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Representations of Palindromic, Prime, and Fibonacci Sequence Patterns

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Abstract

This work brings representations of palindromic and number patterns in terms of single letter "a". Some examples of prime number patterns are also considered. Different classifications of palindromic patterns are considered, such as, palindromic decompositions, double symmetric patterns, number pattern decompositions, etc. Numbers patterns with power are also studied. Study towards Fibonacci sequence and its extensions is also made.

1 Palindromic and Numbers Patterns

Before proceeding further, here below are some general ways of writing palindromic numbers. Odd and even orders are considered separately.

1.1 General Form of Palindromic Numbers

We shall divide palindromic numbers in two parts, *odd and even orders*.

1.1.1 Odd Order Palindromes

Odd order palindromes are those where the number of digits are odd, i.e, aba, $abcba$, etc. See the general form below:

$$\begin{aligned}
aba &:= (10^2+10^0)\times a + 10^1\times b\\
abcba &:= (10^4+10^0)\times a + (10^3+10^1)\times b + 10^2\times c\\
abcdcba &:= (10^6+10^0)\times a + (10^5+10^1)\times b + (10^4+10^2)\times c + 10^3\times d\\
abcdedcba &:= (10^8+10^0)\times a + (10^7+10^1)\times b + (10^6+10^2)\times c + (10^5+10^3)\times d + 10^4\times e.
\end{aligned}$$

equivalently,

$$\begin{aligned}
aba &:= 101\times a + 10\times b\\
abcba &:= 10001\times a + 1010\times b + 100\times c\\
abcdcba &:= 1000001\times a + 100010\times b + 10100\times c + 1000\times d\\
abcdedcba &:= 100000001\times a + 10000010\times b + 1000100\times c + 101000\times d + 10000\times e.
\end{aligned}$$

1.1.2 Even Order Palindromes

Even order palindromes are those where the number of digits are even, i.e, $abba$, $abccba$, etc. See the general form below:

$$\begin{aligned}
abba &:= (10^3+10^0)\times a + (10^2+10^1)\times b\\
abccba &:= (10^5+10^0)\times a + (10^4+10^1)\times b + (10^3+10^2)\times c\\
abcddcba &:= (10^7+10^0)\times a + (10^6+10^1)\times b + (10^5+10^2)\times c + (10^4+10^3)\times d\\
abcdeedcba &:= (10^9+10^0)\times a + (10^8+10^1)\times b + (10^7+10^2)\times c + (10^6+10^3)\times d + (10^5+10^4)\times e.
\end{aligned}$$

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equivalently,

$$\begin{aligned}
abba &:= 1001 \times a + 110 \times b \\
abccba &:= 100001 \times a + 10010 \times b + 1100 \times c \\
abcddcba &:= 10000001 \times a + 1000010 \times b + 100100 \times c + 11000 \times d \\
abcdeedcba &:= 1000000001 \times a + 100000010 \times b + 10000100 \times c + 1001000 \times d + 110000 \times e.
\end{aligned}$$

Combining both orders, we have

$$\begin{aligned}
aba &:= (10^2 + 10^0) \times a + 10^1 \times b \\
abba &:= (10^3 + 10^0) \times a + (10^2 + 10^1) \times b \\
abcba &:= (10^4 + 10^0) \times a + (10^3 + 10^1) \times b + 10^2 \times c \\
abccba &:= (10^5 + 10^0) \times a + (10^4 + 10^1) \times b + (10^3 + 10^2) \times c \\
abcdcba &:= (10^6 + 10^0) \times a + (10^5 + 10^1) \times b + (10^4 + 10^2) \times c + 10^3 \times d \\
abcddcba &:= (10^7 + 10^0) \times a + (10^6 + 10^1) \times b + (10^5 + 10^2) \times c + (10^4 + 10^3) \times d \\
abcdedcba &:= (10^8 + 10^0) \times a + (10^7 + 10^1) \times b + (10^6 + 10^2) \times c + (10^5 + 10^3) \times d + 10^4 \times e \\
abcdeedcba &:= (10^9 + 10^0) \times a + (10^8 + 10^1) \times b + (10^7 + 10^2) \times c + (10^6 + 10^3) \times d + (10^5 + 10^4) \times e \\
&\ldots\ldots
\end{aligned}$$

It is understood that *even order palindromes* are different from *even numbers*. *Even numbers* are multiple of 2, while *even order palindromes* has *even number of digits*, for examples, 99, 1221, 997766, etc. The same is with *odd order palindromes*. We observed that the general way of writing both orders is different.

1.1.3 Particular Palindromes

If we consider some a simplified form of palindromes, for example, 121, 1221, etc. In this case we can write a simplified general form. Here the intermediate values are equal except extremes. The general form is given by

$$\begin{aligned}
aba &:= (9 \times a + b) \times 11 + 2 \times a - b \\
abba &:= (9 \times a + b) \times 111 + 2 \times a - b \\
abbba &:= (9 \times a + b) \times 1111 + 2 \times a - b \\
abbbba &:= (9 \times a + b) \times 11111 + 2 \times a - b \\
abbbbba &:= (9 \times a + b) \times 111111 + 2 \times a - b \\
abbbbbba &:= (9 \times a + b) \times 1111111 + 2 \times a - b \\
abbbbbbba &:= (9 \times a + b) \times 11111111 + 2 \times a - b \\
&\ldots\ldots
\end{aligned}$$

where $a \in \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$, and $b \in \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9\}$.

Before proceeding further, let us clarify, the difference between *"Palindromic patterns"* and *"Number patterns"*.

1.2 Palindromic Patterns

When there is symmetry in representing palindromes, we call as *"palindromic patterns"*. See the examples below:

393	1111	11211	6112116
3993	2222	22322	6223226
39993	3333	33433	6334336
....	....	....	...

1.3 Number Patterns

When there is symmetry in representing numbers similar as palindromic patterns, we call *"number patterns"*. See examples below:

399	1156	123	$23^2 = 549$
3999	111556	1234	$233^2 = 54289$
39999	11115556	12345	$2333^2 = 5442889$
399999	1111155556	123456	$23333^2 = 544428889$
....	...	....	....

2 Single Letter Representations

Let us consider

$$f^n(10) = 10^n + 10^{n-1} + ... + 10^2 + 10 + 10^0,$$

For $a \in \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$, we can write

$$af^n(10) = \underbrace{aaa...a}_{(n+1)-times},$$

In particular,

$$\begin{aligned}
aa &= f^1(10) = a10 + a && \Longrightarrow 11 := \frac{aa}{a} \\
aaa &= f^2(10) = a10^2 + a10 + a && \Longrightarrow 111 := \frac{aaa}{a} \\
aaaa &= f^3(10) = a10^3 + a10^2 + a10 + a && \Longrightarrow 1111 := \frac{aaaa}{a} \\
aaaaa &= f^4(10) = a10^4 + a10^3 + a10^2 + a10 + a && \Longrightarrow 11111 := \frac{aaaaa}{a} \\
&......
\end{aligned}$$

In [13, 15] author wrote natural numbers in terms of single letter "a". See some examples below:

$$\begin{aligned}
5 &:= \frac{aa - a}{a + a} \\
56 &:= \frac{aaa + a}{a + a} \\
582 &:= \frac{aaaaa + aaaa}{aa + aa - a} \\
1233 &:= \frac{aaaa + aaa + aa}{a} \\
4950 &:= \frac{aaaaa - aaaa - aaa + aa}{a + a}.
\end{aligned}$$

The work on representations of natural numbers using *single letter* is first of its kind [10, 15]. Author also worked with representation of numbers using *single digit* for each value from 1 to 9 separately. Representation of numbers using all the digits from 1 to 9 in increasing and decreasing ways is done by author [8]. Comments to this work can be seen at [1, 6]. Different studies on numbers, such as, *selfie numbers, running expressions*, etc. refer to author's work [9, 11, 12, 13, 14].

Our aim is to write palindromic, prime and number patterns in terms of single letter "a". For studies on patterns refer to [2, 3, 5].

3 Representations of Palindromic and Number Patterns

Below are examples of *palindromic* and *number patterns* in terms of single letter "*a*". These examples are divided in subsections. All the examples are followed by their respective decompositions. The following patterns are considered:

3.1 Number Patterns with Palindromic Decompositions;
3.2 Palindromic Patterns with Number Pattern Decompositions;
3.3 Palindromic Patterns with Palindromic Decompositions;
3.4 Squared Number Patterns;
3.5 Repeated Digits Patterns;
3.6 Fibonacci Sequence and its Extensions;
3.7 Doubly Symmetric Patterns;
3.8 Number Patterns with Number Pattern Decompositions;
3.9 Prime Number Patterns.

3.1 Number Patterns with Palindromic Decompositions

Here below are examples of palindromic patterns with palindromic decompositions.

Example 1.

$$\begin{aligned}
11 &= 11 && :=(a \times aa)/(a \times a)\\
121 &= 11 \times 11 && :=(aa \times aa)/(a \times a)\\
12321 &= 111 \times 111 && :=(aaa \times aaa)/(a \times a)\\
1234321 &= 1111 \times 1111 && :=(aaaa \times aaaa)/(a \times a)\\
123454321 &= 11111 \times 11111 && :=(aaaaa \times aaaaa)/(a \times a)\\
12345654321 &= 111111 \times 111111 && :=(aaaaaa \times aaaaaa)/(a \times a)\\
1234567654321 &= 1111111 \times 1111111 && :=(aaaaaaa \times aaaaaaa)/(a \times a)\\
123456787654321 &= 11111111 \times 11111111 && :=(aaaaaaaa \times aaaaaaaa)/(a \times a)\\
12345678987654321 &= 111111111 \times 111111111 && :=(aaaaaaaaa \times aaaaaaaaa)/(a \times a).
\end{aligned}$$

Example 2.

$$\begin{aligned}
11 &= 1 \times 11 && :=(a \times aa)/(a \times a)\\
1221 &= 11 \times 111 && :=(aa \times aaa)/(a \times a)\\
123321 &= 111 \times 1111 && :=(aaa \times aaaa)/(a \times a)\\
12344321 &= 1111 \times 11111 && :=(aaaa \times aaaaa)/(a \times a)\\
1234554321 &= 11111 \times 111111 && :=(aaaaa \times aaaaaa)/(a \times a)\\
123456654321 &= 111111 \times 1111111 && :=(aaaaaa \times aaaaaaa)/(a \times a)\\
12345677654321 &= 1111111 \times 11111111 && :=(aaaaaaa \times aaaaaaaa)/(a \times a)\\
1234567887654321 &= 11111111 \times 111111111 && :=(aaaaaaaa \times aaaaaaaaa)/(a \times a)\\
123456789987654321 &= 111111111 \times 1111111111 && :=(aaaaaaaaa \times aaaaaaaaaa)/(a \times a).
\end{aligned}$$

Example 3. *The following example is represented in two different forms. One with product decomposition and another with potentiation.*

$$\begin{aligned}
1089 &= 11 \times 99 && :=aa \times (aaa - aa - a)/(a \times a)\\
110889 &= 111 \times 999 && :=aaa \times (aaaa - aaa - a)/(a \times a)\\
11108889 &= 1111 \times 9999 && :=aaaa \times (aaaaa - aaaa - a)/(a \times a)\\
1111088889 &= 11111 \times 99999 && :=aaaaa \times (aaaaaa - aaaaa - a)/(a \times a)\\
111110888889 &= 111111 \times 999999 && :=aaaaaa \times (aaaaaaa - aaaaaa - a)/(a \times a).
\end{aligned}$$

The above decomposition can also we written as $11 \times 99 = 33^2$, $111 \times 999 = 333^2$, etc. We can rewrite the above representation using as potentiation:

$$\begin{aligned}
1089 = & \quad 11 \times 99 = 33^2 & :=((aa + aa + aa)/a)^{(a+a)/a} \\
110889 = & \quad 111 \times 999 = 333^2 & :=((aaa + aaa + aaa)/a)^{(a+a)/a} \\
11108889 = & \quad 1111 \times 9999 = 3333^2 & :=((aaaa + aaaa + aaaa)/a)^{(a+a)/a} \\
1111088889 = & \quad 11111 \times 99999 = 33333^2 & :=((aaaaa + aaaaa + aaaaa)/a)^{(a+a)/a}; \\
111110888889 = & \quad 111111 \times 999999 = 333333^2 & :=((aaaaaa + aaaaaa + aaaaaa)/a)^{(a+a)/a}.
\end{aligned}$$

More examples on potentiation are given in section 3.4. See examples 24, 25, 26, 27 and 28.

Example 4.

$$\begin{aligned}
7623 = 11 \times 9 \times 77 & :=aa \times (aaa - aa - a) \times (aa - a - a - a - a)/(a \times a \times a) \\
776223 = 111 \times 9 \times 777 & :=aaa \times (aaaa - aaa - a) \times (aa - a - a - a - a)/(a \times a \times a) \\
77762223 = 1111 \times 9 \times 7777 & :=aaaa \times (aaaaa - aaaa - a) \times (aa - a - a - a - a)/(a \times a \times a) \\
7777622223 = 11111 \times 9 \times 77777 & :=aaaaa \times (aaaaaa - aaaaa - a) \times (aa - a - a - a - a)/(a \times a \times a) \\
777776222223 = 111111 \times 9 \times 777777 & :=aaaaaa \times (aaaaaaa - aaaaaa - a) \times (aa - a - a - a - a)/(a \times a \times a).
\end{aligned}$$

3.2 Palindromic Patterns with Number Pattern Decompositions

Here below are examples palindromic patterns decomposed in number patterns.

Example 5.

$$\begin{aligned}
1 = 0 \times 9 + 1 & :=a/a \\
11 = 1 \times 9 + 2 & :=aa/a \\
111 = 12 \times 9 + 3 & :=aaa/a \\
1111 = 123 \times 9 + 4 & :=aaaa/a \\
11111 = 1234 \times 9 + 5 & :=aaaaa/a \\
111111 = 12345 \times 9 + 6 & :=aaaaaa/a \\
1111111 = 123456 \times 9 + 7 & :=aaaaaaa/a \\
11111111 = 1234567 \times 9 + 8 & :=aaaaaaaa/a \\
111111111 = 12345678 \times 9 + 9 & :=aaaaaaaaa/a \\
1111111111 = 123456789 \times 9 + 10 & :=aaaaaaaaaa/a.
\end{aligned}$$

Example 6.

$$\begin{aligned}
88 = 9 \times 9 + 7 & :=(aa - a - a - a) \times aa/(a \times a) \\
888 = 98 \times 9 + 6 & :=(aa - a - a - a) \times aaa/(a \times a) \\
8888 = 987 \times 9 + 5 & :=(aa - a - a - a) \times aaaa/(a \times a) \\
88888 = 9876 \times 9 + 4 & :=(aa - a - a - a) \times aaaaa/(a \times a) \\
888888 = 98765 \times 9 + 3 & :=(aa - a - a - a) \times aaaaaa/(a \times a) \\
8888888 = 987654 \times 9 + 2 & :=(aa - a - a - a) \times aaaaaaa/(a \times a) \\
88888888 = 9876543 \times 9 + 1 & :=(aa - a - a - a) \times aaaaaaaa/(a \times a) \\
888888888 = 98765432 \times 9 + 0 & :=(aa - a - a - a) \times aaaaaaaaa/(a \times a) \\
8888888888 = 987654321 \times 9 - 1 & :=(aa - a - a - a) \times aaaaaaaaaa/(a \times a) \\
88888888888 = 9876543210 \times 9 - 2 & :=(aa - a - a - a) \times aaaaaaaaaaa/(a \times a).
\end{aligned}$$

Example 7.

$$\begin{aligned}
33 = 12 + 21 \quad &:= (a + a + a) \times aa/(a \times a)\\
444 = 123 + 321 \quad &:= (a + a + a + a) \times aaa/(a \times a)\\
5555 = 1234 + 4321 \quad &:= (a + a + a + a + a) \times aaaa/(a \times a)\\
66666 = 12345 + 54321 \quad &:= (a + a + a + a + a + a) \times aaaaa/(a \times a)\\
777777 = 123456 + 654321 \quad &:= (aa - a - a - a - a) \times aaaaaa/(a \times a)\\
8888888 = 1234567 + 7654321 \quad &:= (aa - a - a - a) \times aaaaaaa/(a \times a)\\
99999999 = 12345678 + 87654321 \quad &:= (aa - a - a) \times aaaaaaaa/(a \times a)
\end{aligned}$$

Example 8.

$$\begin{aligned}
99 = 98 + 1 \quad &:= (aaa - aa - a)/(a \times a)\\
999 = 987 + 12 \quad &:= (aaaa - aaa - a)/(a \times a)\\
9999 = 9876 + 123 \quad &:= (aaaaa - aaaa - a)/(a \times a)\\
99999 = 98765 + 1234 \quad &:= (aaaaaa - aaaaa - a)/(a \times a)\\
999999 = 987654 + 12345 \quad &:= (aaaaaaa - aaaaaa - a)/(a \times a)\\
9999999 = 9876543 + 123456 \quad &:= (aaaaaaaa - aaaaaaa - a)/(a \times a)\\
99999999 = 98765432 + 1234567 \quad &:= (aaaaaaaaa - aaaaaaaa - a)/(a \times a)\\
999999999 = 987654321 + 12345678 \quad &:= (aaaaaaaaaa - aaaaaaaaa - a)/(a \times a)\\
9999999999 = 9876543210 + 123456789 \quad &:= (aaaaaaaaaaa - aaaaaaaaaa - a)/(a \times a).
\end{aligned}$$

Example 9.

$$\begin{aligned}
2772 = 4 \times 693 \quad &:= (aaaaa - aa - aa - a)/(a + a + a + a)\\
27772 = 4 \times 6943 \quad &:= (aaaaaa - aa - aa - a)/(a + a + a + a)\\
277772 = 4 \times 69443 \quad &:= (aaaaaaa - aa - aa - a)/(a + a + a + a)\\
2777772 = 4 \times 694443 \quad &:= (aaaaaaaa - aa - aa - a)/(a + a + a + a)\\
27777772 = 4 \times 6944443 \quad &:= (aaaaaaaaa - aa - aa - a)/(a + a + a + a).
\end{aligned}$$

In this example the previous number $272 := (aaaa - aa - aa - a)/(a + a + a + a)$ is also a palindrome, but its decomposition $272 = 4 \times 68$ is not symmetrical to other values of the patterns.

Example 10. *This example is little irregular in terms of number patterns. But, later making proper choices, we can bring two different regular patterns.*

$$\begin{aligned}
101 = 101 \quad &:= (aaa - aa + a)/a\\
1001 = 11 \times 91 \quad &:= (aaaa - aaa + a)/a\\
10001 = 73 \times 137 \quad &:= (aaaaa - aaaa + a)/a\\
100001 = 11 \times 9091 \quad &:= (aaaaaa - aaaaa + a)/a\\
1000001 = 101 \times 9901 \quad &:= (aaaaaaa - aaaaaa + a)/a\\
10000001 = 11 \times 909091 \quad &:= (aaaaaaaa - aaaaaaa + a)/a\\
100000001 = 17 \times 5882353 \quad &:= (aaaaaaaaa - aaaaaaaa + a)/a\\
1000000001 = 11 \times 90909091 \quad &:= (aaaaaaaaaa - aaaaaaaaa + a)/a\\
10000000001 = 101 \times 99009901 \quad &:= (aaaaaaaaaaa - aaaaaaaaaa + a)/a\\
100000000001 = 11 \times 9090909091 \quad &:= (aaaaaaaaaaaa - aaaaaaaaaaa + a)/a.
\end{aligned}$$

This example shows that it not necessary that every palindromic pattern can be decomposed to number pattern. By considering only even number of terms, i.e., 2nd, 4th, 6th, ..., we get a number pattern decomposition. Also considering 1st, 5th, 9th, ...terms, we get another regular number pattern decomposition.

$$\begin{aligned}
1001 &= 11 \times 91 && :=(aaaa - aaa + a)/a \\
100001 &= 11 \times 9091 && :=(aaaaaa - aaaaa + a)/a \\
10000001 &= 11 \times 909091 && :=(aaaaaaaa - aaaaaaa + a)/a \\
1000000001 &= 11 \times 90909091 && :=(aaaaaaaaaa - aaaaaaaaa + a)/a \\
100000000001 &= 11 \times 9090909091 && :=(aaaaaaaaaaaa - aaaaaaaaaaa + a)/a.
\end{aligned}$$

$$\begin{aligned}
101 &= 101 && :=(aaa - aa + a)/a \\
1000001 &= 101 \times 9901 && :=(aaaaaaa - aaaaaa + a)/a \\
10000000001 &= 101 \times 99009901 && :=(aaaaaaaaaaa - aaaaaaaaaa + a)/a \\
100000000000001 &= 101 \times 990099009901 && :=(aaaaaaaaaaaaaaa - aaaaaaaaaaaaaa + a)/a.
\end{aligned}$$

Another interesting pattern for above palindromic pattern is the following example.

Example 11. *This example brings repeated patterns*

$$\begin{aligned}
11 &= 1 \times 7 + 3 && :=(aa - a + a)/a \\
101 &= 14 \times 7 + 3 && :=(aaa - aa + a)/a \\
1001 &= 142 \times 7 + 7 && :=(aaaa - aaa + a)/a \\
10001 &= 1428 \times 7 + 5 && :=(aaaaa - aaaa + a)/a \\
100001 &= 14285 \times 7 + 6 && :=(aaaaaa - aaaaa + a)/a \\
1000001 &= 142857 \times 7 + 2 && :=(aaaaaaa - aaaaaa + a)/a \\
10000001 &= 1428571 \times 7 + 4 && :=(aaaaaaaa - aaaaaaa + a)/a \\
100000001 &= 14285714 \times 7 + 3 && :=(aaaaaaaaa - aaaaaaaa + a)/a \\
1000000001 &= 142857142 \times 7 + 7 && :=(aaaaaaaaaa - aaaaaaaaa + a)/a \\
10000000001 &= 1428571428 \times 7 + 5 && :=(aaaaaaaaaaa - aaaaaaaaaa + a)/a \\
100000000001 &= 14285714285 \times 7 + 6 && :=(aaaaaaaaaaaa - aaaaaaaaaaa + a)/a \\
1000000000001 &= 142857142857 \times 7 + 2 && :=(aaaaaaaaaaaaa - aaaaaaaaaaaa + a)/a.
\end{aligned}$$

It is interesting to observe that the number 142857 make an interesting triangle in first six lines, and last members in each line make a sequence of numbers 1, 4, 2, 8, 5 and 7. The same repeats again in 7th to 12th line. More properties of the this number can be seen in examples 30 given in section 3.5. Another interesting property of the above palindromic pattern is given by

$$\begin{aligned}
101 \times ab &= ab\ ab \\
1001 \times abc &= abc\ abc \\
10001 \times abcd &= abcd\ abcd \\
100001 \times abcde &= abcde\ abcde \\
1000001 \times abcdef &= abcdef\ abcdef \\
10000001 \times abcdefg &= abcdefg\ abcdefg \\
&\cdots
\end{aligned}$$

where, $abc = a \times 10^2 + b \times 10 + c$, etc. For example, $101 \times 23 = 23\,23$, $1001 \times 347 = 347\,347$, etc.

Example 12.

$$
\begin{aligned}
111111111 &= 12345679 \times 9 \times 1 := aaaaaaaaa \times a/(a \times a)\\
222222222 &= 12345679 \times 9 \times 2 := aaaaaaaaa \times (a+a)/(a \times a)\\
333333333 &= 12345679 \times 9 \times 3 := aaaaaaaaa \times (a+a+a)/(a \times a)\\
444444444 &= 12345679 \times 9 \times 4 := aaaaaaaaa \times (a+a+a+a)/(a \times a)\\
555555555 &= 12345679 \times 9 \times 5 := aaaaaaaaa \times (a+a+a+a+a)/(a \times a)\\
666666666 &= 12345679 \times 9 \times 6 := aaaaaaaaa \times (a+a+a+a+a+a)/(a \times a)\\
777777777 &= 12345679 \times 9 \times 7 := aaaaaaaaa \times (aa-a-a-a-a)/(a \times a)\\
888888888 &= 12345679 \times 9 \times 8 := aaaaaaaaa \times (aa-a-a-a)/(a \times a)\\
999999999 &= 12345679 \times 9 \times 9 := aaaaaaaaa \times (aa-a-a)/(a \times a).
\end{aligned}
$$

The number 12345679 also appears as division of 1/81, i.e.

$$\frac{1}{81} = 0.012345679\ 012345679\ 012345679\ 012345679\ \ldots. = 0.\overline{012345679}.$$

More situations of similar kind with less number of repetitions are given in examples 20, 21 and 22.

Example 13. *Multiplying by 3 the number 12345679, appearing in previous example, we get*

$$12345679 \times 3 = 37037037.$$

The number 37037037 has very interesting properties. Multiplying it from 1 to 27 and reorganizing the values, we get very interesting patterns:

$$
\begin{array}{lll}
 & 37037037 \times 1 = 037\ 037\ 037 & 37037037 \times 5 = 185\ 185\ 185\\
 & 37037037 \times 10 = 370\ 370\ 370 & 37037037 \times 14 = 518\ 518\ 518\\
37037037 \times 3 = 111111111 & 37037037 \times 19 = 703\ 703\ 703 & 37037037 \times 23 = 851\ 851\ 851\\
37037037 \times 6 = 222222222 & & \\
37037037 \times 9 = 333333333 & 37037037 \times 2 = 074\ 074\ 074 & 37037037 \times 7 = 259\ 259\ 259\\
37037037 \times 12 = 444444444 & 37037037 \times 11 = 407\ 407\ 407 & 37037037 \times 16 = 592\ 592\ 592\\
37037037 \times 15 = 555555555 & 37037037 \times 20 = 740\ 740\ 740 & 37037037 \times 25 = 925\ 925\ 925\\
37037037 \times 18 = 666666666 & & \\
37037037 \times 21 = 777777777 & 37037037 \times 4 = 148\ 148\ 148 & 37037037 \times 8 = 296\ 296\ 296\\
37037037 \times 24 = 888888888 & 37037037 \times 13 = 481\ 481\ 481 & 37037037 \times 17 = 629\ 629\ 629\\
37037037 \times 27 = 999999999. & 37037037 \times 22 = 814\ 814\ 814 & 37037037 \times 26 = 962\ 962\ 962.
\end{array}
$$

Example 14. *Dividing 37037037 by 37 we get palindromic number 1001001. Any other number with the similar kind of pattern divided by last two digits always give the same palindromic number. See below*

$$
\begin{aligned}
&17017017/17=1001001\\
&19019019/19=1001001\\
&23023023/23=1001001\\
&45045045/45=1001001.
\end{aligned}
$$

Let us make similar kind of multiplications as in previous example with number 17017017, we get symmetrical values, but not as beautiful as in previous example. See below:

$$
\begin{array}{lll}
17017017 \times 1 = 017\ 017\ 017 & 17017017 \times 10 = 170\ 170\ 170 & 17017017 \times 19 \ = 323\ 323\ 323 \\
17017017 \times 2 = 034\ 034\ 034 & 17017017 \times 11 = 187\ 187\ 187 & 17017017 \times 20 \ = 340\ 340\ 340 \\
17017017 \times 3 = 051\ 051\ 051 & 17017017 \times 12 = 204\ 204\ 204 & 17017017 \times 21 \ = 357\ 357\ 357 \\
17017017 \times 4 = 068\ 068\ 068 & 17017017 \times 13 = 221\ 221\ 221 & 17017017 \times 22 \ = 374\ 374\ 374 \\
17017017 \times 5 = 085\ 085\ 085 & 17017017 \times 14 = 238\ 238\ 238 & 17017017 \times 23 \ = 391\ 391\ 391 \\
17017017 \times 6 = 102\ 102\ 102 & 17017017 \times 15 = 255\ 255\ 255 & 17017017 \times 24 \ = 408\ 408\ 408 \\
17017017 \times 7 = 119\ 119\ 119 & 17017017 \times 16 = 272\ 272\ 272 & 17017017 \times 25 \ = 425\ 425\ 425 \\
17017017 \times 8 = 136\ 1361\ 36 & 17017017 \times 17 = 289\ 289\ 289 & 17017017 \times 26 \ = 442\ 442\ 442 \\
17017017 \times 9 = 153\ 153\ 153 & 17017017 \times 18 = 306\ 306\ 306 & 17017017 \times 27 = 459\ 459\ 459.
\end{array}
$$

Above multiplications are done only up to 27, but it can go up to 58, and still the results remains symmetric, i.e.,

$$17017017 \times 58 = 986\ 986\ 986.$$

Example 15. *The 717 can be decomposed in two different ways, i.e.,* $717 = 88 \times 8 + 13$ *and* $717 = 56 \times 13 - 1$. *The first decomposition is palindromic, while the second is just number pattern. Here below are both decompositions:*

$$
\begin{array}{rcl}
717 = & 88 \times 8 + 13 = 56 \times 13 - 11 & := (aaa + a)/(a + a) \times (aa + a + a)/a - aa/a \\
7117 = & 888 \times 8 + 13 = 556 \times 13 - 111 & := (aaaa + a)/(a + a) \times (aa + a + a)/a - aaa/a \\
71117 = & 8888 \times 8 + 13 = 5556 \times 13 - 1111 & := (aaaaa + a)/(a + a) \times (aa + a + a)/a - aaaa/a \\
711117 = & 88888 \times 8 + 13 = 55556 \times 13 - 11111 & := (aaaaaa + a)/(a + a) \times (aa + a + a)/a - aaaaa/a \\
7111117 = & 888888 \times 8 + 13 = 555556 \times 13 - 111111 & := (aaaaaaa + a)/(a + a) \times (aa + a + a)/a - aaaaaa/a.
\end{array}
$$

The same we can have with many other palindromic patterns.

3.3 Palindromic Patterns with Palindromic Decompositions

In this subsections the palindromic patterns are decomposed in another palindromic patterns.

Example 16.

$$
\begin{array}{rl}
121 = 11 \times 11 & := aa \times aa/(a \times a \times a) \\
1221 = 11 \times 111 & := aa \times aaa/(a \times a \times a) \\
12221 = 11 \times 1111 & := aa \times aaaa/(a \times a \times a) \\
122221 = 11 \times 11111 & := aa \times aaaaa/(a \times a \times a) \\
1222221 = 11 \times 111111 & := aa \times aaaaaa/(a \times a \times a).
\end{array}
$$

Example 17.

$$
\begin{array}{rl}
1331 = 11 \times 11 \times 11 & := aa \times aa \times aa/(a \times a \times a) \\
13431 = 11 \times 11 \times 111 & := aa \times aa \times aaa/(a \times a \times a) \\
134431 = 11 \times 11 \times 1111 & := aa \times aa \times aaaa/(a \times a \times a) \\
1344431 = 11 \times 11 \times 11111 & := aa \times aa \times aaaaa/(a \times a \times a) \\
13444431 = 11 \times 11 \times 111111 & := aa \times aa \times aaaaaa/(a \times a \times a).
\end{array}
$$

Example 18.

$$
\begin{array}{rl}
99 = 9 \times 11 & := (aaa - aa - a)/a \\
999 = 9 \times 111 & := (aaaa - aaa - a)/a \\
9999 = 9 \times 1111 & := (aaaaa - aaaa - a)/a \\
99999 = 9 \times 11111 & := (aaaaaa - aaaaa - a)/a \\
999999 = 9 \times 111111 & := (aaaaaaa - aaaaaa - a)/a.
\end{array}
$$

Example 19.

$$
\begin{aligned}
1001 = 13 \times 77 \quad &:= aa \times (aaaa - aaa + a)/(aa \times a) \\
10101 = 13 \times 777 \quad &:= aaa \times (aaaa - aaa + a)/(aa \times a) \\
101101 = 13 \times 7777 \quad &:= aaaa \times (aaaa - aaa + a)/(aa \times a) \\
1011101 = 13 \times 77777 \quad &:= aaaaa \times (aaaa - aaa + a)/(aa \times a) \\
10111101 = 13 \times 777777 \quad &:= aaaaaa \times (aaaa - aaa + a)/(aa \times a).
\end{aligned}
$$

Example 20.

$$
\begin{aligned}
&11111111 = 11 \times 1010101 := aaaaaaaa \times a/(a \times a) \\
&22222222 = 11 \times 2020202 := aaaaaaaa \times (a + a)/(a \times a) \\
&33333333 = 11 \times 3030303 := aaaaaaaa \times (a + a + a)/(a \times a) \\
&44444444 = 11 \times 4040404 := aaaaaaaa \times (a + a + a + a)/(a \times a) \\
&55555555 = 11 \times 5050505 := aaaaaaaa \times (a + a + a + a + a)/(a \times a) \\
&66666666 = 11 \times 6060606 := aaaaaaaa \times (a + a + a + a + a + a)/(a \times a) \\
&77777777 = 11 \times 7070707 := aaaaaaaa \times (aa - a - a - a - a)/(a \times a) \\
&88888888 = 11 \times 8080808 := aaaaaaaa \times (aa - a - a - a)/(a \times a) \\
&99999999 = 11 \times 9090909 := aaaaaaaa \times (aa - a - a)/(a \times a).
\end{aligned}
$$

Above example 20 brings palindromes on both sides of the expressions, while example 12 is written in terms of number pattern. Here the repetition of same digit is eight times, i.e., 11111111, while in example 12 same digits repeats nine times, i.e., 111111111 . When we work with repetition of even number of digits, i.e, 2, 4, 6, etc., the decomposition can be palindromic or number pattern. In case of odd numbers the situation is different. The following two examples brings patterns working with 3,4, 5, 6 and 7 times repetitions of same digit.

Example 21. *Here below are two even order decompositions resulting again in palindromic patterns.*

$$
\begin{aligned}
1111 &= 11 \times 101 & 111111 &= 11 \times 10101 \\
2222 &= 11 \times 202 & 222222 &= 11 \times 20202 \\
3333 &= 11 \times 303 & 333333 &= 11 \times 30303 \\
4444 &= 11 \times 404 & 444444 &= 11 \times 40404 \\
5555 &= 11 \times 505 & 555555 &= 11 \times 50505 \\
6666 &= 11 \times 606 & 666666 &= 11 \times 60606 \\
7777 &= 11 \times 707 & 777777 &= 11 \times 70707 \\
8888 &= 11 \times 808 & 888888 &= 11 \times 80808 \\
9999 &= 11 \times 909 & 999999 &= 11 \times 90909.
\end{aligned}
$$

Example 22. *Here below are three odd order decompositions, resulting in prime numbers. The third is represented in single letter a.*

$$
\begin{array}{lll}
111 = 37 \times 3 \times 1 & 11111 = 41 \times 271 \times 1 & 1111111 = 239 \times 4649 \times 1 := aaaaaaa \times a/(a \times a) \\
222 = 37 \times 3 \times 2 & 22222 = 41 \times 271 \times 2 & 2222222 = 239 \times 4649 \times 2 := aaaaaaa \times (a + a)/(a \times a) \\
333 = 37 \times 3 \times 3 & 33333 = 41 \times 271 \times 3 & 3333333 = 239 \times 4649 \times 3 := aaaaaaa \times (a + a + a)/(a \times a) \\
444 = 37 \times 3 \times 4 & 44444 = 41 \times 271 \times 4 & 4444444 = 239 \times 4649 \times 4 := aaaaaaa \times (a + a + a + a)/(a \times a) \\
555 = 37 \times 3 \times 5 & 55555 = 41 \times 271 \times 5 & 5555555 = 239 \times 4649 \times 5 := aaaaaaa \times (a + a + a + a + a)/(a \times a) \\
666 = 37 \times 3 \times 6 & 66666 = 41 \times 271 \times 6 & 6666666 = 239 \times 4649 \times 6 := aaaaaaa \times (a + a + a + a + a + a)/(a \times a) \\
777 = 37 \times 3 \times 7 & 77777 = 41 \times 271 \times 7 & 7777777 = 239 \times 4649 \times 7 := aaaaaaa \times (aa - a - a - a - a)/(a \times a) \\
888 = 37 \times 3 \times 8 & 88888 = 41 \times 271 \times 8 & 8888888 = 239 \times 4649 \times 8 := aaaaaaa \times (aa - a - a - a)/(a \times a) \\
999 = 37 \times 3 \times 9 & 99999 = 41 \times 271 \times 9 & 9999999 = 239 \times 4649 \times 9 := aaaaaaa \times (aa - a - a)/(a \times a).
\end{array}
$$

We observe that the examples 20 and 21 are with 3, 6 and 9 times repetitions of same digit bringing palindromic patterns. But it is not always necessary. We can write in the terms of number patterns too. See the example below for 6 digit repetition written in two different number patterns.

Example 23.

$$\begin{aligned}
111111 &= 15873 \times 7 \times 1 = 37037 \times 3 \times 1 \\
222222 &= 15873 \times 7 \times 2 = 37037 \times 3 \times 2 \\
333333 &= 15873 \times 7 \times 3 = 37037 \times 3 \times 3 \\
444444 &= 15873 \times 7 \times 4 = 37037 \times 3 \times 4 \\
555555 &= 15873 \times 7 \times 5 = 37037 \times 3 \times 5 \\
666666 &= 15873 \times 7 \times 6 = 37037 \times 3 \times 6 \\
777777 &= 15873 \times 7 \times 7 = 37037 \times 3 \times 7 \\
888888 &= 15873 \times 7 \times 8 = 37037 \times 3 \times 8 \\
999999 &= 15873 \times 7 \times 9 = 37037 \times 3 \times 9.
\end{aligned}$$

3.4 Squared Number Patterns

Examples 1 and 3 works with square of 11 and 33 respectively. Here we shall bring more examples considering squared number patterns.

Example 24.

$$\begin{aligned}
16^2 &= 256 &&:=((aa+aa+aa-a)/(a+a))^{(a+a)/a} \\
166^2 &= 27556 &&:=((aaa+aaa+aaa-a)/(a+a))^{(a+a)/a} \\
1666^2 &= 2775556 &&:=((aaaa+aaaa+aaaa-a)/(a+a))^{(a+a)/a} \\
16666^2 &= 277755556 &&:=((aaaaa+aaaaa+aaaaa-a)/(a+a))^{(a+a)/a} \\
166666^2 &= 27777555556 &&:=((aaaaaa+aaaaaa+aaaaaa-a)/(a+a))^{(a+a)/a}.
\end{aligned}$$

Example 25.

$$\begin{aligned}
34^2 &= 1156 &&:=((aa+aa+aa+a)/a)^{(a+a)/a} \\
334^2 &= 111556 &&:=((aaa+aaa+aaa+a)/a)^{(a+a)/a} \\
3334^2 &= 11115556 &&:=((aaaa+aaaa+aaaa+a)/a)^{(a+a)/a} \\
33334^2 &= 1111155556 &&:=((aaaaa+aaaaa+aaaaa+a)/a)^{(a+a)/a} \\
333334^2 &= 111111555556 &&:=((aaaaaa+aaaaaa+aaaaaa+a)/a)^{(a+a)/a}.
\end{aligned}$$

Example 26.

$$\begin{aligned}
43^2 &= 1849 &&:=((aa+aa+aa+aa-a)/a)^{(a+a)/a} \\
433^2 &= 187489 &&:=((aaa+aaa+aaa+aaa-aa)/a)^{(a+a)/a} \\
4333^2 &= 18774889 &&:=((aaaa+aaaa+aaaa+aaaa-aaa)/a)^{(a+a)/a} \\
43333^2 &= 1877748889 &&:=((aaaaa+aaaaa+aaaaa+aaaaa-aaaa)/a)^{(a+a)/a} \\
433333^2 &= 187777488889 &&:=((aaaaaa+aaaaaa+aaaaaa+aaaaaa-aaaaa)/a)^{(a+a)/a}.
\end{aligned}$$

Example 27.

$$\begin{aligned}
67^2 &= 4489 &&:=((aaa+aa+aa+a)/(a+a))^{(a+a)/a} \\
667^2 &= 444889 &&:=((aaaa+aaa+aaa+a)/(a+a))^{(a+a)/a} \\
6667^2 &= 44448889 &&:=((aaaaa+aaaa+aaaa+a)/(a+a))^{(a+a)/a} \\
66667^2 &= 4444488889 &&:=((aaaaaa+aaaaa+aaaaa+a)/(a+a))^{(a+a)/a} \\
666667^2 &= 444444888889 &&:=((aaaaaaa+aaaaaa+aaaaaa+a)/(a+a))^{(a+a)/a}.
\end{aligned}$$

Example 28.

$$\begin{aligned}
91^2 &= 828 &&:=((aaa - aa - aa + a + a)/a)^{(a+a)/a}\\
991^2 &= 98208 &&:=((aaaa - aaa - aa + a + a)/a)^{(a+a)/a}\\
9991^2 &= 9982008 &&:=((aaaaa - aaaa - aa + a + a)/a)^{(a+a)/a}\\
99991^2 &= 999820008 &&:=((aaaaaa - aaaaa - aa + a + a)/a)^{(a+a)/a}\\
999991^2 &= 99998200008 &&:=((aaaaaaa - aaaaaa - aa + a + a)/a)^{(a+a)/a}.
\end{aligned}$$

Example 29. *When we work with equal digits, the numbers 11, 33, 66 and 99 give following beautiful patterns:*

$$\begin{array}{cccc}
11^2 = 121 & 33^2 = 1089 & 66^2 = 4356 & 99^2 = 9801\\
111^2 = 12321 & 333^2 = 110889 & 666^2 = 443556 & 999^2 = 998001\\
1111^2 = 1234321 & 3333^2 = 11108889 & 6666^2 = 44435556 & 9999^2 = 99980001\\
11111^2 = 123454321 & 33333^2 = 1111088889 & 66666^2 = 4444355556 & 99999^2 = 9999800001\\
111111^2 = 12345654321 & 333333^2 = 111110888889 & 666666^2 = 444443555556 & 999999^2 = 999998000001.
\end{array}$$

Only in the first case the result is palindromic, others are just number patterns. They can also be brought to palindromic by making proper divisions in each case, i.e., dividing by 3^2, 6^2 *and* 9^2 *respectively. See below*

$$\begin{aligned}
11^2 &= \frac{33^2}{3^2} = \frac{66^2}{6^2} = \frac{99^2}{9^2} = 121\\
111^2 &= \frac{333^2}{3^2} = \frac{666^2}{6^2} = \frac{999^2}{9^2} = 12321\\
1111^2 &= \frac{3333^2}{3^2} = \frac{6666^2}{6^2} = \frac{9999^2}{9^2} = 1234321\\
11111^2 &= \frac{33333^2}{3^2} = \frac{66666^2}{6^2} = \frac{99999^2}{9^2} = 123454321\\
111111^2 &= \frac{333333^2}{3^2} = \frac{666666^2}{6^2} = \frac{999999^2}{9^2} = 12345654321.
\end{aligned}$$

Above process works also for numbers $\frac{22^2}{2^2}$, $\frac{44^2}{4^2}$, $\frac{55^2}{5^2}$, $\frac{77^2}{7^2}$ *and* $\frac{88^2}{8^2}$, *and still the result remains the same.*

3.5 Repeated Digits Patterns

In this subsection, we shall present situations, where the patterns are formed by repetition of digits. In each case, the repetitions are in different forms. We can write

$$999999 = 3 \times 7 \times 11 \times 13 \times 37.$$

Division by 3, 11 and 37 always bring palindromes, i.e.,

$$\begin{aligned}
999999/3 &= 333333\\
999999/11 &= 90909\\
999999/37 &= 9009 \times 3.
\end{aligned}$$

The division by other two numbers, i.e., by 7 and 13 gives two different numbers, i.e.,

$$\begin{aligned}
999999/7 &= 142857\\
999999/13 &= 76923.
\end{aligned}$$

The following three examples are based on the numbers 142857 and 76923 with repetition of digits. The number 142857 also appeared in example 11 with interesting pattern.

Example 30. *This example is based on the number 142857. Multiplying by certain numbers, we get numbers with repetition of same digits. See below*

$$\begin{aligned}
142857\times 1 = 142857 &:= aaaaaa\times(aa-a-a)\times a/(a\times a\times(aa-a-a-a-a))\\
142857\times 3 = 428571 &:= aaaaaa\times(aa-a-a)\times(a+a+a)/(a\times a\times(aa-a-a-a-a))\\
142857\times 2 = 285714 &:= aaaaaa\times(aa-a-a)\times(a+a)/(a\times a\times(aa-a-a-a-a))\\
142857\times 6 = 857142 &:= aaaaaa\times(aa-a-a)\times(a+a+a+a+a+a)/(a\times a\times(aa-a-a-a-a))\\
142857\times 4 = 571428 &:= aaaaaa\times(aa-a-a)\times(a+a+a+a)/(a\times a\times(aa-a-a-a-a))\\
142857\times 5 = 714285 &:= aaaaaa\times(aa-a-a)\times(a+a+a+a+a)/(a\times a\times(aa-a-a-a-a)).
\end{aligned}$$

Multiplying by 7 and dividing by 9, above numbers, we get interesting pattern:

$$\begin{aligned}
142857\times 7/9 &= 111111\\
428571\times 7/9 &= 333333\\
285714\times 7/9 &= 222222\\
857142\times 7/9 &= 666666\\
571428\times 7/9 &= 444444\\
714285\times 7/9 &= 555555
\end{aligned}$$

Also, we can write

$$\frac{1}{7} = 0.142857\ 142857\ 142857\ 142857\ .. = 0.\overline{142857}.$$

Example 31. *This example is based on the number 76923. After multiplication by different numbers, we get a numbers with repetitions of same digits.*

$$\begin{aligned}
76923\times 1\ = 076923 &:= aaaaaa\times(aa-a-a)\times a/(a\times a\times(aa+a+a))\\
76923\times 10 = 769230 &:= aaaaaa\times(aa-a-a)\times(aa-a)/(a\times a\times(aa+a+a))\\
76923\times 9\ = 692307 &:= aaaaaa\times(aa-a-a)\times(aa-a-a)/(a\times a\times(aa+a+a))\\
76923\times 12 = 923076 &:= aaaaaa\times(aa-a-a)\times(aa+a)/(a\times a\times(aa+a+a))\\
76923\times 3\ = 230769 &:= aaaaaa\times(aa-a-a)\times(a+a+a)/(a\times a\times(aa+a+a))\\
76923\times 4\ = 307692 &:= aaaaaa\times(aa-a-a)\times(a+a+a+a)/(a\times a\times(aa+a+a)).
\end{aligned}$$

Here also we have similar property as of previous example. Multiplying by 13 and dividing by 9, above numbers, we get interesting pattern:

$$\begin{aligned}
076923\times 13/9 &= 111111\\
230769\times 13/9 &= 333333\\
307692\times 13/9 &= 444444\\
692307\times 13/9 &= 999999.
\end{aligned}$$

Here we don't have symmetrical values for 769230 and 923076. Also, we have

$$\frac{1}{13} := 0.076923\ 076923\ 076923\ 076923\ \ldots = 0.\overline{076923}.$$

Example 32. *This example also deals with the number 76923. After multiplication by different numbers, we get a numbers with repetitions of same digits.*

$$\begin{aligned}
76923\times 2\ = 153846 &:= aaaaaa\times(aa-a-a)\times(a+a)/(a\times a\times(aa+a+a))\\
76923\times 7\ = 538461 &:= aaaaaa\times(aa-a-a)\times(aa-a-a-a-a)/(a\times a\times(aa+a+a))\\
76923\times 5\ = 384615 &:= aaaaaa\times(aa-a-a)\times(a+a+a+a+a)/(a\times a\times(aa+a+a))\\
76923\times 11 = 846153 &:= aaaaaa\times(aa-a-a)\times aa/(a\times a\times(aa+a+a))\\
76923\times 6\ = 461538 &:= aaaaaa\times(aa-a-a)\times(a+a+a+a+a+a)/(a\times a\times(aa+a+a))\\
76923\times 8\ = 615384 &:= aaaaaa\times(aa-a-a)\times(aa-a-a-a)/(a\times a\times(aa+a+a)).
\end{aligned}$$

Example 33. *The division 1/7 and 1/13 also works, we consider multiplications of 7 and 13. Here below are two situations for multiplication of 13:*

$$\frac{1}{26} = 0.038461\ 538461\ 538461\ 538461\ 538461\ 538461... = 0.038461\ \overline{538461}.$$

and

$$\frac{1}{39} = 0.025641\ 025641\ 025641\ 025641\ 025641 = 0.\overline{025641}.$$

In the second case we have following repetition of digits:

$$\begin{aligned}
025641 \times 1 &= 025641 := aaaaaa \times (a+a+a)/(a \times (aa+a+a)) \\
025641 \times 10 &= 256410 := aaaaaa \times (a+a+a) \times (aa-a)/(a \times a \times (aa+a+a)) \\
025641 \times 22 &= 564102 := aaaaaa \times (a+a+a) \times (aa+aa)/(a \times a \times (aa+a+a)) \\
025641 \times 25 &= 641025 := aaaaaa \times (a+a+a) \times (aa+aa+a+a+a)/(a \times a \times (aa+a+a)) \\
025641 \times 16 &= 410256 := aaaaaa \times (a+a+a) \times (aa+a+a+a+a+a)/(a \times a \times (aa+a+a)) \\
025641 \times 4 &= 102564 := aaaaaa \times (a+a+a) \times (a+a+a+a)/(a \times a \times (aa+a+a)).
\end{aligned}$$

The numbers 142857, 076923, 153846 and 025641 respectively appearing in examples 30, 31, 32 and 33 satisfies common properties given by

$$\begin{array}{llll}
14+28+57=99 & 07+69+23=99 & 15+38+46=99 & 02+56+41=99 \\
42+85+71=99\times 2 & 76+92+30=99\times 2 & 53+84+61=99\times 2 & 25+64+10=99 \\
28+57+14=99 & 69+23+07=99 & 38+46+15=99 & 56+41+02=99 \\
85+71+42=99\times 2 & 92+30+76=99\times 2 & 84+61+53=99\times 2 & 64+10+25=99 \\
57+14+28=99 & 23+07+69=99 & 46+15+38=99 & 41+02+56=99 \\
71+42+85=99 & 30+76+92=99\times 2 & 61+53+84=99\times 2. & 10+25+64=99.
\end{array}$$

and

$$\begin{array}{llll}
142+857=999 & 076+923=999 & 153+846=999 & 025+641=666 \\
428+571=999 & 769+230=999 & 538+461=999 & 256+410=666 \\
285+714=999 & 692+307=999 & 384+615=999 & 564+102=666 \\
857+142=999 & 923+076=999 & 846+153=999 & 641+025=666 \\
571+428=999 & 230+769=999 & 461+538=999 & 410+256=666 \\
714+285=999 & 307+692=999 & 615+384=999 & 102+564=666.
\end{array}$$

Example 34. *Multiplication of 1089 with* $1,2,3,4,5,6,7,8$ *and* 9 *brings very interesting pattern. See below:*

$$\begin{aligned}
1089 \times 1 = 1089 &:= (aaaa-aa-aa) \times a/(a \times a) \\
1089 \times 2 = 2178 &:= (aaaa-aa-aa) \times (a+a)/(a \times a) \\
1089 \times 3 = 3267 &:= (aaaa-aa-aa) \times (a+a+a)/(a \times a) \\
1089 \times 4 = 4356 &:= (aaaa-aa-aa) \times (a+a+a+a)/(a \times a) \\
1089 \times 5 = 5445 &:= (aaaa-aa-aa) \times (a+a+a+a+a)/(a \times a) \\
1089 \times 6 = 6534 &:= (aaaa-aa-aa) \times (a+a+a+a+a+a)/(a \times a) \\
1089 \times 7 = 7623 &:= (aaaa-aa-aa) \times (aa-a-a-a-a)/(a \times a) \\
1089 \times 8 = 8712 &:= (aaaa-aa-aa) \times (aa-a-a-a)/(a \times a) \\
1089 \times 9 = 9801 &:= (aaaa-aa-aa) \times (aa-a-a)/(a \times a).
\end{aligned}$$

In each column, the digits are in consecutive way (increasing or decreasing). In pairs, they are reverse of each other, i.e., (1089, 9801), (2178, 8712), (3267, 7623) and (4356, 6534). Another interesting property of these nine numbers is with magic squares, i.e., considering members of 2, 3 or 4 columns, they always forms magic squares of order 3x3 [2].

Example 35. *Another number having similar kind of properties of previous example is 9109. Multiplying it by 1, 2, 3, 4, 5, 6, 7, 8 and 9, we get*

$$\begin{aligned}
9109 \times 1 = 09109 &:= (aaaaa - aaaaaa \times (a+a)/aaa) \times a/(a \times a)\\
9109 \times 2 = 18218 &:= (aaaaa - aaaaaa \times (a+a)/aaa) \times (a+a)/(a \times a)\\
9109 \times 3 = 27327 &:= (aaaaa - aaaaaa \times (a+a)/aaa) \times (a+a+a)/(a \times a)\\
9109 \times 4 = 36436 &:= (aaaaa - aaaaaa \times (a+a)/aaa) \times (a+a+a+a)/(a \times a)\\
9109 \times 5 = 45545 &:= (aaaaa - aaaaaa \times (a+a)/aaa) \times (a+a+a+a+a)/(a \times a)\\
9109 \times 6 = 54654 &:= (aaaaa - aaaaaa \times (a+a)/aaa) \times (a+a+a+a+a+a)/(a \times a)\\
9109 \times 7 = 63763 &:= (aaaaa - aaaaaa \times (a+a)/aaa) \times (aa-a-a-a-a)/(a \times a)\\
9109 \times 8 = 72872 &:= (aaaaa - aaaaaa \times (a+a)/aaa) \times (aa-a-a-a)/(a \times a)\\
9109 \times 9 = 81981 &:= (aaaaa - aaaaaa \times (a+a)/aaa) \times (aa-a-a)/(a \times a).
\end{aligned}$$

Column members are in increasing and decreasing orders of 0 to 8 or 9. Also first and last two digit of each number are same, and are multiple of 9. Moreover, the number 9109 is a prime number. Here also, all the new nine numbers make a magic square of order 3×3. Another interesting property of above pattern is

$$\begin{aligned}
100 &= \frac{091+09}{1} = \frac{182+18}{2} = \frac{273+27}{3}\\
&= \frac{364+36}{4} = \frac{455+45}{5} = \frac{546+54}{6}\\
&= \frac{637+63}{7} = \frac{728+72}{8} = \frac{819+81}{9}.
\end{aligned}$$

Example 36. *This example is based on the property,* $8712 = 2178 \times 4 = 1089 \times 2 \times 4$, *i.e., after multiplication by 4 the number 2178 becomes its reverse, i.e., 8712. Also it is a multiple of 8 with 1089.*

$$\begin{aligned}
8712 = 2178 \times 4 \quad &:=(aaaa - aa - aa) \times (a+a) \times (a+a+a+a)/(a \times a \times a)\\
87912 = 21978 \times 4 \quad &:=(aaaaa - aaa - aa) \times (a+a) \times (a+a+a+a)/(a \times a \times a)\\
879912 = 219978 \times 4 \quad &:=(aaaaaa - aaaa - aa) \times (a+a) \times (a+a+a+a)/(a \times a \times a)\\
8799912 = 2199978 \times 4 \quad &:=(aaaaaaa - aaaaa - aa) \times (a+a) \times (a+a+a+a)/(a \times a \times a)\\
87999912 = 21999978 \times 4 \quad &:=(aaaaaaaa - aaaaaa - aa) \times (a+a) \times (a+a+a+a)/(a \times a \times a).
\end{aligned}$$

Example 37. *If we multiply 1089 by 9, we get 9801, i.e., reverse of 1089. The same happens with next members of the patterns.*

$$\begin{aligned}
9801 &= 1089 \times 9 = 99 \times 99 &&:=(aaa - aa - a) \times (aaa - aa - a)/(a \times a)\\
98901 &= 10989 \times 9 = 99 \times 999 &&:=(aaa - aa - a) \times (aaaa - aaa - a)/(a \times a)\\
989901 &= 109989 \times 9 = 99 \times 9999 &&:=(aaa - aa - a) \times (aaaaa - aaaa - a)/(a \times a)\\
9899901 &= 1099989 \times 9 = 99 \times 99999 &&:=(aaa - aa - a) \times (aaaaaa - aaaaa - a)/(a \times a)\\
98999901 &= 10999989 \times 9 = 99 \times 999999 &&:=(aaa - aa - a) \times (aaaaaaa - aaaaaa - a)/(a \times a).
\end{aligned}$$

Tricks for Making Pattern. [3] Let us consider numbers of 3, 4, 5 and 6 digits, for example, 183, 3568, 19757 and 876456. Changing first digit with last and vice-versa, in each case, we get, 381, 8563, 79751 and 676458 respectively. Let us consider the difference among the respective values (higher minus lesser), i.e.,

$$\begin{aligned}
381 - 183 &= 198\\
8563 - 3568 &= 4995\\
79751 - 19757 &= 59994\\
876456 - 676458 &= 199998.
\end{aligned}$$

Changing again last digit with first and vice-versa, and adding we get the required pattern, i.e.,

$$\begin{aligned} 198 + 891 &= 1089 \\ 4995 + 5994 &= 10989 \\ 59994 + 49995 &= 109989 \\ 199998 + 899991 &= 1099989. \end{aligned}$$

Proceeding further with higher digits, we get further values of the pattern. Here the condition is that, the difference in each case should be bigger than 1, for example, $3453 - 3453 = 0$ is not valid number. Second condition is that if this differences come to 99, 999, etc, i.e., $201 - 102 = 99$, $4433 - 3434 = 999$, etc. In this situation, we have to sum twice, i.e, $99 + 99 = 198$ and $999 + 999 = 1998$, and then $198 + 891 = 1089$ and $1998 + 8991 = 10989$, etc. Example 36, also brings a general way to bring the same pattern.

3.6 Fibonacci Sequences and Extensions

This subsection deals with the patterns related to *Fibonacci sequences* [4, 7, 16] and its extensions or variations to *Lucas sequences*, *Tribonacci sequence*, *Tetranacci sequence*, etc.

Example 38. ***(Fibonacci sequence)*** *The recurring formula for Fibonacci sequence is given by*

$$F_0 = 1, \quad F_1 = 1, \quad F_{n+1} = F_n + F_{n-1}, \quad n \geq 1.$$

Its values are as follows:

$$1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, 144, 233, 377, 610, 987, 1597, 2584, 4181, 6765, 10946, 17711, 28657,$$
$$46368, 75025, 121393, 196418, 317811, 514229, 832040, \ldots$$

The above values can be obtained by subsequent divisions given below:

$$\begin{aligned} \frac{1}{89} &= 0.01\ 1\ 2\ 3\ 5\ 9550561\ldots \\ \frac{1}{9899} &= 0.0001\ 01\ 02\ 03\ 05\ 08\ 13\ 21\ 34\ 55\ 9046368\ldots \\ \frac{1}{998999} &= 0.000001\ 001\ 002\ 003\ 005\ 008\ 013\ 021\ 034\ 055\ 089\ 144\ 233\ 377\ 610\ 9885995\ldots \\ \frac{1}{99989999} &= 1.0001\ 0002\ 0003\ 0005\ 0008\ 0013\ 0021\ 0034\ 0055\ 0089\ 0144\ 0233\ 0377\ 0610\ 0987\ 1597\ 2584 \\ &\quad\ 4181\ 6766094777 \times 10^{-8}\ldots \\ \frac{1}{9999899999} &= 1.00001\ 00002\ 00003\ 00005\ 00008\ 00013\ 00021\ 00034\ 00055\ 00089\ 00144\ 00233\ 00377\ 00610 \\ &\quad\ 00987\ 01597\ 02584\ 04181\ 06765\ 10946\ 17711\ 28657\ 46368\ 750262139 \times 10^{-10} \\ \frac{1}{999998999999} &= 1.000001\ 000002\ 000003\ 000005\ 000008\ 000013\ 000021\ 000034\ 000055\ 000089\ 000144\ 000233 \\ &\quad\ 000377\ 000610\ 000987\ 001597\ 002584\ 004181\ 006765\ 010946\ 017711\ 028657\ 046368\ 075025 \\ &\quad\ 121393\ 196418\ 317811\ 514229\ 83204134 \times 10^{-12}\ldots \end{aligned}$$

Here we observe that as 9 increases on both sides of the division $1/89$*, the resulting decimal fractions approximates more near to Fibonacci sequence values. Denominator values lead us to following pattern:*

$$\begin{aligned} 89 &= 10^2 - 11 && :=(aaa - aa - aa)/a \\ 9899 &= 10^4 - 101 && :=(aaaaa - aaaa - aaa + aa - a)/a \\ 998999 &= 10^6 - 1001 && :=(aaaaaaa - aaaaaa - aaaa + aaa - a)/a \\ 99989999 &= 10^8 - 10001 && :=(aaaaaaaaa - aaaaaaaa - aaaaa + aaaa - a)/a \\ 9999899999 &= 10^{10} - 100001 && :=(aaaaaaaaaaa - aaaaaaaaaa - aaaaaa + aaaaa - a)/a. \end{aligned}$$

We can also write in terms of *geometric series*:

$$\frac{1}{89} = \frac{1}{10^2} + \frac{11}{10^4} + \frac{11^2}{10^6} + \dots$$
$$\frac{1}{9899} = \frac{1}{10^4} + \frac{101}{10^8} + \frac{101^2}{10^{12}} + \dots$$
$$\frac{1}{998999} = \frac{1}{10^6} + \frac{1001}{10^{12}} + \frac{1001^2}{10^{18}} + \dots$$
$$\frac{1}{99989999} = \frac{1}{10^8} + \frac{1001}{10^{16}} + \frac{1001^2}{10^{24}} + \dots$$

Example 39. *As an extension or variation of Fibonacci sequence is Lucas sequence. It is given by the following recurring formula:*

$$L_0 = 2, \quad L_1 = 1, \quad L_n = L_{n-1} + F_{n-2}, \quad n \geq 2.$$

Its values are as follows

2, 1, 3, 4, 7, 11, 18, 29, 47, 76, 123, 199, 322, 521, 843, 1364, 2207, 3571, 5778, 9349, 15127, 24476, 39603, 64079, 103682, 167761, 271443, 439204, 710647, 1149851, ...

The above values are obtained by subsequent divisions formed by following pattern:

$$\frac{19}{89} = 0.2\ 1\ 3\ 4\ 83146067\dots$$
$$\frac{199}{9899} = 0.02\ 01\ 03\ 04\ 07\ 11\ 18\ 29\ 47\ 77250227\dots$$
$$\frac{1999}{998999} = 0.002\ 001\ 003\ 004\ 007\ 011\ 018\ 029\ 047\ 076\ 123\ 199\ 322\ 521\ 844210576\dots$$
$$\frac{19999}{99989999} = 0.0002\ 0001\ 0003\ 0004\ 000\ 70011\ 0018\ 0029\ 0047\ 0076\ 0123\ 0199\ 0322\ 0521\ 0843\ 1364\ 2207\ 3571\ 5778\ 9350512944\dots$$
$$\frac{199999}{9999899999} = 0.00002\ 00001\ 00003\ 00004\ 00007\ 00011\ 00018\ 00029\ 00047\ 00076\ 00123\ 00199\ 00322\ 00521\ 00843\ 01364\ 02207\ 03571\ 05778\ 09349\ 15127\ 24476\ 39603\ 640800368\dots$$
$$\frac{1999999}{999998999999} = 0.000002\ 000001\ 000003\ 000004\ 000007\ 000011\ 000018\ 000029\ 000047\ 000076\ 000123\ 000199\ 000322\ 000521\ 000843\ 001364\ 002207\ 003571\ 005778\ 009349\ 015127\ 024476\ 039603\ 064079\ 103682\ 167761\ 271443\ 439204\ 71064814985\dots$$

We observe that above expressions are formed by two patterns, i.e., one in numerator and another in denominator. Single letter representations are given by

$$\frac{19}{89} := \frac{aa + aa - a - a - a}{aaa - aa - aa}$$
$$\frac{199}{9899} := \frac{aaa + aaa - aa - aa - a}{aaaaa - aaaa - aaa + aa - a}$$
$$\frac{1999}{998999} := \frac{aaaa + aaaa - aaa - aaa - a}{aaaaaaa - aaaaaa - aaaa + aaa - a}$$
$$\frac{19999}{99989999} := \frac{aaaaa + aaaaa - aaaa - aaaa - a}{aaaaaaaaa - aaaaaaaa - aaaaa + aaaa - a}$$
$$\frac{199999}{9999899999} := \frac{aaaaaa + aaaaaa - aaaaa - aaaaa - a}{aaaaaaaaaaa - aaaaaaaaaa - aaaaaa + aaaaa - a}.$$

Example 40. ***(Tribonacci Sequence)*** *The Tribonacci sequence is a generalization of the Fibonacci sequence defined by the recurrence formula:*

$$T_1 = 1, \quad T_2 = 1, \quad T_3 = 2, \quad T_n = T_{n-1} + T_{n-2} + T_{n-3}, \quad n \geq 4.$$

Its values are given by

$$1, 1, 2, 4, 7, 13, 24, 44, 81, 149, 274, 504, 927, 1705, 3136, 5768, 10609, 19513, 35890, 66012, 121415, 223317,$$
$$410744, 755476, 1389537, 2555757, 4700770, 8646064, \ldots$$

The above sequence values are obtained in little different form of example 38. See below the respective division pattern:

$$\begin{aligned}
\frac{1}{889} &= 0.001\ 1\ 2\ 4\ 8593925\ldots. \\
\frac{1}{989899} &= 0.000001\ 01\ 02\ 04\ 07\ 13\ 24\ 44\ 82517913\ldots \\
\frac{1}{998998999} &= 1.001\ 002\ 004\ 007\ 013\ 024\ 044\ 081\ 149\ 274\ 504\ 92870814 \times 10^{9}\ldots \\
\frac{1}{999899989999} &= 1.0001\ 0002\ 0004\ 0007\ 0013\ 0024\ 0044\ 0081\ 0149\ 0274\ 0504\ 0927\ 1705\ 3136\ 576906109 \times 10^{-12}\ldots. \\
\frac{1}{999989999899999} &= 1.00001\ 00002\ 00004\ 00007\ 00013\ 00024\ 00044\ 00081\ 00149\ 00274\ 00504\ 00927\ 01705 \\
&\qquad 03136\ 05768\ 10609\ 19513\ 35890\ 6601321417 \times 10 \times -15\ldots
\end{aligned}$$

Single digit representation of denominator in division pattern is as follows:

$$\begin{aligned}
889 = 10^3 - 111 &:= (aaaa - aaa - aaa)/a \\
989899 = 10^6 - 10101 &:= (aaaaaaa - aaaaaa - aaaaa + aaaa - aaa + aa - a)/a \\
998998999 = 10^9 - 1001001 &:= (aaaaaaaaaa - aaaaaaaaa - aaaaaaa + aaaaaa - aaaa + aaa - a)/a \\
999899989999 = 10^{12} - 100010001 &:= (aaaaaaaaaaaaa - aaaaaaaaaaaa - aaaaaaaaa + aaaaaaaa - aaaaa + aaaa - a)/a.
\end{aligned}$$

Example 41. ***(Tetranacci Sequence)*** *The Tetranacci sequence is a generalization of the Fibonacci sequence defined by the recurrence formula:*

$$T_0 = 0, \quad T_1 = 1, \quad T_2 = 1, \quad T_3 = 2, \quad T_n = T_{n-1} + T_{n-2} + T_{n-3} + T_{n-4}, \quad n \geq 4.$$

Its values are given by

$$0, 1, 1, 2, 4, 8, 15, 29, 56, 108, 208, 401, 773, 1490, 2872, 5536, 10671, 20569, 39648, 76424, \ldots$$

The above sequence values are obtained in little different form of example 38. See below the respective division pattern:

$$\begin{aligned}
\frac{1}{8889} &= 0.0001\ 1\ 2\ 4\ 98593767\ldots. \\
\frac{1}{98989899} &= 1.01\ 02\ 04\ 08\ 15\ 29\ 5710120888\ldots \\
\frac{1}{998998998999} &= 1.001\ 002\ 004\ 008\ 015\ 029\ 056\ 108\ 208\ 401\ 7744928775\ldots \\
\frac{1}{9998999899989999} &= 1.0001\ 0002\ 0004\ 0008\ 0015\ 0029\ 0056\ 0108\ 0208\ 0401\ 0773\ 1490\ 2872\ 55370673\ldots.
\end{aligned}$$

We can write

$$\begin{aligned}
8889 = 10^4 - \frac{1111}{1} &:= ((aa-a)/a)^{(a+a+a+a)/a} - \frac{aaaa}{a} \\
98989899 = 10^8 - \frac{11111111}{11} &:= ((aa-a)/a)^{(aa-a-a-a)/a} - \frac{aaaaaaaa}{aa} \\
998998998999 = 10^{12} - \frac{111111111111}{111} &:= ((aa-a)/a)^{(aa+a)/a} - \frac{aaaaaaaaaaaa}{aaa} \\
9998999899989999 = 10^{16} - \frac{1111111111111111}{1111} &:= ((aa-a)/a)^{(aa+a+a+a+a+a)/a} - \frac{aaaaaaaaaaaaaaaa}{aaaa}.
\end{aligned}$$

Examples 38 and 39 have the same denominator. The difference is in numerator values; one is 1 and another is pattern 19, 199,.. Thus we have three different *denominators* in above four examples 38, 39, 40, and 41. Here below are these three different patterns:

Fibonacci sequence	**Tribonacci sequence**	**Tetranacci sequence**
$89 = 10^2 - 11$	$889 = 10^3 - 111$	$8889 = 10^4 - 1111$
$9899 = 10^4 - 101$	$989899 = 10^6 - 10101$	$98989899 = 10^8 - 1010101$
$998999 = 10^6 - 1001$	$998998999 = 10^9 - 1001001$	$998998998999 = 10^{12} - 1001001001$
$99989999 = 10^8 - 10001$	$999899989999 = 10^{12} - 100010001$	$9998999899989999 = 10^{16} - 1000100010001.$

3.7 Doubly Symmetric Patterns

Below are three examples of *doubly symmetric patterns*, i.e., we can write, $99 \times 5 = 9 \times 55$, $99 \times 7 = 9 \times 77$ and $99 \times 8 = 9 \times 88$, etc.

Example 42.

$$\begin{aligned}
45 = & \quad 9 \times 5 = 9 \times 5 && :=(aaa - aa - aa + a)/(a + a) \\
495 = & \quad 99 \times 5 = 9 \times 55 && :=(aaaa - aaa - aa + a)/(a + a) \\
4995 = & \quad 999 \times 5 = 9 \times 555 && :=(aaaaa - aaaa - aa + a)/(a + a) \\
49995 = & \quad 9999 \times 5 = 9 \times 5555 && :=(aaaaaa - aaaaa - aa + a)/(a + a) \\
499995 = & \quad 99999 \times 5 = 9 \times 55555 && :=(aaaaaaa - aaaaaa - aa + a)/(a + a).
\end{aligned}$$

Example 43.

$$\begin{aligned}
63 = & \quad 9 \times 7 = 9 \times 7 && :=(a + a + a) \times (aa + aa - a)/(a \times a) \\
693 = & \quad 99 \times 7 = 9 \times 77 && :=(aa + aa + aa) \times (aa + aa - a)/(a \times a) \\
6993 = & \quad 999 \times 7 = 9 \times 777 && :=(aaa + aaa + aaa) \times (aa + aa - a)/(a \times a) \\
69993 = & \quad 9999 \times 7 = 9 \times 7777 && :=(aaaa + aaaa + aaaa) \times (aa + aa - a)/(a \times a) \\
699993 = & \quad 99999 \times 7 = 9 \times 77777 && :=(aaaaa + aaaaa + aaaaa) \times (aa + aa - a)/(a \times a).
\end{aligned}$$

Example 44.

$$\begin{aligned}
72 = & \quad 9 \times 8 = 9 \times 8 && :=(aa + a) \times (aa + a) \times a/(a + a)/(a \times a) \\
792 = & \quad 99 \times 8 = 9 \times 88 && :=(aa + a) \times (aa + a) \times aa/(a + a)/(a \times a) \\
7992 = & \quad 999 \times 8 = 9 \times 888 && :=(aa + a) \times (aa + a) \times aaa/(a + a)/(a \times a) \\
79992 = & \quad 9999 \times 8 = 9 \times 8888 && :=(aa + a) \times (aa + a) \times aaaa/(a + a)/(a \times a) \\
799992 = & \quad 99999 \times 8 = 9 \times 88888 && :=(aa + a) \times (aa + a) \times aaaaa/(a + a)/(a \times a).
\end{aligned}$$

Above examples are written multiplying 9 with 5, 7 and 8. Same can be done multiplying 9 with 2, 3, 4 and 6. This is due to the fact that 11 is a factor of 99 and aa, for all $a \in \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$

Example 45. ***(General way)****. Examples 42, 43 and 44 can be written in a general way. Let us consider a and b, such that*

$$a, b \in \{1, 2, 3, 4, 5, 6, 7, 8, 9\}, a + b = 9, a > b,$$

then, examples 43 and 44 can be summarized as

$$\begin{aligned}
a9b = & \quad 99 \times A = 9 \times AA \\
a99b = & \quad 999 \times A = 9 \times AAA \\
a999b = & \quad 9999 \times A = 9 \times AAAA \\
a9999b = & \quad 99999 \times A = 9 \times AAAAA \\
a99999b = & \quad 999999 \times A = 9 \times AAAAAA.
\end{aligned}$$

where

$$A = a + 1, \textit{ and } AAA = A \times 10^2 + A \times 10^1 + A \times 10^0, \textit{ etc.}$$

If $b > a$ as in case of example 42, writing $B = b + 1$, the above pattern changes as

$$\begin{aligned} a9b &= \quad 99 \times B &= 9 \times BB \\ a99b &= \quad 999 \times B &= 9 \times BBB \\ a999b &= \quad 9999 \times B &= 9 \times BBBB \\ a9999b &= \quad 99999 \times B &= 9 \times BBBBB \\ a99999b &= 999999 \times B &= 9 \times BBBBBB. \end{aligned}$$

We can apply this general way to get the pattern appearing in example 37. Let us change a with b and b with a in the above pattern, then sum both, we get

$$\begin{aligned} a9b + b9a &= 1089 \\ a99b + b99a &= 10989 \\ a999b + b999a &= 109989 \\ a9999b + b9999a &= 1099989 \\ a99999b + b99999a &= 10999989. \end{aligned}$$

This is the same pattern appearing in example 37. Multiplying above pattern with 8 and 9, we get respectively the patterns appearing in examples 36 and 37. Multiplying by 5 we get palindromic numbers. Here below are these three multiplications:

$$\begin{array}{ccc} 1089 \times 5 = 5445 & 1089 \times 8 = 4 \times 2178 = 8712 & 1089 \times 9 = 9801 \\ 10989 \times 5 = 54445 & 10989 \times 8 = 4 \times 21978 = 87912 & 10989 \times 9 = 98901 \\ 109989 \times 5 = 544445 & 109989 \times 8 = 4 \times 219978 = 879912 & 109989 \times 9 = 989901 \\ 1099989 \times 5 = 5444445 & 1099989 \times 8 = 4 \times 2199978 = 8799912 & 1099989 \times 9 = 9899901 \\ 10999989 \times 5 = 54444445 & 10999989 \times 8 = 4 \times 21999978 = 87999912 & 10999989 \times 9 = 98999901. \end{array}$$

In the first case results are palindromic patterns, while second and third brings reverse numbers. Here below is another interesting relations with 1, 6 and 9:

$$\begin{aligned} 1089 &= (916 - 619) + (961 - 169) \\ 10989 &= (9116 - 6119) + (9661 - 1669) \\ 109989 &= (91116 - 61119) + (96661 - 16669) \\ 1099989 &= (911116 - 611119) + (966661 - 166669). \end{aligned}$$

Also we can write

$$\begin{aligned} 1089 &= (996 - 699) + (991 - 199) \\ 1089 &= (966 - 669) + (911 - 119) \\ 1089 &= (916 - 619) + (961 - 169) \\ 1089 &= (1 + 1) \times (611 - 116) + 99 \\ 1089 &= (1 + 1) \times (661 - 166) + 99 \\ 1089 &= (1 + 1) \times (691 - 196) + 99 \\ 1089 &= (6 \times 6 + 6 \times 6 + 6 \times 6) \times (9 + 1) + 9 \\ 1089 &= (1 \times 6 \times 9 + 1 \times 6 \times 9) \times (9 + 1) + 9 \\ 1089 &= 11 \times 11 \times 9. \end{aligned}$$

3.8 Number Patterns with Number Pattern Decompositions

Example 46.

$$\begin{aligned}
11111111101 &= 123456789 \times 9 \times 1 := (aaaaaaaaaaa - aa + a) \times a/(a \times a)\\
22222222202 &= 123456789 \times 9 \times 2 := (aaaaaaaaaaa - aa + a) \times (a+a)/(a \times a)\\
33333333303 &= 123456789 \times 9 \times 3 := (aaaaaaaaaaa - aa + a) \times (a+a+a)/(a \times a)\\
44444444404 &= 123456789 \times 9 \times 4 := (aaaaaaaaaaa - aa + a) \times (a+a+a+a)/(a \times a)\\
55555555505 &= 123456789 \times 9 \times 5 := (aaaaaaaaaaa - aa + a) \times (a+a+a+a+a)/(a \times a)\\
66666666606 &= 123456789 \times 9 \times 6 := (aaaaaaaaaaa - aa + a) \times (a+a+a+a+a+a)/(a \times a)\\
77777777707 &= 123456789 \times 9 \times 7 := (aaaaaaaaaaa - aa + a) \times (aa-a-a-a-a)/(a \times a)\\
88888888808 &= 123456789 \times 9 \times 8 := (aaaaaaaaaaa - aa + a) \times (aa-a-a-a)/(a \times a)\\
99999999909 &= 123456789 \times 9 \times 9 := (aaaaaaaaaaa - aa + a) \times (aa-a-a)/(a \times a).
\end{aligned}$$

This example is an extension of example 12. Instead considering 12345679, we have considered all the digits, i.e., 123456789. We get number pattern on both sides, while in example 12, we have palindromic pattern on one side of the expression.

Example 47.

$$\begin{aligned}
9 &= 1 \times 8 + 1 && := (aa - a - a)/a\\
98 &= 12 \times 8 + 2 && := (aaa - aa - a - a)/a\\
987 &= 123 \times 8 + 3 && := (aaaa - aaa - aa - a - a)/a\\
9876 &= 1234 \times 8 + 4 && := (aaaaa - aaaa - aaa - aa - a - a)/a\\
98765 &= 12345 \times 8 + 5 && := (aaaaaa - aaaaa - aaaa - aaa - aa - a - a)/a\\
987654 &= 123456 \times 8 + 6 && := (aaaaaaa - aaaaaa - aaaaa - aaaa - aaa - aa - a - a)/a\\
9876543 &= 1234567 \times 8 + 7 && := (aaaaaaaa - aaaaaaa - aaaaaa - aaaaa - aaaa - aaa - aa - a - a)/a\\
98765432 &= 12345678 \times 8 + 8 && := (aaaaaaaaa - aaaaaaaa - aaaaaaa - aaaaaa - aaaaa - aaaa - aaa - aa - a - a)/a\\
987654321 &= 123456789 \times 8 + 9 && := (aaaaaaaaaa - aaaaaaaaa - aaaaaaaa - aaaaaaa - aaaaaa - aaaaa - aaaa - aaa - aa - a - a)/a.
\end{aligned}$$

Example 48.

$$\begin{aligned}
9 &= & 9 \times 1 &= 11 - 2 && := (aa - a - a)/a\\
108 &= & 9 \times 12 &= 111 - 3 && := (aaa - a - a - a)/a\\
1107 &= & 9 \times 123 &= 1111 - 4 && := (aaaa - a - a - a - a)/a\\
11106 &= & 9 \times 1234 &= 11111 - 5 && := (aaaaa - a - a - a - a - a)/a\\
111105 &= & 9 \times 12345 &= 111111 - 6 && := (aaaaaa - a - a - a - a - a - a)/a\\
1111104 &= & 9 \times 123456 &= 1111111 - 9 && := (aaaaaaa - aa + a + a + a + a)/a\\
11111103 &= & 9 \times 1234567 &= 11111111 - 8 && := (aaaaaaaa - aa + a + a + a)/a\\
111111102 &= & 9 \times 12345678 &= 111111111 - 9 && := (aaaaaaaaa - aa + a + a)/a\\
1111111101 &= & 9 \times 123456789 &= 1111111111 - 10 && := (aaaaaaaaaa - aa + a)/a.
\end{aligned}$$

Example 49.

$$\begin{aligned}
9 &= & 9 \times 1 &= 1 \times 10 - 1 && := (a \times (aa - a)/a - a)/a\\
189 &= & 9 \times 21 &= 2 \times 100 - 11 && := ((a+a) \times (aaa - aa)/a - aa)/a\\
2889 &= & 9 \times 321 &= 3 \times 1000 - 111 && := ((a+a+a) \times (aaaa - aaa)/a - aaa)/a\\
38889 &= & 9 \times 4321 &= 4 \times 10000 - 1111 && := ((a+a+a+a) \times (aaaaa - aaaa)/a - aaaa)/a\\
488889 &= & 9 \times 54321 &= 5 \times 100000 - 11111 && := ((a+a+a+a+a) \times (aaaaaa - aaaaa)/a - aaaaa)/a\\
5888889 &= & 9 \times 654321 &= 6 \times 1000000 - 111111 && := ((a+a+a+a+a+a) \times (aaaaaaa - aaaaaa)/a - aaaaaa)/a\\
68888889 &= & 9 \times 7654321 &= 7 \times 10000000 - 1111111 && := ((aa-a-a-a-a) \times (aaaaaaaa - aaaaaaa)/a - aaaaaaa)/a\\
788888889 &= & 9 \times 87654321 &= 8 \times 100000000 - 11111111 && := ((aa-a-a-a) \times (aaaaaaaaa - aaaaaaaa)/a - aaaaaaaa)/a\\
8888888889 &= & 9 \times 987654321 &= 9 \times 1000000000 - 111111111 && := ((aa-a-a) \times (aaaaaaaaaa - aaaaaaaaa)/a - aaaaaaaaa)/a.
\end{aligned}$$

Example 50.

$$\begin{aligned}
81 = {} & \quad 9 \times 9 = 88 - 7 && :=((aa - a - a - a) \times aa/a - aa + a + a + a + a)/a \\
882 = {} & \quad 9 \times 98 = 888 - 6 && :=((aa - a - a - a) \times aaa/a - a - a - a - a - a - a)/a \\
8883 = {} & \quad 9 \times 987 = 8888 - 5 && :=((aa - a - a - a) \times aaaa/a - a - a - a - a - a)/a \\
88884 = {} & \quad 9 \times 9876 = 88888 - 4 && :=((aa - a - a - a) \times aaaaa/a - a - a - a - a)/a \\
888885 = {} & \quad 9 \times 98765 = 888888 - 3 && :=((aa - a - a - a) \times aaaaaa/a - a - a - a)/ \\
8888886 = {} & \quad 9 \times 987654 = 8888888 - 2 && :=((aa - a - a - a) \times aaaaaaa/a - a - a)/a \\
88888887 = {} & \quad 9 \times 9876543 = 88888888 - 1 && :=((aa - a - a - a) \times aaaaaaaa/a - a)/a \\
888888888 = {} & \quad 9 \times 98765432 = 888888888 - 0 && :=((aa - a - a - a) \times aaaaaaaaa/a)/a \\
8888888889 = {} & \quad 9 \times 987654321 = 8888888888 + 1 && :=((aa - a - a - a) \times aaaaaaaaaa/a + a)/a.
\end{aligned}$$

Example 51.

$$\begin{aligned}
81 = {} & \quad 9 \times 9 = 91 - 10 && :=((aa - a - a) \times (aa - a)/a + a - aa + a)/a \\
801 = {} & \quad 9 \times 89 = 811 - 10 && :=((aa - a - a - a) \times (aaa - aa)/a + aa - aa + a)/a \\
7101 = {} & \quad 9 \times 789 = 7111 - 10 && :=((aa - a - a - a - a) \times (aaaa - aaa)/a + aaa - aa + a)/a \\
61101 = {} & \quad 9 \times 6789 = 61111 - 10 && :=((a + a + a + a + a + a) \times (aaaaa - aaaa)/a + aaaa - aa + a)/a \\
511101 = {} & \quad 9 \times 56789 = 511111 - 10 && :=((a + a + a + a + a) \times (aaaaaa - aaaaa)/a + aaaaa - aa + a)/a \\
4111101 = {} & \quad 9 \times 456789 = 4111111 - 10 && :=((a + a + a + a) \times (aaaaaaa - aaaaaa)/a + aaaaaa - aa + a)/a \\
31111101 = {} & \quad 9 \times 3456789 = 31111111 - 10 && :=((a + a + a) \times (aaaaaaaa - aaaaaaa)/a + aaaaaaa - aa + a)/a \\
211111101 = {} & \quad 9 \times 23456789 = 211111111 - 10 && :=((a + a) \times (aaaaaaaaa - aaaaaaaa)/a + aaaaaaaa - aa + a)/a \\
1111111101 = {} & \quad 9 \times 123456789 = 1111111111 - 10 && :=(a \times (aaaaaaaaaa - aaaaaaaaa)/a + aaaaaaaaa - aa + a)/a.
\end{aligned}$$

Example 52.

$$\begin{aligned}
91 &= 10^2 - 10^1 + 1 := (aaa - aa - aa + a + a)/a \\
9901 &= 10^4 - 10^2 + 1 := (aaaaa - aaaa - aaa + aa + a)/a \\
999001 &= 10^6 - 10^3 + 1 := (aaaaaaa - aaaaaa - aaaa + aaa + a)/a \\
99990001 &= 10^8 - 10^4 + 1 := (aaaaaaaaa - aaaaaaaa - aaaaa + aaaa + a)/a \\
9999900001 &= 10^{10} - 10^5 + 1 := (aaaaaaaaaaa - aaaaaaaaaa - aaaaaa + aaaaa + a)/a.
\end{aligned}$$

3.9 Prime Number Patterns

Below are some examples of *prime number patterns*. It is not necessary that the further number each example be a prime number.

Example 53.

$$\begin{aligned}
31 &:= (aa + aa + aa - a - a)/a \\
331 &:= (aaa + aaa + aaa - a - a)/a \\
3331 &:= (aaaa + aaaa + aaaa - a - a)/a \\
33331 &:= (aaaaa + aaaaa + aaaaa - a - a)/a \\
333331 &:= (aaaaaa + aaaaaa + aaaaaa - a - a)/a \\
3333331 &:= (aaaaaaa + aaaaaaa + aaaaaaa - a - a)/a \\
33333331 &:= (aaaaaaaa + aaaaaaaa + aaaaaaaa - a - a)/a.
\end{aligned}$$

The next number in this case is not a prime number, i.e., we can write $333333331 = 17 \times 19607843$.

Example 54.

$$
\begin{aligned}
59 &:= ((aa \times aa - a \times a)/(a+a) - a)/a \\
599 &:= ((aaa \times aa - a \times a)/(a+a) - aa)/a \\
59999 &:= ((aaaaa \times aa - a \times a)/(a+a) - aaaa)/a \\
599999 &:= ((aaaaaa \times aa - a \times a)/(a+a) - aaaaa)/a \\
59999999 &:= ((aaaaaaaa \times aa - a \times a)/(a+a) - aaaaaaa)/a \\
59999999999 &:= ((aaaaaaaaaaa \times aa - a \times a)/(a+a) - aaaaaaaaaa)/a.
\end{aligned}
$$

In this example, in between numbers are not primes, for example, $5999 = 7 \times 857$, $5999999 = 1013 \times 5923$, etc.

Example 55.

$$
\begin{aligned}
23 &:= (aa + aa + a)/a \\
233 &:= (aaa + aaa + aa)/a \\
2333 &:= (aaaa + aaaa + aaa)/a \\
23333 &:= (aaaaa + aaaaa + aaaa)/a.
\end{aligned}
$$

Here also the next number is not prime, i.e., $233333 = 353 \times 661$. After this, the next prime number is 23333333333.

Example 56.

$$
\begin{aligned}
19 &:= (aa + aa - a - a - a)/a \\
199 &:= (aaa + aaa - aa - aa - a)/a \\
1999 &:= (aaaa + aaaa - aaa - aaa - a)/a \\
199999 &:= (aaaaaa + aaaaaa - aaaaa - aaaaa - a)/a \\
19999999 &:= (aaaaaaaa + aaaaaaaa - aaaaaaa - aaaaaaa - a)/a.
\end{aligned}
$$

In this example the numbers $19999 = 7 \times 2857$ and $1999999 = 17 \times 71 \times 1657$ are not prime numbers. The next number is also not prime, i.e., $199999999 = 89 \times 1447 \times 1553$.

Example 57. ***Palindromic Prime Pattern***. *Here below are palindromic patterns of prime numbers in different forms. No representations are given, since numbers are too high.*

131	1124243424211
11311	1124363634211
1123211	1124472744211
112434211	1124536354211
11248384211	1124543454211
1124843484211	1124833384211
112486131684211	1124843484211
11248615351684211	1124873784211.

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