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Crazy Representations of Natural Numbers

Inder J. Taneja[1]

SUMMARY

This summary brings author's work on numbers. The study is made in different ways. Specially, towards, *"Crazy Representations of Natural Numbers"*, *Running expressions*, etc. Natural numbers are represented in different situations, such as, writing in terms of 1 to 9 or reverse, flexible power of same digits as bases, single digit, single letter, etc. Expreesions appearing with equalities having 1 to 9 or 9 to 1, calling running expressions are also presented. The work is separated by sections and subsections are as follows:

1 Crazy Representations of Natural Numbers [1];
2 Flexible Power Representations [15];
2.1 Unequal String Lengths [14];
2.2 Equal String Lengths [11];
3 Pyramidical Representations [9, 10, 12, 13];
3.1 Crazy Representations [14];
3.2 Flexible Power [11];
4 Double Sequential Representations [9, 10, 12, 13];
5 Triple Sequential Representations; [16];
6 Single Digit Representations; [2];
7 Single Letter Representations [4, 5];
7.1 Single Letter Power Representations [5];
7.2 Palindromic and Number Patterns [6, 7];
8 Running Expressions. [3].

1 Crazy Representations of Natural Numbers

In 2014, author [1] wrote natural numbers in increasing and decreasing orders of 1 to 9 and 9 to 1. See examples below:

$$
\begin{aligned}
100 &= 1+2+3+4+5+6+7+8\times 9 &&= 9\times 8+7+6+5+4+3+2+1.\\
101 &= 1+2+34+5+6\times 7+8+9 &&= 9\times 8+7+6+5+4+3\times 2+1.\\
102 &= 12+3\times 4\times 5+6+7+8+9 &&= 9+8+7+6+5+4^3+2+1.\\
103 &= 1\times 2\times 34+5+6+7+8+9 &&= 9+8+7\times 6+5\times 4+3+21.\\
104 &= 1+23+4+5+6+7\times 8+9 &&= 9+8+7+65+4\times 3+2+1.\\
105 &= 1+2\times 3\times 4+56+7+8+9 &&= 9+8\times 7+6\times 5+4+3+2+1.\\
106 &= 12+3+4\times 5+6+7\times 8+9 &&= 9+8\times 7+6\times 5+4+3\times 2+1.\\
107 &= 1\times 23+4+56+7+8+9 &&= 9+8+76+5+4+3+2\times 1.\\
108 &= 1+2+3+4+5+6+78+9 &&= 9+8+76+5+4+3+2+1.
\end{aligned}
$$

[1] *Formerly, Professor of Mathematics, Universidade Federal de Santa Catarina, 88.040-900 Florianópolis, SC, Brazil.*
E-mail: *ijtaneja@gmail.com;* ***Web-site:*** *www.numbersmagic.wordpress.com.*

Below are more examples,

$$\begin{aligned}
999 &= 12 \times 3 \times (4+5) + (67+8) \times 9 &&= 9+8+7+654+321.\\
2535 &= 1+2345+(6+7+8) \times 9 &&= 9+87 \times (6+5 \times 4+3)+2+1.\\
2607 &= 123 \times 4 \times 5+6+(7+8) \times 9 &&= 987+6 \times 54 \times (3+2) \times 1.\\
10958 &= 12 \times 3+\sqrt{4}+5! \times (67+8 \times \sqrt{9}) &&= (9+8 \times 7 \times 65+4) \times 3-2+1.\\
11807 &= 1 \times 234 \times (5+6 \times 7)+89 &&= -9+8+7 \times (6+5) \times (4 \times 3)^2 \times 1.
\end{aligned}$$

We observe that the number 10958 is the only number among 0 to 11111, where use *square-root* and *factorial*. All other numbers are just with basic operations. For full work, refer to link below [1] (Jan., 2014):

http://arxiv.org/abs/1302.1479.

For comments on this work see [17, 18, 23, 24].

2 Flexible Power Representations

Let us consider two numbers, 1 and 2. Using the idea of power and the operations of *addition* and *subtraction*, we can write following 3 numbers in terms of 1 and 2, as $1 = -1^2 + 2^1$, $3 = 1^2 + 2^1$ and $5 = 1^1 + 2^2$. In this situation, we observe that *bases* and *exponents* are of same digits. Permutations of exponent values helps in bringing different numbers. In case of repeated values, for example, $3 = 1^2 + 2^1 = -1^1 + 2^2$, only possibilities is considered. There is only one number having single digit, i.e., $1 = 1^1$. For simplicity, let us represent the above procedure as $(1, 2)^{(1,2)}$, resulting in three possible values. The above procedure is with two digits. Instead having two digits, we can work with two letters, such as, $(a, b)^{(a,b)}$, where $a \neq b$ and $a, b \in \{1, 2, 3, 4, 5, 6, 8, 9\}$. This process can be extended for more number of letters. See below:

$$\begin{gathered}
(a, b)^{(a,b)};\\
(a, b, c)^{(a,b,c)};\\
(a, b, c, d)^{(a,b,c,d)};\\
(a, b, c, d, e)^{(a,b,c,d,e)}\\
(a, b, c, d, e, f)^{(a,b,c,d,e,f)};\\
(a, b, c, d, e, f, g)^{(a,b,c,d,e,f,g)};\\
(a, b, c, d, e, f, g, h)^{(a,b,c,d,e,f,g,h)};\\
(a, b, c, d, e, f, g, h, i)^{(a,b,c,d,e,f,g,h,i)}.
\end{gathered}$$

where $a, b, c, d, e, f, g, h, i \in \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$, all distinct.

Based on above procedure, natural numbers are written in two different procedures.

2.1 Unequal String Lengths

$100 := 2^6 + 6^2.$
$101 := 1^1 + 2^6 + 6^2.$
$102 := -2^5 + 3^2 + 5^3.$
$103 := 1^1 - 2^5 + 3^2 + 5^3.$
$104 := -1^1 + 2^3 + 3^4 + 4^2.$
$105 := 2^3 + 3^4 + 4^2.$
$106 := 2^7 + 3^3 - 7^2.$
$107 := -1^2 + 2^7 - 3^3 + 7^1.$
$108 := 1^7 + 2^6 + 6^2 + 7^1.$

$109 := 1^2 + 2^7 - 3^3 + 7^1.$
$110 := 1^9 + 2^6 + 6^2 + 9^1.$
$111 := -1^3 + 2^7 - 3^2 - 7^1.$
$112 := 3^5 - 4^4 + 5^3.$
$113 := -1^5 - 2^1 - 3^2 + 5^3.$
$114 := -2^2 + 3^5 - 5^3.$
$115 := 1^5 - 2^1 - 3^2 + 5^3.$
$116 := 2^2 + 3^5 - 4^4 + 5^3.$
$117 := -1^1 + 3^5 - 5^3.$
$118 := 3^5 - 5^3.$
$119 := 1^1 + 3^5 - 5^3.$
$192 := 2^8 - 8^2.$
$193 := 1^1 + 2^8 - 8^2.$
$194 := -1^3 + 2^8 + 3^1 - 8^2.$
$195 := 1^8 + 2^1 + 4^4 - 8^2.$
$196 := 1^3 + 2^8 + 3^1 - 8^2.$
$197 := -1^2 - 2^6 + 4^4 + 6^1.$
$198 := -1^1 - 2^5 + 4^4 - 5^2.$
$199 := -2^5 + 4^4 - 5^2.$
$200 := 1^1 - 2^5 + 4^4 - 5^2.$
$201 := -1^9 + 2^7 - 7^1 + 9^2.$

By unequal string length, we mean that each number is not represented by same digits. It uses minimum necessary number of expressions. The digits used in the powers are the same as of bases but with different permutations. For complete representations of numbers from 0 to 11111 refer to link [8]:

http://rgmia.org/papers/v19/v19a31.pdf.

2.2 Equal String Lengths

Instead working with unequal strings as of previous section, here we worked with equal string using the digits 0 to 9, i.e., using all the 10 digits, {0,1,2,3,4,5,6,7,8,9}. The results obtained are symmetric, i.e., writing in 0 to 9 or 9 to 0, the resulting number is same. See some examples below,

$201 := 0^3 + 1^9 + 2^4 + 3^7 - 4^8 + 5^1 + 6^6 + 7^5 + 8^2 + 9^0.$
$202 := 0^0 + 1^9 + 2^6 + 3^8 - 4^7 + 5^5 + 6^3 + 7^2 + 8^1 + 9^4.$
$203 := 0^3 - 1^9 + 2^4 + 3^7 - 4^8 + 5^0 + 6^6 + 7^5 + 8^2 + 9^1.$
$204 := 0^8 + 1^9 + 2^5 + 3^7 - 4^6 + 5^1 + 6^4 + 7^2 + 8^0 + 9^3.$
$205 := 0^3 + 1^9 + 2^4 + 3^7 - 4^8 + 5^0 + 6^6 + 7^5 + 8^2 + 9^1.$
$206 := 0^7 - 1^9 - 2^5 - 3^8 + 4^6 + 5^1 + 6^3 + 7^4 + 8^0 + 9^2.$
$207 := 0^8 + 1^9 + 2^5 + 3^7 - 4^6 + 5^0 + 6^4 + 7^2 + 8^1 + 9^3.$
$208 := 0^7 + 1^9 - 2^5 - 3^8 + 4^6 + 5^1 + 6^3 + 7^4 + 8^0 + 9^2.$
$209 := 0^7 - 1^9 - 2^5 - 3^8 + 4^6 + 5^0 + 6^3 + 7^4 + 8^1 + 9^2.$
$210 := 0^5 - 1^7 - 2^8 - 3^9 + 4^1 + 5^6 + 6^0 + 7^3 + 8^4 + 9^2.$
$211 := 0^7 + 1^9 - 2^5 - 3^8 + 4^6 + 5^0 + 6^3 + 7^4 + 8^1 + 9^2.$
$212 := 0^5 + 1^7 - 2^8 - 3^9 + 4^1 + 5^6 + 6^0 + 7^3 + 8^4 + 9^2.$
$213 := 0^5 + 1^8 - 2^7 - 3^9 + 4^1 + 5^6 + 6^3 + 7^0 + 8^4 + 9^2.$
$214 := 0^5 + 1^7 - 2^8 - 3^9 + 4^0 + 5^6 + 6^1 + 7^3 + 8^4 + 9^2.$
$215 := 0^5 + 1^9 + 2^8 + 3^7 - 4^6 + 5^0 + 6^4 + 7^2 + 8^3 + 9^1.$
$216 := 0^1 - 1^7 + 2^8 - 3^9 + 4^5 + 5^6 + 6^0 + 7^4 + 8^3 + 9^2.$
$217 := 0^7 - 1^9 + 2^5 - 3^8 + 4^6 + 5^2 + 6^3 + 7^4 + 8^1 + 9^0.$
$218 := 0^1 + 1^7 + 2^8 - 3^9 + 4^5 + 5^6 + 6^0 + 7^4 + 8^3 + 9^2.$
$219 := 0^7 + 1^9 + 2^5 - 3^8 + 4^6 + 5^2 + 6^3 + 7^4 + 8^1 + 9^0.$
$220 := 0^7 + 1^9 + 2^5 - 3^8 + 4^6 + 5^2 + 6^3 + 7^4 + 8^0 + 9^1.$

Below are more examples,

$$11080 := 0^8 + 1^9 + 2^7 + 3^6 + 4^2 + 5^5 + 6^0 + 7^1 + 8^3 + 9^4.$$
$$11081 := 0^8 - 1^9 + 2^6 + 3^7 + 4^4 + 5^1 + 6^5 + 7^0 + 8^2 + 9^3.$$
$$11082 := 0^8 + 1^9 + 2^6 + 3^7 + 4^1 + 5^4 + 6^5 + 7^3 + 8^0 + 9^2.$$
$$11083 := 0^8 + 1^9 + 2^6 + 3^7 + 4^4 + 5^1 + 6^5 + 7^0 + 8^2 + 9^3.$$
$$11084 := 0^7 + 1^9 + 2^8 + 3^6 + 4^1 + 5^5 + 6^0 + 7^3 + 8^2 + 9^4.$$
$$11085 := 0^8 + 1^9 + 2^6 + 3^7 + 4^4 + 5^0 + 6^5 + 7^1 + 8^2 + 9^3.$$
$$11086 := 0^7 + 1^9 + 2^8 + 3^6 + 4^0 + 5^5 + 6^1 + 7^3 + 8^2 + 9^4.$$
$$11087 := 0^6 + 1^9 - 2^8 + 3^7 + 4^2 + 5^4 + 6^5 + 7^0 + 8^1 + 9^3.$$

For complete representations of numbers from 0 to 11111 refer to link [15]:

http://rgmia.org/papers/v19/v19a131.pdf.

Analysing the procedures given in sections 1 and 2, we observe that in the section 1, all the 9 digits from 1 to 9 or 9 to 1 are used to bring natural numbers, where each digit appears only once. In this case, the operations used are, *addition, subtraction, multiplication, division, potentiation, factorial* and *square-root*. The section 2 works with representations of natural numbers written in a way that we use each digit twice, where *bases* and *exponents* are of same digits with different permutations. Subsection 2.1 choose the digits from 1 to 9, according to necessity, while subsection 2.2 works with all the 10 digits, i.e., 0 to 9, along with the operations of *addition* and *subtraction*.

3 Pyramidical-Type Representations

This section deals with pyramidical-type of representations of natural numbers in two different ways. One is based on the procedure used in section 1 and second is based on procedure given in section 2. This happens because, we can write same number in different ways.

3.1 Crazy Representations

Following the procedure of section 2, we can write the natural numbers in pyramidical forms, for examples,

- $$\begin{aligned} 33 &= 32 + 1 \times 0! \\ &= 4 \times 3 + 21 \times 0! \\ &= 5 + 4 + 3 \times (-2 + 10) \\ &= 6 + 5 + 43 - 21 \times 0! \\ &= 76 - 5 + 4 - 32 - 10 \\ &= 8 + 7 + 65 - 4 \times 3! \times 2 + 1 \times 0! \\ &= -9 - 8 + 7 - 6 + 54 - 3 - 2 \times 1 \times 0!. \end{aligned}$$

- $$\begin{aligned} 48 &= 3! \times (-2 + 10) \\ &= 4! + 3 + 21 \times 0! \\ &= -54 \times 3 + 210 \\ &= 6 + 5 + 4! + 3!/2 + 10 \\ &= 7 \times (6 + 5) + 4 - 32 - 1 \times 0! \\ &= 8 + 7 + 6 - 5 + 4 \times 3 + 2 \times 10 \\ &= -9 + 8 \times 76 - 543 + 2 - 10. \end{aligned}$$

- $$\begin{aligned} 434 &= 432 + 1 + 0! \\ &= 54 \times (3! + 2) + 1 + 0! \\ &= 6 \times 54 + (3 + 2)! - 10 \\ &= -7 + 654 - 3 - 210 \\ &= 8 \times (7 + 6) \times 5 - 4! - 3 \times 21 + 0! \\ &= (9 + 8 + 7 + 6 + 5 - 4) \times (-3! + 2 \times 10). \end{aligned}$$

- $$\begin{aligned} 729 &= 3^{(2+1)! \times 0!} \\ &= (4! + 3)^2 - 1 + 0! \\ &= (5 + 4)^3 \times (2 - 1) \times 0! \\ &= 6! + 5 + 4 + 321 \times 0 \\ &= 765 - 4 \times 3! - 2 - 10 \\ &= (8 + 76 + 5 - 4^3 + 2)^{(1+0!)} \\ &= (9 + 8 + 7 + 6 - 54 - 3)^2 \times 1 \times 0!. \end{aligned}$$

- $$\begin{aligned} 895 &= -5 + (4! + 3!)^2 \times 1 \times 0! \\ &= 6 \times (5 + 4 \times 3!^2 \times 1) + 0! \\ &= 7 \times (65 + 4^3) + 2 - 10 \\ &= -8 - 7 + 65 \times (4 \times 3 + 2) + 1 \times 0 \\ &= 9 + 876 + 5 - 4 + 3^2 \times 1 \times 0!. \end{aligned}$$

- $$\begin{aligned} 947 &= -5! + 43 + 2^{10} \\ &= 6! + 5 + 4 \times 3 + 210 \\ &= 7 + ((6 + 5) \times 4 + 3) \times 2 \times 10 \\ &= (8 + 7) \times 65 - 4! - 3! + 2 + 1 \times 0 \\ &= -9 + 8 - 76 + 5! + 43 \times 21 + 0!. \end{aligned}$$

For complete representations of natural numbers from 0 to 1000 refer to link [13]:

http://rgmia.org/papers/v19/v19a130.pdf.

3.2 Flexible Power

Following the procedure of section 2 , we can write the natural numbers in pyramidical forms, for examples,

- $1 = 0^0$
  $= 0^1 + 1^0$
  $= 0^2 - 1^0 + 2^1$
  $= 0^0 + 1^3 - 2^2 + 3^1$
  $= 0^3 + 1^4 - 2^2 + 3^1 + 4^0$
  $= 0^2 + 1^4 - 2^5 + 3^3 + 4^1 + 5^0$
  $= 0^5 - 1^0 - 2^6 - 3^4 + 4^2 + 5^3 + 6^1$
  $= 0^2 + 1^7 + 2^5 - 3^6 + 4^3 + 5^4 + 6^0 + 7^1$
  $= 0^3 + 1^7 + 2^5 - 3^8 + 4^6 + 5^2 + 6^1 + 7^4 + 8^0$
  $= 0^5 - 1^8 + 2^9 + 3^7 - 4^6 + 5^4 + 6^2 + 7^0 + 8^1 + 9^3.$

- $22 = 0^1 - 1^0 - 2^2 + 3^3$
  $= 0^2 + 1^3 + 2^4 + 3^0 + 4^1$
  $= 0^4 - 1^5 + 2^3 + 3^2 + 4^0 + 5^1$
  $= 0^2 + 1^6 + 2^5 - 3^4 + 4^3 + 5^1 + 60$
  $= 0^5 + 1^7 - 2^6 - 3^4 + 4^1 + 5^3 + 6^2 + 7^0$
  $= 0^1 + 1^4 + 2^8 + 3^5 - 4^7 + 5^6 + 6^3 + 7^0 + 8^2$
  $= 0^6 - 1^9 + 2^8 - 3^7 + 4^5 + 5^4 + 6^3 + 7^1 + 8^0 + 9^2.$

- $666 = 0^1 - 1^3 + 2^5 + 3^2 + 4^0 + 5^4$
  $= 0^0 + 1^5 - 2^6 + 3^1 + 4^3 + 5^4 + 6^2$
  $= 0^5 + 1^7 - 2^6 + 3^1 + 4^3 + 5^4 + 6^2 + 7^0$
  $= 0^2 - 1^7 - 2^6 - 3^8 + 4^3 + 5^5 + 6^1 + 7^0 + 8^4$
  $= 0^7 + 1^9 - 2^5 - 3^8 + 4^6 + 5^2 + 6^1 + 7^4 + 8^0 + 9^3.$

- $1089 = 0^1 + 1^0 + 2^3 + 3^4 + 4^5 - 5^2$
  $= 0^4 - 1^6 + 2^1 + 3^3 + 4^5 + 5^0 + 6^2$
  $= 0^2 + 1^6 - 2^7 + 3^5 + 4^1 + 5^4 + 6^0 + 7^3$
  $= 0^0 - 1^7 + 2^4 - 3^8 + 4^6 + 5^5 + 6^1 + 7^3 + 8^2$
  $= 0^6 - 1^9 + 2^7 - 3^8 + 4^1 + 5^5 + 6^3 + 7^0 + 8^4 + 9^2.$

- $1179 = 0^1 + 1^0 + 2^5 + 3^6 + 4^4 + 5^3 + 6^2$
  $= 0^2 + 1^6 + 2^4 - 3^7 + 4^0 + 5^5 + 6^3 + 7^1$
  $= 0^6 + 1^7 - 2^8 + 3^5 + 4^1 + 5^4 + 6^0 + 7^2 + 8^3$
  $= 0^6 + 1^9 - 2^8 - 3^7 + 4^5 + 5^3 + 6^1 + 7^4 + 8^2 + 9^0.$

We observe that the digits appearing in bases and exponents are same in each number. For complete representations of natural numbers from 0 to 1500 refer to link [11]:

***http://rgmia.org/papers/v19/v19a31.pdf*.**

4 Double Sequential Representations

This section deals with representations of natural numbers written in a sequential way of 3 to 8 digits ending in 0, such as {2,1,0}, {3,2,1,0},..., {9,8,7,6,5,4,32,1,0}. These representations are done combining both the processes given in sections 1 and 2 . It is interesting to observe that the processes given in subsection 1 uses operations such as, *addition, subtraction, multiplication, division, potentiation, square-root* and *factorial* with each digit appearing once. In case of process given in section 2 only addition and subtractions are used but each digit appears twice, once in base and another as potentiation. Below are some examples,

- $1 = 2^1 - 1^0 + 0^2$
  $= 2 - 1 \times 0!.$

- $2 = 2^0 + 1^2 + 0^1$
  $= 2 \times 1 \times 0!.$

- $11 = 3^2 + 2^0 + 1^3 + 0^1$
  $= 3 - 2 + 10.$

- $25 = 3^3 - 2^2 + 1^1 + 0^0$
  $= 3 + 21 + 0!.$

- $20 = 4^2 + 3^0 + 2^1 + 1^4 + 0^3$
  $= 4 + 3 \times 2 + 10.$

- $21 = 4^2 + 3^1 + 2^0 + 1^4 + 0^3$
  $= (4 - 3) \times 21 \times 0!.$

- $116 = 5^2 + 4^0 + 3^4 + 2^3 + 1^5 + 0^1$
  $= 54 + 3 \times 21 - 0!.$

- $120 = 5^2 + 4^1 + 3^4 + 2^3 + 1^5 + 0^0$
  $= (5 + 4321 \times 0)!.$

- $1406 = 6^4 + 5^1 + 4^3 + 3^2 + 2^5 - 1^6 + 0^0$
  $= 6 + 5! + 4 \times 32 \times 10.$

- $1411 = 6^3 + 5^2 + 4^5 + 3^4 + 2^6 + 1^0 + 0^1$
  $= 6! - 5 - 4! + (3 \times 2 \times 1)! \times 0!.$

- $78 = -7^4 + 6^3 - 5^1 + 4^2 + 3^7 + 2^6 + 1^0 + 0^5$
  $= -76 - 54 - 3 + 210.$

- $227 = -7^5 + 6^1 + 5^4 + 4^7 + 3^2 + 2^3 + 1^6 + 0^0$
  $= (765 - 4!)/3 - 2 \times 10.$

- $1048 = -7^3 + 6^4 + 5^2 + 4^1 + 3^0 + 2^6 + 1^7 + 0^5$
  $= 7 - 6 + 5 \times 4 + 3 + 2^{10}.$

- $2016 = 7^3 + 6^4 + 5^0 + 4^1 + 3^5 + 2^7 + 1^6 + 0^2$
  $= (7 + 65) \times (\sqrt{4} \times 3^2 + 10).$

- $661 = 8^2 - 7^3 + 6^1 + 5^4 + 4^0 + 3^5 + 2^6 + 1^8 + 0^7$
  $= 8 + 7 + 654 - 3^2 + 1 \times 0!.$

- $1087 = -8^2 + 7^1 + 6^0 + 5^3 + 4^4 + 3^6 + 2^5 + 1^8 + 0^7$
  $= -87 - 6 + (5 \times 4 \times 3! - 2) \times 10.$

- $192 = 9^2 - 8^4 + 7^1 + 6^3 + 5^5 + 4^0 + 3^6 + 2^7 + 1^9 + 0^8$
  $= 98 + 76 - 5 + 4 - 3 + 21 + 0!.$

- $1417 = -9^0 + 8^3 - 7^4 + 6^1 + 5^2 + 4^5 + 3^7 + 2^6 + 1^9 + 0^8$
  $= 9 \times (8 - 7) + (6 + 5) \times 4 \times 32 \times 1 \times 0!.$

For complete representations of numbers refer to links [9, 10, 12, 13]:

http://rgmia.org/papers/v19/v19a48.pdf.
http://rgmia.org/papers/v19/v19a57.pdf.
http://rgmia.org/papers/v19/v19a128.pdf.
http://rgmia.org/papers/v19/v19a129.pdf.

In the first work, the results are up 7 digits, i.e., from {2,1,0} to {6,5,4,3,2,1,0}. The second paper give the results for 8 digits, i.e., for {7,6,5,4,3,2,1,0}. The third paper give the results for 8 digits, i.e., for {8,7,6,5,4,3,2,1,0}, and finally the forth paper given the results for digits {9,8,7,6,5,4,3,2,1,0}. Since all numbers ending in 0, obviously, all these results are in decreasing order.

5 Triple Representations of Numbers

This section deals with the representations of natural numbers in three different ways. In each case the same digits are used. The first way is based on the representations given in section 2 . The second and third are based on section 1 in increasing and decreasing order of digits. It is not possible to write all the numbers in three ways. Only those numbers are written in the work, when representations are possible in all the three ways. For an idea see some examples below:

- $3 = 1^2 + 2^1$
  $= 1 + 2$
  $= 2 + 1.$

- $1 = -2^3 + 3^2$
  $= -2 + 3$
  $= 3 - 2.$

- $32 = 1^1 + 2^2 + 3^3$
  $= 32 \times 1.$

- $13 = -2^2 + 3^4 - 4^3$
  $= 4 + 3^2.$

- $138 = -3^5 + 4^4 + 5^3$
  $= -3! + 4! + 5!$
  $= 5! + 4! - 3!.$

- $10 = -1^4 - 2^3 + 3^1 + 4^2$
  $= \sqrt{12 \times 3} + 4$
  $= 4 + 3 + 2 + 1.$

- $60 = 2^5 - 3^4 - 4^2 + 5^3$
  $= (2^3 + 4) \times 5$
  $= 54 + 3 \times 2.$

- $4278 = -3^3 - 4^6 + 5^4 + 6^5$
  $= 3! \times (-\sqrt{4} - 5 + 6!)$
  $= (6! - 5 - \sqrt{4}) \times 3!.$

- $31 = 1^1 - 2^3 - 3^5 + 4^4 + 5^2$
  $= 1 \times 2 + 34 - 5$
  $= (5 - 4) \times 32 - 1.$

- $2009 = 3^7 - 4^3 - 5^6 - 6^4 + 7^5$
  $= -3!! + 4! \times (5! - 6) - 7$
  $= -7 + (-6 + 5!) \times 4! - 3!!.$

- $1840 = -4^5 - 5^8 + 6^7 + 7^6 - 8^4$
  $= 4! \times (5 + 6) \times 7 - 8$
  $= -8 + 7 \times (6 + 5) \times 4!.$

- $121 = -1^5 - 2^4 + 3^2 + 4^1 + 5^3$
  $= 1^{234} + 5!$
  $= 5 \times 4 \times 3 \times 2 + 1.$

- $194 = -2^6 + 3^5 + 4^4 - 5^2 - 6^3$
  $= 2 \times 34 + 5! + 6$
  $= -6 + 5 \times (4 + 3!^2).$

- $425 = 1^6 + 2^4 + 3^5 + 4^1 + 5^3 + 6^2$
  $= -1 + 23 \times 4! - 5! - 6$
  $= 6! + 5 \times (4 - 3 \times 21).$

- $905 = 2^6 + 3^2 - 4^7 + 5^4 - 6^3 + 7^5$
  $= 2 + (3 \times 45 - 6) \times 7$
  $= 7!/6 + 5 + 4! + 3!^2.$

For complete detail refer refer to links [16]:

http://rgmia.org/papers/v19/v19a134.pdf.

The above work give the results only up to width 6. We can extend the results for higher width too. See examples below:

- $1008 = 1^6 - 2^7 + 3^5 + 4^1 + 5^4 + 6^3 + 7^2$
  $= (12 - 3) \times (45 + 67)$
  $= (-7 + 6 + 5) \times 4 \times 3 \times 21.$

- $109 = 2^8 - 3^3 - 4^7 - 5^2 + 6^6 + 7^4 - 8^5$
  $= 234 + 5 \times 67 + 8$
  $= 87 + (6 - 5 + 43)/2.$

- $944 = 3^9 + 4^8 - 5^7 + 6^6 - 7^5 + 8^4 - 9^3$
  $= (-3 + 4!) \times 5 + 6! + 7 \times (8 + 9)$
  $= 987 \times (6 - 5) + 43.$

- $288 = -2^3 - 3^8 - 4^9 + 5^4 - 6^7 + 7^5 + 8^2 + 9^6$
  $= 2 \times (34 - 5 \times (67 - 89))$
  $= (-9 + 87 - 6 \times 5) \times 4 \times 3/2.$

- $512 = 1^7 + 2^5 - 3^8 + 4^6 + 5^2 + 6^1 + 7^4 + 8^3$
  $= 1 - 23 + 456 + 78$
  $= 8 \times ((7 - 6)^{54} + 3 \times 21).$

- $1172 = -2^9 - 3^2 + 4^8 - 5^7 - 6^6 + 7^4 - 8^3 + 9^5$
  $= (2 \times 3)^4 + 5 + 6 - (7 + 8) \times 9$
  $= (9 + 8) \times 76 - 5 \times 4 \times 3 \times 2.$

The work on above type of examples is under preparation and shall be dealt later on.

6 Single Digit Representations

In section 1 , all the nine digits are used to write natural numbers. Here the work is done writing numbers for each digit separately. See examples below:

$$\begin{aligned}
717 &= (1+1)^{11} - 11^{(1+1+1)} \\
&= 22^2 + 222 + 22/2 \\
&= 3^{(3+3)} - 3 - 3 \times 3 \\
&= 4 \times (4 \times 44 + 4) - 4 + 4/4 \\
&= (55 \times (55 + 5 + 5) + 5 + 5)/5 \\
&= (6 \times 6/(6+6))^6 - 6 - 6 \\
&= 777 - 7 \times 7 - 77/7 \\
&= 8 \times 88 + (88 + 8 + 8)/8 \\
&= 9 \times 9 \times 9 - (99 + 9)/9.
\end{aligned}$$

$$\begin{aligned}
995 &= (11-1)^{(1+1+1)} - (11-1)/(1+1) \\
&= 22 + 2 \times (22^2 + 2) + 2/2 \\
&= 3 \times 333 - 3 - 3/3 \\
&= 4 \times (4^4 - 4 - 4) + 4 - 4/4 \\
&= 5 \times (5+5) \times (5 \times 5 - 5) - 5 \\
&= 666 + 6 \times 66 - 66 - 6/6 \\
&= (7+7) \times (77 - 7) + 7 + 7 + 7/7 \\
&= 888 + 88 + 8 + 88/8 \\
&= 999 - (9 + 9 + 9 + 9)/9.
\end{aligned}$$

$$\begin{aligned}
666 &= (1+1) \times (1+1+1) \times 111 \\
&= (2 + 2/2) \times 222 \\
&= 3 \times ((3+3)^3 + 3 + 3) \\
&= 444 \times (4 + (4+4)/4)/4 \\
&= 555 + 555/5 \\
&= 666 \\
&= 777 - 777/7 \\
&= 888 \times (8 - (8+8)/8)/8 \\
&= 9 \times (9 \times 9 - 9) + 9 + 9.
\end{aligned}$$

$$\begin{aligned}
1000 &= (11-1)^{(1+1+1)} \\
&= 2 \times (22^2 + 2^{(2+2)}) \\
&= (3 \times 3 + 3/3)^3 \\
&= 4 \times (4^4 - 4) - 4 - 4 \\
&= 5 \times (5+5) \times (5 \times 5 - 5) \\
&= ((66 - 6)/6)^{(6 \times 6/(6+6))} \\
&= (7 + 7 + 7 - 7/7) \times (7 \times 7 + 7/7) \\
&= 888 + 88 + 8 + 8 + 8 \\
&= 999 + 9/9.
\end{aligned}$$

Values are calculated up to 1.000.000, but the work is written only from 0 to 1000. For details, refer to link [2]:

http://arxiv.org/abs/1502.03501.

7 Single Letter Representations

We observe that the numbers written in previous section 6 are not in a symmetric way. But there are numbers, that can be written in a symmetric way, see examples below:

- $5 = \frac{11-1}{1+1} = \frac{22-2}{2+2} = \frac{33-3}{3+3} = \frac{44-4}{4+4} = \frac{55-5}{5+5} = \frac{66-6}{6+6} = \frac{77-7}{7+7} = \frac{88-8}{8+8} = \frac{99-9}{9+9}$.

- $6 = \frac{11+1}{1+1} = \frac{22+2}{2+2} = \frac{33+3}{3+3} = \frac{44+4}{4+4} = \frac{55+5}{5+5} = \frac{66+6}{6+6} = \frac{77+7}{7+7} = \frac{88+8}{8+8} = \frac{99+9}{9+9}$.

- $55 = \frac{111-1}{1+1} = \frac{222-2}{2+2} = \frac{333-3}{3+3} = \frac{444-4}{4+4} = \frac{555-5}{5+5} = \frac{666-6}{6+6} = \frac{777-7}{7+7} = \frac{888-8}{8+8} = \frac{999-9}{9+9}$.

- $56 = \frac{111+1}{1+1} = \frac{222+2}{2+2} = \frac{333+3}{3+3} = \frac{444+4}{4+4} = \frac{555+5}{5+5} = \frac{666+6}{6+6} = \frac{777+7}{7+7} = \frac{888+8}{8+8} = \frac{999+9}{9+9}$.

Motivated by this idea, instead working for each digit separately, we can work with a *single letter "a"* . See examples below:

$$
\begin{aligned}
5 &:= (aa-a)/(a+a).\\
6 &:= (aa+a)/(a+a).\\
55 &:= (aaa-a)/(a+a).\\
56 &:= (aaa+a)/(a+a).\\
561 &:= (aaaa+aa)/(a+a).\\
666 &:= aaa \times (aa+a)/((a+a)\times a).\\
925 &:= (aaaaa-aa)/(aa+a).
\end{aligned}
$$

$$
\begin{aligned}
1089 &:= (aaaa-aa-aa)/a.\\
1991 &:= (aaaaaa/aaa \times (a+a)-aa)/a.\\
2020 &:= (aaaaa-a)/aa \times (a+a)/a.\\
2035 &:= (aaaa-a)/(a+a+a)\times aa/(a+a).\\
4477 &:= (aaa/(a+a+a)\times aa \times aa)/(a\times a).\\
4999 &:= (aaaaa-aaaa-a-a)/(a+a).\\
5000 &:= (aaaaa-aaaa)/(a+a).
\end{aligned}
$$

where $a \in \{1,2,3,4,5,6,7,8,9\}$, and $aa = 10^2 \times a + a$, $aaa = 10^3 \times a + 10^2 \times a + a$, etc.

For full work, refer to links below [4, 5]:

http://rgmia.org/papers/v18/v18a40.pdf

http://rgmia.org/papers/v18/v18a73.pdf

The first link is up to 3000 numbers, while second link extend it to 5000 numbers.

7.1 Single Letter Power Representations

Above there are numbers written in terms of single letter "*a*". Using same idea, below are examples numbers with of exponential values written in terms of letter "*a*":

Power 2.

$$4 := 2^2 = ((a+a)/a)^{(a+a)/a}.$$
$$9 := 3^2 = ((a+a+a)/a)^{(a+a)/a}.$$
$$16 := 4^2 = ((a+a+a+a)/a)^{(a+a)/a}.$$
$$25 := 5^2 = ((aa-a)/(a+a))^{(a+a)/a}.$$
$$36 := 6^2 = ((aa+a)/(a+a))^{(a+a)/a}.$$
$$49 := 7^2 = ((aa-a-a-a-a)/a)^{(a+a)/a}.$$

Power 3.

$$8 := 2^3 = ((a+a)/a)^{(a+a+a)/a}.$$
$$27 := 3^3 = ((a+a+a)/a)^{(a+a+a)/a}.$$
$$64 := 4^3 = ((a+a+a+a)/a)^{(a+a+a)/a}.$$
$$125 := 5^3 = ((aa-a)/(a+a))^{(a+a+a)/a}.$$
$$216 := 6^3 = ((aa+a)/(a+a))^{(a+a+a)/a}.$$
$$343 := 7^3 = ((aa-a-a-a-a)/a)^{(a+a+a)/a}.$$
$$512 := 8^3 = ((aa-a-a-a)/a)^{(a+a+a)/a}.$$

Power of 2.

$$4 := 2^2 = ((a+a)/a)^{(a+a)/a}.$$
$$8 := 2^3 = ((a+a)/a)^{(a+a+a)/a}.$$
$$16 := 2^4 = ((a+a)/a)^{(a+a+a+a)/a}.$$
$$32 := 2^5 = ((a+a)/a)^{(aa-a)/(a+a)}.$$
$$64 := 2^6 = ((a+a)/a)^{(aa+a)/(a+a)}.$$
$$128 := 2^7 = ((a+a)/a)^{(aa-a-a-a-a)/a}.$$

Power of 3.

$$9 := 3^2 = ((a+a+a)/a)^{(a+a)/a}.$$
$$27 := 3^3 = ((a+a+a)/a)^{(a+a+a)/a}.$$
$$81 := 3^4 = ((a+a+a)/a)^{(a+a+a+a)/a}.$$
$$243 := 3^5 = ((a+a+a)/a)^{(aa-a)/(a+a)}.$$
$$729 := 3^6 = ((a+a+a)/a)^{(aa+a)/(a+a)}.$$
$$2187 := 3^7 = ((a+a+a)/a)^{(aa-a-a-a-a)/a}.$$
$$6561 := 3^8 = ((a+a+a)/a)^{(aa-a-a-a)/a}.$$

For full work, refer to link below [5]:

http://rgmia.org/papers/v18/v18a73.pdf

7.2 Palindromic and Number Patterns

The idea of single letter representations of numbers given above can be applied to palindromic and number patterns. The study is also extended to prime patterns, doubly symmetric patterns, etc. See some examples below:

$$11 := (a \times aa)/(a \times a).$$
$$121 := (aa \times aa)/(a \times a).$$
$$12321 := (aaa \times aaa)/(a \times a).$$
$$1234321 := (aaaa \times aaaa)/(a \times a).$$
$$123454321 := (aaaaa \times aaaaa)/(a \times a).$$
$$12345654321 := (aaaaaa \times aaaaaa)/(a \times a).$$
$$1234567654321 := (aaaaaaa \times aaaaaaa)/(a \times a).$$
$$123456787654321 := (aaaaaaaa \times aaaaaaaa)/(a \times a).$$
$$12345678987654321 := (aaaaaaaaa \times aaaaaaaaa)/(a \times a).$$

$$
\begin{aligned}
1156 = 34^2 &:= ((aa + aa + aa + a)/a)^{(a+a)/a}.\\
111556 = 334^2 &:= ((aaa + aaa + aaa + a)/a)^{(a+a)/a}.\\
11115556 = 3334^2 &:= ((aaaa + aaaa + aaaa + a)/a)^{(a+a)/a}.\\
1111155556 = 33334^2 &:= ((aaaaa + aaaaa + aaaaa + a)/a)^{(a+a)/a}.\\
111111555556 = 333334^2 &:= ((aaaaaa + aaaaaa + aaaaaa + a)/a)^{(a+a)/a}.\\
11111115555556 = 3333334^2 &:= ((aaaaaaa + aaaaaaa + aaaaaaa + a)/a)^{(a+a)/a}.\\
1111111155555556 = 33333334^2 &:= ((aaaaaaaa + aaaaaaaa + aaaaaaaa + a)/a)^{(a+a)/a}.
\end{aligned}
$$

$$
\begin{aligned}
99 = 98 + 1 &:= (aaa - aa - a)/(a \times a).\\
999 = 987 + 12 &:= (aaaa - aaa - a)/(a \times a).\\
9999 = 9876 + 123 &:= (aaaaa - aaaa - a)/(a \times a).\\
99999 = 98765 + 1234 &:= (aaaaaa - aaaaa - a)/(a \times a).\\
999999 = 987654 + 12345 &:= (aaaaaaa - aaaaaa - a)/(a \times a).\\
9999999 = 9876543 + 123456 &:= (aaaaaaaa - aaaaaaa - a)/(a \times a).\\
99999999 = 98765432 + 1234567 &:= (aaaaaaaaa - aaaaaaaa - a)/(a \times a).\\
999999999 = 987654321 + 12345678 &:= (aaaaaaaaaa - aaaaaaaaa - a)/(a \times a).\\
9999999999 = 9876543210 + 123456789 &:= (aaaaaaaaaaa - aaaaaaaaaa - a)/(a \times a).
\end{aligned}
$$

$$
\begin{aligned}
33 = 12 + 21 &:= (a + a + a) \times aa/(a \times a)\\
444 = 123 + 321 &:= (a + a + a + a) \times aaa/(a \times a)\\
5555 = 1234 + 4321 &:= (a + a + a + a + a) \times aaaa/(a \times a)\\
66666 = 12345 + 54321 &:= (a + a + a + a + a + a) \times aaaaa/(a \times a)\\
777777 = 123456 + 654321 &:= (aa - a - a - a - a) \times aaaaaa/(a \times a)\\
8888888 = 1234567 + 7654321 &:= (aa - a - a - a) \times aaaaaaa/(a \times a)\\
99999999 = 12345678 + 87654321 &:= (aa - a - a) \times aaaaaaaa/(a \times a)
\end{aligned}
$$

$$
\begin{aligned}
111111111 = 12345679 \times 9 \times 1 &:= aaaaaaaaa \times a/(a \times a)\\
222222222 = 12345679 \times 9 \times 2 &:= aaaaaaaaa \times (a + a)/(a \times a)\\
333333333 = 12345679 \times 9 \times 3 &:= aaaaaaaaa \times (a + a + a)/(a \times a)\\
444444444 = 12345679 \times 9 \times 4 &:= aaaaaaaaa \times (a + a + a + a)/(a \times a)\\
555555555 = 12345679 \times 9 \times 5 &:= aaaaaaaaa \times (a + a + a + a + a)/(a \times a)\\
666666666 = 12345679 \times 9 \times 6 &:= aaaaaaaaa \times (a + a + a + a + a + a)/(a \times a)\\
777777777 = 12345679 \times 9 \times 7 &:= aaaaaaaaa \times (aa - a - a - a - a)/(a \times a)\\
888888888 = 12345679 \times 9 \times 8 &:= aaaaaaaaa \times (aa - a - a - a)/(a \times a)\\
999999999 = 12345679 \times 9 \times 9 &:= aaaaaaaaa \times (aa - a - a)/(a \times a).
\end{aligned}
$$

The number 12345679 also appears as a division of 1/81, i.e.

$$\frac{1}{81} = 0.012345679\ 012345679\ 012345679\ 012345679\ .... = 0.\overline{012345679}$$

For full work, refer to links below [6, 7]:

http://rgmia.org/papers/v18/v18a77.pdf.

http://rgmia.org/papers/v18/v18a99.pdf.

8 Running Expressions

It is well know that one can write, $12 = 3 \times 4$, $56 = 7 \times 8$. Here 9 remains alone. The aim of this work is to see how we can write expressions using 9 digits in a sequence in increasing and/or decreasing way. In the decreasing case, the possibility of 9 to 0 is also considered. The expressions are separated either by single or by double equality signs. See example below:

$$12 = 3 + 4 + (5 \times 6 + 7 + 8)/9$$
$$123 = 4 + 5 + 6 \times 7 + 8 \times 9$$
$$1234 = -5 + 6! + 7 + 8^{\sqrt{9}}$$

$$12 + 3 \times 4 + 5 \times (6 + 7) = 89$$
$$1 + 23 + 45 + 6! = 789$$

$$98 = (7 + 6) \times 5 + 4 \times 3 + 21$$
$$987 = 6! + 5! + (4 + 3) \times 21$$

$$98 - 7 \times (6 + 5) \times (4 - 3) = 21$$
$$\sqrt{9} \times 87 + 6 + 54 = 321$$
$$9 - 8 + 7! - 6 \times 5! = 4321$$

$$9 - 8 + 7 - 6 + 5 + 4 - 3 + 2 = 10$$
$$9 \times (8 + 7) + 6 + 5 + 4^3 = 210$$
$$(9 - 87 + 6!) \times 5!/4! = 3210.$$

Above examples give running expressions in increasing or decreasing orders of 1 to 9 or 9 to 1 or 9 to 0 separated by single equality sign. But there are numbers, that can be separated by more equality signs, for example,

$$\begin{aligned} 16 &:= 12/3 \times 4 && = 5 + 6 + (7 + 8)/\sqrt{9} \\ &:= (9 + 87)/6 && = 5 + 4 + 3 \times 2 + 1. \end{aligned}$$

$$\begin{aligned} 18 &:= 12 + 3! && = \sqrt{4 + 5} \times 6 && = 7 + 8 + \sqrt{9} \\ &:= \sqrt{9} + 8 + 7 && = \sqrt{6 \times 54} && = -3 + 21 = 3! + 2 + 10. \end{aligned}$$

$$\begin{aligned} 24 &:= 1 + 23 && = 4 + 5!/6 && = 7 + 8 + 9 \\ &:= 9 + 8 + 7 && = (6 - 5) \times 4! && = 3 + 21 \end{aligned}$$

$$\begin{aligned} 120 &:= 1 \times (2 + 3)! && = 4 + 5!/6 + 7 + 89 \\ &:= 98 + 7 + 6 + 5 + 4 && = (3 + 2)! \times 1 \\ &:= \sqrt{9} + 87 + 6 \times 5 && = \sqrt{4} \times 3 \times 2 \times 10. \end{aligned}$$

For full work, refer to the link below [3]:

http://rgmia.org/papers/v18/v18a27.pdf.

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