Received 29/01/17

# Hardy-Ramanujan Number - 1729

**Inder J. Taneja**[1]

**Abstract**

*This paper brings representations of 1729, a famous **Hardy-Ramanujan number** in different situations. These representations are with single digit, single letter, Selfie-Type, Running Expressions, Equivalent Fractions, Triangular, Fibonacci, Fixed Digits Repetitions Prime Numbers Patterns , Palindromic-Type, Polygonal-Type, Prime Numbers, Embedded, Repeated, etc. Ideas toward magic squares are also extended. Some quotes and historical notes on Ramanujan's life and work are also given.*

**On a special day: January 29, 17 (17-29)**

***An equation means nothing to me unless it expresses a thought of God.***
**- S. Ramanujan**

Another famous quote of Ramanujan on his dreams:

*While asleep, I had an unusual experience. There was a red screen formed by flowing blood, as it were. I was observing it. Suddenly a hand began to write on the screen. I became all attention. That hand wrote a number of elliptic integrals.*

*They stuck to my mind. As soon as I woke up, I committed them to writing.*

**S. Ramanujan**

Confirmations to Ramanujan's dreams:

*Ono and his colleagues (Emory University, Atlanta, GA, USA) drew on modern mathematical tools that had not been developed before Ramanujan's death to prove this theory was correct. We proved that Ramanujan was right. We've solved the problems from his last mysterious letters. For people who work in this area of math, the problem has been open for 90 years.*

**We found the formula explaining one of the visions that he believed came from his goddess.**

**K. Ono**

Read more at:

1. *http://www.dailymail.co.uk/sciencetech/article-2254352/Deathbed-dream-puzzles-renowned-Indian-mathematician-Srinivasa-finally-solved--100-years-died.html.*
2. *http://www.livescience.com/25597-ramanujans-math-theories-proved.html.*
3. *https://www.quantamagazine.org/20160519-ken-ono-mathematician-inspired-by-ramanujan/*

[1] *Formerly, Professor of Mathematics, Universidade Federal de Santa Catarina, 88.040-900 Florianópolis, SC, Brazil.*
***E-mail:*** *ijtaneja@gmail.com;* ***Twitter:*** *@IJTANEJA* ***Web-site:*** *https://numbersmagic.wordpress.com.*

Below are few more **quotes** on Ramanujan's work:

*I had never seen anything in the least like them before. A single look at them is enough to show that they could only be written by a mathematician of the highest class. They must be true because, if they were not true,* ***"no one would have the imagination to invent them"****.*
*He also said, like other great men* ***"he invented himself"****.*
**– G.H. Hardy**

*Every positive integer is one of Ramanujan's personal friends.*
**– J. Littlewood**

*I still don't understand it all. I may be able to prove it, but I don't know where it comes from and where it fits into the rest of mathematics.*
**– B.C. Berndt**

*The enigma of Ramanujan's creative process is still covered by a curtain that has barely been drawn.*
*The Man Who Knew Infinity, 1991.*
**– R. Kanigel**

*When we comprehend some of Ramanujan's equations, we realize that he was a true artist, expressing deep and beautiful mathematical truth in familiar symbols.*
*Mathematical Mysteries, 1996.*
**– C. Clawson**.

*That was the wonderful thing about Ramanujan. He discovered so much, and yet he left so much more in his garden for other people to discover.*
**– F. Dyson**.

*He discovered bizarre and strange set of modular mathematical functions which are out of this world ... Civilization has to wait another one hundred years to understand remotely Ramanujan's mind".*
**– K. Ono**.

Read more quotes at

1. *https://www.famousscientists.org/srinivasa-ramanujan/.*
2. *http://quoteaddicts.com/author/srinivasa-ramanujan.*
3. *http://www.azquotes.com/quotes/topics/ramanujan.html*

## Index

The work is divided in 29 sections according to details below:

# 1 Historical Notes

The history of S. Ramanujan is well-known in the literature. There are so many sites on Internet relating about him. Below are some of them:

1. *http://www-groups.dcs.st-and.ac.uk/history/Biographies/Ramanujan.html.*
2. *https://en.wikipedia.org/wiki/Srinivasa-Ramanujan.*
3. *http://www.thebetterindia.com/52974/srinivasa-ramanujan-mathematician-biopic.*
4. *http://www.biography.com/people/srinivasa-ramanujan-082515.*
5. *http://www.livescience.com/25597-ramanujans-math-theories-proved.html.*

Besides from his excellent work on mathematics, the Taxicab number 1729 is famous due to his instantaneous combinations with minimum cubes done by him, such as

$$1729 := 1^3 + 12^3 = 9^3 + 10^3.$$

Even though this number is famous as **"Hardy-Ramanujan number"**, historically it has been studied before in 1657 (Boyer, 2008).

1. *http://esciencecommons.blogspot.com.br/2015/10/mathematicians-find-magic-key-to-drive.html.*
2. *http://www.zmescience.com/science/math/taxi-number-ramanujan-03213.*
3. ***C. Boyer**, New upper bounds for Taxicab and Cabtaxi Numbers, J. Integer Sequences, **11**(2008), Art. 08.1.6, https://cs.uwaterloo.ca/journals/JIS/VOL11/Boyer/boyer-new.pdf.*

Ramanujan also worked on magic squares. One of his famous magic square is based on his date of birth. Below are some sites talking about his magic square:

1. *http://www.math.mcgill.ca/styan/Beamer3-18jan12-opt.pdf.*
2. *http://mathstimes.com/wp-content/uploads/2014/10/Birthday-magic-square.pdf.*
3. *http://slideplayer.com/slide/9142307/.*
4. *http://jollymaths.com/blog/srinivasa-ramanujan-16x16-biography-magic-square/*

Ramanujan's whole work is composed as **"Notebooks"**. See the following links:

1. ***B. C. Berndt**, Ramanujan's Notebooks, Parts I-V. Springer-Verlag, New York, 1985, 1989, 1991, 1994, 1998.*
2. ***G. E. Andrews** and **B. C. Berndt**, Ramanujan's Lost Notebook, Part I-IV, Springer, New York, 2005, 2009, 2012, 2013.*

Some of his work and biography can be seen in following books:

1. **C.C. Clawson** *Mathematical Mysteries - The Beauty and Magic of Numbers. Springer, New York, 1996.*
2. **R. Kanigel** *The Man Who Knew Infinity: A Life of the Genius Ramanujan, Washington Square Press, New York, 1991*
3. **C.A. Pickover**, *A Passion for Mathematics, John Wiley & Sons, New Jersey, 2005.*
4. **K. Ono** and **A. D. Aczel**, *My Search for Ramanujan - How I Learned to Count, Springer, New York, 2016.*

The aim of this work is to present different representations of Hardy-Ramanujan number 1729. Also to present new types of magic squares with more details of his biography with the presence of number 1729.

## 2 Magic Squares with 1729

This section deals with the magic squares of order 4 and 5 are made in such a way that they brings more details of life of Ramanujan.

## 2.1 Magic Square of Order 4

Ramanujan himself constructed a magic square of order 4 containing details of his date of birth, i.e., 22.12.1887. Below is his classical magic square

| | | | | *139* |
|---|---|---|---|---|
| **22** | **12** | **18** | **87** | *139* |
| 88 | 17 | 9 | 25 | *139* |
| 10 | 24 | 89 | 16 | *139* |
| 19 | 86 | 23 | 11 | *139* |
| *139* | *139* | *139* | *139* | *139* |

Ramanujan's Magic Square
Figure 1

The magic sum $S := 139$ is a prime number:

$$22 + 12 + 18 + 87 = 139$$

The above magic square (Figure 1) constructed by Ramanujan brings only his date of birth. Below is modified version of above magic square where we brought *Hardy-Ramanujan number* 1729:

| | | | | *139* |
|---|---|---|---|---|
| **22** | **12** | **18** | **87** | *139* |
| 88 | **17** | **29** | 5 | *139* |
| 20 | 14 | 79 | 26 | *139* |
| 9 | 96 | 13 | 21 | *139* |
| *139* | *139* | *139* | *139* | *139* |

Figure 2

Interestingly, if we sum the numbers with his date of death (26.04.1920) in little different way, i.e., considering sum four in four instead two and two digits, we get much more options to put in a magic square. See below

$$2212 + 1887 + 2604 + 1920 = 8623.$$

Again we have a prime number. Below is a magic square with followings details of **Ramanujan**:

Date of Birth : 22.12.1887
Date of Death : 26.04.1920
Servived from Smallpox : 1889
Got Job at Madras Port Trust : 1912
Year of Entering England : 1914
Fellow Royal Society, London : 1918
Hardy-Ramanujan Number : 1729.

| | | | | |
|---|---|---|---|---|
| | | | | *8623* |
| **2212** | **1887** | **2604** | **1920** | *8623* |
| **1914** | 2610 | **1729** | 2370 | *8623* |
| **1918** | 2395 | **1889** | 2421 | *8623* |
| 2579 | 1731 | 2401 | **1912** | *8623* |
| *8623* | *8623* | *8623* | *8623* | *8623* |

Figure 3

We observe that the above magic squares (Figure 1, Figure 2 and Figure 3) are not *pan diagonal*. In all the examples above the magic sum is prime, i.e., odd numbers. It is impossible to construct pan diagonal magic square of order 4 for odd number magic square sum. If we want pan diagonal magic square of order 4 with some Ramanujan's details, we have to make little change, i.e., to consider magic sum as even number. See below:

Date of Birth : 22.12.1887
Date of Death : 26.04.1920
Hardy-Ramanujan Number : 1729.

| | | | | | |
|---|---|---|---|---|---|
| | | *8432* | *8432* | *8432* | *8432* |
| | **2212** | **1887** | **1729** | **2604** | *8432* |
| *8432* | 1696 | 2637 | 2179 | **1920** | *8432* |
| *8432* | 2487 | 1612 | 2004 | 2329 | *8432* |
| *8432* | 2037 | 2296 | 2520 | 1579 | *8432* |
| | *8432* | *8432* | *8432* | *8432* | *8432* |

Figure 4

In this case the magic sum, $S = 8432$ is not a prime number, but the magic square is pan diagonal. Coincidentally, the number 8432 divided in two by two i.e, 84 and 32 give 84-gonal 7th value is 1729, i.e., $P_{84}(7) := 1729$ and 32 the age Ramanujan died. According to Hindu philosophy *"The number 84 is a "whole" or "perfect" number. Thus the eighty-four siddhas can be seen as archetypes representing the thousands of exemplars and adept of the tantric way"*

*http://keithdowman.net/essays/introduction-mahasiddhas-and-tantra.html*

## 2.2 Magic Squares of Order 5

The magic square appearing in Figure 4 contains less details as being pan diagonal. Still, we can make pan diagonal magic square of order 5 with more details. See below:

Date of Birth : 22.12.1887
Date of Death : 26.04.1920
Age at the time of Death : 32
Got Job at Madrs Port Trust : 1912
Year of Entering England : 1914
Fellow Royal Society, London : 1918
Hardy-Ramanujan Number : 1729.

| | | | | | | |
|---|---|---|---|---|---|---|
| | | *8655* | *8655* | *8655* | *8655* | *8655* |
| | **2212** | **1887** | 32 | **2604** | **1920** | *8655* |
| *8655* | 1754 | **1914** | 3062 | **1918** | 7 | *8655* |
| *8655* | 2768 | 38 | **1729** | 1064 | 3056 | *8655* |
| *8655* | 1039 | 2206 | 2762 | 888 | 1760 | *8655* |
| *8655* | 882 | 2610 | 1070 | 2181 | **1912** | *8655* |
| | *8655* | *8655* | *8655* | *8655* | *8655* | *8655* |

Figure 5

Since our work is more concentrated towards number 1729. Below is another pan diagonal magic square of order 5 with sum $5 \times 1729 = 8645$ with similar details as of Figure 5.

Date of Birth : 22.12.1887
Date of Death : 26.04.1920
Worked on Mathematics : 22 years
Age at the time of Death : 32 years
Most Struggling Year : 1908
Year of Entering England : 1914
Fellow Royal Society, London : 1918
Hardy-Ramanujan Number : 1729
Magic Sum : $8645 = 5 \times 1729$.

| | | | | | | |
|---|---|---|---|---|---|---|
| | | $5 \times 1729$ | $5 \times 1729$ | $5 \times 1729$ | $5 \times 1729$ | $5 \times 1729$ |
| | **2212** | **1887** | **22** | **2604** | **1920** | $5 \times 1729$ |
| $5 \times 1729$ | 1750 | **1914** | 3062 | **1918** | 1 | $5 \times 1729$ |
| $5 \times 1729$ | 2768 | **32** | **1729** | 1060 | 3056 | $5 \times 1729$ |
| $5 \times 1729$ | 1039 | 2202 | 2762 | 882 | 1760 | $5 \times 1729$ |
| $5 \times 1729$ | 876 | 2610 | 1070 | 2181 | **1908** | $5 \times 1729$ |
| | $5 \times 1729$ | $5 \times 1729$ | $5 \times 1729$ | $5 \times 1729$ | $5 \times 1729$ | $5 \times 1729$ |

Figure 6

It is interesting to observe that the number "22" appearing in the middle of first line is the span period of time Ramanujan worked on mathematics. Also it is famous that Professor Bruce C. Berndt (University of Illinois at Urbana-Champaign, Illinois, USA) spent about 22 years to solve 3254 problems from Ramanujan's notebooks (*http://ramanujans.blogspot.com.br/*). Another number appearing in magic square is 1908. As compared to other years, this year don't brings any this special to Ramanujan's life. His marriage happened in 1909. On the other hand, according to the book "**R. Kanigel** *The Man Who Knew Infinity: A Life of the Genius Ramanujan, Washington Square Press, New York, 1991*", on page 55, it is written that year 1908 was very difficult for him as he was, *out of school, without job, without food, etc.*

The approach applied to construct above four squares is different from the one used by Ramanujan (The Notebooks of Ramanujan, Tata Institute of Fundamental Research, Bombay, 1957). More details on constructions of these magic squares shall be dealt elsewhere. For study on magic squares in different aspects see the the author's work.

*https://arxiv.org/pdf/1206.2220v1.pdf.*
*http://rgmia.org/papers/v18/v18a98.pdf.*
*http://rgmia.org/papers/v18/v18a127.pdf.*

*http://rgmia.org/papers/v18/v18a140.pdf.*
*http://rgmia.org/papers/v18/v18a151.pdf.*

## 3 Single Digit Representations

Below are representations of the number 1729 written in terms of each digit separately using 1 to 9. Even though it can be written using 0 but it requires use of factorials.

$$\begin{aligned}
1729 &:= (11+1)^{1+1+1}+1\\
&:= (2/2+2)\times(22+2)^2+2/2\\
&:= (3\times 3+3)^3+3/3\\
&:= 4\times(4\times 44+4^4)+4/4\\
&:= 55\times(5\times 5-5)+(5^5-5)/5+5\\
&:= 6\times 6\times(6\times 6+6+6)+6/6\\
&:= 7\times 7\times(7\times 7-7-7)+7+7\\
&:= 8\times(8\times(8+8)+88)+8/8\\
&:= 9\times 9\times 9+999+9/9.
\end{aligned}$$

*https://arxiv.org/abs/1502.03501.*

## 4 Single Letter Representation

In Section 3 the number 1729 is written in terms of each digit separately. Instead using each digit seperately as of Section 3 , one can write this number using single letter "$a$". It remains true for any value of $a$ from 1 to 9.

$$1729 := \frac{(aaa+aa+aa)\times(aa+a+a)}{a\times a}.$$

where, $aaa = a10^2+a10+a$,

$aa = a10+a,\ a\in\{1,2,3,4,5,6,7,8,9\}$.

*http://rgmia.org/papers/v18/v18a73.pdf.*

# 5 Crazy Representations

## 5.1 Increasing and Decreasing

The number 1729 is written in terms of 1 to 5, 6, 7, 8 and 9 and reverse.

$$\begin{aligned}
1729 &:= 12^3 - 4 + 5 && = 54 \times 32 + 1\\
&:= 12^3 + (-4+5)^6 && = 6 \times (5+4) \times 32 + 1\\
&:= 123 \times (4 \times 5 - 6) + 7 && = (7-6) \times (54 \times 32 + 1)\\
&:= -1 + (2 + 34 + 5) \times 6 \times 7 + 8 && = 8 - 7 + 6 \times (5+4) \times 32 \times 1\\
&:= 12 - 3 + 4^5 - 6 + 78 \times 9 && = (98-7) \times (6 \times 5 - 4 \times 3 + 2 - 1).
\end{aligned}$$

*https://arxiv.org/abs/1302.1479.*

## 5.2 Ending in Zero

The number 1729 is written with ending in 0, starting with 5, 6, 7, 8 and 9.

$$\begin{aligned}
1729 &:= (4 \times 3)^{2+1} + 0!\\
&:= 54 \times 32 + 1 \times 0!\\
&:= 6! - 5 - 4 - 3! + 2^{10}\\
&:= (7+6) \times (-5 - 4! \times 3 + 210)\\
&:= 8 - 7 + (6+5) \times 4^3 + 2^{10}\\
&:= 9 + 8 \times (7 \times 6 \times 5 - 4 - 3 + 2 + 10).
\end{aligned}$$

*http://rgmia.org/papers/v19/v19a129.pdf.*

## 5.3 Numbers from 1 to 10 and Reverse

Instead using 1 to 9 or 9 to 1, here the numbers 1 to 10 are used in both ways, i.e., increasing and decreasing.

$$\begin{aligned}
1729 &:= 10 \times (98 + 7 + 65 + \sqrt{4}) + 3^2 \times 1\\
&:= 1 + 2^3 + (-4 + 56 + ((7+8)/\sqrt{9})!) \times 10.
\end{aligned}$$

# 6 Powers and Bases: Same Digits

This section deals with the representations of 1729 in such a way that the powers and based are with same set of digits. Initially in sequential way starting with 0 and 1. The non sequential representation is also given.

### 6.1 Sequential: Starting With 0

$$\begin{aligned}1729 &:= 0^3 + 1^0 + 2^6 + 3^2 + 4^5 + 5^4 + 6^1 \\ &:= 0^4 + 1^7 + 2^9 - 3^8 + 4^6 + 5^5 + 6^2 + 7^1 + 8^3 + 9^0.\end{aligned}$$

### 6.2 Sequential: Starting With 1

$$\begin{aligned}1729 &:= 1^3 + 2^6 + 3^2 + 4^5 + 5^4 + 6^1 \\ &:= 1^5 + 2^8 + 3^9 + 4^4 + 5^1 - 6^7 + 7^2 + 8^6 - 9^3.\end{aligned}$$

### 6.3 Non Sequential

$$\begin{aligned}1729 &:= 1^1 + 2^7 + 4^5 + 5^4 - 7^2 \\ &:= 1^9 - 2^1 + 4^5 + 5^4 + 9^2.\end{aligned}$$

*http://rgmia.org/papers/v19/v19a31.pdf.*
*http://rgmia.org/papers/v19/v19a131.pdf*

### 6.4 Power Patterns

Based on the idea of same set of digits in powers and bases, seperated by addition and subtraction, below are some patterns containing the number 1729.

$$\begin{aligned}1729 \times 10 + 0 &:= -1^7 + 2^6 + 3^1 + 4^4 + 5^3 + 6^2 + 7^5. \\ 1729 \times 10 + 2 &:= 1^7 + 2^6 + 3^1 + 4^4 + 5^3 + 6^2 + 7^5. \\ 1729 \times 10 + 4 &:= 1^7 + 2^6 + 3^2 + 4^1 + 5^4 - 6^3 + 7^5. \\ 1729 \times 10 + 6 &:= -1^5 + 2^4 + 3^6 + 4^7 + 5^3 + 6^2 + 7^1 \\ 1729 \times 10 + 8 &:= 1^5 + 2^4 + 3^6 + 4^7 + 5^3 + 6^2 + 7^1.\end{aligned}$$

$$\begin{aligned}1729 \times 10 + 0 &:= 1^7 + 2^8 - 3^3 - 4^5 + 5^6 - 6^1 + 7^4 + 8^2. \\ 1729 \times 10 + 2 &:= 1^8 + 2^6 + 3^2 - 4^7 + 5^4 + 6^3 - 7^1 + 8^5. \\ 1729 \times 10 + 4 &:= 1^3 + 2^4 + 3^2 + 4^8 - 5^7 + 6^6 - 7^5 + 8^1. \\ 1729 \times 10 + 6 &:= 1^6 + 2^8 - 3^5 + 4^7 + 5^4 + 6^3 + 7^2 + 8^1. \\ 1729 \times 10 + 8 &:= 1^8 + 2^4 + 3^6 - 4^7 + 5^3 + 6^2 + 7^1 + 8^5.\end{aligned}$$

*http://rgmia.org/papers/v19/v19a31.pdf.*
*http://rgmia.org/papers/v19/v19a131.pdf*

# 7 Same Digits Both Sides

This section deals with the representations of 1729 in such way the we have almost same digits on both sides of the expressions. These representations are given in different situations.

## 7.1 Selfie-Type

$$\mathbf{1729}665 := 17^2 \times 9 \times 665$$
$$494\mathbf{1729} := (494 + 1729)^2.$$

In the second case, we have power 2 extra. More study of this kind is given in Section 24 along with Fibonacci and Triangular sequence values.

## 7.2 Multiplication

The representations below are in such a way that we have same digits with each multiplicative factor on both sides.

$$1729 \times 3584 = 1792 \times 3458.$$
$$1729 \times 3854 = 1927 \times 3458.$$
$$1729 \times 4358 = 2179 \times 3458.$$
$$1729 \times 4732 = 2197 \times 3724.$$
$$1729 \times 5438 = 2719 \times 3458.$$
$$1729 \times 5781 = 1927 \times 5187.$$

## 7.3 Addition

The representations below are in such a way that we have same digits on sides, where the one there the digits are as a power.

$$\begin{aligned} 1729 &:= 2^7 + 40^2 + 130^0 \qquad = 27 + 402 + 1300. \\ &:= 2^6 + 40^2 + 64^1 + 66^0 = 26 + 402 + 641 + 660. \\ &:= 1^6 + 41^2 + 46^1 + 84^0 = 16 + 412 + 461 + 840. \end{aligned}$$

$$\begin{aligned} 1729 &:= 3^5 + 3^5 + 3^6 + 4^0 + 8^3 + 150^0 = 35 + 35 + 36 + 40 + 83 + 1500. \\ &:= 2^4 + 2^4 + 3^5 + 6^4 + 8^1 + 150^1 = 24 + 24 + 35 + 64 + 81 + 1501. \\ &:= 2^4 + 2^8 + 3^1 + 6^4 + 8^1 + 150^1 = 24 + 28 + 31 + 64 + 81 + 1501. \\ &:= 2^4 + 2^8 + 4^1 + 6^4 + 7^1 + 150^1 = 24 + 28 + 41 + 64 + 71 + 1501. \\ &:= 2^4 + 2^8 + 5^1 + 6^1 + 6^4 + 150^1 = 24 + 28 + 51 + 61 + 64 + 1501. \\ &:= 3^4 + 3^5 + 5^1 + 5^4 + 5^4 + 150^1 = 34 + 35 + 51 + 54 + 54 + 1501. \end{aligned}$$

$$1729 := 1^0 + 2^3 + 4^3 + 4^4 + 5^4 + 5^4 + 150^1 = 10 + 23 + 43 + 44 + 54 + 54 + 1501.$$
$$:= 1^0 + 2^9 + 4^2 + 4^5 + 5^0 + 5^2 + 150^1 = 10 + 29 + 42 + 45 + 50 + 52 + 1501.$$
$$:= 1^0 + 3^4 + 3^5 + 4^1 + 5^4 + 5^4 + 150^1 = 10 + 34 + 35 + 41 + 54 + 54 + 1501.$$
$$:= 1^1 + 2^9 + 3^0 + 4^5 + 5^1 + 6^2 + 150^1 = 11 + 29 + 30 + 45 + 51 + 62 + 1501.$$
$$:= 1^8 + 2^4 + 2^9 + 4^5 + 5^2 + 6^0 + 150^1 = 18 + 24 + 29 + 45 + 52 + 60 + 1501.$$
$$:= 1^8 + 2^9 + 2^9 + 2^9 + 6^1 + 6^2 + 150^1 = 18 + 29 + 29 + 29 + 61 + 62 + 1501.$$
$$:= 2^0 + 2^9 + 4^0 + 4^2 + 4^5 + 5^2 + 150^1 = 20 + 29 + 40 + 42 + 45 + 52 + 1501.$$
$$:= 2^1 + 2^9 + 3^0 + 4^1 + 4^5 + 6^2 + 150^1 = 21 + 29 + 30 + 41 + 45 + 62 + 1501.$$
$$:= 2^5 + 2^6 + 2^7 + 3^6 + 5^4 + 6^0 + 150^1 = 25 + 26 + 27 + 36 + 54 + 60 + 1501.$$
$$:= 2^9 + 3^0 + 3^0 + 4^2 + 4^5 + 5^2 + 150^1 = 29 + 30 + 30 + 42 + 45 + 52 + 1501.$$
$$:= 2^9 + 3^0 + 3^1 + 3^1 + 4^5 + 6^2 + 150^1 = 29 + 30 + 31 + 31 + 45 + 62 + 1501.$$

$$1729 := 1^0 + 2^8 + 2^9 + 3^6 + 4^0 + 4^2 + 4^3 + 150^1 := 10 + 28 + 29 + 36 + 40 + 42 + 43 + 1501.$$
$$:= 1^1 + 2^6 + 2^9 + 3^6 + 4^0 + 4^2 + 4^4 + 150^1 := 11 + 26 + 29 + 36 + 40 + 42 + 44 + 1501.$$
$$:= 1^2 + 2^4 + 2^9 + 3^6 + 4^0 + 4^3 + 4^4 + 150^1 := 12 + 24 + 29 + 36 + 40 + 43 + 44 + 1501.$$
$$:= 1^4 + 2^0 + 2^9 + 3^6 + 4^2 + 4^3 + 4^4 + 150^1 := 14 + 20 + 29 + 36 + 42 + 43 + 44 + 1501.$$
$$:= 1^4 + 2^5 + 2^8 + 3^2 + 4^0 + 4^4 + 4^5 + 150^1 := 14 + 25 + 28 + 32 + 40 + 44 + 45 + 1501.$$
$$:= 1^4 + 2^8 + 3^3 + 3^3 + 3^5 + 4^0 + 4^5 + 150^1 := 14 + 28 + 33 + 33 + 35 + 40 + 45 + 1501.$$
$$:= 1^5 + 1^9 + 2^9 + 3^6 + 4^2 + 4^3 + 4^4 + 150^1 := 15 + 19 + 29 + 36 + 42 + 43 + 44 + 1501.$$

$$1729 := 1^6 + 1^8 + 2^9 + 3^6 + 4^2 + 4^3 + 4^4 + 150^1 := 16 + 18 + 29 + 36 + 42 + 43 + 44 + 1501.$$
$$:= 1^6 + 2^4 + 2^9 + 3^2 + 4^0 + 4^2 + 4^5 + 150^1 := 16 + 24 + 29 + 32 + 40 + 42 + 45 + 1501.$$
$$:= 1^6 + 2^6 + 3^1 + 3^5 + 3^5 + 4^0 + 4^5 + 150^1 := 16 + 26 + 31 + 35 + 35 + 40 + 45 + 1501.$$
$$:= 1^6 + 2^7 + 2^7 + 3^4 + 3^6 + 4^4 + 4^4 + 150^1 := 16 + 27 + 27 + 34 + 36 + 44 + 44 + 1501.$$
$$:= 1^6 + 2^7 + 2^9 + 3^4 + 3^6 + 4^3 + 4^3 + 150^1 := 16 + 27 + 29 + 34 + 36 + 43 + 43 + 1501.$$
$$:= 1^6 + 2^9 + 2^9 + 3^5 + 3^5 + 4^1 + 4^3 + 150^1 := 16 + 29 + 29 + 35 + 35 + 41 + 43 + 1501.$$
$$:= 1^6 + 2^9 + 3^4 + 3^5 + 3^5 + 3^5 + 4^4 + 150^1 := 16 + 29 + 34 + 35 + 35 + 35 + 44 + 1501.$$

$$1729 := 1^7 + 1^7 + 2^9 + 3^6 + 4^2 + 4^3 + 4^4 + 150^1 := 17 + 17 + 29 + 36 + 42 + 43 + 44 + 1501.$$
$$:= 1^8 + 2^0 + 2^5 + 3^2 + 4^4 + 4^4 + 4^5 + 150^1 := 18 + 20 + 25 + 32 + 44 + 44 + 45 + 1501.$$
$$:= 1^8 + 2^0 + 2^9 + 3^2 + 4^2 + 4^2 + 4^5 + 150^1 := 18 + 20 + 29 + 32 + 42 + 42 + 45 + 1501.$$
$$:= 1^8 + 2^0 + 3^3 + 3^3 + 3^5 + 4^4 + 4^5 + 150^1 := 18 + 20 + 33 + 33 + 35 + 44 + 45 + 1501.$$
$$:= 1^9 + 1^9 + 2^5 + 3^2 + 4^4 + 4^4 + 4^5 + 150^1 := 19 + 19 + 25 + 32 + 44 + 44 + 45 + 1501.$$
$$:= 1^9 + 1^9 + 2^9 + 3^2 + 4^2 + 4^2 + 4^5 + 150^1 := 19 + 19 + 29 + 32 + 42 + 42 + 45 + 1501.$$
$$:= 1^9 + 1^9 + 3^3 + 3^3 + 3^5 + 4^4 + 4^5 + 150^1 := 19 + 19 + 33 + 33 + 35 + 44 + 45 + 1501.$$

$$
\begin{aligned}
1729 &:= 1^9+2^0+3^1+3^5+3^5+4^3+4^5+150^1 := 19+20+31+35+35+43+45+1501.\\
&:= 2^0+2^8+2^9+3^0+3^6+4^2+4^3+150^1 := 20+28+29+30+36+42+43+1501.\\
&:= 2^0+2^9+3^3+3^3+3^3+3^6+4^4+150^1 := 20+29+33+33+33+36+44+1501.\\
&:= 2^2+2^9+3^2+3^4+3^5+3^6+4^0+150^1 := 22+29+32+34+35+36+40+1501.\\
&:= 2^3+2^5+2^6+3^6+3^6+4^0+4^2+150^1 := 23+25+26+36+36+40+42+1501.\\
&:= 2^5+2^8+2^8+3^0+3^2+4^0+4^5+150^1 := 25+28+28+30+32+40+45+1501.\\
&:= 2^7+2^7+2^7+2^7+3^3+4^2+4^5+150^1 := 27+27+27+27+33+42+45+1501.
\end{aligned}
$$

There are much more representations of of this kind, but only few are written.

## 8 Narcissistic-Type: Flexible Power with Division

Narcissistic numbers are famous in the literature, when there are same digits on both sides with fixed power and operations of addition. Below is a representation of 1729 in terms division with flexible power instead of fixed.

$$1729 := \frac{1^0+7^0+2^{15}+9^2}{1+7+2+9}.$$

*http://rgmia.org/papers/v19/v19a32.pdf.*

## 9 Power Representations

As explained in Introduction, the idea of power representation of 1729 is given by Ramanujan. Here there are much more possibilities of wring this number as powers of 2 and 3.

### 9.1 Power 2

$$
\begin{aligned}
1729 &:= 6^2+18^2+37^2\\
&:= 8^2+12^2+39^2\\
&:= 8^2+24^2+33^2\\
&:= 10^2+27^2+30^2\\
&:= 12^2+17^2+36^2\\
&:= 18^2+26^2+27^2.
\end{aligned}
$$

### 9.2 Power 3

$$\begin{aligned}1729 &:= 1^3 + 12^3\\ &:= 9^3 + 10^3\\ &:= 1^3 + 6^3 + 8^3 + 10^3\\ &:= 1^3 + 3^3 + 4^3 + 5^3 + 8^3 + 10^3.\end{aligned}$$

### 9.3 Power 3 Multiplication

This subsection give product decomposition of 1779 using power 3.

$$\begin{aligned}1729 &:= (6^3 - 5^3) \times (3^3 - 2^3)\\ &:= (4^3 + 3^3) \times (3^3 - 2^3).\end{aligned}$$

## 10 Patterns with 1729: Power 3

The idea of power representation of 1729 given in Section 9 is extended here to give some patterns with number 1729.

### 10.1 First Pattern

$$\begin{aligned}1729 \times 10 + 0 &:= 13^3 + 18^3 + 21^3\\ 1729 \times 10 + 1 &:= 1^3 + 13^3 + 18^3 + 21^3\\ 1729 \times 10 + 2 &:= 2^3 + 7^3 + 13^3 + 16^3 + 22^3\\ &:= 7^3 + 9^3 + 13^3 + 15^3 + 22^3\\ 1729 \times 10 + 3 &:= 1^3 + 2^3 + 7^3 + 13^3 + 16^3 + 22^3\\ &:= 1^3 + 7^3 + 9^3 + 13^3 + 15^3 + 22^3\\ 1729 \times 10 + 4 &:= 5^3 + 8^3 + 10^3 + 14^3 + 17^3 + 20^3\\ 1729 \times 10 + 5 &:= 5^3 + 9^3 + 11^3 + 13^3 + 17^3 + 20^3\\ 1729 \times 10 + 6 &:= 6^3 + 17^3 + 23^3\\ 1729 \times 10 + 7 &:= 9^3 + 14^3 + 24^3\\ 1729 \times 10 + 8 &:= 1^3 + 9^3 + 14^3 + 24^3\\ &:= 2^3 + 13^3 + 18^3 + 21^3\\ 1729 \times 10 + 9 &:= 7^3 + 11^3 + 25^3,\end{aligned}$$

The last two numbers can also be written as power 4:

$$1729 \times 10 + 8 := 4^4 + 7^4 + 11^4$$
$$1729 \times 10 + 9 := 1^4 + 4^4 + 7^4 + 11^4.$$

### 10.2 Second Pattern

$$\begin{aligned}
10000 + 1729 &:= 1^3 + 2^3 + 7^3 + 9^3 + 22^3 \\
&:= 6^3 + 9^3 + 12^3 + 13^3 + 19^3 \\
20000 + 1729 &:= 1^3 + 5^3 + 16^3 + 19^3 + 22^3 \\
&:= 1^3 + 10^3 + 12^3 + 15^3 + 25^3 \\
&:= 4^3 + 6^3 + 12^3 + 16^3 + 25^3 \\
&:= 7^3 + 13^3 + 16^3 + 18^3 + 21^3 \\
30000 + 1729 &:= 1^3 + 10^3 + 13^3 + 16^3 + 19^3 + 26^3 \\
&:= 3^3 + 4^3 + 10^3 + 16^3 + 19^3 + 27^3 \\
&:= 6^3 + 10^3 + 13^3 + 18^3 + 19^3 + 25^3 \\
40000 + 1729 &:= 2^3 + 5^3 + 15^3 + 18^3 + 20^3 + 29^3 \\
&:= 2^3 + 8^3 + 11^3 + 13^3 + 17^3 + 32^3 \\
&:= 8^3 + 9^3 + 17^3 + 18^3 + 23^3 + 26^3 \\
50000 + 1729 &:= 5^3 + 9^3 + 20^3 + 35^3 \\
&:= 14^3 + 17^3 + 27^3 + 29^3 \\
60000 + 1729 &:= 6^3 + 12^3 + 17^3 + 38^3 \\
&:= 6^3 + 20^3 + 26^3 + 33^3 \\
70000 + 1729 &:= 4^3 + 14^3 + 41^3 \\
80000 + 1729 &:= 10^3 + 12^3 + 17^3 + 42^3 \\
90000 + 1729 &:= 9^3 + 30^3 + 40^3.
\end{aligned}$$

The last two numbers can also be written as power 4:

$$80000 + 1729 := 4^4 + 6^4 + 11^4 + 16^4$$
$$90000 + 1729 := 4^4 + 6^4 + 10^4 + 11^4 + 16^4.$$

## 11 Functional Representations

This deals with representations of 1729 in two different ways. One in terms of Fibonacci sequence values. Second as particular cases of *s-sides of polygons*.

### 11.1 Fibonacci Sequences

$$F(0) = F(1) = 1,\ F(n) = F(n-1) + F(n-2),\ \ n \geq 2,$$
$$0, 1, 1, 2, 3, 5, 8, 13, ..$$

Then,

$$1729 := F(2) + F(6) + F(9) + F(11) + F(17).$$

### 11.2 S-gonol Values

The general formula for *s-sides of a polygon* (**s-gonal**) is known as

$$P_s(n) := \frac{n(n-1)(s-2)}{2} + n, \quad s > 2.$$

Below are particular cases:

**Triangle (3-gonal):** $P_3(n) = n(n+1)/2 \rightarrow 1729 := P_3(26) + P_3(52).$
**Square (4-gonal):** $P_4(n) = n^2 \rightarrow 1729 := P_4(6) + P_4(18) + P_4(37).$
**Pentagonal (5-gonal):** $P_5(n) = n(3n-1)/2 \rightarrow 1729 := P_5(3) + P_5(34).$
**Hexagonal (6-gonal):** $P_6(n) = n(2n-1) \rightarrow 1729 := P_6(9) + P_6(18) + P_6(22).$
**Heptagonal (7-gonal):** $P_7(n) = n(5n-3)/2 \rightarrow 1729 := P_7(9) + P_7(14) + P_7(21).$
**Octagonal (8-gonal):** $P_8(n) = n(3n-2) \rightarrow 1729 := P_8(4) + P_8(12) + P_8(21).$
**Nonagonal (9-gonal):** $P_9(n) = n(7n-5)/2 \rightarrow 1729 := P_9(1) + P_9(2) + P_9(15) + P_9(17).$
**Decagonal (10-gonal):** $P_{10}(n) = n(4n-3) \rightarrow 1729 := P_{10}(1) + P_{10}(3) + P_{10}(21).$
**Hendecagonal (11-gonal):** $P_{11}(n) = n(9n-7)/2 \rightarrow 1729 := P_{11}(1) + P_{11}(9) + P_{11}(18).$

Calculating further values, the exact values are for 12-gonal, 24-gonal and 84-gonal. See below:

**12-gonal:** $P_{12}(n) = n(5n-4) \rightarrow 1729 := P_{12}(19).$
**24-gonal:** $P_{24}(n) = n(11n-10) \rightarrow 1729 := P_{24}(13).$
**84-gonal:** $P_{84}(n) = n(41n-40) \rightarrow 1729 := P_{84}(7).$

Interestingly, 7, 13 and 19 are the *multiplicative factors* of 1729.

In case of *decagonal (10-gonal)*, Ramanujan proved that

$$1^3 + 3^3 \times \frac{n-1}{n+1} + 5^3 \times \frac{(n-1)(n-2)}{(n+1)(n+2)} + 7^3 \times \frac{(n-1)(n-2)(n-3)}{(n+1)(n+2)(n+3)} + ... = n(4n-3).$$

*https://oeis.org/A001107*

# 12 Special Value Numbers

## 12.1 Carmichael number

This section deals with the value of 1729 in different forms. These values are obtained in different situations with different kind of sequence values. All the values given here are already study previous by different authors as specified in each subsection.

Considering Carmichael (Charmick, 1939) numbers of the form

$$(6n+1)(12n+1)(18n+1).$$

where $6n+1$, $12n+1$ and $18n+1$ are primes, then

$$7 \times 13 \times 19 = 1729.$$

*J. Chernick (1939), On Fermat's simple theorem, Bull. Amer. Math. Soc.* ***45:*** *269-274.*

*https://oeis.org/A033502*

Also, Carmichael numbers of the form

$$(6n+1)(12n+1)(18n+1)$$

are known as *Zeisel numbers*.

*https://oeis.org/A051015.*

## 12.2 Centered cube number

Centered cube numbers are defined as

$$C(n) := n^3 + (n+1)^3 = (2n+1) \times (n^2+n+1), \ n \geq 0.$$

This gives

$$C(9) := 9^3 + 10^3 = 1729$$

*https://oeis.org/A005898*

## 12.3 Generalized Heptagonal (7-gonal)

Generalized heptagonal number is defined as

$$G_7(n) = n(5n-3)/2, \ n = 0, \pm 1, \pm 2, ...$$

This gives

$$G_7(-6) := 1729.$$

*https://oeis.org/A085787*

### 12.4 Third Spoke of a Hexagonal Spiral

Third spoke of a hexagonal spiral is defined as

$$M(n) := 3n^2 + 1, \ \ n = 0, 1, 2, ...$$

This gives

$$M(24) := 1729$$

*https://oeis.org/A056107*

## 13 Stair-Type Formula

This sections deals with triangular values written in such way that it brings sum in deslocating sequences after each two values, and then summing them. For all $n > 2, S(1) = 0, S(2) = 1$, we have

$$\begin{aligned} S(n) &:= S(n-2) + \frac{n(n-1)}{2} \\ &:= \frac{4n^3 + 6n^2 - 4n + 3(-1)^n - 3}{48} \end{aligned}$$

This gives

$$S(27) := 1729$$

The "*stair-type values*" are given by

| | | | | | |
|---|---|---|---|---|---|
| 1 | | | | | 1 |
| 3 | | | | | 3 |
| 6 | 1 | | | | 7 |
| 10 | 3 | | | | 13 |
| 15 | 6 | 1 | | | 22 |
| 21 | 10 | 3 | | | 34 |
| 28 | 15 | 6 | 1 | | 50 |
| 36 | 21 | 10 | 3 | | 70 |
| 45 | 28 | 15 | 6 | 1 | 95 |
| ... | ... | ... | ... | ... | ... |

*https://oeis.org/A002623*

This idea of *stair-type* numbers can be applied to other functions given in Section 11.2. This shell be dealt elsewhere.

# 14 Selfie-Fractions

Selfie numbers are understood as numbers having their representations with same digits or reverse with some operations. For examples,

$$26364 = 26^3 \times 6/4.$$
$$34425 = 3^4 \times 425.$$
$$35721 = 3^5 \times 7 \times 21.$$

The above examples are with multiplication, division and exponential. There are lot of numbers this kind with more operations. See the reference at the end of this work. The same kind of idea can be applied to fractions, where then numerators and denominators are with same digits as of fractions separated by basic operations. Below are some examples containing the numbers 1729 either in numerator or in denominator.

- $\frac{364}{1729} = \frac{(3+6)\times 4}{(17+2)\times 9}$.
- $\frac{546}{1729} = \frac{(5+4)\times 6}{(17+2)\times 9}$.
- $\frac{1729}{4368} = \frac{1+7+2+9}{4+36+8}$.
- $\frac{1729}{8463} = \frac{1+7+2+9}{84+6+3}$.
- $\frac{1729}{6384} = \frac{1+7\times(2+9)}{(6+3)\times 8\times 4}$.
- $\frac{1729}{58604} = \frac{1+7+2+9}{5\times 8+604}$.

*http://rgmia.org/papers/v19/v19a113.pdf*
*http://rgmia.org/papers/v19/v19a114.pdf*
*http://rgmia.org/papers/v19/v19a115.pdf.*

## 14.1 Equivalent Selfie Fractions

Above we have written only one selife representation for each fraction, but there are many fraction those have more than one "selfie-representation". This we shall call as "equivalent selfie fraction. Below are some of these having the number 1729 in the numerator.

- $\frac{1729}{3458} = \frac{17+29}{34+58} = \frac{17-29}{34-58} = \frac{17+2+9}{3+45+8} = \frac{1+7+2\times 9}{3\times 4+5\times 8}$
  $= \frac{17+2\times 9}{3\times 4+58} = \frac{1+7+29}{34+5\times 8} = \frac{1\times 7\times 29}{(3+4)\times 58} = \frac{1+7\times(2+9)}{3\times 4\times(5+8)}$.
- $\frac{1729}{3640} = \frac{(17+2)\times 9}{(3+6)\times 40} = \frac{1+7+2+9}{36+4+0}$.

- $\dfrac{1729}{5460} = \dfrac{(17+2)\times 9}{(5+4)\times 60} = \dfrac{1+7+2+9}{54+6+0}$.

- $\dfrac{1729}{8645} = \dfrac{1\times 7+2+9}{(8+6+4)\times 5} = \dfrac{1+7+2+9}{86+4+5} = \dfrac{1+7\times 2+9}{8\times(6+4+5)}$
$= \dfrac{(1+7+2)\times 9}{(86+4)\times 5} = \dfrac{1\times 7\times 2\times 9}{(8+6)\times 45} = \dfrac{1^7\times 2^9}{8\times 64\times 5}$.

*http://rgmia.org/papers/v19/v19a116.pdf*
*http://rgmia.org/papers/v19/v19a117.pdf*

# 15 Equivalent Fractions

Above Section give selfie fractions with the property that there are same digits at numerator and denominators. But there are fractions that are equivalent to each other without use of the basic operations such as $\frac{1}{2} = \frac{2}{4}$, $\frac{2}{3} = \frac{6}{9}$, etc. Below are some equivalent fractions with the digits 1729 in the numerator. Initially are few with triple representations and then with double representations.

## 15.1 Triple Representations

- $\dfrac{1729}{38456} = \dfrac{3458}{76912} = \dfrac{4368}{97152}$.
- $\dfrac{1729}{53846} = \dfrac{1267}{39458} = \dfrac{2534}{78916}$.
- $\dfrac{1729}{45836} = \dfrac{1976}{52384} = \dfrac{3458}{91672}$.
- $\dfrac{1729}{308465} = \dfrac{2639}{470815} = \dfrac{5278}{941630}$.

## 15.2 Double Representations

- $\dfrac{1729}{34586} = \dfrac{3458}{69172}$.
- $\dfrac{1729}{43586} = \dfrac{2457}{61938}$.
- $\dfrac{1729}{48356} = \dfrac{3458}{96712}$.
- $\dfrac{1729}{34856} = \dfrac{3458}{69712}$.
- $\dfrac{1729}{45638} = \dfrac{3458}{91276}$.
- $\dfrac{1729}{48563} = \dfrac{3458}{97126}$.
- $\dfrac{1729}{35648} = \dfrac{3458}{71296}$.
- $\dfrac{1729}{45863} = \dfrac{3458}{91726}$.
- $\dfrac{1729}{304586} = \dfrac{3458}{609172}$.
- $\dfrac{1729}{35846} = \dfrac{3458}{71692}$.
- $\dfrac{1729}{46358} = \dfrac{3458}{92716}$.
- $\dfrac{1729}{304856} = \dfrac{3458}{609712}$.
- $\dfrac{1729}{36458} = \dfrac{3458}{72916}$.
- $\dfrac{1729}{46835} = \dfrac{2639}{71485}$.
- $\dfrac{1729}{305486} = \dfrac{3458}{610972}$.
- $\dfrac{1729}{305864} = \dfrac{4921}{870536}$.

- $\frac{1729}{306485} = \frac{3458}{612970}$.
- $\frac{1729}{308546} = \frac{3458}{617092}$.
- $\frac{1729}{308645} = \frac{3458}{617290}$.
- $\frac{1729}{345086} = \frac{3458}{690172}$.
- $\frac{1729}{346085} = \frac{3458}{692170}$.
- $\frac{1729}{348506} = \frac{3458}{697012}$.
- $\frac{1729}{348605} = \frac{3458}{697210}$.
- $\frac{1729}{350648} = \frac{3458}{701296}$.
- $\frac{1729}{350846} = \frac{3458}{701692}$.
- $\frac{1729}{354608} = \frac{3458}{709216}$.
- $\frac{1729}{354806} = \frac{3458}{709612}$.
- $\frac{1729}{356048} = \frac{3458}{712096}$.
- $\frac{1729}{358046} = \frac{3458}{716092}$.
- $\frac{1729}{360458} = \frac{3458}{720916}$.
- $\frac{1729}{360548} = \frac{3458}{721096}$.
- $\frac{1729}{360845} = \frac{3458}{721690}$.
- $\frac{1729}{364508} = \frac{3458}{729016}$.
- $\frac{1729}{364805} = \frac{3458}{729610}$.
- $\frac{1729}{380456} = \frac{3458}{760912}$.
- $\frac{1729}{380546} = \frac{3458}{761092}$.
- $\frac{1729}{380645} = \frac{3458}{761290}$.
- $\frac{1729}{384506} = \frac{3458}{769012}$.
- $\frac{1729}{384605} = \frac{3458}{769210}$.
- $\frac{1729}{450638} = \frac{3458}{901276}$.
- $\frac{1729}{450836} = \frac{3458}{901672}$.
- $\frac{1729}{450863} = \frac{3458}{901726}$.
- $\frac{1729}{453086} = \frac{3458}{906172}$.
- $\frac{1729}{453608} = \frac{3458}{907216}$.
- $\frac{1729}{453806} = \frac{3458}{907612}$.
- $\frac{1729}{456038} = \frac{3458}{912076}$.
- $\frac{1729}{458036} = \frac{3458}{916072}$.
- $\frac{1729}{458603} = \frac{3458}{917206}$.
- $\frac{1729}{460358} = \frac{3458}{920716}$.
- $\frac{1729}{460538} = \frac{3458}{921076}$.
- $\frac{1729}{460835} = \frac{3458}{921670}$.
- $\frac{1729}{460853} = \frac{3458}{921706}$.
- $\frac{1729}{463085} = \frac{3458}{926170}$.
- $\frac{1729}{463508} = \frac{3458}{927016}$.
- $\frac{1729}{463805} = \frac{3458}{927610}$.
- $\frac{1729}{480356} = \frac{3458}{960712}$.
- $\frac{1729}{480536} = \frac{3458}{961072}$.
- $\frac{1729}{480635} = \frac{3458}{961270}$.
- $\frac{1729}{483506} = \frac{3458}{967012}$.
- $\frac{1729}{483605} = \frac{3458}{967210}$.
- $\frac{1729}{485063} = \frac{3458}{970126}$.
- $\frac{1729}{485306} = \frac{3458}{970612}$.
- $\frac{1729}{485603} = \frac{3458}{971206}$.
- $\frac{1729}{486053} = \frac{3458}{972106}$.
- $\frac{1729}{486305} = \frac{3458}{972610}$.
- $\frac{1729}{540683} = \frac{1792}{560384}$.
- $\frac{1729}{568043} = \frac{1092}{358764}$.
- $\frac{1729}{605834} = \frac{1365}{478290}$.
- $\frac{1729}{643058} = \frac{1064}{395728}$.

*http://rgmia.org/papers/v19/v19a148.pdf*
*http://rgmia.org/papers/v19/v19a149.pdf*
*http://rgmia.org/papers/v19/v19a150.pdf.*

## 16 Upside Down and Mirror Looking

It is well known that the digits 0, 1 and 8 are always mirror looking and upside down. The digits 6 and 9 are only upside down but not mirror looking. If we write 2 and 5 in digital ways as appears in lifts, watchs etc., then they becomes upside down and mirror looking. The only difference is that in case of mirror looking 2 becomes 5 and 5 as 2. Some studies in this direction can be seen in author's work. Later Macau Post Office, China produced a stamp on this work.

*https://arxiv.org/ftp/arxiv/papers/1005/1005.1384.pdf*

Below are some representations of 1729 upside down and mirror looking.

### 16.1 Upside Down

$$1729 := 1001 + 619 + 101 + 8.$$

### 16.2 Upside Down and Mirror Looking

$$\begin{aligned} 1729 &:= 1001 + 512 + 215 + 1. \\ &:= 1111 + (1+1+1+1+1) \times (101 + 11 + 11) + 1 + 1 + 1. \end{aligned}$$

*In case of 2 and 5, the numbers are written in digital form. Looking in mirror, 2 becomes 5 and 5 becomes 2.*

*https://arxiv.org/ftp/arxiv/papers/1005/1005.1384.pdf*

## 17 Prime Numbers with Digits 1, 2, 7, 9

Below are prime numbers obtained from the digits of 1729 without repetition.

$$17, 29, 71, 127, 179, 197, 271, 719, 971, 2917, 7219, 9721.$$

## 18 Palindromic Representation

The number 1729271 is palindrome made from the digits of 1729. The representation below is in terms of digits of this palindrome.

$$1729 := 1 + 72 \times (9 \times 2 + 7 - 1).$$

## 19 Multiplicative Factors

It is well known that

$$1729 := 13 \times 133 = 7 \times 13 \times 19.$$

The representations below are in terms of multiplicative factors using the digit in order of 1729 and reverse 9271.

$$\begin{aligned} 1729 &:= (1+7+2+\sqrt{9}) \times ((\sqrt{9})! + 2^7 - 1). \\ &:= ((1+7) \times 2 - 9) \times (1+7+2+\sqrt{9}) \times (1+7+2+9). \end{aligned}$$

## 20 Special Digits

Two date are very important. Once date of birth and second date of death of S. Ramanjuan. The 1729 is written in terms of both of these dates in increasing order.

### 20.1 Birth Day: 22.12.1887

$$1729 := (-2+21) \times (-2-1+8+8) \times 7.$$

### 20.2 Death Day: 26.04.1920

$$1729 := (-(2+60) \times 4 + 1) \times (-9+2+0).$$

## 21 Digits of 2017 and 1729

This work is written in 2017. Below are numbers 1729 and 2017 are written in terms of other, i.e., 1729 in terms of digit of 2017 and 2017 in terms of digits of 1729 both written twice.

$$\begin{aligned} 1729 &:= 2 + 01720 + 1 \times 7. \\ 2017 &:= 1 + 72 \times (-9+1+7+29) \end{aligned}$$

## 22 Numbers From 1 to 99 in Terms of Twice 1729

Below are numbers from 1 to 99 in terms of digits of 1729 used twice, only with the operations *addition, subtraction* and *multiplication*.

$1 := 17 - 29 - 1 + 7 - 2 + 9.$
$2 := 17 - (29 - 17) \times 2 + 9.$
$3 := 1 \times 72 - 91 - 7 + 29.$
$4 := 1 + 72 - 91 - 7 + 29.$
$5 := 17 - 2 - 91 + 72 + 9.$

$6 := 17 \times 2 - 91 + 72 - 9.$
$7 := 1 + 7 + 2 + 9 + 17 - 29.$
$8 := (1 + 7) \times (-2 - 9 - 17 + 29).$
$9 := 17 + 2 - 91 + 72 + 9.$
$10 := -1 + 7 \times 2 + 9 + 17 - 29.$

$11 := 17 + 29 - 17 - 2 \times 9.$
$12 := 17 - 2 + 9 + 17 - 29.$
$13 := 1 \times 7 - 29 - 1 + 7 + 29.$
$14 := 1 + 7 - 29 - 1 + 7 + 29.$
$15 := 17 \times 2 - 9 + 1 + 7 - 2 \times 9.$

$16 := 17 + 2 + 9 + 17 - 29.$
$17 := -17 + 2 + 9 + 1 - 7 + 29.$
$18 := 17 - 2 - 9 - 17 + 29.$
$19 := 17 \times 2 - 9 - 17 + 2 + 9.$
$20 := 17 - 2 \times 9 - 1 - 7 + 29.$

$21 := -1 + 7 \times 2 + 9 + 17 - 2 \times 9.$
$22 := 17 + 29 - 17 + 2 - 9.$
$23 := 17 - 29 + 17 + 2 \times 9.$
$24 := 17 \times 2 - 91 + 72 + 9.$
$25 := 1 + 7 + 29 + 17 - 29.$

$26 := 17 \times 2 - 9 - 17 + 2 \times 9.$
$27 := 17 - 2 - 9 - 1 - 7 + 29.$
$28 := 1 + 7 - 2 \times (9 - 1) + 7 + 29.$
$29 := 17 + 2 + 91 - 72 - 9.$
$30 := (1 - 7 + 2 + 9) \times (17 - 2 - 9).$

$31 := 1 + 7 + 2 + 9 - 17 + 29.$
$32 := 17 - 2 \times (-9 + 1 + 7 - 2) + 9.$
$33 := -17 + 29 - 1 - 7 + 29.$
$34 := 17 + 29 + 17 - 29.$
$35 := 1 - 7 + 29 - 17 + 29.$

$36 := 17 + 29 - 17 - 2 + 9.$
$37 := 1 \times 7 + 2 + 9 + 1 + 7 + 2 + 9.$
$38 := 17 - (2 + 9 - 17) \times 2 + 9.$
$39 := 1 + 7 - 2 + 9 + 1 + 7 \times 2 + 9.$
$40 := 17 + 29 - 17 + 2 + 9.$

$41 := -1 + 7 - 2 - 9 + 17 + 29.$
$42 := 17 + 29 \times 1 + 7 - 2 - 9.$
$43 := 17 - 2 + 91 - 72 + 9.$
$44 := (-17 + 2 \times 9 + 1) \times (-7 + 29).$

$45 := 17 + 2 - 9 + 17 + 2 \times 9.$
$46 := 17 - 2 + 9 \times 1 - 7 + 29.$
$47 := 1 + 72 + 9 + 1 - 7 - 29.$
$48 := (1 + 7) \times (-29 + 17 + 2 \times 9).$
$49 := 17 \times 2 - 9 + 17 - 2 + 9.$
$50 := 17 + (29 - 17) \times 2 + 9.$

$51 := 17 - 29 \times 1 + 72 - 9.$
$52 := 17 + 29 + 17 - 2 - 9.$
$53 := 17 + 2 - 9 + 17 \times 2 + 9.$
$54 := 17 + 2 - 9 - 1 + (7 - 2) \times 9.$
$55 := 17 + 2 \times (9 + 1) + 7 + 2 + 9.$

$56 := 17 + 2 - 9 + 17 + 29.$
$57 := (1 + 7) \times 2 \times (9 - 1 - 7 + 2) + 9.$
$58 := 17 + 29 - 17 + 29.$
$59 := -1 + 7 + 29 + 17 - 2 + 9.$
$60 := 1 + 7 + 29 + 1 - 7 + 29.$

$61 := 1 + 7 - 29 + 1 + 72 + 9.$
$62 := 1 + 7 \times (-2 + 9) - 17 + 29.$
$63 := 1 + 72 - 9 - 1 - 7 - 2 + 9.$
$64 := (1 + 72 - 9) \times (1 + 7 + 2 - 9).$
$65 := 1 + 72 - 9 + 1 + 7 + 2 - 9.$

$66 := 1 + 7 \times (2 + 9) + 17 - 29.$
$67 := -1 - 7 + 29 + 17 + 29.$
$68 := -1 + 7 \times 2 + 9 + 17 + 29.$
$69 := 17 - 29 \times 1 + 72 + 9.$
$70 := 17 + 29 + 17 - 2 + 9.$

$71 := 17 \times 2 - 9 + 17 + 29.$
$72 := 1 - 7 - 2 + 91 + 7 - 2 \times 9.$
$73 := -1 - 7 + 2 \times 9 \times 1 + 72 - 9.$
$74 := 17 + 29 + 17 + 2 + 9.$
$75 := 1 \times 72 - 9 - 17 + 29.$

$76 := 172 - 91 - 7 \times 2 + 9.$
$77 := 1 + (-72 + 91) \times (-7 + 2 + 9).$
$78 := 1 \times 72 + 9 + 1 + 7 - 2 - 9.$
$79 := 1 + 7 - 2 - 9 + 1 + 72 + 9.$
$80 := 1 \times 72 - 9 - 1 + 7 + 2 + 9.$

$81 := 17 + 2 \times 9 + 17 + 29.$
$82 := (1 + 72 + 9) \times (1 + 7 + 2 - 9).$
$83 := 1 + 72 + 9 + 1 + 7 + 2 - 9.$
$84 := 1 \times 72 + 9 - 1 - 7 + 2 + 9.$
$85 := -1 \times 7 + 2 + (-9 + 17 + 2) \times 9.$

$86 := -1 \times 7 + 2 + 9 + 1 + 72 + 9.$
$87 := 1 + 7 \times 2 - 9 \times 1 + 72 + 9.$
$88 := 1 + 72 - 9 - 1 + 7 + 2 \times 9.$
$89 := (-17 + 29) \times (1 + 7) + 2 - 9.$
$90 := (-1 + 7) \times ((29 - 17) \times 2 - 9).$

$91 := 1 \times 72 - 9 + 17 + 2 + 9.$
$92 := 17 + 29 + 17 + 29.$
$93 := 17 + (2 \times 9 + 1) \times (-7 + 2 + 9).$
$94 := 1 + 72 + 9 - 17 + 29.$
$95 := -1 + 7 - 2 + 9 + 1 + 72 + 9.$

$96 := 1 \times 7 - 2 + 91 + 7 + 2 - 9.$
$97 := 1 + 72 + 9 + 1 + 7 - 2 + 9.$
$98 := 17 + 2 \times (9 + 17) + 29.$
$99 := 1 \times 72 - 9 \times 1 + 7 + 29.$
.... ...

Further numbers from 100 onwards can also written in the same way. In some cases we need to use more operations such as division, exponential, factorial, square-root etc.

## 23 Running Expressions: Triangular and Fibonacci

By running expressions we understand as when the numbers are used in a sequence 1 to 9 and 9 to 1 or 9 to 0. The vaules appearing in between are separated by equality sign, for example,

$$\begin{aligned} 120 &:= 1 \times (2+3)! && = 4 + 5!/6 + 7 + 89 \\ &:= 98 + 7 + 6 + 5 + 4 && = (3+2)! \times 1 \\ &:= \sqrt{9} + 87 + 6 \times 5 && = \sqrt{4} \times 3 \times 2 \times 10. \end{aligned}$$

More details can be seen at author's work:

*http://rgmia.org/papers/v18/v18a27.pdf.*

In case of 1729 we are unable to write as above. Extra operations like, Fibonacci sequence and Triangular values are used.

## 23.1 Increasing Order: 1 to 9

$$1729 := -1 + T(-2 + T(T(3))) + T(T(T(4))) = T(5) + 6 + 7 + T(T(8)) + T(T(9)).$$

$$1729 := 12^3 - 4 + 5 \qquad = T(6) + 7 + T(T(8)) + T(T(9)).$$

$$\begin{aligned} 1729 &:= F(12) \times 3 \times 4 - 5 + 6 && = T(7) + T(T(8)) + T(T(9)). \\ 1729 &:= 12^3 - T(4) + 5 + 6 && = T(7) + T(T(8)) + T(T(9)). \end{aligned}$$

## 23.2 Decreasing Order: 9 to 1 or 9 to 0

$$1729 := -9 + T(T(8)) + T(T(7)) + T(T(6) + T(5)) = T(T(T(4))) + T((T(T(3)) - 2)) - 1$$

$$1729 := 98 + 7 \times F(F(6) + 5) \qquad = T(T(T(4))) - T(T(3)) + 210.$$

$$1729 := T(T(9)) + T(T(8)) + 7 + T(6) = 54 \times 32 + 1.$$

$$\begin{aligned} 1729 := T(T(9)) + T(T(8)) + T(7) \qquad &= -6 \times 5 + T(T(4)) \times 32 - 1 \\ &= F(6) \times 54 \times F(3) \times 2 + 1 \\ &= -65 + 43^2 - F(10). \end{aligned}$$

## 23.3 Increasing and Decreasing

$$\begin{aligned} 1729 := T(9) + 8 \times 7 \times 6 \times 5 + 4 &= (F(F(4)) + 5) \times (F(6) + F(F(7))) + 8 + F(9) \\ &= 4 - T(56) + T(T(7) + 8 + T(9). \end{aligned}$$

$$1729 := 12^3 - 4 + 5 = 5! \times T(4) + T(32) + 1.$$

$$\begin{aligned} 1729 := 12^3 + (-4+5)^6 &= T(F(6) + 5 \times T(4)) - 3 + 21. \\ &= (F(6) + 5) \times (T(4! - F(3!)) - 2 - 1 \times 0!). \end{aligned}$$

$$1729 := 123 \times (4 \times 5 - 6) + 7 = F(7) \times (6 + (5 - F(4)) \times 3 \times 21 + 0!).$$

$$1729 := T(12) + 3 - 4 + 56 + T(7 \times 8) = 876 + F(T(5)) + 4 \times 3 + T(21).$$
$$= \sqrt{\sqrt{87 - 6}} + 54 \times 32 - 1 - 0!.$$

$$1729 := 12 - 3 + 4^5 - 6 + 78 \times 9 = (9 \times F(8) + 7 + T(F(6)) \times 5) \times 4 - 3! + T(21).$$
$$= (9 - 8) \times 7 \times (6 + (5 \times 4 \times F(3)) \times (2 + 1)! + 0!).$$

# 24 Selfie Representations

In Sections 11 we have given the idea of selfie numbers and selfie fractions. More detailed study can be seen at author's work given reference list. Here we have written the number 1729 as selfie number by using difference sequence values, such as, *Fibonacci, Triangular, Polygon-type,* etc.

## 24.1 Fibonacci and Triangular

Below are selfie representation of 1729 in terms of Fibonacci sequence and Triangular values in order of digits and reverse.

$$1729 := 1 + (F(7) - F(2))^{\sqrt{9}}.$$
$$:= T(-1 + T(7)) \times T(2) + T(F(9)).$$

$$1729 := 9!/T\left(\sqrt{-T(T(2)) + T(T(7))}\right) + 1.$$
$$:= T(F(9)) + T(2) \times T(T(7) - 1).$$

## 24.2 Polygonal-Type

According to *s-ogonal* values given in subsection 11.2, below are selfie representations of 1729:

### 24.2.1 Digit's Order

$$
\begin{aligned}
1729 &:= 1 \times 7 \times (P_4(P_4(P_4(2))) - 9). \\
&:= 1 + (7 + P_5(2))^{\sqrt{9}}. \\
&:= 1 \times P_7\left(\sqrt{P_7(7) \times P_7(2)}\right) - P_7(9). \\
&:= 1 \times P_8(7) \times \left(-P_8(2) + P_8(\sqrt{9})\right). \\
&:= 1 + 72 \times P_9(\sqrt{9}) \\
&:= -1 - P_{10}(7 + P_{10}(2)) + P_{10}(P_{10}(\sqrt{9})). \\
&:= -1^7 + P_{11}(P_{11}(2) + 9). \\
&:= P_{12}(1 + 7 + 2 + 9). \\
&:= P_{24}(-1 + 7 - 2 + 9). \\
&:= P_{84}((1 + 7) \times 2 - 9).
\end{aligned}
$$

### 24.2.2 Reverse Order of Digits

$$
\begin{aligned}
1729 &:= (-9 + P_4(P_4(P_4(2)))) \times 7 \times 1. \\
&:= P_5(\sqrt{9}) + P_5(P_5(2) \times 7 - 1). \\
&:= -P_7(9) + P_7(27 + 1). \\
&:= P_8(9) \times P_8(2) - 71. \\
&:= P_9(\sqrt{9}) \times (P_9(2))!/7! + 1. \\
&:= P_{10}(P_{10}(\sqrt{9})) - P_{10}(P_{10}(2) + 7) - 1. \\
&:= P_{11}(9) + P_{11}(P_{11}(2) + 7)) + 1. \\
&:= P_{12}(9 + 2 + 7 + 1). \\
&:= P_{24}(9 - 2 + 7 - 1). \\
&:= P_{84}(-9 + 2 \times (7 + 1)).
\end{aligned}
$$

Combining the results given above with given in Subsection 11.2, we have following unified values:

$$
\begin{aligned}
1729 &:= P_{12}(19) = P_{12}(1 + 7 + 2 + 9) \quad = P_{12}(9 + 2 + 7 + 1). \\
&:= P_{24}(13) = P_{24}(-1 + 7 - 2 + 9) = P_{24}(9 - 2 + 7 - 1). \\
&:= P_{84}(7) \ = P_{84}((1 + 7) \times 2 - 9) \ = P_{84}(-9 + 2 \times (7 + 1)).
\end{aligned}
$$

# 25 Prime Numbers Patterns: Fixed Digits Repetitions

This section deals with patterns with prime numbers with 1729. These patterns are considered in such a way that after second prime number there is fixed digit or digits repeats in subsequent primes. But the approach don't go much longer. Below are examples of patterns of 6, 7 and 8 prime numbers, i.e., 6-patterns, 7-patterns and 8-patterns respectively. These patterns are always with with 1929. The detailed work shall be given elsewhere.

## 25.1 6-Patterns

► 261**1729**
261(0)**1729**
261(0)(0)**1729**
261(0)(0)(0)**1729**
261(0)(0)(0)(0)**1729**
261(0)(0)(0)(0)(0)**1729**.

► **172**987
(939)**172**987
(939)(939)**172**987
(939)(939)(939)**172**987
(939)(939)(939)(939)**172**987
(939)(939)(939)(939)(939)**172**987.

► 10**1729**3
(87)10**1729**3
(87)(87)10**1729**3
(87)(87)(87)10**1729**3
(87)(87)(87)(87)10**1729**3
(87)(87)(87)(87)(87)10**1729**3.

► 10**1729**9
1(435)0**1729**9
1(435)(435)0**1729**9
1(435)(435)(435)0**1729**9
1(435)(435)(435)(435)0**1729**9
1(435)(435)(435)(435)(435)0**1729**9.

► 237**1729**
237(99)**1729**
237(99)(99)**1729**
237(99)(99)(99)**1729**
237(99)(99)(99)(99)**1729**
237(99)(99)(99)(99)(99)**1729**.

► 12**1729**9
(861)12**1729**9
(861)(861)12**1729**9
(861)(861)(861)12**1729**9
(861)(861)(861)(861)12**1729**9
(861)(861)(861)(861)(861)12**1729**9.

► **172**973
**1729**7(444)3
**1729**7(444)(444)3
**1729**7(444)(444)(444)3
**1729**7(444)(444)(444)(444)3
**1729**7(444)(444)(444)(444)(444)3.

► 153**1729**
15(348)3**1729**
15(348)(348)3**1729**
15(348)(348)(348)3**1729**
15(348)(348)(348)(348)3**1729**
15(348)(348)(348)(348)(348)3**1729**.

► 153**1729**
153(483)**1729**
153(483)(483)**1729**
153(483)(483)(483)**1729**
153(483)(483)(483)(483)**1729**
153(483)(483)(483)(483)(483)**1729**.

► **1729**237
(954)**1729**237
(954)(954)**1729**237
(954)(954)(954)**1729**237
(954)(954)(954)(954)**1729**237
(954)(954)(954)(954)(954)**1729**237.

► **1729**477
**1729**47(966)7
**1729**47(966)(966)7
**1729**47(966)(966)(966)7
**1729**47(966)(966)(966)(966)7
**1729**47(966)(966)(966)(966)(966)7.

► **1729**493
(519)**1729**493
(519)(519)**1729**493
(519)(519)(519)**1729**493
(519)(519)(519)(519)**1729**493
(519)(519)(519)(519)(519)**1729**493.

► **1729**543
**1729**(783)543
**1729**(783)(783)543
**1729**(783)(783)(783)543
**1729**(783)(783)(783)(783)543
**1729**(783)(783)(783)(783)(783)543.

► **1729**723
**1729**(327)723
**1729**(327)(327)723
**1729**(327)(327)(327)723
**1729**(327)(327)(327)(327)723
**1729**(327)(327)(327)(327)(327)723.

► **1729**901
(330)**1729**901
(330)(330)**1729**901
(330)(330)(330)**1729**901
(330)(330)(330)(330)**1729**901
(330)(330)(330)(330)(330)**1729**901.

► **1729**909
**1729**90(663)9
**1729**90(663)(663)9
**1729**90(663)(663)(663)9
**1729**90(663)(663)(663)(663)9
**1729**90(663)(663)(663)(663)(663)9.

► 2**1729**01
2(576)**1729**01
2(576)(576)**1729**01
2(576)(576)(576)**1729**01
2(576)(576)(576)(576)**1729**01
2(576)(576)(576)(576)(576)**1729**01.

► 2**1729**79
(789)2**1729**79
(789)(789)2**1729**79
(789)(789)(789)2**1729**79
(789)(789)(789)(789)2**1729**79
(789)(789)(789)(789)(789)2**1729**79.

► 32**1729**7
32(450)**1729**7
32(450)(450)**1729**7
32(450)(450)(450)**1729**7
32(450)(450)(450)(450)**1729**7
32(450)(450)(450)(450)(450)**1729**7.

► 36**1729**3
36**1729**(183)3
36**1729**(183)(183)3
36**1729**(183)(183)(183)3
36**1729**(183)(183)(183)(183)3
36**1729**(183)(183)(183)(183)(183)3.

▶ 39**1729**7
(534)39**1729**7
(534)(534)39**1729**7
(534)(534)(534)39**1729**7
(534)(534)(534)(534)39**1729**7
(534)(534)(534)(534)(534)39**1729**7.

▶ 56**1729**1
56**1729**1(363)
56**1729**1(363)(363)
56**1729**1(363)(363)(363)
56**1729**1(363)(363)(363)(363)
56**1729**1(363)(363)(363)(363)(363).

▶ 58**1729**3
(840)58**1729**3
(840)(840)58**1729**3
(840)(840)(840)58**1729**3
(840)(840)(840)(840)58**1729**3
(840)(840)(840)(840)(840)58**1729**3.

▶ 604**1729**
604**1729**(579)
604**1729**(579)(579)
604**1729**(579)(579)(579)
604**1729**(579(579)(579)(579))
604**1729**(579)(579)(579)(579)(579).

▶ 6**1729**27
6**1729**(723)27
6**1729**(723)(723)27
6**1729**(723)(723)(723)27
6**1729**(723)(723)(723)(723)27
6**1729**(723)(723)(723)(723)(723)27.

▶ 621**1729**
621(219)**1729**
621(219)(219)**1729**
621(219)(219)(219)**1729**
621(219)(219)(219)(219)**1729**
621(219)(219)(219)(219)(219)**1729**.

▶ 691729
6(654)91729
6(654)(654)91729
6(654)(654)(654)91729
6(654)(654)(654)(654)91729
6(654)(654)(654)(654)(654)91729.

▶ 691729
691729(948)
691729(948)(948)
691729(948)(948)(948)
691729(948)(948)(948)(948)
691729(948)(948)(948)(948)(948).

▶ 691729
691729(489)
691729(489)(489)
691729(489)(489)(489)
691729(489)(489)(489)(489)
691729(489)(489)(489)(489)(489).

▶ 7051729
70(186)51729
70(186)(186)51729
70(186)(186)(186)51729
70(186)(186)(186)(186)51729
70(186)(186)(186)(186)(186)51729.

▶ 761729
7(963)61729
7(963)(963)61729
7(963)(963)(963)61729
7(963)(963)(963)(963)61729
7(963)(963)(963)(963)(963)61729.

▶ 8101729
810(660)1729
810(660)(660)1729
810(660)(660)(660)1729
810(660)(660)(660)(660)1729
810(660)(660)(660)(660)(660)1729.

► 8**1729**11
8**1729**1(840)1
8**1729**1(840)(840)1
8**1729**1(840)(840)(840)1
8**1729**1(840)(840)(840)(840)1
8**1729**1(840)(840)(840)(840)(840)1.

► 8**617291**
(588)8**617291**
(588)(588)8**617291**
(588)(588)(588)8**617291**
(588)(588)(588)(588)8**617291**
(588)(588)(588)(588)(588)8**617291**.

► 9**172957**
9**172957**(603)
9**172957**(603)(603)
9**172957**(603)(603)(603)
9**172957**(603)(603)(603)(603)
9**172957**(603)(603)(603)(603)(603).

► **172973**
(6891)**172973**
(6891)(6891)**172973**
(6891)(6891)(6891)**172973**
(6891)(6891)(6891)(6891)**172973**
(6891)(6891)(6891)(6891)(6891)**172973**.

► 55**1729**
5(3036)5**1729**
5(3036)(3036)5**1729**
5(3036)(3036)(3036)5**1729**
5(3036)(3036)(3036)(3036)5**1729**
5(3036)(3036)(3036)(3036)(3036)5**1729**.

► 61**1729**
61**1729**(8151)
61**1729**(8151)(8151)
61**1729**(8151)(8151)(8151)
61**1729**(8151)(8151)(8151)(8151)
61**1729**(8151)(8151)(8151)(8151)(8151).

► 8**17291**
8(9354)**17291**
8(9354)(9354)**17291**
8(9354)(9354)(9354)**17291**
8(9354)(9354)(9354)(9354)**17291**
8(9354)(9354)(9354)(9354)(9354)**17291**.

► 93**1729**
93(3288)**1729**
93(3288)(3288)**1729**
93(3288)(3288)(3288)**1729**
93(3288)(3288)(3288)(3288)**1729**
93(3288)(3288)(3288)(3288)(3288)**1729**.

## 25.2 7-Patterns

► 225**1729**
225(0)**1729**
225(0)(0)**1729**
225(0)(0)(0)**1729**
225(0)(0)(0)(0)**1729**
225(0)(0)(0)(0)(0)**1729**
225(0)(0)(0)(0)(0)(0)**1729**.

► 61**1729**
61**1729**(81)
61**1729**(81)(81)
61**1729**(81)(81)(81)
61**1729**(81)(81)(81)(81)
61**1729**(81)(81)(81)(81)(81)
61**1729**(81)(81)(81)(81)(81)(81).

► 732**1729**7
7(15)32**1729**7
7(15)(15)32**1729**7
7(15)(15)(15)32**1729**7
7(15)(15)(15)(15)32**1729**7
7(15)(15)(15)(15)(15)32**1729**7
7(15)(15)(15)(15)(15)(15)32**1729**7.

► 267**1729**
267(714)**1729**
267(714)(714)**1729**
267(714)(714)(714)**1729**
267(714)(714)(714)(714)**1729**
267(714)(714)(714)(714)(714)**1729**
267(714)(714)(714)(714)(714)(714)**1729**.

► 3**1729**07
3(615)**1729**07
3(615)(615)**1729**07
3(615)(615)(615)**1729**07
3(615)(615)(615)(615)**1729**07
3(615)(615)(615)(615)(615)**1729**07
3(615)(615)(615)(615)(615)(615)**1729**07.

► 501**1729**
501(222)**1729**
501(222)(222)**1729**
501(222)(222)(222)**1729**
501(222)(222)(222)(222)**1729**
501(222)(222)(222)(222)(222)**1729**
501(222)(222)(222)(222)(222)(222)**1729**.

► 549**1729**
5(423)49**1729**
5(423)(423)49**1729**
5(423)(423)(423)49**1729**
5(423)(423)(423)(423)49**1729**
5(423)(423)(423)(423)(423)49**1729**
5(423)(423)(423)(423)(423)(423)49**1729**.

► 549**1729**
54(234)9**1729**
54(234)(234)9**1729**
54(234)(234)(234)9**1729**
54(234)(234)(234)(234)9**1729**
54(234)(234)(234)(234)(234)9**1729**
54(234)(234)(234)(234)(234)(234)9**1729**.

► 565**1729**
5(126)65**1729**
5(126)(126)65**1729**
5(126)(126)(126)65**1729**
5(126)(126)(126)(126)65**1729**
5(126)(126)(126)(126)(126)65**1729**
5(126)(126)(126)(126)(126)(126)65**1729**.

► 631729**3**
631**1729**(897)3
631**1729**(897)(897)3
631**1729**(897)(897)(897)3
631**1729**(897)(897)(897)(897)3
631**1729**(897)(897)(897)(897)(897)3
631**1729**(897)(897)(897)(897)(897)(897)3.

### 25.3 8-Patterns

► 13**1729**83
1(6)3**1729**83
1(6)(6)3**1729**83
1(6)(6)(6)3**1729**83
1(6)(6)(6)(6)3**1729**83
1(6)(6)(6)(6)(6)3**1729**83
1(6)(6)(6)(6)(6)(6)3**1729**83
1(6)(6)(6)(6)(6)(6)(6)3**1729**83.

► 78**1729**93
78(768)**1729**93
78(768)(768)**1729**93
78(768)(768)(768)**1729**93
78(768)(768)(768)(768)**1729**93
78(768)(768)(768)(768)(768)**1729**93
78(768)(768)(768)(768)(768)(768)**1729**93
78(768)(768)(768)(768)(768)(768)(768)**1729**93.

► 78**1729**93
7(876)8**1729**93
7(876)(876)8**1729**93
7(876)(876)(876)8**1729**93
7(876)(876)(876)(876)8**1729**93
7(876)(876)(876)(876)(876)8**1729**93
7(876)(876)(876)(876)(876)(876)8**1729**93
7(876)(876)(876)(876)(876)(876)(876)8**1729**93.

► 90391**729**
(141)90391**729**
(141)(141)90391**729**
(141)(141)(141)90391**729**
(141)(141)(141)(141)90391**729**
(141)(141)(141)(141)(141)90391**729**
(141)(141)(141)(141)(141)(141)90391**729**
(141)(141)(141)(141)(141)(141)(141)90391**729**.

## 26 Embedded Palindromic Prime Numbers

Embedded or Nested palindromic prime numbers are well known in literature (ref. *https://oeis.org/A158089*). In this section we have given embedded palindromic numbers in such a way that it has the number 1729. The study is done in two different ways. One considering all the digits, and second considering only the digits 1, 7, 2 and 9. In both the situations the we have the number 1729.

### 26.1 All Digits With 1729

131
11311
92711311**1729**
339271131**1729**33
9339271131**1729**339.

131
71317
1592713**1729**51
31592713**1729**513
331592713**1729**5133.

191
71917
3092719**1729**03
73092719**1729**037
973092719**1729**0379.

191
71917
79271**9172**97
1977927**9172**97791
11977927**9172**977911.

191
71917
79271**9172**97
9279271**9172**9729
79279271**9172**97297.

191
71917
79271**9172**97
9279271**9172**9729
99279271**9172**97299.

191
9919199
72991919927
17**1729**91919927171
117**1729**919199271711.

313
93139
7299931399927
1**1729**993139992711
11**1729**9931399927111.

313
93139
9931399
1**1729**993139992711
11**1729**9931399927111.

313
93139
9931399
7299931399927
1**1729**993139992711
11**1729**9931399927111.

313
9931399
7299931399927
1**1729**993139992711
11**1729**9931399927111.

787
194787491
71947874917
927194787491**1729**
9927194787491**1729**9.

797
17971
711797117
19927117971**1729**91
119927117971**1729**911.

797
77977
9779779
7**1729**7797792717
17**1729**77977927171.

797
77977
9779779
7**1729**7797792717
97**1729**77977927179.

93139
9931399
7299931399927
1**1729**993139992711
11**1729**9931399927111.

93239
9932399
199323991
927199323991**729**
1192719932399**1729**11.

97379
729737927
**17297**379271
3**17297**3792713
113**17297**379271311.

97379
729737927
**17297**379271
3**17297**3792713
753**17297**379271357.

97379
729737927
**17297**379271
3**17297**3792713
783**17297**379271387.

97379
729737927
**17297**379271
3**17297**3792713
903**17297**379271309.

97379
729737927
**1729**7379271
92**1729**737927129
992**1729**7379271299.

97379
729737927
**1729**7379271
95**1729**737927159
995**1729**7379271599.

729272927
**1729**2729271
3**1729**27292713
93**1729**272927139
1193**1729**27292713911.

927**1729**
11927**1729**11
311927**1729**113
1311927**1729**1131
11131192**1729**113111.

313
93139
9931399
7299931399927
1**1729**993139992711
11**1729**9931399927111.

## 26.2 Digits 1, 7, 2, 9 and 1729

927**1729**
912927**1729**219
9291292**1729**21929
19291292**1729**219291
111929129**1729**21929111.

927**1729**
77927**1729**77
917792**1729**7719
7791779**1729**771977.

927**1729**
92927**1729**29
19292**1729**291
119192927**1729**291911.

927**1729**
11792**1729**711
797117927**1729**711797
9797117927**1729**7117979.

927**1729**
19292**1729**291
119192927**1729**291911
9797117927**1729**7117979.

927**1729**
97292**1729**279
997292**1729**2799
1199729**1729**279911.

927**1729**
191992**1729**9191
12191992**1729**919121
712191992**1729**9191217.

729272927
**1729**2729271
12**1729**272927121
112712**1729**2729271217211.

729272927
**1729**2729271
77**1729**272927177
12177**1729**272927177121.

**1729**2129271
77**1729**212927177
777**1729**2129271777
92777**1729**212927177729.

**1729**9299271
9**1729**92992719
719**1729**9299271917
1719**1729**92992719171.

72997979927
**1729**979799271
11**1729**97979927111
7211**1729**9797992711127.

99**1729**27199
999**1729**271999
99999**1729**27199999
1999999**1729**2719999991.

79271117297
997927111729799
12997927111729799 21
1212997927111729 7992121.

927**1729**
11927**1729**11
1197119271729117911

927**1729**
927**1729**27**1729**
12992**1729**27**1729**921.

927**1729**
122192**1729**1221
9912219**1729**122199

**1729**9299271
9**1729**92992719
129**1729**9299271921.

927**1729**
1979927**1729**9791
191979927**1729**979191.

77927**1729**77
9177927**1729**7719
7791779**1729**771977.

971222179
7297122217927
1**1729**712221792711.

92927**1729**29
1929**1729**291
11919292**1729**291911.

11927**1729**11
9711927**1729**1179
119711927**1729**117911.

92711911**1729**
119271191**1729**11
1771192711911**1729**11771.

**1729**2729271
12**1729**272927121
112712**1729**2729271217211.

79271917297
1279271**1729**721
11112792719**1729**721111.

## 27 Palindromic Prime Numbers

Palindromic prime numbers are well known in the literature. Below are some palindromic prime numbers using only the digits of 1729. The number 1729 appears twice in first subsection and trice in second subsection. Obviously, there are much more numbers with 1729, below are written only few.

### 27.1 Double 1729

927**1729**27**1729**

1299271729271729921
772927**1729**27**1729**277
9729271729271729279
92292717292717292229

11779271729271729711
1122927**1729**27**1729**2211
1717927**1729**27**1729**7171
1719927**1729**27**1729**9171
17719271729271729**1**771

19929271729271729**2**991
7192927**1729**27**1729**2917
7777927**1729**27**1729**7777
7779927**1729**27**1729**9777
7722927**1729**27**1729**2277
7272927**1729**27**1729**2727

7221927**1729**27**1729**1227
7921927**1729**27**1729**1297
9111927**1729**27**1729**1119
9721927**1729**27**1729**1279
9277927**1729**27**1729**7729
9991927**1729**27**1729**1999

1117992**1729**27**1729**97111
1119792**1729**27**1729**79111
1121292**1729**27**1729**21211
1122992**1729**27**1729**92211
1719792**1729**27**1729**79171

1772292**1729**27**1729**22771
1217792**1729**27**1729**77121
1212192**1729**27**1729**12121
1272792**1729**27**1729**72721
1227992**1729**27**1729**97221

**1729172**9127772192719271
**1729172**9217171292719271
**1729172**9229792292719271
**1729172**9279797292719271
**1729172**9727172792719271

**1729172**9799199792719271
**1729172**9911911992719271
**1729172**9927772992719271
**1729172**9977277992719271
**1729172**9991719992719271

**1729117**2919291927119271
**1729117**2922922927119271
**1729117**2992729927119271
**1729217**2911111927129271
**1729217**2912921927129271

**1729217**2929292927129271
**1729217**2999999927129271
**1729717**2929792927179271
**1729717**2977177927179271
**1729917**2929292927199271

**1729111729**9999271119271
**1729191729**2729271919271
**1729191729**7279271919271
**1729291729**1119271929271
**1729711729**9999271179271

**1729771729**1119271779271
**1729771729**7979271779271
**1729911729**9199271199271
**1729971729**1219271799271
**1729991729**1119271999271

**1729991729**1219271999271
**1729991729**7779271999271
**1729117172**9992717119271
**1729121172**9192711219271
**1729171172**9792711719271

**1729172172**9792712719271
**1729192172**9192712919271
**1729222172**9992712229271
**1729711172**9792711179271
**1729712172**9792712179271

**1729727172**9192717279271
**1729799172**9192719979271
**1729971172**9292711799271
**1729979172**9792719799271
**1729129917**2927199219271

**1729221172**9271112279271
**1729221917**2927191229271
**1729722117**2927112279271
**1729771217**2927121779271
**1729771917**2927191779271

**1729792917**2927192979271
**1729797717**2927177979271
**1729111292717**2921119271
**1729212292717**2922129271
**1729277192717**2917729271

**1729**2912927**1729**21929271
**1729**2917927**1729**71929271
**1729**7199927**1729**99179271
**1729**7229927**1729**92279271
**1729**7711927**1729**11779271

**1729**1199271217**1729**9119271
**1729**1299271117**1729**9219271
**1729**1999271217**1729**9919271
**1729**2919271917**1729**1929271
**1729**7129271717**1729**2179271

**1729**7299271717**1729**9279271
**1729**7929271717**1729**2979271
**1729**9229271117**1729**2299271
**1729**9729271917**1729**2799271
**1729**1992711911**1729**919271
**1729**2292712721**1729**229271

**1729**2292717271**1729**229271
**1729**2792711111**1729**729271
**1729**2792717271**1729**729271
**1729**7792717171**1729**779271
**1729**7792717971**1729**779271

**1729**9192719291**1729**199271
**1729**9792711211**1729**799271
**1729**9992717271**1729**999271
**1729**1927171117**1729**19271
**1729**1927172727**1729**19271

**1729**2927129292**1729**29271
**1729**2927177177**1729**29271
**1729**2927192229**1729**29271
**1729**7927127172**1729**79271
**1729**7927172227**1729**79271

**1729**7927179997**1729**79271
**1729**9927197279**1729**99271
**1729**9927197779**1729**99271
**1729**9271111111**1729**9271
**1729**9271121712**1729**9271

**1729**9271199799**1729**9271
**1729**9271217171**1729**9271
**1729**9271221912**1729**9271
**1729**9271299999**1729**9271
**1729**9271779997**1729**9271

**1729**9271791219**1729**9271
**1729**9271792929**1729**9271
**1729**9271921712**1729**9271
**1729**9271972729**1729**9271
**1729**9271979797**1729**9271

**1729**2711117171**1729**271
**1729**2711122122**1729**271
**1729**2711179797**1729**271
**1729**2711212921**1729**271
**1729**2711221912**1729**271

**1729**2711711211**1729**271
**1729**2711719791**1729**271
**1729**2711721212**1729**271
**1729**2711779997**1729**271
**1729**2711911111**1729**271

**1729**2711912121**1729**271
**1729**2711917971**1729**271
**1729**2711992929**1729**271
**1729**2712111211**1729**271
**1729**2712112721**1729**271

**1729**2712122222**1729**271
**1729**2712171217**1729**271
**1729**2712172127**1729**271
**1729**2712177977**1729**271
**1729**2712199199**1729**271

**1729**2712222222**1729**271
**1729**2712297279**1729**271
**1729**2712719791**1729**271
**1729**2712722222**1729**271
**1729**2712727772**1729**271

**1729**2712729992**1729**271
**1729**2712777277**1729**271
**1729**2712779797**1729**271
**1729**2712797279**1729**271
**1729**2712799199**1729**271

**1729**2712997779**1729**271
**1729**2717121212**1729**271
**1729**2717127972**1729**271
**1729**2717179197**1729**271
**1729**2717217271**1729**271

**1729**2717721212**1729**271
**1729**2717729792**1729**271
**1729**2717771217**1729**271
**1729**2717917171**1729**271
**1729**2717992729**1729**271

**1729**2719129992**1729**271
**1729**2719192929**1729**271
**1729**2719199199**1729**271
**1729**2719291119**1729**271
**1729**2719297779**1729**271

**1729**2719297929**1729**271
**1729**2719299999**1729**271
**1729**2719711111**1729**271
**1729**2719711911**1729**271

**1729**2719729192**1729**271
**1729**2719791219**1729**271
**1729**2719919191**1729**271
**1729**2719921112**1729**271

### 27.2 Triple 1729

**1729**27**1729**9799271**1729**271
**1729**271**1729**792711**1729**271
**1729**9927**1729**27**1729**99271

## 28 Palindromic-Type: Multiplications

Palindromic numbers are famous by their property that when reading in reverse order the number remains the same. This deals with the numbers such that they are non palindromic but are separated by multiplication. These separations are in such a way

that if we remove the operations the remaining numbers turns palindromic. These are called as "palindromic-type". Below are palindromic-type number with 1729 only on the one side of the expression. obviously, there are much more numbers of this kind. Only few are written.

$$\mathbf{1729}821 \times 128\mathbf{9271} = 1183881 \times 1883811.$$

$$2\mathbf{1729}12 \times 21\mathbf{9271}2 = 1163904 \times 4093611.$$
$$2\mathbf{1729}06 \times 60\mathbf{9271}2 = 3091824 \times 4281903.$$

$$\mathbf{1729}9221 \times 1229\mathbf{9271} = 11293881 \times 18839211.$$
$$\mathbf{1729}9921 \times 1299\mathbf{9271} = 11839981 \times 18993811.$$
$$1\mathbf{1729}622 \times 226\mathbf{9271}1 = 12773802 \times 20837721.$$
$$2\mathbf{1729}162 \times 261\mathbf{9271}2 = 13291824 \times 42819231.$$
$$2\mathbf{1729}232 \times 232\mathbf{9271}2 = 12363904 \times 40936321.$$
$$2\mathbf{1729}264 \times 462\mathbf{9271}2 = 23491824 \times 42819432.$$
$$2\mathbf{1729}344 \times 443\mathbf{9271}2 = 23563904 \times 40936532.$$
$$2\mathbf{1729}366 \times 663\mathbf{9271}2 = 33691824 \times 42819633.$$
$$2\mathbf{1729}456 \times 654\mathbf{9271}2 = 34763904 \times 40936743.$$
$$6\mathbf{1729}332 \times 233\mathbf{9271}6 = 35273904 \times 40937253.$$
$$6\mathbf{1729}353 \times 353\mathbf{9271}6 = 35273916 \times 61937253.$$
$$2\mathbf{1729}568 \times 865\mathbf{9271}2 = 40936954 \times 45963904.$$
$$2\mathbf{1729}468 \times 864\mathbf{9271}2 = 42819834 \times 43891824.$$
$$6\mathbf{1729}164 \times 461\mathbf{9271}6 = 35273808 \times 80837253.$$
$$6\mathbf{1729}374 \times 473\mathbf{9271}6 = 35273928 \times 82937253.$$
$$22\mathbf{1729}13 \times 31\mathbf{9271}22 = 11251926 \times 62915211.$$
$$25\mathbf{1729}33 \times 33\mathbf{9271}52 = 13361936 \times 63916331.$$
$$33\mathbf{1729}23 \times 32\mathbf{9271}33 = 30707553 \times 35570703.$$
$$36\mathbf{1729}43 \times 34\mathbf{9271}63 = 33708473 \times 37480733.$$
$$90\mathbf{1729}81 \times 18\mathbf{9271}09 = 17208199 \times 99180271.$$
$$180\mathbf{1729}7 \times 7\mathbf{9271}081 = 19817027 \times 72071891.$$
$$262\mathbf{1729}8 \times 8\mathbf{9271}262 = 28836118 \times 81163882.$$
$$344\mathbf{1729}9 \times 9\mathbf{9271}443 = 37855209 \times 90255873.$$

$$\mathbf{1729}93221 \times 12239\mathbf{9271} = 112393881 \times 188393211.$$

$$1\mathbf{1729}1351 \times 1531\mathbf{9271}1 = 126708141 \times 141807621.$$
$$1\mathbf{1729}2472 \times 2742\mathbf{9271}1 = 126709352 \times 253907621.$$
$$1\mathbf{1729}8887 \times 7888\mathbf{9271}1 = 128898717 \times 717898821.$$
$$1\mathbf{1729}7231 \times 1327\mathbf{9271}1 = 121937721 \times 127739121.$$
$$1\mathbf{1729}7711 \times 1177\mathbf{9271}1 = 107287821 \times 128782701.$$
$$2\mathbf{1729}5241 \times 1425\mathbf{9271}2 = 130936632 \times 236639031.$$
$$12\mathbf{1729}642 \times 246\mathbf{9271}21 = 145781602 \times 206187541.$$
$$10\mathbf{1729}853 \times 358\mathbf{9271}01 = 110785923 \times 329587011.$$
$$11\mathbf{1729}654 \times 456\mathbf{9271}11 = 121675914 \times 419576121.$$
$$12\mathbf{1729}455 \times 554\mathbf{9271}21 = 132565905 \times 509565231.$$
$$22\mathbf{1729}331 \times 133\mathbf{9271}22 = 112519362 \times 263915211.$$
$$23\mathbf{1729}031 \times 130\mathbf{9271}32 = 111618172 \times 271816111.$$
$$25\mathbf{1729}541 \times 145\mathbf{9271}52 = 133619472 \times 274916331.$$
$$34\mathbf{1729}011 \times 110\mathbf{9271}43 = 101707273 \times 372707101.$$
$$81\mathbf{1729}821 \times 128\mathbf{9271}18 = 117218298 \times 892812711.$$
$$114\mathbf{1729}12 \times 21\mathbf{9271}411 = 116390512 \times 215093611.$$
$$115\mathbf{1729}02 \times 20\mathbf{9271}511 = 105171922 \times 229171501.$$
$$167\mathbf{1729}52 \times 25\mathbf{9271}761 = 147518392 \times 293815741.$$
$$168\mathbf{1729}42 \times 24\mathbf{9271}861 = 147708382 \times 283807741.$$
$$1033\mathbf{1729}4 \times 4\mathbf{9271}3301 = 123937014 \times 410739321.$$
$$1081\mathbf{1729}7 \times 7\mathbf{9271}1801 = 118917027 \times 720719811.$$
$$1163\mathbf{1729}8 \times 8\mathbf{9271}3611 = 127936118 \times 811639721.$$
$$1245\mathbf{1729}9 \times 9\mathbf{9271}5421 = 136955209 \times 902559631.$$
$$11138\mathbf{9271} \times \mathbf{1729}83111 = 102283881 \times 188382201.$$

# 29 Palindromic-Type: Addition and Multiplication

This section deals with palindromic-type numbers in little different form. In the above section we worked with the operation multiplication. Here both addition and multiplications are used. These operations are used in such way that we have 1729 on both sides of the expression.

## 29.1 Sequential Patterns: 1729 Both Sides

Even though there are much more values we have considered only few, where 1729 appears in both sides and forming a sequential pattern.

$$\mathbf{1729}1 \times 10001 + 10001 \times 1\mathbf{9271} = \mathbf{1729}27291 + 1927\mathbf{29271}.$$
$$\mathbf{1729}2 \times 10001 + 10001 \times 2\mathbf{9271} = \mathbf{1729}37292 + 2927\mathbf{39271}.$$
$$\mathbf{1729}3 \times 10001 + 10001 \times 3\mathbf{9271} = \mathbf{1729}47293 + 3927\mathbf{49271}.$$
$$\mathbf{1729}4 \times 10001 + 10001 \times 4\mathbf{9271} = \mathbf{1729}57294 + 4927\mathbf{59271}.$$
$$\mathbf{1729}5 \times 10001 + 10001 \times 5\mathbf{9271} = \mathbf{1729}67295 + 5927\mathbf{69271}.$$
$$\mathbf{1729}6 \times 10001 + 10001 \times 6\mathbf{9271} = \mathbf{1729}77296 + 6927\mathbf{79271}.$$
$$\mathbf{1729}7 \times 10001 + 10001 \times 7\mathbf{9271} = \mathbf{1729}87297 + 7927\mathbf{89271}.$$
$$\mathbf{1729}8 \times 10001 + 10001 \times 8\mathbf{9271} = \mathbf{1729}97298 + 8927\mathbf{99271}.$$

$$\mathbf{17290}1 \times 100001 + 100001 \times 10\mathbf{9271} = \mathbf{17290}272901 + 1092720\mathbf{9271}.$$
$$\mathbf{17290}2 \times 100001 + 100001 \times 20\mathbf{9271} = \mathbf{17290}372902 + 2092730\mathbf{9271}.$$
$$\mathbf{17290}3 \times 100001 + 100001 \times 30\mathbf{9271} = \mathbf{17290}472903 + 3092740\mathbf{9271}.$$
$$\mathbf{17290}4 \times 100001 + 100001 \times 40\mathbf{9271} = \mathbf{17290}572904 + 4092750\mathbf{9271}.$$
$$\mathbf{17290}5 \times 100001 + 100001 \times 50\mathbf{9271} = \mathbf{17290}672905 + 5092760\mathbf{9271}.$$
$$\mathbf{17290}6 \times 100001 + 100001 \times 60\mathbf{9271} = \mathbf{17290}772906 + 6092770\mathbf{9271}.$$
$$\mathbf{17290}7 \times 100001 + 100001 \times 70\mathbf{9271} = \mathbf{17290}872907 + 7092780\mathbf{9271}.$$
$$\mathbf{17290}8 \times 100001 + 100001 \times 80\mathbf{9271} = \mathbf{17290}972908 + 8092790\mathbf{9271}.$$

$$\mathbf{17291}1 \times 100001 + 100001 \times 11\mathbf{9271} = \mathbf{17291}272911 + 1192721\mathbf{9271}.$$
$$\mathbf{17291}2 \times 100001 + 100001 \times 21\mathbf{9271} = \mathbf{17291}372912 + 2192731\mathbf{9271}.$$
$$\mathbf{17291}3 \times 100001 + 100001 \times 31\mathbf{9271} = \mathbf{17291}472913 + 3192741\mathbf{9271}.$$
$$\mathbf{17291}4 \times 100001 + 100001 \times 41\mathbf{9271} = \mathbf{17291}572914 + 4192751\mathbf{9271}.$$
$$\mathbf{17291}5 \times 100001 + 100001 \times 51\mathbf{9271} = \mathbf{17291}672915 + 5192761\mathbf{9271}.$$
$$\mathbf{17291}6 \times 100001 + 100001 \times 61\mathbf{9271} = \mathbf{17291}772916 + 6192771\mathbf{9271}.$$
$$\mathbf{17291}7 \times 100001 + 100001 \times 71\mathbf{9271} = \mathbf{17291}872917 + 7192781\mathbf{9271}.$$
$$\mathbf{17291}8 \times 100001 + 100001 \times 81\mathbf{9271} = \mathbf{17291}972918 + 8192791\mathbf{9271}.$$

$$\mathbf{17292}1 \times 100001 + 100001 \times 12\mathbf{9271} = \mathbf{17292}272921 + 1292722\mathbf{9271}.$$
$$\mathbf{17292}2 \times 100001 + 100001 \times 22\mathbf{9271} = \mathbf{17292}372922 + 2292732\mathbf{9271}.$$
$$\mathbf{17292}3 \times 100001 + 100001 \times 32\mathbf{9271} = \mathbf{17292}472923 + 3292742\mathbf{9271}.$$
$$\mathbf{17292}4 \times 100001 + 100001 \times 42\mathbf{9271} = \mathbf{17292}572924 + 4292752\mathbf{9271}.$$
$$\mathbf{17292}5 \times 100001 + 100001 \times 52\mathbf{9271} = \mathbf{17292}672925 + 5292762\mathbf{9271}.$$
$$\mathbf{17292}6 \times 100001 + 100001 \times 62\mathbf{9271} = \mathbf{17292}772926 + 6292772\mathbf{9271}.$$
$$\mathbf{17292}7 \times 100001 + 100001 \times 72\mathbf{9271} = \mathbf{17292}872927 + 7292782\mathbf{9271}.$$
$$\mathbf{17292}8 \times 100001 + 100001 \times 82\mathbf{9271} = \mathbf{17292}972928 + 8292792\mathbf{9271}.$$

$$\mathbf{1729}31 \times 100001 + 100001 \times 13\mathbf{9271} = \mathbf{1729}3272931 + 1392723\mathbf{9271}.$$
$$\mathbf{1729}32 \times 100001 + 100001 \times 23\mathbf{9271} = \mathbf{1729}3372932 + 2392733\mathbf{9271}.$$
$$\mathbf{1729}33 \times 100001 + 100001 \times 33\mathbf{9271} = \mathbf{1729}3472933 + 3392743\mathbf{9271}.$$
$$\mathbf{1729}34 \times 100001 + 100001 \times 43\mathbf{9271} = \mathbf{1729}3572934 + 4392753\mathbf{9271}.$$
$$\mathbf{1729}35 \times 100001 + 100001 \times 53\mathbf{9271} = \mathbf{1729}3672935 + 5392763\mathbf{9271}.$$
$$\mathbf{1729}36 \times 100001 + 100001 \times 63\mathbf{9271} = \mathbf{1729}3772936 + 6392773\mathbf{9271}.$$
$$\mathbf{1729}37 \times 100001 + 100001 \times 73\mathbf{9271} = \mathbf{1729}3872937 + 7392783\mathbf{9271}.$$
$$\mathbf{1729}38 \times 100001 + 100001 \times 83\mathbf{9271} = \mathbf{1729}3972938 + 8392793\mathbf{9271}.$$

$$\mathbf{1729}41 \times 100001 + 100001 \times 14\mathbf{9271} = \mathbf{1729}4272941 + 1492724\mathbf{9271}.$$
$$\mathbf{1729}42 \times 100001 + 100001 \times 24\mathbf{9271} = \mathbf{1729}4372942 + 2492734\mathbf{9271}.$$
$$\mathbf{1729}43 \times 100001 + 100001 \times 34\mathbf{9271} = \mathbf{1729}4472943 + 3492744\mathbf{9271}.$$
$$\mathbf{1729}44 \times 100001 + 100001 \times 44\mathbf{9271} = \mathbf{1729}4572944 + 4492754\mathbf{9271}.$$
$$\mathbf{1729}45 \times 100001 + 100001 \times 54\mathbf{9271} = \mathbf{1729}4672945 + 5492764\mathbf{9271}.$$
$$\mathbf{1729}46 \times 100001 + 100001 \times 64\mathbf{9271} = \mathbf{1729}4772946 + 6492774\mathbf{9271}.$$
$$\mathbf{1729}47 \times 100001 + 100001 \times 74\mathbf{9271} = \mathbf{1729}4872947 + 7492784\mathbf{9271}.$$
$$\mathbf{1729}48 \times 100001 + 100001 \times 84\mathbf{9271} = \mathbf{1729}4972948 + 8492794\mathbf{9271}.$$

$$\mathbf{1729}51 \times 100001 + 100001 \times 15\mathbf{9271} = \mathbf{1729}5272951 + 1592725\mathbf{9271}.$$
$$\mathbf{1729}52 \times 100001 + 100001 \times 25\mathbf{9271} = \mathbf{1729}5372952 + 2592735\mathbf{9271}.$$
$$\mathbf{1729}53 \times 100001 + 100001 \times 35\mathbf{9271} = \mathbf{1729}5472953 + 3592745\mathbf{9271}.$$
$$\mathbf{1729}54 \times 100001 + 100001 \times 45\mathbf{9271} = \mathbf{1729}5572954 + 4592755\mathbf{9271}.$$
$$\mathbf{1729}55 \times 100001 + 100001 \times 55\mathbf{9271} = \mathbf{1729}5672955 + 5592765\mathbf{9271}.$$
$$\mathbf{1729}56 \times 100001 + 100001 \times 65\mathbf{9271} = \mathbf{1729}5772956 + 6592775\mathbf{9271}.$$
$$\mathbf{1729}57 \times 100001 + 100001 \times 75\mathbf{9271} = \mathbf{1729}5872957 + 7592785\mathbf{9271}.$$
$$\mathbf{1729}58 \times 100001 + 100001 \times 85\mathbf{9271} = \mathbf{1729}5972958 + 8592795\mathbf{9271}.$$

$$\mathbf{1729}61 \times 100001 + 100001 \times 16\mathbf{9271} = \mathbf{1729}6272961 + 1692726\mathbf{9271}.$$
$$\mathbf{1729}62 \times 100001 + 100001 \times 26\mathbf{9271} = \mathbf{1729}6372962 + 2692736\mathbf{9271}.$$
$$\mathbf{1729}63 \times 100001 + 100001 \times 36\mathbf{9271} = \mathbf{1729}6472963 + 3692746\mathbf{9271}.$$
$$\mathbf{1729}64 \times 100001 + 100001 \times 46\mathbf{9271} = \mathbf{1729}6572964 + 4692756\mathbf{9271}.$$
$$\mathbf{1729}65 \times 100001 + 100001 \times 56\mathbf{9271} = \mathbf{1729}6672965 + 5692766\mathbf{9271}.$$
$$\mathbf{1729}66 \times 100001 + 100001 \times 66\mathbf{9271} = \mathbf{1729}6772966 + 6692776\mathbf{9271}.$$
$$\mathbf{1729}67 \times 100001 + 100001 \times 76\mathbf{9271} = \mathbf{1729}6872967 + 7692786\mathbf{9271}.$$
$$\mathbf{1729}68 \times 100001 + 100001 \times 86\mathbf{9271} = \mathbf{1729}6972968 + 8692796\mathbf{9271}.$$

$$\mathbf{1729}71 \times 100001 + 100001 \times 17\mathbf{9271} = \mathbf{1729}7272971 + 17927279\mathbf{271}.$$
$$\mathbf{1729}72 \times 100001 + 100001 \times 27\mathbf{9271} = \mathbf{1729}7372972 + 27927379\mathbf{271}.$$
$$\mathbf{1729}73 \times 100001 + 100001 \times 37\mathbf{9271} = \mathbf{1729}7472973 + 37927479\mathbf{271}.$$
$$\mathbf{1729}74 \times 100001 + 100001 \times 47\mathbf{9271} = \mathbf{1729}7572974 + 47927579\mathbf{271}.$$
$$\mathbf{1729}75 \times 100001 + 100001 \times 57\mathbf{9271} = \mathbf{1729}7672975 + 57927679\mathbf{271}.$$
$$\mathbf{1729}76 \times 100001 + 100001 \times 67\mathbf{9271} = \mathbf{1729}7772976 + 67927779\mathbf{271}.$$
$$\mathbf{1729}77 \times 100001 + 100001 \times 77\mathbf{9271} = \mathbf{1729}7872977 + 77927879\mathbf{271}.$$
$$\mathbf{1729}78 \times 100001 + 100001 \times 87\mathbf{9271} = \mathbf{1729}7972978 + 87927979\mathbf{271}.$$

$$\mathbf{1729}81 \times 100001 + 100001 \times 18\mathbf{9271} = \mathbf{1729}8272981 + 18927289\mathbf{271}.$$
$$\mathbf{1729}82 \times 100001 + 100001 \times 28\mathbf{9271} = \mathbf{1729}8372982 + 28927389\mathbf{271}.$$
$$\mathbf{1729}83 \times 100001 + 100001 \times 38\mathbf{9271} = \mathbf{1729}8472983 + 38927489\mathbf{271}.$$
$$\mathbf{1729}84 \times 100001 + 100001 \times 48\mathbf{9271} = \mathbf{1729}8572984 + 48927589\mathbf{271}.$$
$$\mathbf{1729}85 \times 100001 + 100001 \times 58\mathbf{9271} = \mathbf{1729}8672985 + 58927689\mathbf{271}.$$
$$\mathbf{1729}86 \times 100001 + 100001 \times 68\mathbf{9271} = \mathbf{1729}8772986 + 68927789\mathbf{271}.$$
$$\mathbf{1729}87 \times 100001 + 100001 \times 78\mathbf{9271} = \mathbf{1729}8872987 + 78927889\mathbf{271}.$$
$$\mathbf{1729}88 \times 100001 + 100001 \times 88\mathbf{9271} = \mathbf{1729}8972988 + 88927989\mathbf{271}.$$

$$\mathbf{1729}91 \times 100001 + 100001 \times 19\mathbf{9271} = \mathbf{1729}9272991 + 19927299\mathbf{271}.$$
$$\mathbf{1729}92 \times 100001 + 100001 \times 29\mathbf{9271} = \mathbf{1729}9372992 + 29927399\mathbf{271}.$$
$$\mathbf{1729}93 \times 100001 + 100001 \times 39\mathbf{9271} = \mathbf{1729}9472993 + 39927499\mathbf{271}.$$
$$\mathbf{1729}94 \times 100001 + 100001 \times 49\mathbf{9271} = \mathbf{1729}9572994 + 49927599\mathbf{271}.$$
$$\mathbf{1729}95 \times 100001 + 100001 \times 59\mathbf{9271} = \mathbf{1729}9672995 + 59927699\mathbf{271}.$$
$$\mathbf{1729}96 \times 100001 + 100001 \times 69\mathbf{9271} = \mathbf{1729}9772996 + 69927799\mathbf{271}.$$
$$\mathbf{1729}97 \times 100001 + 100001 \times 79\mathbf{9271} = \mathbf{1729}9872997 + 79927899\mathbf{271}.$$
$$\mathbf{1729}98 \times 100001 + 100001 \times 89\mathbf{9271} = \mathbf{1729}9972998 + 89927999\mathbf{271}.$$

$$10\mathbf{9271} \times 100001 + 100001 \times \mathbf{1729}01 = 10927209\mathbf{271} + \mathbf{1729}0272901.$$
$$11\mathbf{9271} \times 100001 + 100001 \times \mathbf{1729}11 = 11927219\mathbf{271} + \mathbf{1729}1272911.$$
$$12\mathbf{9271} \times 100001 + 100001 \times \mathbf{1729}21 = 12927229\mathbf{271} + \mathbf{1729}2272921.$$
$$13\mathbf{9271} \times 100001 + 100001 \times \mathbf{1729}31 = 13927239\mathbf{271} + \mathbf{1729}3272931.$$
$$14\mathbf{9271} \times 100001 + 100001 \times \mathbf{1729}41 = 14927249\mathbf{271} + \mathbf{1729}4272941.$$
$$15\mathbf{9271} \times 100001 + 100001 \times \mathbf{1729}51 = 15927259\mathbf{271} + \mathbf{1729}5272951.$$
$$16\mathbf{9271} \times 100001 + 100001 \times \mathbf{1729}61 = 16927269\mathbf{271} + \mathbf{1729}6272961.$$
$$17\mathbf{9271} \times 100001 + 100001 \times \mathbf{1729}71 = 17927279\mathbf{271} + \mathbf{1729}7272971.$$
$$18\mathbf{9271} \times 100001 + 100001 \times \mathbf{1729}81 = 18927289\mathbf{271} + \mathbf{1729}8272981.$$
$$19\mathbf{9271} \times 100001 + 100001 \times \mathbf{1729}91 = 19927299\mathbf{271} + \mathbf{1729}9272991.$$

$$20\mathbf{9271}\times 100001+100001\times \mathbf{1729}02=20927309271+\mathbf{1729}0372902.$$
$$21\mathbf{9271}\times 100001+100001\times \mathbf{1729}12=21927319271+\mathbf{1729}1372912.$$
$$22\mathbf{9271}\times 100001+100001\times \mathbf{1729}22=22927329271+\mathbf{1729}2372922.$$
$$23\mathbf{9271}\times 100001+100001\times \mathbf{1729}32=23927339271+\mathbf{1729}3372932.$$
$$24\mathbf{9271}\times 100001+100001\times \mathbf{1729}42=24927349271+\mathbf{1729}4372942.$$
$$25\mathbf{9271}\times 100001+100001\times \mathbf{1729}52=25927359271+\mathbf{1729}5372952.$$
$$26\mathbf{9271}\times 100001+100001\times \mathbf{1729}62=26927369271+\mathbf{1729}6372962.$$
$$27\mathbf{9271}\times 100001+100001\times \mathbf{1729}72=27927379271+\mathbf{1729}7372972.$$
$$28\mathbf{9271}\times 100001+100001\times \mathbf{1729}82=28927389271+\mathbf{1729}8372982.$$
$$29\mathbf{9271}\times 100001+100001\times \mathbf{1729}92=29927399271+\mathbf{1729}9372992.$$

$$30\mathbf{9271}\times 100001+100001\times \mathbf{1729}03=30927409271+\mathbf{1729}0472903.$$
$$31\mathbf{9271}\times 100001+100001\times \mathbf{1729}13=31927419271+\mathbf{1729}1472913.$$
$$32\mathbf{9271}\times 100001+100001\times \mathbf{1729}23=32927429271+\mathbf{1729}2472923.$$
$$33\mathbf{9271}\times 100001+100001\times \mathbf{1729}33=33927439271+\mathbf{1729}3472933.$$
$$34\mathbf{9271}\times 100001+100001\times \mathbf{1729}43=34927449271+\mathbf{1729}4472943.$$
$$35\mathbf{9271}\times 100001+100001\times \mathbf{1729}53=35927459271+\mathbf{1729}5472953.$$
$$36\mathbf{9271}\times 100001+100001\times \mathbf{1729}63=36927469271+\mathbf{1729}6472963.$$
$$37\mathbf{9271}\times 100001+100001\times \mathbf{1729}73=37927479271+\mathbf{1729}7472973.$$
$$38\mathbf{9271}\times 100001+100001\times \mathbf{1729}83=38927489271+\mathbf{1729}8472983.$$

$$10\mathbf{1729}\times 10001+10001\times \mathbf{9271}01=1017391729+\mathbf{9271}937101.$$
$$20\mathbf{1729}\times 10001+10001\times \mathbf{9271}02=2017491729+\mathbf{9271}947102.$$

$$\mathbf{1729}01\times 10001+10001\times 10\mathbf{9271}=\mathbf{1729}182901+109281\mathbf{9271}.$$
$$\mathbf{1729}02\times 10001+10001\times 20\mathbf{9271}=\mathbf{1729}192902+209291\mathbf{9271}.$$
$$\mathbf{1729}11\times 10001+10001\times 11\mathbf{9271}=\mathbf{1729}282911+119282\mathbf{9271}.$$
$$\mathbf{1729}12\times 10001+10001\times 21\mathbf{9271}=\mathbf{1729}292912+219292\mathbf{9271}.$$
$$\mathbf{1729}21\times 10001+10001\times 12\mathbf{9271}=\mathbf{1729}382921+129283\mathbf{9271}.$$
$$\mathbf{1729}22\times 10001+10001\times 22\mathbf{9271}=\mathbf{1729}392922+229293\mathbf{9271}.$$

$$\mathbf{1729}31\times 10001+10001\times 13\mathbf{9271}=\mathbf{1729}482931+139284\mathbf{9271}.$$
$$\mathbf{1729}32\times 10001+10001\times 23\mathbf{9271}=\mathbf{1729}492932+239294\mathbf{9271}.$$
$$\mathbf{1729}41\times 10001+10001\times 14\mathbf{9271}=\mathbf{1729}582941+149285\mathbf{9271}.$$
$$\mathbf{1729}42\times 10001+10001\times 24\mathbf{9271}=\mathbf{1729}592942+249295\mathbf{9271}.$$
$$\mathbf{1729}51\times 10001+10001\times 15\mathbf{9271}=\mathbf{1729}682951+159286\mathbf{9271}.$$
$$\mathbf{1729}52\times 10001+10001\times 25\mathbf{9271}=\mathbf{1729}692952+259296\mathbf{9271}.$$

$$\mathbf{1729}61 \times 10001 + 10001 \times 16\mathbf{9271} = \mathbf{1729}782961 + 1692879271.$$
$$\mathbf{1729}62 \times 10001 + 10001 \times 26\mathbf{9271} = \mathbf{1729}792962 + 2692979271.$$
$$\mathbf{1729}71 \times 10001 + 10001 \times 17\mathbf{9271} = \mathbf{1729}882971 + 1792889271.$$
$$\mathbf{1729}72 \times 10001 + 10001 \times 27\mathbf{9271} = \mathbf{1729}892972 + 2792989271.$$
$$\mathbf{1729}81 \times 10001 + 10001 \times 18\mathbf{9271} = \mathbf{1729}982981 + 1892899271.$$
$$\mathbf{1729}82 \times 10001 + 10001 \times 28\mathbf{9271} = \mathbf{1729}992982 + 2892999271.$$

$$109271 \times 10001 + 10001 \times \mathbf{1729}01 = 1092819271 + \mathbf{1729}182901.$$
$$119271 \times 10001 + 10001 \times \mathbf{1729}11 = 1192829271 + \mathbf{1729}282911.$$
$$129271 \times 10001 + 10001 \times \mathbf{1729}21 = 1292839271 + \mathbf{1729}382921.$$
$$139271 \times 10001 + 10001 \times \mathbf{1729}31 = 1392849271 + \mathbf{1729}482931.$$
$$149271 \times 10001 + 10001 \times \mathbf{1729}41 = 1492859271 + \mathbf{1729}582941.$$
$$159271 \times 10001 + 10001 \times \mathbf{1729}51 = 1592869271 + \mathbf{1729}682951.$$

$$169271 \times 10001 + 10001 \times \mathbf{1729}61 = 1692879271 + \mathbf{1729}782961.$$
$$179271 \times 10001 + 10001 \times \mathbf{1729}71 = 1792889271 + \mathbf{1729}882971.$$
$$189271 \times 10001 + 10001 \times \mathbf{1729}81 = 1892899271 + \mathbf{1729}982981.$$
$$209271 \times 10001 + 10001 \times \mathbf{1729}02 = 2092919271 + \mathbf{1729}192902.$$
$$219271 \times 10001 + 10001 \times \mathbf{1729}12 = 2192929271 + \mathbf{1729}292912.$$
$$229271 \times 10001 + 10001 \times \mathbf{1729}22 = 2292939271 + \mathbf{1729}392922.$$

$$239271 \times 10001 + 10001 \times \mathbf{1729}32 = 2392949271 + \mathbf{1729}492932.$$
$$249271 \times 10001 + 10001 \times \mathbf{1729}42 = 2492959271 + \mathbf{1729}592942.$$
$$259271 \times 10001 + 10001 \times \mathbf{1729}52 = 2592969271 + \mathbf{1729}692952.$$
$$269271 \times 10001 + 10001 \times \mathbf{1729}62 = 2692979271 + \mathbf{1729}792962.$$
$$279271 \times 10001 + 10001 \times \mathbf{1729}72 = 2792989271 + \mathbf{1729}892972.$$
$$289271 \times 10001 + 10001 \times \mathbf{1729}82 = 2892999271 + \mathbf{1729}992982.$$

$$\mathbf{1729}1 \times 100001 + 100001 \times 19271 = \mathbf{1729}11729\mathbf{1} + 1927119271.$$
$$\mathbf{1729}2 \times 100001 + 100001 \times 29271 = \mathbf{1729}21729\mathbf{2} + 2927129271.$$
$$\mathbf{1729}3 \times 100001 + 100001 \times 39271 = \mathbf{1729}31729\mathbf{3} + 3927139271.$$
$$\mathbf{1729}4 \times 100001 + 100001 \times 49271 = \mathbf{1729}41729\mathbf{4} + 4927149271.$$
$$\mathbf{1729}5 \times 100001 + 100001 \times 59271 = \mathbf{1729}51729\mathbf{5} + 5927159271.$$
$$\mathbf{1729}6 \times 100001 + 100001 \times 69271 = \mathbf{1729}61729\mathbf{6} + 6927169271.$$
$$\mathbf{1729}7 \times 100001 + 100001 \times 79271 = \mathbf{1729}71729\mathbf{7} + 7927179271.$$
$$\mathbf{1729}8 \times 100001 + 100001 \times 89271 = \mathbf{1729}81729\mathbf{8} + 8927189271.$$
$$\mathbf{1729}9 \times 100001 + 100001 \times 99271 = \mathbf{1729}91729\mathbf{9} + 9927199271.$$

$$1\mathbf{17291} \times 100001 + 100001 \times 1\mathbf{92711} = 1\mathbf{1729217291} + 1\mathbf{9271292711}.$$
$$1\mathbf{17292} \times 100001 + 100001 \times 2\mathbf{92711} = 1\mathbf{1729317292} + 2\mathbf{9271392711}.$$
$$1\mathbf{17293} \times 100001 + 100001 \times 3\mathbf{92711} = 1\mathbf{1729417293} + 3\mathbf{9271492711}.$$
$$1\mathbf{17294} \times 100001 + 100001 \times 4\mathbf{92711} = 1\mathbf{1729517294} + 4\mathbf{9271592711}.$$
$$1\mathbf{17295} \times 100001 + 100001 \times 5\mathbf{92711} = 1\mathbf{1729617295} + 5\mathbf{9271692711}.$$
$$1\mathbf{17296} \times 100001 + 100001 \times 6\mathbf{92711} = 1\mathbf{1729717296} + 6\mathbf{9271792711}.$$
$$1\mathbf{17297} \times 100001 + 100001 \times 7\mathbf{92711} = 1\mathbf{1729817297} + 7\mathbf{9271892711}.$$
$$1\mathbf{17298} \times 100001 + 100001 \times 8\mathbf{92711} = 1\mathbf{1729917298} + 8\mathbf{9271992711}.$$

$$2\mathbf{17291} \times 100001 + 100001 \times 1\mathbf{92712} = 2\mathbf{1729317291} + 1\mathbf{9271392712}.$$
$$2\mathbf{17292} \times 100001 + 100001 \times 2\mathbf{92712} = 2\mathbf{1729417292} + 2\mathbf{9271492712}.$$
$$2\mathbf{17293} \times 100001 + 100001 \times 3\mathbf{92712} = 2\mathbf{1729517293} + 3\mathbf{9271592712}.$$
$$2\mathbf{17294} \times 100001 + 100001 \times 4\mathbf{92712} = 2\mathbf{1729617294} + 4\mathbf{9271692712}.$$
$$2\mathbf{17295} \times 100001 + 100001 \times 5\mathbf{92712} = 2\mathbf{1729717295} + 5\mathbf{9271792712}.$$
$$2\mathbf{17296} \times 100001 + 100001 \times 6\mathbf{92712} = 2\mathbf{1729817296} + 6\mathbf{9271892712}.$$
$$2\mathbf{17297} \times 100001 + 100001 \times 7\mathbf{92712} = 2\mathbf{1729917297} + 7\mathbf{9271992712}.$$

$$3\mathbf{17291} \times 100001 + 100001 \times 1\mathbf{92713} = 3\mathbf{1729417291} + 1\mathbf{9271492713}.$$
$$3\mathbf{17292} \times 100001 + 100001 \times 2\mathbf{92713} = 3\mathbf{1729517292} + 2\mathbf{9271592713}.$$
$$3\mathbf{17293} \times 100001 + 100001 \times 3\mathbf{92713} = 3\mathbf{1729617293} + 3\mathbf{9271692713}.$$
$$3\mathbf{17294} \times 100001 + 100001 \times 4\mathbf{92713} = 3\mathbf{1729717294} + 4\mathbf{9271792713}.$$
$$3\mathbf{17295} \times 100001 + 100001 \times 5\mathbf{92713} = 3\mathbf{1729817295} + 5\mathbf{9271892713}.$$
$$3\mathbf{17296} \times 100001 + 100001 \times 6\mathbf{92713} = 3\mathbf{1729917296} + 6\mathbf{9271992713}.$$

*http://rgmia.org/papers/v19/v19a159.pdf.*

## 29.2 Non Sequential Patterns

Below are two patterns different from the previous subsection. Here they are in such a way that increasing one zero in each line becomes very interesting pattern. Also, the number 1729 is on both sides of the expression.

$$\begin{aligned}
\mathbf{1729} \times 10001 + 10001 \times \mathbf{9271} &= \mathbf{17291729} + \mathbf{92719271} \\
\mathbf{1729} \times 100001 + 100001 \times \mathbf{9271} &= \mathbf{172901729} + \mathbf{9271\,09271} \\
\mathbf{1729} \times 1000001 + 1000001 \times \mathbf{9271} &= \mathbf{1729001729} + \mathbf{9271\,009271} \\
\mathbf{1729} \times 10000001 + 10000001 \times \mathbf{9271} &= \mathbf{17290001729} + \mathbf{9271\,0009271}.
\end{aligned}$$

$$\begin{aligned}
1\mathbf{1729} \times 100001 + 100001 \times \mathbf{92711} &= 1\mathbf{172911729} + \mathbf{9271\,192711} \\
1\mathbf{1729} \times 1000001 + 1000001 \times \mathbf{92711} &= 1\mathbf{1729011729} + \mathbf{9271\,1092711} \\
1\mathbf{1729} \times 10000001 + 10000001 \times \mathbf{92711} &= 1\mathbf{17290011729} + \mathbf{9271\,10092711}.
\end{aligned}$$

Still there is one more non sequential pattern giving same digits on both sides except number 0:

$$10001 \times 1729 + 9271 \times 10001 = 1729\,1729 + 7291\,7291.$$
$$10001 \times 1927 + 7291 \times 10001 = 1927\,1927 + 7291\,7291.$$
$$10001 \times 2719 + 9172 \times 10001 = 2719\,2719 + 9172\,9172.$$
$$10001 \times 2917 + 7192 \times 10001 = 2917\,2917 + 7192\,7192.$$

*http://rgmia.org/papers/v19/v19a159.pdf.*

## Reference

Most of the references are given in each section. The reference below is a summary of author's work on numbers in different situations:

**I.J. TANEJA**, Crazy Representations of Natural Numbers, Selfie Numbers, Fibonacci Sequence, and Selfie Fractions, *RGMIA Research Report Collection*, **19**(2016), Article 179, pp.1-37, *http://rgmia.org/papers/v19/v19a179.pdf*.