \times 9 - 1 \quad &:= (aa - a - a - a) \times aaaaaaaaaa/(a \times a).\\
88888888888 = 9876543210 \times 9 - 2 \quad &:= (aa - a - a - a) \times aaaaaaaaaaa/(a \times a).
\end{aligned}
$$

Example 7.

$$
\begin{aligned}
99 = 98 + 1 &:=(aaa - aa - a)/(a \times a).\\
999 = 987 + 12 &:=(aaaa - aaa - a)/(a \times a).\\
9999 = 9876 + 123 &:=(aaaaa - aaaa - a)/(a \times a).\\
99999 = 98765 + 1234 &:=(aaaaaa - aaaaa - a)/(a \times a).\\
999999 = 987654 + 12345 &:=(aaaaaaa - aaaaaa - a)/(a \times a).\\
9999999 = 9876543 + 123456 &:=(aaaaaaaa - aaaaaaa - a)/(a \times a).\\
99999999 = 98765432 + 1234567 &:=(aaaaaaaaa - aaaaaaaa - a)/(a \times a).\\
999999999 = 987654321 + 12345678 &:=(aaaaaaaaaa - aaaaaaaaa - a)/(a \times a).\\
9999999999 = 9876543210 + 123456789 &:=(aaaaaaaaaaa - aaaaaaaaaa - a)/(a \times a).
\end{aligned}
$$

Example 8.

$$
\begin{aligned}
2772 = 4 \times 693 &:=(aaaaa - aa - aa - a)/(a + a + a + a).\\
27772 = 4 \times 6943 &:=(aaaaaa - aa - aa - a)/(a + a + a + a).\\
277772 = 4 \times 69443 &:=(aaaaaaa - aa - aa - a)/(a + a + a + a).\\
2777772 = 4 \times 694443 &:=(aaaaaaaa - aa - aa - a)/(a + a + a + a).\\
27777772 = 4 \times 6944443 &:=(aaaaaaaaa - aa - aa - a)/(a + a + a + a).\\
277777772 = 4 \times 69444443 &:=(aaaaaaaaaa - aa - aa - a)/(a + a + a + a).\\
2777777772 = 4 \times 694444443 &:=(aaaaaaaaaaa - aa - aa - a)/(a + a + a + a).\\
27777777772 = 4 \times 6944444443 &:=(aaaaaaaaaaaa - aa - aa - a)/(a + a + a + a).
\end{aligned}
$$

In this example the previous number $272 := (aaaa - aa - aa - a)/(a + a + a + a)$ is also a palindrome, but its decomposition $272 = 4 \times 68$ is not symmetrical to other values of the patterns.

Example 9. *This example is little irregular in terms of number patterns. But, later making proper choices, we can bring two different regular patterns.*

$$
\begin{aligned}
101 = 101 &:=(aaa - aa + a)/a.\\
1001 = 11 \times 91 &:=(aaaa - aaa + a)/a.\\
10001 = 73 \times 137 &:=(aaaaa - aaaa + a)/a.\\
100001 = 11 \times 9091 &:=(aaaaaa - aaaaa + a)/a.\\
1000001 = 101 \times 9901 &:=(aaaaaaa - aaaaaa + a)/a.\\
10000001 = 11 \times 909091 &:=(aaaaaaaa - aaaaaaa + a)/a.\\
100000001 = 17 \times 5882353 &:=(aaaaaaaaa - aaaaaaaa + a)/a.\\
1000000001 = 11 \times 90909091 &:=(aaaaaaaaaa - aaaaaaaaa + a)/a.\\
10000000001 = 101 \times 99009901 &:=(aaaaaaaaaaa - aaaaaaaaaa + a)/a.\\
100000000001 = 11 \times 9090909091 &:=(aaaaaaaaaaaa - aaaaaaaaaaa + a)/a.
\end{aligned}
$$

This example shows that it not necessary that every palindromic pattern can be decomposed to number pattern. By considering only even number of terms, i.e., 2nd, 4th, 6th, ..., we get a number pattern decomposition. Also considering 1st, 5th, 9th, 13th, ...terms, we get another regular number pattern decomposition.

$$
\begin{aligned}
1001 &= 11 \times 91 &&:=(aaaa - aaa + a)/a.\\
100001 &= 11 \times 9091 &&:=(aaaaaa - aaaaa + a)/a.\\
10000001 &= 11 \times 909091 &&:=(aaaaaaaa - aaaaaaa + a)/a.\\
1000000001 &= 11 \times 90909091 &&:=(aaaaaaaaaa - aaaaaaaaa + a)/a.\\
100000000001 &= 11 \times 9090909091 &&:=(aaaaaaaaaaaa - aaaaaaaaaaa + a)/a.
\end{aligned}
$$

$$
\begin{aligned}
101 &= 101 &&:=(aaa - aa + a)/a.\\
1000001 &= 101 \times 9901 &&:=(aaaaaaa - aaaaaa + a)/a.\\
10000000001 &= 101 \times 99009901 &&:=(aaaaaaaaaaa - aaaaaaaaaa + a)/a.\\
100000000000001 &= 101 \times 990099009901 &&:=(aaaaaaaaaaaaaaa - aaaaaaaaaaaaaa + a)/a.
\end{aligned}
$$

Example 10. *Here we have considered nine times the same value and the decomposition is palindromic.*

$$
\begin{aligned}
111111111 &= 12345679 \times 9 \times 1 :=aaaaaaaaa \times a/(a \times a).\\
222222222 &= 12345679 \times 9 \times 2 :=aaaaaaaaa \times (a+a)/(a \times a).\\
333333333 &= 12345679 \times 9 \times 3 :=aaaaaaaaa \times (a+a+a)/(a \times a).\\
444444444 &= 12345679 \times 9 \times 4 :=aaaaaaaaa \times (a+a+a+a)/(a \times a).\\
555555555 &= 12345679 \times 9 \times 5 :=aaaaaaaaa \times (a+a+a+a+a)/(a \times a).\\
666666666 &= 12345679 \times 9 \times 6 :=aaaaaaaaa \times (a+a+a+a+a+a)/(a \times a).\\
777777777 &= 12345679 \times 9 \times 7 :=aaaaaaaaa \times (aa-a-a-a-a)/(a \times a).\\
888888888 &= 12345679 \times 9 \times 8 :=aaaaaaaaa \times (aa-a-a-a)/(a \times a).\\
999999999 &= 12345679 \times 9 \times 9 :=aaaaaaaaa \times (aa-a-a)/(a \times a).
\end{aligned}
$$

More situations of similar kind with less number of repetitions are given in examples 18, 19 and 20.

Example 11. *Multiplying by 3 the number 12345679, appearing in previous example, we get*

$$12345679 \times 3 = 37037037.$$

The number 37037037 has very interesting properties. Multiplying it from 1 to 27 and reorganizing the values, we get very interesting patterns:

$$
\begin{aligned}
37037037 \times 3 &= 111111111\\
37037037 \times 6 &= 222222222\\
37037037 \times 9 &= 333333333\\
37037037 \times 12 &= 444444444\\
37037037 \times 15 &= 555555555\\
37037037 \times 18 &= 666666666\\
37037037 \times 21 &= 777777777\\
37037037 \times 24 &= 888888888\\
37037037 \times 27 &= 999999999
\end{aligned}
$$

$$
\begin{aligned}
37037037 \times 1 &= 037\ 037\ 037\\
37037037 \times 10 &= 370\ 370\ 370\\
37037037 \times 19 &= 703\ 703\ 703
\end{aligned}
$$

$$
\begin{aligned}
37037037 \times 2 &= 074\ 074\ 074\\
37037037 \times 11 &= 407\ 407\ 407\\
37037037 \times 20 &= 740\ 740\ 740
\end{aligned}
$$

$$
\begin{aligned}
37037037 \times 4 &= 148\ 148\ 148\\
37037037 \times 13 &= 481\ 481\ 481\\
37037037 \times 22 &= 814\ 814\ 814
\end{aligned}
$$

$$
\begin{aligned}
37037037 \times 5 &= 185\ 185\ 185\\
37037037 \times 14 &= 518\ 518\ 518\\
37037037 \times 23 &= 851\ 851\ 851
\end{aligned}
$$

$$
\begin{aligned}
37037037 \times 7 &= 259\ 259\ 259\\
37037037 \times 16 &= 592\ 592\ 592\\
37037037 \times 25 &= 925\ 925\ 925
\end{aligned}
$$

$$
\begin{aligned}
37037037 \times 8 &= 296\ 296\ 296\\
37037037 \times 17 &= 629\ 629\ 629\\
37037037 \times 26 &= 962\ 962\ 962
\end{aligned}
$$

Example 12. *Dividing 37037037 by 37 we get palindromic number 1001001. Any other number with the similar kind of pattern divided by last two digits always give the same palindromic number. See below*

$$\begin{aligned}
&17017017/17 = 1001001,\\
&19019019/19 = 1001001,\\
&23023023/23 = 1001001,\\
&45045045/45 = 1001001,\\
&\ldots\ldots \qquad \ldots\ldots
\end{aligned}$$

Let us make similar kind of multiplications as in previous example with number 17017017, we get symmetrical values, but not as beautiful as in previous example. See below:

$$\begin{aligned}
17017017 \times 1 &= 017017017\\
17017017 \times 2 &= 034034034\\
17017017 \times 3 &= 051051051\\
17017017 \times 4 &= 068068068\\
17017017 \times 5 &= 085085085\\
17017017 \times 6 &= 102102102\\
17017017 \times 7 &= 119119119\\
17017017 \times 8 &= 136136136\\
17017017 \times 9 &= 153153153
\end{aligned}$$

$$\begin{aligned}
17017017 \times 10 &= 170170170\\
17017017 \times 11 &= 187187187\\
17017017 \times 12 &= 204204204\\
17017017 \times 13 &= 221221221\\
17017017 \times 14 &= 238238238\\
17017017 \times 15 &= 255255255\\
17017017 \times 16 &= 272272272\\
17017017 \times 17 &= 289289289\\
17017017 \times 18 &= 306306306
\end{aligned}$$

$$\begin{aligned}
17017017 \times 19 &= 323323323\\
17017017 \times 20 &= 340340340\\
17017017 \times 21 &= 357357357\\
17017017 \times 22 &= 374374374\\
17017017 \times 23 &= 391391391\\
17017017 \times 24 &= 408408408\\
17017017 \times 25 &= 425425425\\
17017017 \times 26 &= 442442442\\
17017017 \times 27 &= 459459459
\end{aligned}$$

Above multiplications are done only up to 27, but it can go up to 58, and still the results remains symmetric, i.e.,

$$17017017 \times 58 = 986986986.$$

Example 13. *This example is written in two different decompositions for the same palindromic pattern. One is* $717 = 88 \times 8 + 13$ *and another is* $717 = 56 \times 13 - 1$. *The first decomposition is in terms of "palindromic numbers", while the second representation is in terms of "number patterns". In second case, less number of letters "a" are used.*

$$\begin{aligned}
717 = 88 \times 8 + 13 \quad &:= (aaa - aa - aa - a) \times (aa - a - a - a)/(a \times a) + (aa + a + a)/a.\\
7117 = 888 \times 8 + 13 \quad &:= (aaaa - aaa - aaa - a) \times (aa - a - a - a)/(a \times a) + (aa + a + a)/a.\\
71117 = 8888 \times 8 + 13 \quad &:= (aaaaa - aaaa - aaaa - a) \times (aa - a - a - a)/(a \times a) + (aa + a + a)/a.\\
711117 = 88888 \times 8 + 13 \quad &:= (aaaaaa - aaaaa - aaaaa - a) \times (aa - a - a - a)/(a \times a) + (aa + a + a)/a.\\
7111117 = 888888 \times 8 + 13 \quad &:= (aaaaaaa - aaaaaa - aaaaaa - a) \times (aa - a - a - a)/(a \times a) + (aa + a + a)/a.\\
71111117 = 8888888 \times 8 + 13 \quad &:= (aaaaaaaa - aaaaaaa - aaaaaaa - a) \times (aa - a - a - a)/(a \times a) + (aa + a + a)/a.\\
711111117 = 88888888 \times 8 + 13 \quad &:= (aaaaaaaaa - aaaaaaaa - aaaaaaaa - a) \times (aa - a - a - a)/(a \times a) + (aa + a + a)/a.
\end{aligned}$$

$$\begin{aligned}
717 = 56 \times 13 - 11 \quad &:= (aaa + a)/(a + a) \times (aa + a + a)/a - aa/a.\\
7117 = 556 \times 13 - 111 \quad &:= (aaaa + a)/(a + a) \times (aa + a + a)/a - aaa/a.\\
71117 = 5556 \times 13 - 1111 \quad &:= (aaaaa + a)/(a + a) \times (aa + a + a)/a - aaaa/a.\\
711117 = 55556 \times 13 - 11111 \quad &:= (aaaaaa + a)/(a + a) \times (aa + a + a)/a - aaaaa/a.\\
7111117 = 555556 \times 13 - 111111 \quad &:= (aaaaaaa + a)/(a + a) \times (aa + a + a)/a - aaaaaa/a.\\
71111117 = 5555556 \times 13 - 1111111 \quad &:= (aaaaaaaa + a)/(a + a) \times (aa + a + a)/a - aaaaaaa/a.\\
711111117 = 55555556 \times 13 - 11111111 \quad &:= (aaaaaaaaa + a)/(a + a) \times (aa + a + a)/a - aaaaaaaa/a.
\end{aligned}$$

3.3 Palindromic Patterns with Palindromic Decompositions

Here palindromic patterns with palindromic decompositions are considered.

Example 14.

$$
\begin{aligned}
121 &= 11 \times 11 &&:= aa \times aa/(a \times a \times a).\\
1221 &= 11 \times 111 &&:= aa \times aaa/(a \times a \times a).\\
12221 &= 11 \times 1111 &&:= aa \times aaaa/(a \times a \times a).\\
122221 &= 11 \times 11111 &&:= aa \times aaaaa/(a \times a \times a).\\
1222221 &= 11 \times 111111 &&:= aa \times aaaaaa/(a \times a \times a).\\
12222221 &= 11 \times 1111111 &&:= aa \times aaaaaaa/(a \times a \times a).\\
122222221 &= 11 \times 11111111 &&:= aa \times aaaaaaaa/(a \times a \times a).\\
1222222221 &= 11 \times 111111111 &&:= aa \times aaaaaaaaa/(a \times a \times a).
\end{aligned}
$$

Example 15.

$$
\begin{aligned}
1331 &= 11 \times 11 \times 11 &&:= aa \times aa \times aa/(a \times a \times a).\\
13431 &= 11 \times 11 \times 111 &&:= aa \times aa \times aaa/(a \times a \times a).\\
134431 &= 11 \times 11 \times 1111 &&:= aa \times aa \times aaaa/(a \times a \times a).\\
1344431 &= 11 \times 11 \times 11111 &&:= aa \times aa \times aaaaa/(a \times a \times a).\\
13444431 &= 11 \times 11 \times 111111 &&:= aa \times aa \times aaaaaa/(a \times a \times a).\\
134444431 &= 11 \times 11 \times 1111111 &&:= aa \times aa \times aaaaaaa/(a \times a \times a).\\
1344444431 &= 11 \times 11 \times 11111111 &&:= aa \times aa \times aaaaaaaa/(a \times a \times a).\\
13444444431 &= 11 \times 11 \times 111111111 &&:= aa \times aa \times aaaaaaaaa/(a \times a \times a).
\end{aligned}
$$

Example 16.

$$
\begin{aligned}
99 &= 9 \times 11 &&:= (aaa - aa - a)/a.\\
999 &= 9 \times 111 &&:= (aaaa - aaa - a)/a.\\
9999 &= 9 \times 1111 &&:= (aaaaa - aaaa - a)/a.\\
99999 &= 9 \times 11111 &&:= (aaaaaa - aaaaa - a)/a.\\
999999 &= 9 \times 111111 &&:= (aaaaaaa - aaaaaa - a)/a.\\
9999999 &= 9 \times 1111111 &&:= (aaaaaaaa - aaaaaaa - a)/a.\\
99999999 &= 9 \times 11111111 &&:= (aaaaaaaaa - aaaaaaaa - a)/a.\\
999999999 &= 9 \times 111111111 &&:= (aaaaaaaaaa - aaaaaaaaa - a)/a.
\end{aligned}
$$

Example 17.

$$
\begin{aligned}
1001 &= 13 \times 77 &&:= aa \times (aaaa - aaa + a)/(aa \times a).\\
10101 &= 13 \times 777 &&:= aaa \times (aaaa - aaa + a)/(aa \times a).\\
101101 &= 13 \times 7777 &&:= aaaa \times (aaaa - aaa + a)/(aa \times a).\\
1011101 &= 13 \times 77777 &&:= aaaaa \times (aaaa - aaa + a)/(aa \times a).\\
10111101 &= 13 \times 777777 &&:= aaaaaa \times (aaaa - aaa + a)/(aa \times a).\\
101111101 &= 13 \times 7777777 &&:= aaaaaaa \times (aaaa - aaa + a)/(aa \times a).\\
1011111101 &= 13 \times 77777777 &&:= aaaaaaaa \times (aaaa - aaa + a)/(aa \times a).\\
10111111101 &= 13 \times 777777777 &&:= aaaaaaaaa \times (aaaa - aaa + a)/(aa \times a).
\end{aligned}
$$

Example 18.

$$11111111 = 11 \times 1010101 := aaaaaaaa \times a/(a \times a).$$
$$22222222 = 11 \times 2020202 := aaaaaaaa \times (a + a)/(a \times a).$$
$$33333333 = 11 \times 3030303 := aaaaaaaa \times (a + a + a)/(a \times a).$$
$$44444444 = 11 \times 4040404 := aaaaaaaa \times (a + a + a + a)/(a \times a).$$
$$55555555 = 11 \times 5050505 := aaaaaaaa \times (a + a + a + a + a)/(a \times a).$$
$$66666666 = 11 \times 6060606 := aaaaaaaa \times (a + a + a + a + a + a)/(a \times a).$$
$$77777777 = 11 \times 7070707 := aaaaaaaa \times (aa - a - a - a - a)/(a \times a).$$
$$88888888 = 11 \times 8080808 := aaaaaaaa \times (aa - a - a - a)/(a \times a).$$
$$99999999 = 11 \times 9090909 := aaaaaaaa \times (aa - a - a)/(a \times a).$$

This example brings palindromes on both sides of the expressions. While example 10, brings number pattern decomposition. Here we have considered same digits repeating eight times, while example 10 has having 9 digits. When we work with even number of digits, i.e, 2, 4, 6, etc. with repetition of same digit, the decomposition is always palindromic. In case of odd numbers, it is different. The following two examples give how patterns will take their form working with 3,4, 5, 6 and 7 repetitions of same digit.

Example 19. *Here below are two even order decompositions resulting again in palindromic patterns.*

$$1111 := 11 \times 101.$$
$$2222 := 11 \times 202.$$
$$3333 := 11 \times 303.$$
$$4444 := 11 \times 404.$$
$$5555 := 11 \times 505.$$
$$6666 := 11 \times 606.$$
$$7777 := 11 \times 707.$$
$$8888 := 11 \times 808.$$
$$9999 := 11 \times 909.$$

$$111111 := 37037 \times 3 \times 1 = 11 \times 10101.$$
$$222222 := 37037 \times 3 \times 2 = 11 \times 20202.$$
$$333333 := 37037 \times 3 \times 3 = 11 \times 30303.$$
$$444444 := 37037 \times 3 \times 4 = 11 \times 40404.$$
$$555555 := 37037 \times 3 \times 5 = 11 \times 50505.$$
$$666666 := 37037 \times 3 \times 6 = 11 \times 60606.$$
$$777777 := 37037 \times 3 \times 7 = 11 \times 70707.$$
$$888888 := 37037 \times 3 \times 8 = 11 \times 80808.$$
$$999999 := 37037 \times 3 \times 9 = 11 \times 90909.$$

Example 20. *Here below are three odd order decomposition, where result is always in terms of prime numbers The third example is given with letter "a" representations.*

$$111 := 37 \times 3 \times 1.$$
$$222 := 37 \times 3 \times 2.$$
$$333 := 37 \times 3 \times 3.$$
$$444 := 37 \times 3 \times 4.$$
$$555 := 37 \times 3 \times 5.$$
$$666 := 37 \times 3 \times 6.$$
$$777 := 37 \times 3 \times 7.$$
$$888 := 37 \times 3 \times 8.$$
$$999 := 37 \times 3 \times 9.$$

$$11111 := 41 \times 271 \times 1.$$
$$22222 := 41 \times 271 \times 2.$$
$$33333 := 41 \times 271 \times 3.$$
$$44444 := 41 \times 271 \times 4.$$
$$55555 := 41 \times 271 \times 5.$$
$$66666 := 41 \times 271 \times 6.$$
$$77777 := 41 \times 271 \times 7.$$
$$88888 := 41 \times 271 \times 8.$$
$$99999 := 41 \times 271 \times 9.$$

$1111111 := 239 \times 4649 \times 1 := aaaaaaa \times a/(a \times a).$
$2222222 := 239 \times 4649 \times 2 := aaaaaaa \times (a+a)/(a \times a).$
$3333333 := 239 \times 4649 \times 3 := aaaaaaa \times (a+a+a)/(a \times a).$
$4444444 := 239 \times 4649 \times 4 := aaaaaaa \times (a+a+a+a)/(a \times a).$
$5555555 := 239 \times 4649 \times 5 := aaaaaaa \times (a+a+a+a+a)/(a \times a).$
$6666666 := 239 \times 4649 \times 6 := aaaaaaa \times (a+a+a+a+a+a)/(a \times a).$
$7777777 := 239 \times 4649 \times 7 := aaaaaaa \times (aa-a-a-a-a)/(a \times a).$
$8888888 := 239 \times 4649 \times 8 := aaaaaaa \times (aa-a-a-a)/(a \times a).$
$9999999 := 239 \times 4649 \times 9 := aaaaaaa \times (aa-a-a)/(a \times a).$

3.4 Number Patterns with Power

Here we shall consider *number patterns with power* and their decompositions are also number patterns.

Example 21.

$$
\begin{aligned}
16^2 &= 256 && := ((aa+aa+aa-a)/(a+a))^{(a+a)/a}.\\
166^2 &= 27556 && := ((aaa+aaa+aaa-a)/(a+a))^{(a+a)/a}.\\
1666^2 &= 2775556 && := ((aaaa+aaaa+aaaa-a)/(a+a))^{(a+a)/a}.\\
16666^2 &= 277755556 && := ((aaaaa+aaaaa+aaaaa-a)/(a+a))^{(a+a)/a}.\\
166666^2 &= 27777555556 && := ((aaaaaa+aaaaaa+aaaaaa-a)/(a+a))^{(a+a)/a}.\\
1666666^2 &= 2777775555556 && := ((aaaaaaa+aaaaaaa+aaaaaaa-a)/(a+a))^{(a+a)/a}.\\
16666666^2 &= 277777755555556 && := ((aaaaaaaa+aaaaaaaa+aaaaaaaa-a)/(a+a))^{(a+a)/a}.
\end{aligned}
$$

Example 22.

$$
\begin{aligned}
34^2 &= 1156 && := ((aa+aa+aa+a)/a)^{(a+a)/a}.\\
334^2 &= 111556 && := ((aaa+aaa+aaa+a)/a)^{(a+a)/a}.\\
3334^2 &= 11115556 && := ((aaaa+aaaa+aaaa+a)/a)^{(a+a)/a}.\\
33334^2 &= 1111155556 && := ((aaaaa+aaaaa+aaaaa+a)/a)^{(a+a)/a}.\\
333334^2 &= 111111555556 && := ((aaaaaa+aaaaaa+aaaaaa+a)/a)^{(a+a)/a}.\\
3333334^2 &= 11111115555556 && := ((aaaaaaa+aaaaaaa+aaaaaaa+a)/a)^{(a+a)/a}.\\
33333334^2 &= 1111111155555556 && := ((aaaaaaaa+aaaaaaaa+aaaaaaaa+a)/a)^{(a+a)/a}.
\end{aligned}
$$

Example 23.

$$
\begin{aligned}
43^2 &= 1849 && := ((aa+aa+aa+aa-a)/a)^{(a+a)/a}.\\
433^2 &= 187489 && := ((aaa+aaa+aaa+aaa-aa)/a)^{(a+a)/a}.\\
4333^2 &= 18774889 && := ((aaaa+aaaa+aaaa+aaaa-aaa)/a)^{(a+a)/a}.\\
43333^2 &= 1877748889 && := ((aaaaa+aaaaa+aaaaa+aaaaa-aaaa)/a)^{(a+a)/a}.\\
433333^2 &= 187777488889 && := ((aaaaaa+aaaaaa+aaaaaa+aaaaaa-aaaaa)/a)^{(a+a)/a}.\\
4333333^2 &= 18777774888889 && := ((aaaaaaa+aaaaaaa+aaaaaaa+aaaaaaa-aaaaaa)/a)^{(a+a)/a}.\\
43333333^2 &= 1877777748888889 && := ((aaaaaaaa+aaaaaaaa+aaaaaaaa+aaaaaaaa-aaaaaaa)/a)^{(a+a)/a}.
\end{aligned}
$$

Example 24.

$$\begin{aligned}
67^2 &= 4489 &&:=((aaa + aa + aa + a)/(a + a))^{(a+a)/a}.\\
667^2 &= 444889 &&:=((aaaa + aaa + aaa + a)/(a + a))^{(a+a)/a}.\\
6667^2 &= 44448889 &&:=((aaaaa + aaaa + aaaa + a)/(a + a))^{(a+a)/a}.\\
66667^2 &= 4444488889 &&:=((aaaaaa + aaaaa + aaaaa + a)/(a + a))^{(a+a)/a}.\\
666667^2 &= 444444888889 &&:=((aaaaaaa + aaaaaa + aaaaaa + a)/(a + a))^{(a+a)/a}.\\
6666667^2 &= 44444448888889 &&:=((aaaaaaaa + aaaaaaa + aaaaaaa + a)/(a + a))^{(a+a)/a}.\\
66666667^2 &= 4444444488888889 &&:=((aaaaaaaaa + aaaaaaaa + aaaaaaaa + a)/(a + a))^{(a+a)/a}.
\end{aligned}$$

Example 25.

$$\begin{aligned}
91^2 &= 828 &&:=((aaa - aa - aa + a + a)/a)^{(a+a)/a}.\\
991^2 &= 98208 &&:=((aaaa - aaa - aa + a + a)/a)^{(a+a)/a}.\\
9991^2 &= 9982008 &&:=((aaaaa - aaaa - aa + a + a)/a)^{(a+a)/a}.\\
99991^2 &= 999820008 &&:=((aaaaaa - aaaaa - aa + a + a)/a)^{(a+a)/a}.\\
999991^2 &= 99998200008 &&:=((aaaaaaa - aaaaaa - aa + a + a)/a)^{(a+a)/a}.\\
9999991^2 &= 9999982000008 &&:=((aaaaaaaa - aaaaaaa - aa + a + a)/a)^{(a+a)/a}.\\
99999991^2 &= 999999820000008 &&:=((aaaaaaaaa - aaaaaaaa - aa + a + a)/a)^{(a+a)/a}.
\end{aligned}$$

3.5 Repeated Digits Patterns

In this subsection, we shall present situations, where the patterns are formed by repetition of digits. In each case, the repetitions are in different forms.

We can write

$$999999 = 3 \times 7 \times 11 \times 13 \times 37.$$

Division by 3, 11 and 37 always bring palindromes, i.e.,

$$\begin{aligned}
999999/3 &= 333333\\
999999/11 &= 90909\\
999999/37 &= 9009 \times 3.
\end{aligned}$$

The division by other two numbers, i.e., by 7 and 13 gives two different numbers, i.e.,

$$\begin{aligned}
999999/7 &= 142857\\
999999/13 &= 76923.
\end{aligned}$$

The following three examples are based on the numbers 142857 and 76923 with repetition of digits.

Example 26. *This example is based on the number 142857.*

$$\begin{aligned}
142857 \times 1 = 142857 &:= aaaaaa \times (aa - a - a) \times a/(a \times a \times (aa - a - a - a - a)).\\
142857 \times 3 = 428571 &:= aaaaaa \times (aa - a - a) \times (a + a + a)/(a \times a \times (aa - a - a - a - a)).\\
142857 \times 2 = 285714 &:= aaaaaa \times (aa - a - a) \times (a + a)/(a \times a \times (aa - a - a - a - a)).\\
142857 \times 6 = 857142 &:= aaaaaa \times (aa - a - a) \times (a + a + a + a + a + a)/(a \times a \times (aa - a - a - a - a)).\\
142857 \times 4 = 571428 &:= aaaaaa \times (aa - a - a) \times (a + a + a + a)/(a \times a \times (aa - a - a - a - a)).\\
142857 \times 5 = 714285 &:= aaaaaa \times (aa - a - a) \times (a + a + a + a + a)/(a \times a \times (aa - a - a - a - a)).
\end{aligned}$$

Multiplying by 7 and dividing by 9, above numbers, we get interesting pattern:

$$142857 \times 7/9 = 111111$$
$$428571 \times 7/9 = 333333$$
$$285714 \times 7/9 = 222222$$
$$857142 \times 7/9 = 666666$$
$$571428 \times 7/9 = 444444$$
$$714285 \times 7/9 = 555555$$

Example 27. *This example is based on the number 76923.*

$$76923 \times 1 = 076923 := aaaaaa \times (aa - a - a) \times a/(a \times a \times (aa + a + a)).$$
$$76923 \times 10 = 769230 := aaaaaa \times (aa - a - a) \times (aa - a)/(a \times a \times (aa + a + a)).$$
$$76923 \times 9 = 692307 := aaaaaa \times (aa - a - a) \times (aa - a - a)/(a \times a \times (aa + a + a)).$$
$$76923 \times 12 = 923076 := aaaaaa \times (aa - a - a) \times (aa + a)/(a \times a \times (aa + a + a)).$$
$$76923 \times 3 = 230769 := aaaaaa \times (aa - a - a) \times (a + a + a)/(a \times a \times (aa + a + a)).$$
$$76923 \times 4 = 307692 := aaaaaa \times (aa - a - a) \times (a + a + a + a)/(a \times a \times (aa + a + a)).$$

Example 28. *This example also deals with the number 76923. After multiplication by different numbers, we get a numbers with repetitions of same digits.*

$$76923 \times 2 = 153846 := aaaaaa \times (aa - a - a) \times (a + a)/(a \times a \times (aa + a + a)).$$
$$76923 \times 7 = 538461 := aaaaaa \times (aa - a - a) \times (aa - a - a - a - a)/(a \times a \times (aa + a + a)).$$
$$76923 \times 5 = 384615 := aaaaaa \times (aa - a - a) \times (a + a + a + a + a)/(a \times a \times (aa + a + a)).$$
$$76923 \times 11 = 846153 := aaaaaa \times (aa - a - a) \times aa/(a \times a \times (aa + a + a)).$$
$$76923 \times 6 = 461538 := aaaaaa \times (aa - a - a) \times (a + a + a + a + a + a)/(a \times a \times (aa + a + a)).$$
$$76923 \times 8 = 615384 := aaaaaa \times (aa - a - a) \times (aa - a - a - a)/(a \times a \times (aa + a + a)).$$

Example 29. *Multiplication of 1089 with* $1, 2, 3, 4, 5, 6, 7, 8$ *and* 9 *brings interesting pattern:*

$$1089 \times 1 = 1089 := (aaaa - aa - aa) \times a/(a \times a).$$
$$1089 \times 2 = 2178 := (aaaa - aa - aa) \times (a + a)/(a \times a).$$
$$1089 \times 3 = 3267 := (aaaa - aa - aa) \times (a + a + a)/(a \times a).$$
$$1089 \times 4 = 4356 := (aaaa - aa - aa) \times (a + a + a + a)/(a \times a).$$
$$1089 \times 5 = 5445 := (aaaa - aa - aa) \times (a + a + a + a + a)/(a \times a).$$
$$1089 \times 6 = 6534 := (aaaa - aa - aa) \times (a + a + a + a + a + a)/(a \times a).$$
$$1089 \times 7 = 7623 := (aaaa - aa - aa) \times (aa - a - a - a - a)/(a \times a).$$
$$1089 \times 8 = 8712 := (aaaa - aa - aa) \times (aa - a - a - a)/(a \times a).$$
$$1089 \times 9 = 9801 := (aaaa - aa - aa) \times (aa - a - a)/(a \times a).$$

In each column, the digits are in consecutive way (increasing or decreasing). In pairs, they are reverse of each other, i.e., (1089, 9801), (2178, 8712), (3267, 7623) and (4356, 6534). Another interesting property of these nine numbers is connected with magic squares, i.e., members of columns considered 2, 3 or 4 columns, they always forms magic squares order 3x3 [2].

Example 30. *Another number having similar kind of properties of previous example is 9109. Multiplying it by 1, 2, 3, 4, 5, 6, 7, 8 and 9, we get*

$$
\begin{aligned}
9109 \times 1 = 09109 &:= (aaaaa - aaaaaa \times (a+a)/aaa) \times a/(a \times a).\\
9109 \times 2 = 18218 &:= (aaaaa - aaaaaa \times (a+a)/aaa) \times (a+a)/(a \times a).\\
9109 \times 3 = 27327 &:= (aaaaa - aaaaaa \times (a+a)/aaa) \times (a+a+a)/(a \times a).\\
9109 \times 4 = 36436 &:= (aaaaa - aaaaaa \times (a+a)/aaa) \times (a+a+a+a)/(a \times a).\\
9109 \times 5 = 45545 &:= (aaaaa - aaaaaa \times (a+a)/aaa) \times (a+a+a+a+a)/(a \times a).\\
9109 \times 6 = 54654 &:= (aaaaa - aaaaaa \times (a+a)/aaa) \times (a+a+a+a+a+a)/(a \times a).\\
9109 \times 7 = 63763 &:= (aaaaa - aaaaaa \times (a+a)/aaa) \times (aa-a-a-a-a)/(a \times a).\\
9109 \times 8 = 72872 &:= (aaaaa - aaaaaa \times (a+a)/aaa) \times (aa-a-a-a)/(a \times a).\\
9109 \times 9 = 81981 &:= (aaaaa - aaaaaa \times (a+a)/aaa) \times (aa-a-a)/(a \times a).
\end{aligned}
$$

Column members are in increasing and decreasing orders of 0 to 8 or 9. Also first and last two digit of each number are same, and are multiple of 9. Moreover, the number 9109 is prime number.

Example 31. *This example is based on the property,* $8712 = 2178 \times 4 = 1089 \times 2 \times 4$, *i.e., after multiplication by 4 the number 2178 becomes its reverse, i.e., 8712. Also it is a multiple of 8 with 1089.*

$$
\begin{aligned}
8712 = 2178 \times 4 &:= (aaaa - aa - aa) \times (a+a) \times (a+a+a+a)/(a \times a \times a).\\
87912 = 21978 \times 4 &:= (aaaaa - aaa - aa) \times (a+a) \times (a+a+a+a)/(a \times a \times a).\\
879912 = 219978 \times 4 &:= (aaaaaa - aaaa - aa) \times (a+a) \times (a+a+a+a)/(a \times a \times a).\\
8799912 = 2199978 \times 4 &:= (aaaaaaa - aaaaa - aa) \times (a+a) \times (a+a+a+a)/(a \times a \times a).\\
87999912 = 21999978 \times 4 &:= (aaaaaaaa - aaaaaa - aa) \times (a+a) \times (a+a+a+a)/(a \times a \times a).\\
879999912 = 219999978 \times 4 &:= (aaaaaaaaa - aaaaaaa - aa) \times (a+a) \times (a+a+a+a)/(a \times a \times a).\\
8799999912 = 2199999978 \times 4 &:= (aaaaaaaaaa - aaaaaaaa - aa) \times (a+a) \times (a+a+a+a)/(a \times a \times a).
\end{aligned}
$$

Example 32. *If we multiply 1089 by 9, we get 9801, i.e., reverse of 1089. The same happens with next members of the patterns.*

$$
\begin{aligned}
9801 = 1089 \times 9 = 99 \times 99 &:= (aaa - aa - a) \times (aaa - aa - a)/(a \times a).\\
98901 = 10989 \times 9 = 99 \times 999 &:= (aaa - aa - a) \times (aaaa - aaa - a)/(a \times a).\\
989901 = 109989 \times 9 = 99 \times 9999 &:= (aaa - aa - a) \times (aaaaa - aaaa - a)/(a \times a).\\
9899901 = 1099989 \times 9 = 99 \times 99999 &:= (aaa - aa - a) \times (aaaaaa - aaaaa - a)/(a \times a).\\
98999901 = 10999989 \times 9 = 99 \times 999999 &:= (aaa - aa - a) \times (aaaaaaa - aaaaaa - a)/(a \times a).\\
989999901 = 109999989 \times 9 = 99 \times 9999999 &:= (aaa - aa - a) \times (aaaaaaaa - aaaaaaa - a)/(a \times a).\\
9899999901 = 1099999989 \times 9 = 99 \times 99999999 &:= (aaa - aa - a) \times (aaaaaaaaa - aaaaaaaa - a)/(a \times a).
\end{aligned}
$$

Tricks for Making Pattern. [3] Let us consider numbers of 3, 4, 5 and 6 digits, for example, 183, 3568, 19757 and 876456. Changing first digit with last and vice-versa, in each case, we get, 381, 8563, 79751 and 676458 respectively. Let us consider the difference among the respective values (higher minus lesser), i.e.,

$$
\begin{aligned}
381 - 183 &= 198\\
8563 - 3568 &= 4995\\
79751 - 19757 &= 59994\\
876456 - 676458 &= 199998
\end{aligned}
$$

Changing again last digit with first and vice-versa, and adding we get the required pattern, i.e.,

$$198 + 891 = 1089$$
$$4995 + 5994 = 10989$$
$$59994 + 49995 = 109989$$
$$199998 + 899991 = 1099989$$

Proceeding further with higher digits, we get further values of the pattern. Here the condition is that, the difference in each case should be bigger than 1, for example, $3453 - 3453 = 0$ is not valid number. Second condition is that if this differences come to 99, 999, etc, i.e., $201 - 102 = 99$, $4433 - 3434 = 999$, etc. In this situation, we have to sum twice, i.e, $99 + 99 = 198$ and $999 + 999 = 1998$, and then $198 + 891 = 1089$ and $1998 + 8991 = 10989$, etc.

3.6 Doubly Symmetric Patterns

Here, below are three examples of *doubly symmetric patterns*, i.e., we can write, $99 \times 5 = 9 \times 55$, $99 \times 7 = 9 \times 77$ and $99 \times 8 = 9 \times 88$, etc.

Example 33.

$$\begin{aligned}
45 &= 9 \times 5 = 9 \times 5 &&:= (aaa - aa - aa + a)/(a + a).\\
495 &= 99 \times 5 = 9 \times 55 &&:= (aaaa - aaa - aa + a)/(a + a).\\
4995 &= 999 \times 5 = 9 \times 555 &&:= (aaaaa - aaaa - aa + a)/(a + a).\\
49995 &= 9999 \times 5 = 9 \times 5555 &&:= (aaaaaa - aaaaa - aa + a)/(a + a).\\
499995 &= 99999 \times 5 = 9 \times 55555 &&:= (aaaaaaa - aaaaaa - aa + a)/(a + a).\\
4999995 &= 999999 \times 5 = 9 \times 555555 &&:= (aaaaaaaa - aaaaaaa - aa + a)/(a + a).\\
49999995 &= 9999999 \times 5 = 9 \times 5555555 &&:= (aaaaaaaaa - aaaaaaaa - aa + a)/(a + a).
\end{aligned}$$

Example 34.

$$\begin{aligned}
63 &= 9 \times 7 = 9 \times 7 &&:= (a + a + a) \times (aa + aa - a)/(a \times a).\\
693 &= 99 \times 7 = 9 \times 77 &&:= (aa + aa + aa) \times (aa + aa - a)/(a \times a).\\
6993 &= 999 \times 7 = 9 \times 777 &&:= (aaa + aaa + aaa) \times (aa + aa - a)/(a \times a).\\
69993 &= 9999 \times 7 = 9 \times 7777 &&:= (aaaa + aaaa + aaaa) \times (aa + aa - a)/(a \times a).\\
699993 &= 99999 \times 7 = 9 \times 77777 &&:= (aaaaa + aaaaa + aaaaa) \times (aa + aa - a)/(a \times a).\\
6999993 &= 999999 \times 7 = 9 \times 777777 &&:= (aaaaaa + aaaaaa + aaaaaa) \times (aa + aa - a)/(a \times a).\\
69999993 &= 9999999 \times 7 = 9 \times 7777777 &&:= (aaaaaaa + aaaaaaa + aaaaaaa) \times (aa + aa - a)/(a \times a).
\end{aligned}$$

Example 35.

$$\begin{aligned}
72 &= 9 \times 8 = 9 \times 8 &&:= (aa + a) \times (aa + a) \times a/(a + a)/(a \times a).\\
792 &= 99 \times 8 = 9 \times 88 &&:= (aa + a) \times (aa + a) \times aa/(a + a)/(a \times a).\\
7992 &= 999 \times 8 = 9 \times 888 &&:= (aa + a) \times (aa + a) \times aaa/(a + a)/(a \times a).\\
79992 &= 9999 \times 8 = 9 \times 8888 &&:= (aa + a) \times (aa + a) \times aaaa/(a + a)/(a \times a).\\
799992 &= 99999 \times 8 = 9 \times 88888 &&:= (aa + a) \times (aa + a) \times aaaaa/(a + a)/(a \times a).\\
7999992 &= 999999 \times 8 = 9 \times 888888 &&:= (aa + a) \times (aa + a) \times aaaaaa/(a + a)/(a \times a).\\
79999992 &= 9999999 \times 8 = 9 \times 8888888 &&:= (aa + a) \times (aa + a) \times aaaaaaa/(a + a)/(a \times a).
\end{aligned}$$

Above examples are written multiplying 9 with 5, 7 and 8. Same can be done multiplying 9 with 2, 3, 4 and 6. This is due to the fact that 11 is a factor of 99 and aa, for all $a \in \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$

Example 36. *(**General way**). Examples 33, 34 and 35 can be written in general way. Let us consider a and b, such that*

$$a, b \in \{1, 2, 3, 4, 5, 6, 7, 8, 9\}, a + b = 9, a > b,$$

then, examples 32, 33 and 34 can be summarized as

$$\begin{aligned}
a9b = 99 \times (a+1) &\quad =9 \times (a+1)(a+1) \\
a99b = 999 \times (a+1) &\quad =9 \times (a+1)(a+1)(a+1) \\
a999b = 9999 \times (a+1) &\quad =9 \times (a+1)(a+1)(a+1)(a+1) \\
a9999b = 99999 \times (a+1) &\quad =9 \times (a+1)(a+1)(a+1)(a+1)(a+1) \\
a99999b = 999999 \times (a+1) &\quad =9 \times (a+1)(a+1)(a+1)(a+1)(a+1)(a+1) \\
.... &\quad ....
\end{aligned}$$

where

$$(a+1)(a+1)(a+1) = (a+1) \times 10^2 + (a+1) \times 10^1 + (a+1) \times 10^0, etc.$$

We can apply this general way to get the pattern appearing in example 32. Let change a with b and b with a in the above pattern, then sum both, we get

$$\begin{aligned}
a9b + b9a &= 1089 \\
a99b + b99a &= 10989 \\
a999b + b999a &= 109989 \\
a9999b + b9999a &= 1099989 \\
a99999b + b99999a &= 10999989
\end{aligned}$$

This is the same pattern appearing in example 32.

3.7 Number Patterns with Number Pattern Decompositions

Example 37.

$$\begin{aligned}
&11111111101 = 123456789 \times 9 \times 1 :=(aaaaaaaaaaa - aa + a) \times a/(a \times a). \\
&22222222202 = 123456789 \times 9 \times 2 :=(aaaaaaaaaaa - aa + a) \times (a + a)/(a \times a). \\
&33333333303 = 123456789 \times 9 \times 3 :=(aaaaaaaaaaa - aa + a) \times (a + a + a)/(a \times a). \\
&44444444404 = 123456789 \times 9 \times 4 :=(aaaaaaaaaaa - aa + a) \times (a + a + a + a)/(a \times a). \\
&55555555505 = 123456789 \times 9 \times 5 :=(aaaaaaaaaaa - aa + a) \times (a + a + a + a + a)/(a \times a). \\
&66666666606 = 123456789 \times 9 \times 6 :=(aaaaaaaaaaa - aa + a) \times (a + a + a + a + a + a)/(a \times a). \\
&77777777707 = 123456789 \times 9 \times 7 :=(aaaaaaaaaaa - aa + a) \times (aa - a - a - a - a)/(a \times a). \\
&88888888808 = 123456789 \times 9 \times 8 :=(aaaaaaaaaaa - aa + a) \times (aa - a - a - a)/(a \times a). \\
&99999999909 = 123456789 \times 9 \times 9 :=(aaaaaaaaaaa - aa + a) \times (aa - a - a)/(a \times a).
\end{aligned}$$

This example is an extension of example 9. Instead considering 12345679, we have considered all the digits, i.e., 123456789. We get number pattern on both sides, while in example 9, we have palindromic pattern on one side of the expression.

Example 38.

$$\begin{aligned}
9 &= 1 \times 8 + 1 &&:=(aa - a - a)/a.\\
98 &= 12 \times 8 + 2 &&:=(aaa - aa - a - a)/a.\\
987 &= 123 \times 8 + 3 &&:=(aaaa - aaa - aa - a - a)/a.\\
9876 &= 1234 \times 8 + 4 &&:=(aaaaa - aaaa - aaa - aa - a - a)/a.\\
98765 &= 12345 \times 8 + 5 &&:=(aaaaaa - aaaaa - aaaa - aaa - aa - a - a)/a.\\
987654 &= 123456 \times 8 + 6 &&:=(aaaaaaa - aaaaaa - aaaaa - aaaa - aaa - aa - a - a)/a.\\
9876543 &= 1234567 \times 8 + 7 &&:=(aaaaaaaa - aaaaaaa - aaaaaa - aaaaa - aaaa - aaa - aa - a - a)/a.\\
98765432 &= 12345678 \times 8 + 8 &&:=(aaaaaaaaa - aaaaaaaa - aaaaaaa - aaaaaa - aaaaa - aaaa - aaa - aa - a - a)/a.\\
987654321 &= 123456789 \times 8 + 9 &&:=(aaaaaaaaaa - aaaaaaaaa - aaaaaaaa - aaaaaaa - aaaaaa - aaaaa - aaaa - aaa - aa - a - a)/a.
\end{aligned}$$

Example 39.

$$\begin{aligned}
9 &= 9 \times 1 = 11 - 2 &&:=(aa - a - a)/a.\\
108 &= 9 \times 12 = 111 - 3 &&:=(aaa - a - a - a)/a.\\
1107 &= 9 \times 123 = 1111 - 4 &&:=(aaaa - a - a - a - a)/a.\\
11106 &= 9 \times 1234 = 11111 - 5 &&:=(aaaaa - a - a - a - a - a)/a.\\
111105 &= 9 \times 12345 = 111111 - 6 &&:=(aaaaaa - a - a - a - a - a - a)/a.\\
1111104 &= 9 \times 123456 = 1111111 - 9 &&:=(aaaaaaa - aa + a + a + a + a)/a.\\
11111103 &= 9 \times 1234567 = 11111111 - 8 &&:=(aaaaaaaa - aa + a + a + a)/a.\\
111111102 &= 9 \times 12345678 = 111111111 - 9 &&:=(aaaaaaaaa - aa + a + a)/a.\\
1111111101 &= 9 \times 123456789 = 1111111111 - 10 &&:=(aaaaaaaaaa - aa + a)/a.
\end{aligned}$$

Example 40.

$$\begin{aligned}
9 &= 9 \times 1 = 1 \times 10 - 1 &&:=(a \times (aa - a)/a - a)/a.\\
189 &= 9 \times 21 = 2 \times 100 - 11 &&:=((a + a) \times (aaa - aa)/a - aa)/a.\\
2889 &= 9 \times 321 = 3 \times 1000 - 111 &&:=((a + a + a) \times (aaaa - aaa)/a - aaa)/a.\\
38889 &= 9 \times 4321 = 4 \times 10000 - 1111 &&:=((a + a + a + a) \times (aaaaa - aaaa)/a - aaaa)/a.\\
488889 &= 9 \times 54321 = 5 \times 100000 - 11111 &&:=((a + a + a + a + a) \times (aaaaaa - aaaaa)/a - aaaaa)/a.\\
5888889 &= 9 \times 654321 = 6 \times 1000000 - 111111 &&:=((a + a + a + a + a + a) \times (aaaaaaa - aaaaaa)/a - aaaaaa)/a.\\
68888889 &= 9 \times 7654321 = 7 \times 10000000 - 1111111 &&:=((aa - a - a - a - a) \times (aaaaaaaa - aaaaaaa)/a - aaaaaaa)/a.\\
788888889 &= 9 \times 87654321 = 8 \times 100000000 - 11111111 &&:=((aa - a - a - a) \times (aaaaaaaaa - aaaaaaaa)/a - aaaaaaaa)/a.\\
8888888889 &= 9 \times 987654321 = 9 \times 1000000000 - 111111111 &&:=((aa - a - a) \times (aaaaaaaaaa - aaaaaaaaa)/a - aaaaaaaaa)/a.
\end{aligned}$$

Example 41.

$$\begin{aligned}
81 &= 9 \times 9 = 88 - 7 &&:=((aa - a - a - a) \times aa/a - aa + a + a + a + a)/a.\\
882 &= 9 \times 98 = 888 - 6 &&:=((aa - a - a - a) \times aaa/a - a - a - a - a - a - a)/a.\\
8883 &= 9 \times 987 = 8888 - 5 &&:=((aa - a - a - a) \times aaaa/a - a - a - a - a - a)/a.\\
88884 &= 9 \times 9876 = 88888 - 4 &&:=((aa - a - a - a) \times aaaaa/a - a - a - a - a)/a.\\
888885 &= 9 \times 98765 = 888888 - 3 &&:=((aa - a - a - a) \times aaaaaa/a - a - a - a)/.\\
8888886 &= 9 \times 987654 = 8888888 - 2 &&:=((aa - a - a - a) \times aaaaaaa/a - a - a)/a.\\
88888887 &= 9 \times 9876543 = 88888888 - 1 &&:=((aa - a - a - a) \times aaaaaaaa/a - a)/a.\\
888888888 &= 9 \times 98765432 = 888888888 - 0 &&:=((aa - a - a - a) \times aaaaaaaaa/a)/a.\\
8888888889 &= 9 \times 987654321 = 8888888888 + 1 &&:=((aa - a - a - a) \times aaaaaaaaaa/a + a)/a.
\end{aligned}$$

Example 42.

$$\begin{aligned}
81 &= & 9\times 9 = 91-10 & :=((aa-a-a)\times(aa-a)/a+a-aa+a)/a.\\
801 &= & 9\times 89 = 811-10 & :=((aa-a-a-a)\times(aaa-aa)/a+aa-aa+a)/a.\\
7101 &= & 9\times 789 = 7111-10 & :=((aa-a-a-a-a)\times(aaaa-aaa)/a+aaa-aa+a)/a.\\
61101 &= & 9\times 6789 = 61111-10 & :=((a+a+a+a+a+a)\times(aaaaa-aaaa)/a+aaaa-aa+a)/a.\\
511101 &= & 9\times 56789 = 511111-10 & :=((a+a+a+a+a)\times(aaaaaa-aaaaa)/a+aaaaa-aa+a)/a.\\
4111101 &= & 9\times 456789 = 4111111-10 & :=((a+a+a+a)\times(aaaaaaa-aaaaaa)/a+aaaaaa-aa+a)/a.\\
31111101 &= & 9\times 3456789 = 31111111-10 & :=((a+a+a)\times(aaaaaaaa-aaaaaaa)/a+aaaaaaa-aa+a)/a.\\
211111101 &= & 9\times 23456789 = 211111111-10 & :=((a+a)\times(aaaaaaaaa-aaaaaaaa)/a+aaaaaaaa-aa+a)/a.\\
1111111101 &= & 9\times 123456789 = 1111111111-10 & :=(a\times(aaaaaaaaaa-aaaaaaaaa)/a+aaaaaaaaa-aa+a)/a.
\end{aligned}$$

Example 43.

$$\begin{aligned}
91 &= 10^2-10^1+1 := (aaa-aa-aa+a+a)/a.\\
9901 &= 10^4-10^2+1 := (aaaaa-aaaa-aaa+aa+a)/a.\\
999001 &= 10^6-10^3+1 := (aaaaaaa-aaaaaa-aaaa+aaa+a)/a.\\
99990001 &= 10^8-10^4+1 := (aaaaaaaaa-aaaaaaaa-aaaaa+aaaa+a)/a.\\
9999900001 &= 10^{10}-10^5+1 := (aaaaaaaaaaa-aaaaaaaaaa-aaaaaa+aaaaa+a)/a.\\
999999000001 &= 10^{12}-10^6+1 := (aaaaaaaaaaaaa-aaaaaaaaaaaa-aaaaaaa+aaaaaa+a)/a.\\
99999990000001 &= 10^{14}-10^7+1 := (aaaaaaaaaaaaaaa-aaaaaaaaaaaaaa-aaaaaaaa+aaaaaaa+a)/a.
\end{aligned}$$

3.8 Prime Number Patterns

Below are some examples of *prime number patterns*. It is not necessary that the further number each example be a prime number.

Example 44.

$$\begin{aligned}
31 &:= (aa+aa+aa-a-a)/a.\\
331 &:= (aaa+aaa+aaa-a-a)/a.\\
3331 &:= (aaaa+aaaa+aaaa-a-a)/a.\\
33331 &:= (aaaaa+aaaaa+aaaaa-a-a)/a.\\
333331 &:= (aaaaaa+aaaaaa+aaaaaa-a-a)/a.\\
3333331 &:= (aaaaaaa+aaaaaaa+aaaaaaa-a-a)/a.\\
33333331 &:= (aaaaaaaa+aaaaaaaa+aaaaaaaa-a-a)/a.
\end{aligned}$$

The next number in this case is not a prime number, i.e., we can write $333333331 = 17\times 19607843$.

Example 45.

$$\begin{aligned}
59 &:= ((aa\times aa-a\times a)/(a+a)-a)/a.\\
599 &:= ((aaa\times aa-a\times a)/(a+a)-aa)/a.\\
59999 &:= ((aaaaa\times aa-a\times a)/(a+a)-aaaa)/a.\\
599999 &:= ((aaaaaa\times aa-a\times a)/(a+a)-aaaaa)/a.\\
59999999 &:= ((aaaaaaaa\times aa-a\times a)/(a+a)-aaaaaaa)/a.\\
59999999999 &:= ((aaaaaaaaaaa\times aa-a\times a)/(a+a)-aaaaaaaaaa)/a.
\end{aligned}$$

In this example, in between numbers are not primes, for example, $5999 = 7 \times 857$, $5999999 = 1013 \times 5923$, etc.

Example 46.

$$
\begin{aligned}
23 &:= (aa + aa + a)/a. \\
233 &:= (aaa + aaa + aa)/a. \\
2333 &:= (aaaa + aaaa + aaa)/a. \\
23333 &:= (aaaaa + aaaaa + aaaa)/a.
\end{aligned}
$$

Here also the next number is not prime, i.e., $233333 = 353 \times 661$. After this, the next prime number is 23333333333.

Example 47.

$$
\begin{aligned}
19 &:= (aa + aa - a - a - a)/a. \\
199 &:= (aaa + aaa - aa - aa - a)/a. \\
1999 &:= (aaaa + aaaa - aaa - aaa - a)/a. \\
199999 &:= (aaaaaa + aaaaaa - aaaaa - aaaaa - a)/a. \\
19999999 &:= (aaaaaaaa + aaaaaaaa - aaaaaaa - aaaaaaa - a)/a.
\end{aligned}
$$

In this example the numbers $19999 = 7 \times 2857$ and $1999999 = 17 \times 71 \times 1657$ are not prime numbers. The next number is also not prime, i.e., $199999999 = 89 \times 1447 \times 1553$.

Example 48. ***Palindromic Prime Pattern***. *Here below are palindromic patterns of prime numbers in different forms. No representations are given, since numbers are too high.*

131	1124243424211
11311	1124363634211
1123211	1124472744211
112434211	1124536354211
11248384211	1124543454211
1124843484211	1124833384211
112486131684211	1124843484211

Example 49. ***Unsymmetrical Prime Pattern***. *This example brings unsymmetrical prime pattern in terms of 1, 4 and 9. The only thing common is that all the numbers begins with 41. This we have written just as curiosity without any representation.*

41	4191919
419	41994191
4111	411919111
41411	4149191911
419999	41491919111

References

[1] ABRAHAMS, M, Lots more numbers, deemed "crazy consecutive", IMPROBABLE RESEACH, http://www.improbable.com/2013/06/08/lots-more-numbers-deemed-crazy-consecutive.

[2] HEINZ, H., "Number Patterns. *http://www.magic-squares.net* and *http://www.magic-squares.net/square-update.htm*.

[3] KAUFMAN, J.F., Math e Magic, Dover Publications, Inc., 1953.

[4] MADACHY, J.S., Mathematics on Vacations, Charlers Scriber's Son, New York, 1966.

[5] NEBUS, J., Counting From 52 to 11,108, nebusresearch, *http://nebusresearch.wordpress.com/2013/06/10/counting-from-52-to-11108/*.

[6] TANEJA, I.J., Crazy Sequential Representation: Numbers from 0 to 11111 in terms of Increasing and Decreasing Orders of 1 to 9, *http://arxiv.org/abs/1302.1479*.

[7] TANEJA, I.J., Selfie Numbers: Consecutive Representations in Increasing and Decreasing Orders, RGMIA Research Report Collection, **17**(2014), Article 140, pp. 1-57. *http://rgmia.org/papers/v17/v17a140.pdf*, 2014.

[8] TANEJA, I.J., Single Digit Representations of Natural Numbers, *http://arxiv.org/abs/1502.03501*. Also at *http://rgmia.org/papers/v18/v18a15.pdf*.

[9] TANEJA, I.J., Running Expressions in Increasing and Decreasing Orders of Natural Numbers Separated by Equality Signs, RGMIA Research Report Collection, **18**(2015), Article 27, pp. 1-54. *http://rgmia.org/papers/v18/v18a27.pdf*, 2015.

[10] TANEJA, I.J., Different Types of Pretty Wild Narcissistic Numbers: Selfie Representations – I, RGMIA Research Report Collection, **18**(2015), Article 32, pp. 1-43. *http://rgmia.org/papers/v18/v18a32.pdf*, 2015.

[11] TANEJA, I.J., Single Letter Representations of Natural Numbers, Palindromic Symmetries and Number Patterns, RGMIA Research Report Collection, **18**(2015), Article 40, pp. 1-30. *http://rgmia.org/papers/v18/v18a70.pdf*, 2015.

[12] TANEJA, I.J., Selfie Numbers: Representations in Increasing and Decreasing Orders of Non Consecutive Digits, RGMIA Research Report Collection, **18**(2015), Article 70, pp. 1-104. *http://rgmia.org/papers/v18/v18a70.pdf*, 2015.

[13] TANEJA, I.J., Single Letter Representations of Natural Numbers, RGMIA Research Report Collection, **18**(2015), Article 73, pp. 1-44. *http://rgmia.org/papers/v18/v18a73.pdf*, 2015.

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