Pythagorean Triples and Perfect Square Sum Magic Squares

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Abstract

*This work brings the idea how we can achieve prefect square sum magic squares using **primitive and non primitive Pythagorean triples**. By perfect square sum magic square, we understand that the total sum of entries of a magic square is a perfect square. The work is divided in two parts, one on primitive triples and another on non primitive triples.*

On a special day: August 15, 17

Pythagoras Theorem: $8^2 + 15^2 = 17^2$

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1 Pythagoras's Theorem

Pythagoras theorem is well known in literature. It has the property that

$$a^2 + b^2 = c^2 \tag{1}$$

where a, b are the sides of a right angle triangles, and c is hypotenuse of the triangle. For simplicity let's represent these letters as triples (a, b, c), then the triple (3,4,5) is understood as

$$3^2 + 4^2 = 5^2. \tag{2}$$

Note 1.1. ***Primitive and Non Primitive Triples:*** *Let's consider the triples (a, b, c) such that $a < c$ and $b < c$, where $a, b, c \in \mathbf{N}_+$ with the condition that $a^2 + b^2 = c^2$, and call them **Pythagorean triples**. The triples (a, b, c) are primitive if there are no common factors among a, b and c, for example, (3, 4, 5). The non primitive triples are those when there are common factors among a, b and c, for example, (6, 8, 10). In this case 2 is a common factor among all the entries, i.e., $(6, 8, 10) = 2 \times (3, 4, 5)$. Throughout, it is understood that, whenever we write **triples**, we are talking about **PythagoreanExamples: Non Primitive Triples triples**.*

The aim of this work is to generate magic squares from **primitive** and **non primitive Pythagorean triples** in such a way that the sum of all entries of a magic square is always a perfect sum.

2 Magic Square Generated by Triple (8,15,17)

Since this work is dedicated to day 15th August 17, i.e., 15.8.17 or 8.15.17, let's analyse this triple. According to equation (1), it meets the conditions because

$$8^2 + 15^2 = 17^2$$

We know that

$$F_n := 1 + 3 + 5 + \dots + (2n - 1) = n^2, n \geq 1 \tag{3}$$

This implies that

$$F_8 := 1 + 3 + 5 + \dots + 15 = 8^2;$$
$$F_{15} := 1 + 3 + 5 + \dots + 29 = 15^2;$$
$$F_{17} := 1 + 3 + 5 + \dots + 33 = 17^2.$$

We can write

$$15^2 = 17^2 - 8^2$$

This gives

$$F_{15} := 1 + 3 + 5 + \dots + 33 - (1 + 3 + 5 + \dots + 15)$$
$$:= 17 + 19 + 21 + 23 + 25 + 27 + 29 + 31 + 33.$$

Thus we have sequence of 9 even numbers starting from 17 and ending at 33, i.e., $(17, 19, 21, 23, 25, 27, 29, 31, 33)$.

We know that the numbers in a sequence that sums to a perfect square always allow us to write a magic square. In this case of order 3×3. The magic square of order 3×3 using numbers $(17, 19, 21, 23, 25, 27, 29, 31, 33)$ is given by

			75
27	17	31	75
29	25	21	75
19	33	23	75
75	75	75	75

The above magic square is of magic sum $S_{3\times3} = 75$ and the total number of elements gives us a perfect square sum, i.e., $225 := 15^2 = 17^2 - 8^2$.

3   Test for Generating Magic Squares from Pythagorean Triples

Let see how we can check that a given primitive Pythagorean triple generates a perfect square sum magic square.

From (3), we know that

$$F_n := 1 + 3 + 5 + ... + (2n - 1) = n^2, n \geq 1$$

Let us consider,

$$F_m := 1 + 3 + 5 + ... + (2m - 1) = n^2, m \geq 1, m \geq n.$$

The the difference is given by

$$F_m - F_n := 1 + 3 + 5 + ... + (2m - 1) - (1 + 3 + 5 + ... + (2n - 1)) = m^2 - n^2$$

After simplifications,

$$F_m - F_n := (2n + 1) + (2n + 3) + ... + (2m - 1) = m^2 - n^2$$

The total number of terms are given by

$$T_{terms} := \frac{(2m - 1) - (2n + 3)}{2} + 1 = m - n$$

Thus, if total number of terms i.e., $m - n$ is a perfect square and greater than or equal to 9, then, we can write a magic square of order $\sqrt{m - n} \geq 3$. And, the magic square is formed by the terms $(2n + 1, 2n + 3, ..., 2m - 1)$ resulting in a perfect square sum magic squares giving sum as $m^2 - n^2$. By **perfect square sum magic square**, we understand that the total number of members of a magic square is a perfect square.

Result 3.1. *In a Pythagorean triple, (a, b, c) if any of the difference*

$$c - b \text{ or } c - a, \ c > b, \ c > a,$$

*is a perfect square greater than or equal to 9, then we can always write a **perfect square sum magic square**.*

Let us consider

$$\{a, b, c\} \Rightarrow \{\text{order of magic square, first member of sequence,}$$
$$\text{last member of sequence, magic sum, sum of all members of a magic square}\}.$$

For example,

$$\{8, 15, 17\} \Rightarrow \{3, 17, 33, 75, 225\}. \tag{4}$$

The above representations of numbers is as follows:

$$3 \rightarrow \text{Order of a magic square;}$$
$$17 \rightarrow \text{First member of the sequence;}$$
$$33 \rightarrow \text{Last member of the squence;}$$
$$75 \rightarrow \text{Magic sum;}$$
$$225 \rightarrow \text{Sum of all members of the magic square, is a perfect square, } 225 = 15^2.$$

In this case, 9 consecutive odd numbers that generate magic square are $(17, 19, 21, 23, 25, 27, 29, 31, 33)$.

Result 3.2. *To reach the result appearing in equation (4) we used the following formula:*

$$\{a, b, c\} \Rightarrow \left\{ \left\{ \frac{c^2 - a^2}{\sqrt{c - a}}, c^2 - a^2, 2a + 1, 2c - 1, \sqrt{c - a} \right\}, \left\{ \frac{c^2 - b^2}{\sqrt{c - b}}, c^2 - b^2, 2b + 1, 2c - 1, \sqrt{c - b} \right\} \right\} \tag{5}$$

Result 3.1 is easy to use for testing the existence of magic square whilst Result 3.2 is useful for antecipating all the possible values appearing in magic square. We can use Result 3.2 to test also, but it is more work.

4   Primitive Triples Generating Magic Squares

Testing only up to 3-digits triples, i.e., less than 1000, there are in total 158 primitive triples [2, 3, 4]. Applying the procedure given in 3.1, it can be found that there are total 137 possible primitive triples that generate magic squares. Below is a list of possible primitive triples generating magic squares, calculated according to formula (5).

1. $\{8, 15, 17\} \Rightarrow \{3, 17, 33, 75, 225\}$.

2. $\{12, 35, 37\} \Rightarrow \{5, 25, 73, 245, 1225\}$.

3. $\{16, 63, 65\} \Rightarrow \{7, 33, 129, 567, 3969\}$.

4. $\{20, 21, 29\} \Rightarrow \{3, 41, 57, 147, 441\}$.

5. $\{20, 99, 101\} \Rightarrow \{9, 41, 201, 1089, 9801\}$.

6. $\{24, 143, 145\} \Rightarrow \{11, 49, 289, 1859, 20449\}$.

7. $\{28, 45, 53\} \Rightarrow \{5, 57, 105, 405, 2025\}$.

8. $\{28, 195, 197\} \Rightarrow \{13, 57, 393, 2925, 38025\}$.

9. $\{32, 255, 257\} \Rightarrow \{15, 65, 513, 4335, 65025\}$.

10. $\{33, 56, 65\} \Rightarrow \{3, 113, 129, 363, 1089\}$.

11. $\{36, 77, 85\} \Rightarrow \{7, 73, 169, 847, 5929\}$.

12. $\{36, 323, 325\} \Rightarrow \{17, 73, 649, 6137, 104329\}$.

13. $\{39, 80, 89\} \Rightarrow \{3, 161, 177, 507, 1521\}$.

14. $\{40, 399, 401\} \Rightarrow \{19, 81, 801, 8379, 159201\}$.

15. $\{44, 117, 125\} \Rightarrow \{9, 89, 249, 1521, 13689\}$.

16. $\{44, 483, 485\} \Rightarrow \{21, 89, 969, 11109, 233289\}$.

17. $\{48, 55, 73\} \Rightarrow \{5, 97, 145, 605, 3025\}$.

18. $\{48, 575, 577\} \Rightarrow \{23, 97, 1153, 14375, 330625\}$.

19. $\{51, 140, 149\} \Rightarrow \{3, 281, 297, 867, 2601\}$.

20. $\{52, 165, 173\} \Rightarrow \{11, 105, 345, 2475, 27225\}$.

21. $\{52, 675, 677\} \Rightarrow \{25, 105, 1353, 18225, 455625\}$.

22. $\{56, 783, 785\} \Rightarrow \{27, 113, 1569, 22707, 613089\}$.

23. $\{57, 176, 185\} \Rightarrow \{3, 353, 369, 1083, 3249\}$.

24. $\{60, 91, 109\} \Rightarrow \{7, 121, 217, 1183, 8281\}$.

25. $\{60, 221, 229\} \Rightarrow \{13, 121, 457, 3757, 48841\}$.

26. $\{60, 899, 901\} \Rightarrow \{29, 121, 1801, 27869, 808201\}$.

27. $\{65, 72, 97\} \Rightarrow \{5, 145, 193, 845, 4225\}$.

28. $\{68, 285, 293\} \Rightarrow \{15, 137, 585, 5415, 81225\}$.

29. $\{69, 260, 269\} \Rightarrow \{3, 521, 537, 1587, 4761\}$.

30. $\{75, 308, 317\} \Rightarrow \{3, 617, 633, 1875, 5625\}$.

31. $\{76, 357, 365\} \Rightarrow \{17, 153, 729, 7497, 127449\}$.

32. $\{84, 187, 205\} \Rightarrow \{11, 169, 409, 3179, 34969\}$.

33. $\{84, 437, 445\} \Rightarrow \{19, 169, 889, 10051, 190969\}$.

34. $\{85, 132, 157\} \Rightarrow \{5, 265, 313, 1445, 7225\}$.

35. $\{87, 416, 425\} \Rightarrow \{3, 833, 849, 2523, 7569\}$.

36. $\{88, 105, 137\} \Rightarrow \{7, 177, 273, 1575, 11025\}$.

37. $\{92, 525, 533\} \Rightarrow \{21, 185, 1065, 13125, 275625\}$.

38. $\{93, 476, 485\} \Rightarrow \{3, 953, 969, 2883, 8649\}$.

39. $\{95, 168, 193\} \Rightarrow \{5, 337, 385, 1805, 9025\}$.

40. $\{96, 247, 265\} \Rightarrow \{13, 193, 529, 4693, 61009\}$.

41. $\{100, 621, 629\} \Rightarrow \{23, 201, 1257, 16767, 385641\}$.

42. $\{104, 153, 185\} \Rightarrow \{9, 209, 369, 2601, 23409\}$.

43. $\{105, 208, 233\} \Rightarrow \{5, 417, 465, 2205, 11025\}$.

44. $\{105, 608, 617\} \Rightarrow \{3, 1217, 1233, 3675, 11025\}$.

45. $\{108, 725, 733\} \Rightarrow \{25, 217, 1465, 21025, 525625\}$.

46. $\{111, 680, 689\} \Rightarrow \{3, 1361, 1377, 4107, 12321\}$.

47. $\{115, 252, 277\} \Rightarrow \{5, 505, 553, 2645, 13225\}$.

48. $\{116, 837, 845\} \Rightarrow \{27, 233, 1689, 25947, 700569\}$.

49. $\{119, 120, 169\} \Rightarrow \{7, 241, 337, 2023, 14161\}$.

50. $\{120, 209, 241\} \Rightarrow \{11, 241, 481, 3971, 43681\}$.

51. $\{120, 391, 409\} \Rightarrow \{17, 241, 817, 8993, 152881\}$.

52. $\{123, 836, 845\} \Rightarrow \{3, 1673, 1689, 5043, 15129\}$.

53. $\{124, 957, 965\} \Rightarrow \{29, 249, 1929, 31581, 915849\}$.

54. $\{129, 920, 929\} \Rightarrow \{3, 1841, 1857, 5547, 16641\}$.

55. $\{132, 475, 493\} \Rightarrow \{19, 265, 985, 11875, 225625\}$.

56. $\{133, 156, 205\} \Rightarrow \{7, 313, 409, 2527, 17689\}$.

57. $\{135, 352, 377\} \Rightarrow \{5, 705, 753, 3645, 18225\}$.

58. $\{136, 273, 305\} \Rightarrow \{13, 273, 609, 5733, 74529\}$.

59. $\{140, 171, 221\} \Rightarrow \{9, 281, 441, 3249, 29241\}$.

60. $\{145, 408, 433\} \Rightarrow \{5, 817, 865, 4205, 21025\}$.

61. $\{152, 345, 377\} \Rightarrow \{15, 305, 753, 7935, 119025\}$.

62. $\{155, 468, 493\} \Rightarrow \{5, 937, 985, 4805, 24025\}$.

63. $\{156, 667, 685\} \Rightarrow \{23, 313, 1369, 19343, 444889\}$.

64. $\{160, 231, 281\} \Rightarrow \{11, 321, 561, 4851, 53361\}$.

65. $\{161, 240, 289\} \Rightarrow \{7, 481, 577, 3703, 25921\}$.

66. $\{165, 532, 557\} \Rightarrow \{5, 1065, 1113, 5445, 27225\}$.

67. $\{168, 425, 457\} \Rightarrow \{17, 337, 913, 10625, 180625\}$.

68. $\{168, 775, 793\} \Rightarrow \{25, 337, 1585, 24025, 600625\}$.

69. $\{175, 288, 337\} \Rightarrow \{7, 577, 673, 4375, 30625\}$.

70. $\{180, 299, 349\} \Rightarrow \{13, 361, 697, 6877, 89401\}$.

71. $\{184, 513, 545\} \Rightarrow \{19, 369, 1089, 13851, 263169\}$.

72. $\{185, 672, 697\} \Rightarrow \{5, 1345, 1393, 6845, 34225\}$.

73. $\{189, 340, 389\} \Rightarrow \{7, 681, 777, 5103, 35721\}$.

74. $\{195, 748, 773\} \Rightarrow \{5, 1497, 1545, 7605, 38025\}$.

75. $\{200, 609, 641\} \Rightarrow \{21, 401, 1281, 17661, 370881\}$.

76. $\{203, 396, 445\} \Rightarrow \{7, 793, 889, 5887, 41209\}$.

77. $\{204, 253, 325\} \Rightarrow \{11, 409, 649, 5819, 64009\}$.

78. $\{205, 828, 853\} \Rightarrow \{5, 1657, 1705, 8405, 42025\}$.

79. $\{207, 224, 305\} \Rightarrow \{9, 449, 609, 4761, 42849\}$.

80. $\{215, 912, 937\} \Rightarrow \{5, 1825, 1873, 9245, 46225\}$.

81. $\{216, 713, 745\} \Rightarrow \{23, 433, 1489, 22103, 508369\}$.

82. $\{217, 456, 505\} \Rightarrow \{7, 913, 1009, 6727, 47089\}$.

83. $\{220, 459, 509\} \Rightarrow \{17, 441, 1017, 12393, 210681\}$.

84. $\{225, 272, 353\} \Rightarrow \{9, 545, 705, 5625, 50625\}$.

85. $\{228, 325, 397\} \Rightarrow \{13, 457, 793, 8125, 105625\}$.

86. $\{231, 520, 569\} \Rightarrow \{7, 1041, 1137, 7623, 53361\}$.

87. $\{232, 825, 857\} \Rightarrow \{25, 465, 1713, 27225, 680625\}$.

88. $\{240, 551, 601\} \Rightarrow \{19, 481, 1201, 15979, 303601\}$.

89. $\{248, 945, 977\} \Rightarrow \{27, 497, 1953, 33075, 893025\}$.

90. $\{252, 275, 373\} \Rightarrow \{11, 505, 745, 6875, 75625\}$.

91. $\{259, 660, 709\} \Rightarrow \{7, 1321, 1417, 9583, 67081\}$.

92. $\{260, 651, 701\} \Rightarrow \{21, 521, 1401, 20181, 423801\}$.

93. $\{261, 380, 461\} \Rightarrow \{9, 761, 921, 7569, 68121\}$.

94. $\{273, 736, 785\} \Rightarrow \{7, 1473, 1569, 10647, 74529\}$.

95. $\{276, 493, 565\} \Rightarrow \{17, 553, 1129, 14297, 243049\}$.

96. $\{279, 440, 521\} \Rightarrow \{9, 881, 1041, 8649, 77841\}$.

97. $\{280, 351, 449\} \Rightarrow \{13, 561, 897, 9477, 123201\}$.

98. $\{280, 759, 809\} \Rightarrow \{23, 561, 1617, 25047, 576081\}$.

99. $\{287, 816, 865\} \Rightarrow \{7, 1633, 1729, 11767, 82369\}$.

100. $\{297, 304, 425\} \Rightarrow \{11, 609, 849, 8019, 88209\}$.

101. $\{300, 589, 661\} \Rightarrow \{19, 601, 1321, 18259, 346921\}$.

102. $\{301, 900, 949\} \Rightarrow \{7, 1801, 1897, 12943, 90601\}$.

103. $\{308, 435, 533\} \Rightarrow \{15, 617, 1065, 12615, 189225\}$.

104. $\{315, 572, 653\} \Rightarrow \{9, 1145, 1305, 11025, 99225\}$.

105. $\{319, 360, 481\} \Rightarrow \{11, 721, 961, 9251, 101761\}$.

106. $\{333, 644, 725\} \Rightarrow \{9, 1289, 1449, 12321, 110889\}$.

107. $\{336, 377, 505\} \Rightarrow \{13, 673, 1009, 10933, 142129\}$.

108. $\{336, 527, 625\} \Rightarrow \{17, 673, 1249, 16337, 277729\}$.

109. $\{341, 420, 541\} \Rightarrow \{11, 841, 1081, 10571, 116281\}$.

110. $\{348, 805, 877\} \Rightarrow \{23, 697, 1753, 28175, 648025\}$.

111. $\{364, 627, 725\} \Rightarrow \{19, 729, 1449, 20691, 393129\}$.

112. $\{368, 465, 593\} \Rightarrow \{15, 737, 1185, 14415, 216225\}$.

113. $\{369, 800, 881\} \Rightarrow \{9, 1601, 1761, 15129, 136161\}$.

114. $\{372, 925, 997\} \Rightarrow \{25, 745, 1993, 34225, 855625\}$.

115. $\{385, 552, 673\} \Rightarrow \{11, 1105, 1345, 13475, 148225\}$.

116. $\{387, 884, 965\} \Rightarrow \{9, 1769, 1929, 16641, 149769\}$.

117. $\{396, 403, 565\} \Rightarrow \{13, 793, 1129, 12493, 162409\}$.

118. $\{400, 561, 689\} \Rightarrow \{17, 801, 1377, 18513, 314721\}$.

119. $\{407, 624, 745\} \Rightarrow \{11, 1249, 1489, 15059, 165649\}$.

120. $\{420, 851, 949\} \Rightarrow \{23, 841, 1897, 31487, 724201\}$.

121. $\{429, 460, 629\} \Rightarrow \{13, 921, 1257, 14157, 184041\}$.

122. $\{429, 700, 821\} \Rightarrow \{11, 1401, 1641, 16731, 184041\}$.

123. $\{432, 665, 793\} \Rightarrow \{19, 865, 1585, 23275, 442225\}$.

124. $\{451, 780, 901\} \Rightarrow \{11, 1561, 1801, 18491, 203401\}$.

125. $\{455, 528, 697\} \Rightarrow \{13, 1057, 1393, 15925, 207025\}$.

126. $\{464, 777, 905\} \Rightarrow \{21, 929, 1809, 28749, 603729\}$.

127. $\{468, 595, 757\} \Rightarrow \{17, 937, 1513, 20825, 354025\}$.

128. $\{473, 864, 985\} \Rightarrow \{11, 1729, 1969, 20339, 223729\}$.

129. $\{481, 600, 769\} \Rightarrow \{13, 1201, 1537, 17797, 231361\}$.

130. $\{504, 703, 865\} \Rightarrow \{19, 1009, 1729, 26011, 494209\}$.

131. $\{533, 756, 925\} \Rightarrow \{13, 1513, 1849, 21853, 284089\}$.

132. $\{540, 629, 829\} \Rightarrow \{17, 1081, 1657, 23273, 395641\}$.

133. $\{555, 572, 797\} \Rightarrow \{15, 1145, 1593, 20535, 308025\}$.

134. $\{580, 741, 941\} \Rightarrow \{19, 1161, 1881, 28899, 549081\}$.

135. $\{615, 728, 953\} \Rightarrow \{15, 1457, 1905, 25215, 378225\}$.

136. $\{616, 663, 905\} \Rightarrow \{17, 1233, 1809, 25857, 439569\}$.

137. $\{696, 697, 985\} \Rightarrow \{17, 1393, 1969, 28577, 485809\}$.

Remark 4.1. *Out of 158 primitive triples, we have magic square with 137 of these. From the triple* $\{44, 117, 125\}$ *onwards, all of the primitive triples generate magic squares. Also* $\{8, 15, 17\}$ *is the first triple to generate a magic square. This test is only up to 3-digits triples, i.e., less than 1000. For higher numbers, i.e., from 1000-2000 (value of c) are given in Appendix 7.*

4.1 Examples: Primitive Triples

Based on some of these primitive triples, further examples of magic squares are found below. The first example has already been shown in section 2. Let's see the examples 2, 3, 5 and 6 given in section 4. These examples give magic squares of order 5, 7, 9 and 11 respectively. In all the cases, pan diagonal magic squares are constructed.

In the order 9 case, an extra example of a **bimagic square** is also given. The Pythagorean triples considered are:

$$\{12, 35, 37\} \Rightarrow \{5, 25, 73, 245, 1225\}$$
$$\{16, 63, 65\} \Rightarrow \{7, 33, 129, 567, 3969\}$$
$$\{20, 99, 101\} \Rightarrow \{9, 41, 201, 1089, 9801\}$$
$$\{24, 143, 145\} \Rightarrow \{11, 49, 289, 1859, 20449\}.$$

4.1.1 Magic Square Generated by Triple (12, 35, 37)

According to triple $\{12, 35, 37\} \Rightarrow \{5, 25, 73, 245, 1225\}$, we have a magic square of order 5 with 25 odd consecutive numbers starting from 25 and ending at 73, i.e., (25, 27, 29, ..., 71, 73). The magic square of order 5 is given by

		245	245	245	245	245
	25	37	49	61	73	245
245	59	71	33	35	47	245
245	43	45	57	69	31	245
245	67	29	41	53	55	245
245	51	63	65	27	39	245
	245	245	245	245	245	245

The above magic square is pan diagonal with a magic sum of $S_{5\times5} = 245$ and sum of all entries give a perfect square $1225 := 35^2 = 37^2 - 12^2$.

4.1.2 Magic Square Generated by Triple (16, 63, 65)

According to triple $\{16, 63, 65\} \Rightarrow \{7, 33, 129, 567, 3969\}$, we have a magic square of order 5 with 49 odd consecutive numbers starting from 33 and ending at 129, i.e., (33, 35, 37, ..., 127, 129). The magic square of order 7 is given by

		567	567	567	567	567	567	567
	33	49	65	81	97	113	129	567
567	111	127	45	47	63	79	95	567
567	77	93	109	125	43	59	61	567
567	57	73	75	91	107	123	41	567
567	121	39	55	71	87	89	105	567
567	101	103	119	37	53	69	85	567
567	67	83	99	115	117	35	51	567
	567	567	567	567	567	567	567	567

The above magic square is pan diagonal with a magic sum of $S_{7\times7} = 567$, and the sum of all entries give a perfect square $3969 := 63^2 = 65^2 - 16^2$.

4.1.3 Magic Square Generated by Triple (20, 99, 101)

According to triple $\{20, 99, 101\} \Rightarrow \{9, 41, 201, 1089, 9801\}$, we have a magic square of order 9 with 81 odd consecutive numbers starting from 41 and ending at 201, i.e., (41, 43, 45, ..., 199, 201). Below are two magic squares of order 9. One is with normal values and another is a bimagic square with square values for each member:

									1089
41	75	85	109	119	135	159	169	197	*1089*
105	115	143	149	183	193	55	65	81	*1089*
163	173	189	51	61	89	95	129	139	*1089*
93	49	59	137	99	127	187	161	177	*1089*
133	107	123	201	157	167	83	45	73	*1089*
191	153	181	79	53	69	147	103	113	*1089*
67	77	57	117	145	101	179	195	151	*1089*
125	141	97	175	185	165	63	91	47	*1089*
171	199	155	71	87	43	121	131	111	*1089*
1089	*1089*	*1089*	*1089*	*1089*	*1089*	*1089*	*1089*	*1089*	*1089*

The above magic square is with a magic sum of $S_{9 \times 9} = 1089$, and the sum of all entries give a perfect square $9801 := 99^2 = 101^2 - 20^2$. The bimagic sum is $Sb_{9 \times 9} = 151449$. Interestingly, 9801 is the reverse of 1089. The bimagic square is given by

									151449
1681	5625	7225	11881	14161	18225	25281	28561	38809	*151449*
11025	13225	20449	22201	33489	37249	3025	4225	6561	*151449*
26569	29929	35721	2601	3721	7921	9025	16641	19321	*151449*
8649	2401	3481	18769	9801	16129	34969	25921	31329	*151449*
17689	11449	15129	40401	24649	27889	6889	2025	5329	*151449*
36481	23409	32761	6241	2809	4761	21609	10609	12769	*151449*
4489	5929	3249	13689	21025	10201	32041	38025	22801	*151449*
15625	19881	9409	30625	34225	27225	3969	8281	2209	*151449*
29241	39601	24025	5041	7569	1849	14641	17161	12321	*151449*
151449	*151449*	*151449*	*151449*	*151449*	*151449*	*151449*	*151449*	*151449*	*151449*

Below is an example of another magic square of order 9 with consecutive odd numbers (41, 43, 45, ..., 199, 201). This is not a bimagic square but it is a pan diagonal.

		1089	1089	1089	1089	1089	1089	1089	1089	1089
	41	65	89	99	123	147	151	175	199	1089
1089	97	121	145	149	173	197	45	69	93	1089
1089	153	177	201	43	67	91	95	119	143	1089
1089	83	53	59	141	111	117	193	163	169	1089
1089	139	109	115	191	161	167	87	57	63	1089
1089	195	165	171	85	55	61	137	107	113	1089
1089	71	77	47	129	135	105	181	187	157	1089
1089	127	133	103	179	185	155	75	81	51	1089
1089	183	189	159	73	79	49	125	131	101	1089
	1089	1089	1089	1089	1089	1089	1089	1089	1089	1089

4.1.4   Magic Square Generated by Triple (24, 143, 145)

According to triple $\{24, 143, 145\} \Rightarrow \{11, 49, 289, 1859, 20449\}$, we have a magic square of order 11 with 121 odd consecutive numbers starting from 49 and ending at 289, i.e., (49, 51, 53, ..., 287, 289). The pan diagonal magic square of order 11 is given by

		1859	1859	1859	1859	1859	1859	1859	1859	1859	1859	1859
	49	89	107	125	143	161	201	219	237	255	273	1859
1859	249	289	65	83	101	119	137	177	195	213	231	1859
1859	207	225	265	283	59	77	95	135	153	171	189	1859
1859	165	183	223	241	259	277	53	71	111	129	147	1859
1859	123	141	159	199	217	235	253	271	69	87	105	1859
1859	81	99	117	157	175	193	211	229	247	287	63	1859
1859	281	57	75	93	133	151	169	187	205	245	263	1859
1859	239	257	275	51	91	109	127	145	163	181	221	1859
1859	197	215	233	251	269	67	85	103	121	139	179	1859
1859	155	173	191	209	227	267	285	61	79	97	115	1859
1859	113	131	149	167	185	203	243	261	279	55	73	1859
	1859	1859	1859	1859	1859	1859	1859	1859	1859	1859	1859	1859

The above pan diagonal magic square with a magic sum of $S_{11 \times 11} = 1859$, and the sum of all entries gives a perfect square $20449 := 143^2 = 145^2 - 24^2$.

The four examples given above show the general idea of constructing magic squares based on primitive triples. Using a similar procedure we can always construct a magic square for the primitive triples given in section 4.

5   Non Primitive Triples Generating Magic Squares

Testing only up to 3–digits triples, i.e., less than 1000, there are in total 878 Pythagorena triples [5]. 158 of these are primitive. We are left with only 720 non primitive triples. Applying the procedure given in 3.1, it is found only 186 possible non primitive triples for the generation of magic squares. Based on equation (5), below is to found a list of possible non primitive triples for the generation of magic squares:

1. $\{10, 24, 26\} \Rightarrow \{4, 21, 51, 144, 576\}$.

2. $\{14, 48, 50\} \Rightarrow \{6, 29, 99, 384, 2304\}$.

3. $\{18, 80, 82\} \Rightarrow \{8, 37, 163, 800, 6400\}$.

4. $\{22, 120, 122\} \Rightarrow \{10, 45, 243, 1440, 14400\})$.

5. $\{24, 32, 40\} \Rightarrow \{4, 49, 79, 256, 1024\}$.

6. $\{26, 168, 170\} \Rightarrow \{12, 53, 339, 2352, 28224\}$.

7. $\{27, 36, 45\} \Rightarrow \{3, 73, 89, 243, 729\}$.

8. $\{30, 224, 226\} \Rightarrow \{14, 61, 451, 3584, 50176\}$.

9. $\{32, 60, 68\} \Rightarrow \{6, 65, 135, 600, 3600\}$.

10. $\{34, 288, 290\} \Rightarrow \{16, 69, 579, 5184, 82944\}$.

11. $\{38, 360, 362\} \Rightarrow \{18, 77, 723, 7200, 129600\}$.

12. $\{40, 42, 58\} \Rightarrow \{4, 85, 115, 400, 1600\}$.

13. $\{40, 96, 104\} \Rightarrow \{8, 81, 207, 1152, 9216\}$.

14. $\{42, 440, 442\} \Rightarrow \{20, 85, 883, 9680, 193600\}$.

15. $\{45, 108, 117\} \Rightarrow \{3, 217, 233, 675, 2025\}$.

16. $\{46, 528, 530\} \Rightarrow \{22, 93, 1059, 12672, 278784\}$.

17. $\{48, 140, 148\} \Rightarrow \{10, 97, 295, 1960, 19600\}$.

18. $\{48, 64, 80\} \Rightarrow \{4, 129, 159, 576, 2304\}$.

19. $\{50, 624, 626\} \Rightarrow \{24, 101, 1251, 16224, 389376\}$.

20. $\{54, 72, 90\} \Rightarrow \{6, 109, 179, 864, 5184\}$.

21. $\{54, 728, 730\} \Rightarrow \{26, 109, 1459, 20384, 529984\}$.

22. $\{56, 192, 200\} \Rightarrow \{12, 113, 399, 3072, 36864\}$.

23. $\{56, 90, 106\} \Rightarrow \{4, 181, 211, 784, 3136\}$.

24. $\{58, 840, 842\} \Rightarrow \{28, 117, 1683, 25200, 705600\}$.

25. $\{62, 960, 962\} \Rightarrow \{30, 125, 1923, 30720, 921600\}$.

26. $\{63, 216, 225\} \Rightarrow \{3, 433, 449, 1323, 3969\}$.

27. $\{64, 120, 136\} \Rightarrow \{4, 241, 271, 1024, 4096\}$.

28. $\{64, 252, 260\} \Rightarrow \{14, 129, 519, 4536, 63504\}$.

29. $\{640, 672, 928\} \Rightarrow \{16, 1345, 1855, 25600, 409600\}$.

30. $\{66, 112, 130\} \Rightarrow \{8, 133, 259, 1568, 12544\}$.

31. $\{72, 135, 153\} \Rightarrow \{9, 145, 305, 2025, 18225\}$.

32. $\{72, 154, 170\} \Rightarrow \{4, 309, 339, 1296, 5184\}$.

33. $\{72, 320, 328\} \Rightarrow \{16, 145, 655, 6400, 102400\}$.

34. $\{75, 100, 125\} \Rightarrow \{5, 201, 249, 1125, 5625\}$.

35. $\{78, 160, 178\} \Rightarrow \{10, 157, 355, 2560, 25600\}$.

36. $\{80, 192, 208\} \Rightarrow \{4, 385, 415, 1600, 6400\}$.

37. $\{80, 396, 404\} \Rightarrow \{18, 161, 807, 8712, 156816\}$.

38. $\{80, 84, 116\} \Rightarrow \{6, 161, 231, 1176, 7056\}$.

39. $\{81, 360, 369\} \Rightarrow \{3, 721, 737, 2187, 6561\}$.

40. $\{88, 234, 250\} \Rightarrow \{4, 469, 499, 1936, 7744\}$.

41. $\{88, 480, 488\} \Rightarrow \{20, 177, 975, 11520, 230400\}$.

42. $\{90, 216, 234\} \Rightarrow \{12, 181, 467, 3888, 46656\}$.

43. $\{96, 110, 146\} \Rightarrow \{6, 221, 291, 1536, 9216\}$.

44. $\{96, 128, 160\} \Rightarrow \{8, 193, 319, 2048, 16384\}$.

45. $\{96, 280, 296\} \Rightarrow \{4, 561, 591, 2304, 9216\}$.

46. $\{96, 572, 580\} \Rightarrow \{22, 193, 1159, 14872, 327184\}$.

47. $\{99, 540, 549\} \Rightarrow \{3, 1081, 1097, 3267, 9801\}$.

48. $\{102, 280, 298\} \Rightarrow \{14, 205, 595, 5600, 78400\}$.

49. $\{104, 330, 346\} \Rightarrow \{4, 661, 691, 2704, 10816\}$.

50. $\{104, 672, 680\} \Rightarrow \{24, 209, 1359, 18816, 451584\}$.

51. $\{108, 144, 180\} \Rightarrow \{6, 289, 359, 1944, 11664\}$.

52. $\{108, 315, 333\} \Rightarrow \{15, 217, 665, 6615, 99225\}$.

53. $\{112, 180, 212\} \Rightarrow \{10, 225, 423, 3240, 32400\}$.

54. $\{112, 384, 400\} \Rightarrow \{4, 769, 799, 3136, 12544\}$.

55. $\{112, 780, 788\} \Rightarrow \{26, 225, 1575, 23400, 608400\}$.

56. $\{114, 352, 370\} \Rightarrow \{16, 229, 739, 7744, 123904\}$.

57. $\{117, 756, 765\} \Rightarrow \{3, 1513, 1529, 4563, 13689\}$.

58. $\{120, 182, 218\} \Rightarrow \{6, 365, 435, 2400, 14400\}$.

59. $\{120, 442, 458\} \Rightarrow \{4, 885, 915, 3600, 14400\}$.

60. $\{120, 896, 904\} \Rightarrow \{28, 241, 1807, 28672, 802816\}$.

61. $\{125, 300, 325\} \Rightarrow \{5, 601, 649, 3125, 15625\}$.

62. $\{126, 432, 450\} \Rightarrow \{18, 253, 899, 10368, 186624\}$.

63. $\{128, 240, 272\} \Rightarrow \{12, 257, 543, 4800, 57600\}$.

64. $\{128, 504, 520\} \Rightarrow \{4, 1009, 1039, 4096, 16384\}$.

65. $\{130, 144, 194\} \Rightarrow \{8, 261, 387, 2592, 20736\}$.

66. $\{132, 224, 260\} \Rightarrow \{6, 449, 519, 2904, 17424\}$.

67. $\{136, 570, 586\} \Rightarrow \{4, 1141, 1171, 4624, 18496\}$.

68. $\{138, 520, 538\} \Rightarrow \{20, 277, 1075, 13520, 270400\}$.

69. $\{144, 270, 306\} \Rightarrow \{6, 541, 611, 3456, 20736\}$.

70. $\{144, 308, 340\} \Rightarrow \{14, 289, 679, 6776, 94864\}$.

71. $\{144, 567, 585\} \Rightarrow \{21, 289, 1169, 15309, 321489\}$.

72. $\{144, 640, 656\} \Rightarrow \{4, 1281, 1311, 5184, 20736\}$.

73. $\{147, 196, 245\} \Rightarrow \{7, 393, 489, 3087, 21609\}$.

74. $\{150, 200, 250\} \Rightarrow \{10, 301, 499, 4000, 40000\}$.

75. $\{150, 616, 634\} \Rightarrow \{22, 301, 1267, 17248, 379456\}$.

76. $\{152, 714, 730\} \Rightarrow \{4, 1429, 1459, 5776, 23104\}$.

77. $\{156, 320, 356\} \Rightarrow \{6, 641, 711, 4056, 24336\}$.

78. $\{160, 168, 232\} \Rightarrow \{8, 337, 463, 3200, 25600\}$.

79. $\{160, 384, 416\} \Rightarrow \{16, 321, 831, 9216, 147456\}$.

80. $\{160, 792, 808\} \Rightarrow \{4, 1585, 1615, 6400, 25600\}$.

81. $\{162, 720, 738\} \Rightarrow \{24, 325, 1475, 21600, 518400\}$.

82. $\{168, 374, 410\} \Rightarrow \{6, 749, 819, 4704, 28224\}$.

83. $\{168, 874, 890\} \Rightarrow \{4, 1749, 1779, 7056, 28224\}$.

84. $\{170, 264, 314\} \Rightarrow \{12, 341, 627, 5808, 69696\}$.

85. $\{174, 832, 850\} \Rightarrow \{26, 349, 1699, 26624, 692224\}$.

86. $\{175, 600, 625\} \Rightarrow \{5, 1201, 1249, 6125, 30625\}$.

87. $\{176, 210, 274\} \Rightarrow \{8, 421, 547, 3872, 30976\}$.

88. $\{176, 468, 500\} \Rightarrow \{18, 353, 999, 12168, 219024\}$.

89. $\{176, 960, 976\} \Rightarrow \{4, 1921, 1951, 7744, 30976\}$.

90. $\{180, 189, 261\} \Rightarrow \{9, 361, 521, 3969, 35721\}$.

91. $\{180, 432, 468\} \Rightarrow \{6, 865, 935, 5400, 32400\}$.

92. $\{180, 891, 909\} \Rightarrow \{27, 361, 1817, 29403, 793881\}$.

93. $\{186, 952, 970\} \Rightarrow \{28, 373, 1939, 32368, 906304\}$.

94. $\{190, 336, 386\} \Rightarrow \{14, 381, 771, 8064, 112896\}$.

95. $\{192, 220, 292\} \Rightarrow \{10, 385, 583, 4840, 48400\}$.

96. $\{192, 256, 320\} \Rightarrow \{8, 513, 639, 4608, 36864\}$.

97. $\{192, 494, 530\} \Rightarrow \{6, 989, 1059, 6144, 36864\}$.

98. $\{192, 560, 592\} \Rightarrow \{20, 385, 1183, 15680, 313600\}$.

99. $\{200, 375, 425\} \Rightarrow \{15, 401, 849, 9375, 140625\}$.

100. $\{204, 560, 596\} \Rightarrow \{6, 1121, 1191, 6936, 41616\}$.

101. $\{208, 306, 370\} \Rightarrow \{8, 613, 739, 5408, 43264\}$.

102. $\{208, 660, 692\} \Rightarrow \{22, 417, 1383, 19800, 435600\}$.

103. $\{210, 416, 466\} \Rightarrow \{16, 421, 931, 10816, 173056\}$.

104. $\{216, 288, 360\} \Rightarrow \{12, 433, 719, 6912, 82944\}$.

105. $\{216, 630, 666\} \Rightarrow \{6, 1261, 1331, 7776, 46656\}$.

106. $\{224, 360, 424\} \Rightarrow \{8, 721, 847, 6272, 50176\}$.

107. $\{224, 768, 800\} \Rightarrow \{24, 449, 1599, 24576, 589824\}$.

108. $\{228, 704, 740\} \Rightarrow \{6, 1409, 1479, 8664, 51984\}$.

109. $\{230, 504, 554\} \Rightarrow \{18, 461, 1107, 14112, 254016\}$.

110. $\{238, 240, 338\} \Rightarrow \{10, 477, 675, 5760, 57600\}$.

111. $\{240, 364, 436\} \Rightarrow \{14, 481, 871, 9464, 132496\}$.

112. $\{240, 418, 482\} \Rightarrow \{8, 837, 963, 7200, 57600\}$.

113. $\{240, 782, 818\} \Rightarrow \{6, 1565, 1635, 9600, 57600\}$.

114. $\{240, 884, 916\} \Rightarrow \{26, 481, 1831, 30056, 781456\}$.

115. $\{243, 324, 405\} \Rightarrow \{9, 649, 809, 6561, 59049\}$.

116. $\{245, 588, 637\} \Rightarrow \{7, 1177, 1273, 8575, 60025\}$.

117. $\{250, 600, 650\} \Rightarrow \{20, 501, 1299, 18000, 360000\}$.

118. $\{252, 405, 477\} \Rightarrow \{15, 505, 953, 10935, 164025\}$.

119. $\{252, 864, 900\} \Rightarrow \{6, 1729, 1799, 10584, 63504\}$.

120. $\{256, 480, 544\} \Rightarrow \{8, 961, 1087, 8192, 65536\}$.

121. $\{260, 288, 388\} \Rightarrow \{10, 577, 775, 6760, 67600\}$.

122. $\{264, 448, 520\} \Rightarrow \{16, 529, 1039, 12544, 200704\}$.

123. $\{264, 950, 986\} \Rightarrow \{6, 1901, 1971, 11616, 69696\}$.

124. $\{266, 312, 410\} \Rightarrow \{12, 533, 819, 8112, 97344\}$.

125. $\{270, 704, 754\} \Rightarrow \{22, 541, 1507, 22528, 495616\}$.

126. $\{272, 546, 610\} \Rightarrow \{8, 1093, 1219, 9248, 73984\}$.

127. $\{280, 342, 442\} \Rightarrow \{10, 685, 883, 7840, 78400\}$.

128. $\{288, 540, 612\} \Rightarrow \{18, 577, 1223, 16200, 291600\}$.

129. $\{288, 616, 680\} \Rightarrow \{8, 1233, 1359, 10368, 82944\}$.

130. $\{290, 816, 866\} \Rightarrow \{24, 581, 1731, 27744, 665856\}$.

131. $\{294, 392, 490\} \Rightarrow \{14, 589, 979, 10976, 153664\}$.

132. $\{297, 504, 585\} \Rightarrow \{9, 1009, 1169, 9801, 88209\}$.

133. $\{300, 400, 500\} \Rightarrow \{10, 801, 999, 9000, 90000\}$.

134. $\{300, 875, 925\} \Rightarrow \{25, 601, 1849, 30625, 765625\}$.

135. $\{304, 690, 754\} \Rightarrow \{8, 1381, 1507, 11552, 92416\}$.

136. $\{310, 936, 986\} \Rightarrow \{26, 621, 1971, 33696, 876096\}$.

137. $\{312, 640, 712\} \Rightarrow \{20, 625, 1423, 20480, 409600\}$.

138. $\{320, 336, 464\} \Rightarrow \{12, 641, 927, 9408, 112896\}$.

139. $\{320, 462, 562\} \Rightarrow \{10, 925, 1123, 10240, 102400\}$.

140. $\{320, 768, 832\} \Rightarrow \{8, 1537, 1663, 12800, 102400\}$.

141. $\{322, 480, 578\} \Rightarrow \{16, 645, 1155, 14400, 230400\}$.

142. $\{324, 693, 765\} \Rightarrow \{21, 649, 1529, 22869, 480249\}$.

143. $\{336, 748, 820\} \Rightarrow \{22, 673, 1639, 25432, 559504\}$.

144. $\{336, 850, 914\} \Rightarrow \{8, 1701, 1827, 14112, 112896\}$.

145. $\{340, 528, 628\} \Rightarrow \{10, 1057, 1255, 11560, 115600\}$.

146. $\{350, 576, 674\} \Rightarrow \{18, 701, 1347, 18432, 331776\}$.

147. $\{351, 720, 801\} \Rightarrow \{9, 1441, 1601, 13689, 123201\}$.

148. $\{352, 420, 548\} \Rightarrow \{14, 705, 1095, 12600, 176400\}$.

149. $\{360, 378, 522\} \Rightarrow \{12, 757, 1043, 10800, 129600\}$.

150. $\{360, 598, 698\} \Rightarrow \{10, 1197, 1395, 12960, 129600\}$.

151. $\{360, 864, 936\} \Rightarrow \{24, 721, 1871, 31104, 746496\}$.

152. $\{363, 484, 605\} \Rightarrow \{11, 969, 1209, 11979, 131769\}$.

153. $\{378, 680, 778\} \Rightarrow \{20, 757, 1555, 23120, 462400\}$.

154. $\{380, 672, 772\} \Rightarrow \{10, 1345, 1543, 14440, 144400\}$.

155. $\{384, 440, 584\} \Rightarrow \{12, 881, 1167, 12288, 147456\}$.

156. $\{384, 512, 640\} \Rightarrow \{16, 769, 1279, 16384, 262144\}$.

157. $\{392, 735, 833\} \Rightarrow \{21, 785, 1665, 25725, 540225\}$.

158. $\{400, 750, 850\} \Rightarrow \{10, 1501, 1699, 16000, 160000\}$.

159. $\{406, 792, 890\} \Rightarrow \{22, 813, 1779, 28512, 627264\}$.

160. $\{408, 506, 650\} \Rightarrow \{12, 1013, 1299, 13872, 166464\}$.

161. $\{414, 448, 610\} \Rightarrow \{14, 829, 1219, 14336, 200704\}$.

162. $\{416, 612, 740\} \Rightarrow \{18, 833, 1479, 20808, 374544\}$.

163. $\{420, 832, 932\} \Rightarrow \{10, 1665, 1863, 17640, 176400\}$.

164. $\{432, 495, 657\} \Rightarrow \{15, 865, 1313, 16335, 245025\}$.

165. $\{432, 576, 720\} \Rightarrow \{12, 1153, 1439, 15552, 186624\}$.

166. $\{448, 720, 848\} \Rightarrow \{20, 897, 1695, 25920, 518400\}$.

167. $\{450, 544, 706\} \Rightarrow \{16, 901, 1411, 18496, 295936\}$.

168. $\{456, 650, 794\} \Rightarrow \{12, 1301, 1587, 17328, 207936\}$.

169. $\{476, 480, 676\} \Rightarrow \{14, 961, 1351, 16184, 226576\}$.

170. $\{480, 728, 872\} \Rightarrow \{12, 1457, 1743, 19200, 230400\}$.

171. $\{480, 836, 964\} \Rightarrow \{22, 961, 1927, 31768, 698896\}$.

172. $\{486, 648, 810\} \Rightarrow \{18, 973, 1619, 23328, 419904\}$.

173. $\{500, 525, 725\} \Rightarrow \{15, 1001, 1449, 18375, 275625\}$.

174. $\{504, 550, 746\} \Rightarrow \{14, 1101, 1491, 18144, 254016\}$.

175. $\{504, 810, 954\} \Rightarrow \{12, 1621, 1907, 21168, 254016\}$.

176. $\{507, 676, 845\} \Rightarrow \{13, 1353, 1689, 19773, 257049\}$.

177. $\{520, 576, 776\} \Rightarrow \{16, 1041, 1551, 20736, 331776\}$.

178. $\{522, 760, 922\} \Rightarrow \{20, 1045, 1843, 28880, 577600\}$.

179. $\{532, 624, 820\} \Rightarrow \{14, 1249, 1639, 20216, 283024\}$.

180. $\{540, 819, 981\} \Rightarrow \{21, 1081, 1961, 31941, 670761\}$.

181. $\{560, 684, 884\} \Rightarrow \{18, 1121, 1767, 25992, 467856\}$.

182. $\{560, 702, 898\} \Rightarrow \{14, 1405, 1795, 22400, 313600\}$.

183. $\{585, 648, 873\} \Rightarrow \{15, 1297, 1745, 22815, 342225\}$.

184. $\{588, 784, 980\} \Rightarrow \{14, 1569, 1959, 24696, 345744\}$.

185. $\{594, 608, 850\} \Rightarrow \{16, 1189, 1699, 23104, 369664\}$.

186. $\{638, 720, 962\} \Rightarrow \{18, 1277, 1923, 28800, 518400\}$.

More possible primitive and non primitive triples having maximum value 2000 are given in Appendix 7.

5.1 Examples: Non Primitive Triples

Let's see how to use the first four examples to write magic squares:

$$\{10, 24, 26\} \Rightarrow \{4, 21, 51, 144, 576\}\,.$$
$$\{14, 48, 50\} \Rightarrow \{6, 29, 99, 384, 2304\}\,.$$
$$\{18, 80, 82\} \Rightarrow \{8, 37, 163, 800, 6400\}\,.$$
$$\{22, 120, 122\} \Rightarrow \{10, 45, 243, 1440, 14400\}).$$

In case of non primitive triples, we produce either even or odd order magic squares. Here, these four examples are of even order magic squares.

5.1.1   Magic Square Generated by Triple (10, 24, 26)

According to triple $\{10, 24, 26\} \Rightarrow \{4, 21, 51, 144, 576\}$, we have a magic square of order 4 with 16 odd consecutive numbers starting from 21 and ending at 51, i.e., (21, 23, 25, ...,49, 51). The pan diagonal magic square of order 4 is given by

		144	144	144	144
	33	43	21	47	144
144	23	45	35	41	144
144	51	25	39	29	144
144	37	31	49	27	144
	144	144	144	144	144

The above pan diagonal magic square has a magic sum $S_{4 \times 4} = 144$, and the sum of all entries gives a perfect square $576 := 24^2 = 26^2 - 10^2$.

5.1.2   Magic Square Generated by Triple (14, 48, 50)

According to triple $\{14, 48, 50\} \Rightarrow \{6, 29, 99, 384, 2304\}$, we have a magic square of order 6 with 36 odd consecutive numbers starting from 29 and ending at 99, i.e., (29, 31, 33, ..., 97, 99). The magic square of order 6 is given by

						384
29	73	83	95	61	43	384
85	41	97	55	69	37	384
51	39	53	81	89	71	384
91	59	35	75	47	77	384
65	93	49	33	87	57	384
63	79	67	45	31	99	384
384	384	384	384	384	384	384

The above magic square has a magic sum $S_{6 \times 6} = 384$, and the sum of all entries gives a perfect square $2304 := 48^2 = 50^2 - 14^2$.

5.1.3   Magic Square Generated by Triple (18, 80, 82)

According to triple $\{18, 80, 82\} \Rightarrow \{8, 37, 163, 800, 6400\}$, we have a magic square of order 8 with 64 odd consecutive numbers starting from 37 and ending at 163, i.e., (37, 39, 41, ..., 161, 163). The pan diagonal magic square of order 8 is given by

		800	800	800	800	800	800	800	800
	67	117	107	45	89	159	145	71	800
800	87	161	143	73	61	123	101	51	800
800	37	115	125	59	79	137	151	97	800
800	81	135	153	95	43	109	131	53	800
800	111	41	55	129	133	83	93	155	800
800	139	77	99	149	113	39	57	127	800
800	121	63	49	103	163	85	75	141	800
800	157	91	69	147	119	65	47	105	800
	800	800	800	800	800	800	800	800	800

The above pan diagonal magic square has a magic sum $S_{8\times8} = 800$, and the sum of all entries gives a perfect square $6400 := 80^2 = 82^2 - 18^2$. Moreover, the above magic square is bimagic and its bimagic sum is $Sb_{8\times8} = 90920$.

								90920
4489	13689	11449	2025	7921	25281	21025	5041	90920
7569	25921	20449	5329	3721	15129	10201	2601	90920
1369	13225	15625	3481	6241	18769	22801	9409	90920
6561	18225	23409	9025	1849	11881	17161	2809	90920
12321	1681	3025	16641	17689	6889	8649	24025	90920
19321	5929	9801	22201	12769	1521	3249	16129	90920
14641	3969	2401	10609	26569	7225	5625	19881	90920
24649	8281	4761	21609	14161	4225	2209	11025	90920
90920	90920	90920	90920	90920	90920	90920	90920	90920

Below is another example of a pan diagonal magic square of order 8 with the same entries as in the above example, but the difference being that it is not bimagic but the totals of many four entries, read symmetrically or grouped gives the same sum, i.e, 800:

		800	*800*	*800*	*800*	*800*	*800*	*800*	*800*
	37	157	91	115	53	141	75	131	*800*
800	99	107	45	149	83	123	61	133	*800*
800	109	85	163	43	125	69	147	59	*800*
800	155	51	101	93	139	67	117	77	*800*
800	39	159	89	113	55	143	73	129	*800*
800	97	105	47	151	81	121	63	135	*800*
800	111	87	161	41	127	71	145	57	*800*
800	153	49	103	95	137	65	119	79	*800*
	800	*800*	*800*	*800*	*800*	*800*	*800*	*800*	*800*

Below are few situations shown in coloured groups four by four, where the resulting sum is 400. Obviously there are many much more possibilities.

5.1.4   Magic Square Generated by Triple (22, 120, 122)

According to triple $\{22, 120, 122\} \Rightarrow \{10, 45, 243, 1440, 14400\}$), we have a magic square of order 10 with 100 odd consecutive numbers starting from 45 and ending at 243, i.e., (45, 47, 49, ..., 241, 243). The magic square of order 10 is given by

										1440
45	203	173	237	121	87	139	215	149	71	*1440*
239	67	61	175	223	191	153	109	125	97	*1440*
137	205	89	201	75	113	231	163	167	59	*1440*
183	157	219	111	47	225	101	73	195	129	*1440*
211	241	147	65	133	179	189	57	103	115	*1440*
69	119	131	63	197	155	207	85	233	181	*1440*
193	135	123	209	99	81	177	227	51	145	*1440*
161	91	235	127	165	49	83	199	117	213	*1440*
95	53	77	159	229	143	105	171	221	187	*1440*
107	169	185	93	151	217	55	141	79	243	*1440*
1440	*1440*	*1440*	*1440*	*1440*	*1440*	*1440*	*1440*	*1440*	*1440*	*1440*

The above magic square has a magic sum $S_{10\times10} = 1440$, and the sum of all entries gives a perfect square $14400 := 120^2 = 122^2 - 22^2$.

6   Summary

In conclusion, we have given total 9 examples, five with primitive triples and another four with non primitive triples. These nine examples show resulting magic squares of order 3 to 11. In the primitive cases, we always have magic squares of odd orders, while in non primitive cases, we can have both the situations, i.e., even as well as odd orders magic squares. The examples given above show the general idea of constructing magic squares based on Pythagorean triples. Using a similar procedure we can always construct a magic square for the Pythagorean triples given in sections 4 and 5. More study on magic squares can be seen in author's work [15]–[20]. In [20] a similar kind of procedure is used to find perfect square sum magic squares. In [12] a similar kind of work is done connecting to Pythagoras's theorem and digital letters representations. For more studies on specific days connecting to digital letters representations of magic squares refer [7]–[11]. For block–wise construction of bimagic squares up to order 4096 refer [13]. Some studies on area–wise magic squares can be seen in [6].

The numbers 1089 and 9801 appearing in magic square of order 9 given in example 4.1.3 are very famous for their numerical and magic square properties [1, 14]. Moreover this example gives a very interesting Pythagorean pattern [2]. See below:

$$20^2 + 99^2 = 101^2 \qquad := 10201$$
$$200^2 + 9999^2 = 1001^2 \qquad := 100020001$$
$$2000^2 + 999999^2 = 100001^2 \quad := 1000002000001$$
$$20000^2 + 99999999^2 = 10000001^2 := 10000000200000001$$
$$\cdots \quad \cdots \qquad \cdots \qquad \cdots$$

7   Appendix

Generating on line [5], total we have 1103 primitive and non primitive triples considering c between 1000 and 2000, where $c^2 = a^2 + b^2$. Among these there are 365 primitive and non primitive triples those generate magic squares with sum of all entries a perfect square. Below is a list of these 365 triples written in increasing order of" a, calculated according to formula given in 5:

1. $\{78, 1520, 1522\} \Rightarrow \{38, 157, 3043, 60800, 2310400\}$.
2. $\{80, 1599, 1601\} \Rightarrow \{39, 161, 3201, 65559, 2556801\}$.
3. $\{82, 1680, 1682\} \Rightarrow \{40, 165, 3363, 70560, 2822400\}$.
4. $\{84, 1763, 1765\} \Rightarrow \{41, 169, 3529, 75809, 3108169\}$.
5. $\{88, 1935, 1937\} \Rightarrow \{43, 177, 3873, 87075, 3744225\}$.
6. $\{128, 1020, 1028\} \Rightarrow \{30, 257, 2055, 34680, 1040400\}$.
7. $\{132, 1085, 1093\} \Rightarrow \{31, 265, 2185, 37975, 1177225\}$.
8. $\{135, 1008, 1017\} \Rightarrow \{3, 2017, 2033, 6075, 18225\}$.
9. $\{136, 1152, 1160\} \Rightarrow \{32, 273, 2319, 41472, 1327104\}$.
10. $\{140, 1221, 1229\} \Rightarrow \{33, 281, 2457, 45177, 1490841\}$.
11. $\{141, 1100, 1109\} \Rightarrow \{3, 2201, 2217, 6627, 19881\}$.
12. $\{144, 1292, 1300\} \Rightarrow \{34, 289, 2599, 49096, 1669264\}$.
13. $\{147, 1196, 1205\} \Rightarrow \{3, 2393, 2409, 7203, 21609\}$.
14. $\{148, 1365, 1373\} \Rightarrow \{35, 297, 2745, 53235, 1863225\}$.
15. $\{152, 1440, 1448\} \Rightarrow \{36, 305, 2895, 57600, 2073600\}$.
16. $\{153, 1296, 1305\} \Rightarrow \{3, 2593, 2609, 7803, 23409\}$.
17. $\{156, 1517, 1525\} \Rightarrow \{37, 313, 3049, 62197, 2301289\}$.
18. $\{159, 1400, 1409\} \Rightarrow \{3, 2801, 2817, 8427, 25281\}$.
19. $\{160, 1596, 1604\} \Rightarrow \{38, 321, 3207, 67032, 2547216\}$.
20. $\{164, 1677, 1685\} \Rightarrow \{39, 329, 3369, 72111, 2812329\}$.
21. $\{165, 1508, 1517\} \Rightarrow \{3, 3017, 3033, 9075, 27225\}$.
22. $\{168, 1760, 1768\} \Rightarrow \{40, 337, 3535, 77440, 3097600\}$.
23. $\{171, 1620, 1629\} \Rightarrow \{3, 3241, 3257, 9747, 29241\}$.
24. $\{172, 1845, 1853\} \Rightarrow \{41, 345, 3705, 83025, 3404025\}$.
25. $\{176, 1932, 1940\} \Rightarrow \{42, 353, 3879, 88872, 3732624\}$.

26. $\{177, 1736, 1745\} \Rightarrow \{3, 3473, 3489, 10443, 31329\}$.

27. $\{183, 1856, 1865\} \Rightarrow \{3, 3713, 3729, 11163, 33489\}$.

28. $\{184, 1050, 1066\} \Rightarrow \{4, 2101, 2131, 8464, 33856\}$.

29. $\{189, 1980, 1989\} \Rightarrow \{3, 3961, 3977, 11907, 35721\}$.

30. $\{192, 1015, 1033\} \Rightarrow \{29, 385, 2065, 35525, 1030225\}$.

31. $\{192, 1144, 1160\} \Rightarrow \{4, 2289, 2319, 9216, 36864\}$.

32. $\{198, 1080, 1098\} \Rightarrow \{30, 397, 2195, 38880, 1166400\}$.

33. $\{200, 1242, 1258\} \Rightarrow \{4, 2485, 2515, 10000, 40000\}$.

34. $\{204, 1147, 1165\} \Rightarrow \{31, 409, 2329, 42439, 1315609\}$.

35. $\{208, 1344, 1360\} \Rightarrow \{4, 2689, 2719, 10816, 43264\}$.

36. $\{210, 1216, 1234\} \Rightarrow \{32, 421, 2467, 46208, 1478656\}$.

37. $\{216, 1287, 1305\} \Rightarrow \{33, 433, 2609, 50193, 1656369\}$.

38. $\{216, 1450, 1466\} \Rightarrow \{4, 2901, 2931, 11664, 46656\}$.

39. $\{222, 1360, 1378\} \Rightarrow \{34, 445, 2755, 54400, 1849600\}$.

40. $\{224, 1560, 1576\} \Rightarrow \{4, 3121, 3151, 12544, 50176\}$.

41. $\{225, 1000, 1025\} \Rightarrow \{5, 2001, 2049, 10125, 50625\}$.

42. $\{228, 1435, 1453\} \Rightarrow \{35, 457, 2905, 58835, 2059225\}$.

43. $\{232, 1674, 1690\} \Rightarrow \{4, 3349, 3379, 13456, 53824\}$.

44. $\{234, 1512, 1530\} \Rightarrow \{36, 469, 3059, 63504, 2286144\}$.

45. $\{235, 1092, 1117\} \Rightarrow \{5, 2185, 2233, 11045, 55225\}$.

46. $\{240, 1591, 1609\} \Rightarrow \{37, 481, 3217, 68413, 2531281\}$.

47. $\{240, 1792, 1808\} \Rightarrow \{4, 3585, 3615, 14400, 57600\}$.

48. $\{245, 1188, 1213\} \Rightarrow \{5, 2377, 2425, 12005, 60025\}$.

49. $\{246, 1672, 1690\} \Rightarrow \{38, 493, 3379, 73568, 2795584\}$.

50. $\{248, 1914, 1930\} \Rightarrow \{4, 3829, 3859, 15376, 61504\}$.

51. $\{252, 1755, 1773\} \Rightarrow \{39, 505, 3545, 78975, 3080025\}$.

52. $\{255, 1288, 1313\} \Rightarrow \{5, 2577, 2625, 13005, 65025\}$.

53. $\{256, 1008, 1040\} \Rightarrow \{28, 513, 2079, 36288, 1016064\}$.

54. $\{258, 1840, 1858\} \Rightarrow \{40, 517, 3715, 84640, 3385600\}$.

55. $\{264, 1073, 1105\} \Rightarrow \{29, 529, 2209, 39701, 1151329\}$.

56. $\{264, 1927, 1945\} \Rightarrow \{41, 529, 3889, 90569, 3713329\}$.

57. $\{265, 1392, 1417\} \Rightarrow \{5, 2785, 2833, 14045, 70225\}$.

58. $\{272, 1140, 1172\} \Rightarrow \{30, 545, 2343, 43320, 1299600\}$.

59. $\{275, 1500, 1525\} \Rightarrow \{5, 3001, 3049, 15125, 75625\}$.

60. $\{276, 1040, 1076\} \Rightarrow \{6, 2081, 2151, 12696, 76176\}$.

61. $\{280, 1209, 1241\} \Rightarrow \{31, 561, 2481, 47151, 1461681\}$.

62. $\{285, 1612, 1637\} \Rightarrow \{5, 3225, 3273, 16245, 81225\}$.

63. $\{288, 1134, 1170\} \Rightarrow \{6, 2269, 2339, 13824, 82944\}$.

64. $\{288, 1280, 1312\} \Rightarrow \{32, 577, 2623, 51200, 1638400\}$.

65. $\{295, 1728, 1753\} \Rightarrow \{5, 3457, 3505, 17405, 87025\}$.

66. $\{296, 1353, 1385\} \Rightarrow \{33, 593, 2769, 55473, 1830609\}$.

67. $\{300, 1232, 1268\} \Rightarrow \{6, 2465, 2535, 15000, 90000\}$.

68. $\{304, 1428, 1460\} \Rightarrow \{34, 609, 2919, 59976, 2039184\}$.

69. $\{305, 1848, 1873\} \Rightarrow \{5, 3697, 3745, 18605, 93025\}$.

70. $\{312, 1334, 1370\} \Rightarrow \{6, 2669, 2739, 16224, 97344\}$.

71. $\{312, 1505, 1537\} \Rightarrow \{35, 625, 3073, 64715, 2265025\}$.

72. $\{315, 1972, 1997\} \Rightarrow \{5, 3945, 3993, 19845, 99225\}$.

73. $\{315, 988, 1037\} \Rightarrow \{7, 1977, 2073, 14175, 99225\}$.

74. $\{320, 1584, 1616\} \Rightarrow \{36, 641, 3231, 69696, 2509056\}$.

75. $\{320, 999, 1049\} \Rightarrow \{27, 641, 2097, 36963, 998001\}$.

76. $\{324, 1440, 1476\} \Rightarrow \{6, 2881, 2951, 17496, 104976\}$.

77. $\{328, 1665, 1697\} \Rightarrow \{37, 657, 3393, 74925, 2772225\}$.

78. $\{329, 1080, 1129\} \Rightarrow \{7, 2161, 2257, 15463, 108241\}$.

79. $\{330, 1064, 1114\} \Rightarrow \{28, 661, 2227, 40432, 1132096\}$.

80. $\{336, 1550, 1586\} \Rightarrow \{6, 3101, 3171, 18816, 112896\}$.

81. $\{336, 1748, 1780\} \Rightarrow \{38, 673, 3559, 80408, 3055504\}$.

82. $\{340, 1131, 1181\} \Rightarrow \{29, 681, 2361, 44109, 1279161\}$.

83. $\{343, 1176, 1225\} \Rightarrow \{7, 2353, 2449, 16807, 117649\}$.

84. $\{344, 1833, 1865\} \Rightarrow \{39, 689, 3729, 86151, 3359889\}$.

85. $\{348, 1664, 1700\} \Rightarrow \{6, 3329, 3399, 20184, 121104\}$.

86. $\{350, 1200, 1250\} \Rightarrow \{30, 701, 2499, 48000, 1440000\}$.

87. $\{352, 1920, 1952\} \Rightarrow \{40, 705, 3903, 92160, 3686400\}$.

88. $\{352, 936, 1000\} \Rightarrow \{8, 1873, 1999, 15488, 123904\}$.

89. $\{357, 1276, 1325\} \Rightarrow \{7, 2553, 2649, 18207, 127449\}$.

90. $\{360, 1271, 1321\} \Rightarrow \{31, 721, 2641, 52111, 1615441\}$.

91. $\{360, 1782, 1818\} \Rightarrow \{6, 3565, 3635, 21600, 129600\}$.

92. $\{368, 1026, 1090\} \Rightarrow \{8, 2053, 2179, 16928, 135424\}$.

93. $\{370, 1344, 1394\} \Rightarrow \{32, 741, 2787, 56448, 1806336\}$.

94. $\{371, 1380, 1429\} \Rightarrow \{7, 2761, 2857, 19663, 137641\}$.

95. $\{372, 1904, 1940\} \Rightarrow \{6, 3809, 3879, 23064, 138384\}$.

96. $\{380, 1419, 1469\} \Rightarrow \{33, 761, 2937, 61017, 2013561\}$.

97. $\{384, 1120, 1184\} \Rightarrow \{8, 2241, 2367, 18432, 147456\}$.

98. $\{384, 988, 1060\} \Rightarrow \{26, 769, 2119, 37544, 976144\}$.

99. $\{385, 1488, 1537\} \Rightarrow \{7, 2977, 3073, 21175, 148225\}$.

100. $\{390, 1496, 1546\} \Rightarrow \{34, 781, 3091, 65824, 2238016\}$.

101. $\{396, 1053, 1125\} \Rightarrow \{27, 793, 2249, 41067, 1108809\}$.

102. $\{399, 1600, 1649\} \Rightarrow \{7, 3201, 3297, 22743, 159201\}$.

103. $\{400, 1218, 1282\} \Rightarrow \{8, 2437, 2563, 20000, 160000\}$.

104. $\{400, 1575, 1625\} \Rightarrow \{35, 801, 3249, 70875, 2480625\}$.

105. $\{405, 972, 1053\} \Rightarrow \{9, 1945, 2105, 18225, 164025\}$.

106. $\{408, 1120, 1192\} \Rightarrow \{28, 817, 2383, 44800, 1254400\}$.

107. $\{410, 1656, 1706\} \Rightarrow \{36, 821, 3411, 76176, 2742336\}$.

108. $\{413, 1716, 1765\} \Rightarrow \{7, 3433, 3529, 24367, 170569\}$.

109. $\{416, 1320, 1384\} \Rightarrow \{8, 2641, 2767, 21632, 173056\}$.

110. $\{420, 1189, 1261\} \Rightarrow \{29, 841, 2521, 48749, 1413721\}$.

111. $\{420, 1739, 1789\} \Rightarrow \{37, 841, 3577, 81733, 3024121\}$.

112. $\{423, 1064, 1145\} \Rightarrow \{9, 2129, 2289, 19881, 178929\}$.

113. $\{427, 1836, 1885\} \Rightarrow \{7, 3673, 3769, 26047, 182329\}$.

114. $\{430, 1824, 1874\} \Rightarrow \{38, 861, 3747, 87552, 3326976\}$.

115. $\{432, 1260, 1332\} \Rightarrow \{30, 865, 2663, 52920, 1587600\}$.

116. $\{432, 1426, 1490\} \Rightarrow \{8, 2853, 2979, 23328, 186624\}$.

117. $\{434, 912, 1010\} \Rightarrow \{24, 869, 2019, 34656, 831744\}$.

118. $\{440, 1911, 1961\} \Rightarrow \{39, 881, 3921, 93639, 3651921\}$.

119. $\{440, 918, 1018\} \Rightarrow \{10, 1837, 2035, 19360, 193600\}$.

120. $\{441, 1160, 1241\} \Rightarrow \{9, 2321, 2481, 21609, 194481\}$.

121. $\{444, 1333, 1405\} \Rightarrow \{31, 889, 2809, 57319, 1776889\}$.

122. $\{448, 1536, 1600\} \Rightarrow \{8, 3073, 3199, 25088, 200704\}$.

123. $\{448, 975, 1073\} \Rightarrow \{25, 897, 2145, 38025, 950625\}$.

124. $\{456, 1408, 1480\} \Rightarrow \{32, 913, 2959, 61952, 1982464\}$.

125. $\{459, 1260, 1341\} \Rightarrow \{9, 2521, 2681, 23409, 210681\}$.

126. $\{460, 1008, 1108\} \Rightarrow \{10, 2017, 2215, 21160, 211600\}$.

127. $\{462, 1040, 1138\} \Rightarrow \{26, 925, 2275, 41600, 1081600\}$.

128. $\{464, 1650, 1714\} \Rightarrow \{8, 3301, 3427, 26912, 215296\}$.

129. $\{468, 1485, 1557\} \Rightarrow \{33, 937, 3113, 66825, 2205225\}$.

130. $\{476, 1107, 1205\} \Rightarrow \{27, 953, 2409, 45387, 1225449\}$.

131. $\{477, 1364, 1445\} \Rightarrow \{9, 2729, 2889, 25281, 227529\}$.

132. $\{480, 1102, 1202\} \Rightarrow \{10, 2205, 2403, 23040, 230400\}$.

133. $\{480, 1564, 1636\} \Rightarrow \{34, 961, 3271, 71944, 2446096\}$.

134. $\{480, 1768, 1832\} \Rightarrow \{8, 3537, 3663, 28800, 230400\}$.

135. $\{490, 1176, 1274\} \Rightarrow \{28, 981, 2547, 49392, 1382976\}$.

136. $\{492, 1645, 1717\} \Rightarrow \{35, 985, 3433, 77315, 2706025\}$.

137. $\{495, 1472, 1553\} \Rightarrow \{9, 2945, 3105, 27225, 245025\}$.

138. $\{495, 952, 1073\} \Rightarrow \{11, 1905, 2145, 22275, 245025\}$.

139. $\{496, 1890, 1954\} \Rightarrow \{8, 3781, 3907, 30752, 246016\}$.

140. $\{496, 897, 1025\} \Rightarrow \{23, 993, 2049, 34983, 804609\}$.

141. $\{500, 1200, 1300\} \Rightarrow \{10, 2401, 2599, 25000, 250000\}$.

142. $\{504, 1247, 1345\} \Rightarrow \{29, 1009, 2689, 53621, 1555009\}$.

143. $\{504, 1728, 1800\} \Rightarrow \{36, 1009, 3599, 82944, 2985984\}$.

144. $\{512, 960, 1088\} \Rightarrow \{24, 1025, 2175, 38400, 921600\}$.

145. $\{513, 1584, 1665\} \Rightarrow \{9, 3169, 3329, 29241, 263169\}$.

146. $\{516, 1813, 1885\} \Rightarrow \{37, 1033, 3769, 88837, 3286969\}$.

147. $\{517, 1044, 1165\} \Rightarrow \{11, 2089, 2329, 24299, 267289\}$.

148. $\{518, 1320, 1418\} \Rightarrow \{30, 1037, 2835, 58080, 1742400\}$.

149. $\{520, 1302, 1402\} \Rightarrow \{10, 2605, 2803, 27040, 270400\}$.

150. $\{528, 1025, 1153\} \Rightarrow \{25, 1057, 2305, 42025, 1050625\}$.

151. $\{528, 1900, 1972\} \Rightarrow \{38, 1057, 3943, 95000, 3610000\}$.

152. $\{528, 896, 1040\} \Rightarrow \{12, 1793, 2079, 23232, 278784\}$.

153. $\{531, 1700, 1781\} \Rightarrow \{9, 3401, 3561, 31329, 281961\}$.

154. $\{532, 1395, 1493\} \Rightarrow \{31, 1065, 2985, 62775, 1946025\}$.

155. $\{539, 1140, 1261\} \Rightarrow \{11, 2281, 2521, 26411, 290521\}$.

156. $\{540, 1408, 1508\} \Rightarrow \{10, 2817, 3015, 29160, 291600\}$.

157. $\{544, 1092, 1220\} \Rightarrow \{26, 1089, 2439, 45864, 1192464\}$.

158. $\{546, 1472, 1570\} \Rightarrow \{32, 1093, 3139, 67712, 2166784\}$.

159. $\{549, 1820, 1901\} \Rightarrow \{9, 3641, 3801, 33489, 301401\}$.

160. $\{552, 986, 1130\} \Rightarrow \{12, 1973, 2259, 25392, 304704\}$.

161. $\{558, 880, 1042\} \Rightarrow \{22, 1117, 2083, 35200, 774400\}$.

162. $\{559, 840, 1009\} \Rightarrow \{13, 1681, 2017, 24037, 312481\}$.

163. $\{560, 1161, 1289\} \Rightarrow \{27, 1121, 2577, 49923, 1347921\}$.

164. $\{560, 1518, 1618\} \Rightarrow \{10, 3037, 3235, 31360, 313600\}$.

165. $\{560, 1551, 1649\} \Rightarrow \{33, 1121, 3297, 72897, 2405601\}$.

166. $\{561, 1240, 1361\} \Rightarrow \{11, 2481, 2721, 28611, 314721\}$.

167. $\{574, 1632, 1730\} \Rightarrow \{34, 1149, 3459, 78336, 2663424\}$.

168. $\{576, 1080, 1224\} \Rightarrow \{12, 2161, 2447, 27648, 331776\}$.

169. $\{576, 1232, 1360\} \Rightarrow \{28, 1153, 2719, 54208, 1517824\}$.

170. $\{576, 943, 1105\} \Rightarrow \{23, 1153, 2209, 38663, 889249\}$.

171. $\{580, 1632, 1732\} \Rightarrow \{10, 3265, 3463, 33640, 336400\}$.

172. $\{583, 1344, 1465\} \Rightarrow \{11, 2689, 2929, 30899, 339889\}$.

173. $\{585, 928, 1097\} \Rightarrow \{13, 1857, 2193, 26325, 342225\}$.

174. $\{588, 1715, 1813\} \Rightarrow \{35, 1177, 3625, 84035, 2941225\}$.

175. $\{592, 1305, 1433\} \Rightarrow \{29, 1185, 2865, 58725, 1703025\}$.

176. $\{594, 1008, 1170\} \Rightarrow \{24, 1189, 2339, 42336, 1016064\}$.

177. $\{600, 1178, 1322\} \Rightarrow \{12, 2357, 2643, 30000, 360000\}$.

178. $\{600, 1750, 1850\} \Rightarrow \{10, 3501, 3699, 36000, 360000\}$.

179. $\{600, 800, 1000\} \Rightarrow \{20, 1201, 1999, 32000, 640000\}$.

180. $\{602, 1800, 1898\} \Rightarrow \{36, 1205, 3795, 90000, 3240000\}$.

181. $\{605, 1452, 1573\} \Rightarrow \{11, 2905, 3145, 33275, 366025\}$.

182. $\{608, 1380, 1508\} \Rightarrow \{30, 1217, 3015, 63480, 1904400\}$.

183. $\{611, 1020, 1189\} \Rightarrow \{13, 2041, 2377, 28717, 373321\}$.

184. $\{612, 1075, 1237\} \Rightarrow \{25, 1225, 2473, 46225, 1155625\}$.

185. $\{616, 1887, 1985\} \Rightarrow \{37, 1233, 3969, 96237, 3560769\}$.

186. $\{616, 870, 1066\} \Rightarrow \{14, 1741, 2131, 27104, 379456\}$.

187. $\{620, 1872, 1972\} \Rightarrow \{10, 3745, 3943, 38440, 384400\}$.

188. $\{620, 861, 1061\} \Rightarrow \{21, 1241, 2121, 35301, 741321\}$.

189. $\{624, 1280, 1424\} \Rightarrow \{12, 2561, 2847, 32448, 389376\}$.

190. $\{624, 1457, 1585\} \Rightarrow \{31, 1249, 3169, 68479, 2122849\}$.

191. $\{627, 1564, 1685\} \Rightarrow \{11, 3129, 3369, 35739, 393129\}$.

192. $\{630, 1144, 1306\} \Rightarrow \{26, 1261, 2611, 50336, 1308736\}$.

193. $\{637, 1116, 1285\} \Rightarrow \{13, 2233, 2569, 31213, 405769\}$.

194. $\{64, 1023, 1025\} \Rightarrow \{31, 129, 2049, 33759, 1046529\}$.

195. $\{640, 1536, 1664\} \Rightarrow \{32, 1281, 3327, 73728, 2359296\}$.

196. $\{640, 924, 1124\} \Rightarrow \{22, 1281, 2247, 38808, 853776\}$.

197. $\{644, 960, 1156\} \Rightarrow \{14, 1921, 2311, 29624, 414736\}$.

198. $\{645, 812, 1037\} \Rightarrow \{15, 1625, 2073, 27735, 416025\}$.

199. $\{648, 1215, 1377\} \Rightarrow \{27, 1297, 2753, 54675, 1476225\}$.

200. $\{648, 1386, 1530\} \Rightarrow \{12, 2773, 3059, 34992, 419904\}$.

201. $\{649, 1680, 1801\} \Rightarrow \{11, 3361, 3601, 38291, 421201\}$.

202. $\{656, 1617, 1745\} \Rightarrow \{33, 1313, 3489, 79233, 2614689\}$.

203. $\{66, 1088, 1090\} \Rightarrow \{32, 133, 2179, 36992, 1183744\}$.

204. $\{660, 779, 1021\} \Rightarrow \{19, 1321, 2041, 31939, 606841\}$.

205. $\{660, 989, 1189\} \Rightarrow \{23, 1321, 2377, 42527, 978121\}$.

206. $\{663, 1216, 1385\} \Rightarrow \{13, 2433, 2769, 33813, 439569\}$.

207. $\{666, 1288, 1450\} \Rightarrow \{28, 1333, 2899, 59248, 1658944\}$.

208. $\{671, 1800, 1921\} \Rightarrow \{11, 3601, 3841, 40931, 450241\}$.

209. $\{672, 1054, 1250\} \Rightarrow \{14, 2109, 2499, 32256, 451584\}$.

210. $\{672, 1496, 1640\} \Rightarrow \{12, 2993, 3279, 37632, 451584\}$.

211. $\{672, 1700, 1828\} \Rightarrow \{34, 1345, 3655, 85000, 2890000\}$.

212. $\{672, 754, 1010\} \Rightarrow \{16, 1509, 2019, 28224, 451584\}$.

213. $\{675, 900, 1125\} \Rightarrow \{15, 1801, 2249, 30375, 455625\}$.

214. $\{68, 1155, 1157\} \Rightarrow \{33, 137, 2313, 40425, 1334025\}$.

215. $\{680, 1056, 1256\} \Rightarrow \{24, 1361, 2511, 46464, 1115136\}$.

216. $\{682, 840, 1082\} \Rightarrow \{20, 1365, 2163, 35280, 705600\}$.

217. $\{684, 1363, 1525\} \Rightarrow \{29, 1369, 3049, 64061, 1857769\}$.

218. $\{688, 1785, 1913\} \Rightarrow \{35, 1377, 3825, 91035, 3186225\}$.

219. $\{689, 1320, 1489\} \Rightarrow \{13, 2641, 2977, 36517, 474721\}$.

220. $\{696, 1610, 1754\} \Rightarrow \{12, 3221, 3507, 40368, 484416\}$.

221. $\{70, 1224, 1226\} \Rightarrow \{34, 141, 2451, 44064, 1498176\}$.

222. $\{700, 1125, 1325\} \Rightarrow \{25, 1401, 2649, 50625, 1265625\}$.

223. $\{700, 1152, 1348\} \Rightarrow \{14, 2305, 2695, 35000, 490000\}$.

224. $\{702, 1440, 1602\} \Rightarrow \{30, 1405, 3203, 69120, 2073600\}$.

225. $\{704, 1872, 2000\} \Rightarrow \{36, 1409, 3999, 97344, 3504384\}$.

226. $\{704, 840, 1096\} \Rightarrow \{16, 1681, 2191, 30976, 495616\}$.

227. $\{704, 903, 1145\} \Rightarrow \{21, 1409, 2289, 38829, 815409\}$.

228. $\{705, 992, 1217\} \Rightarrow \{15, 1985, 2433, 33135, 497025\}$.

229. $\{715, 1428, 1597\} \Rightarrow \{13, 2857, 3193, 39325, 511225\}$.

230. $\{72, 1295, 1297\} \Rightarrow \{35, 145, 2593, 47915, 1677025\}$.

231. $\{720, 1196, 1396\} \Rightarrow \{26, 1441, 2791, 55016, 1430416\}$.

232. $\{720, 1519, 1681\} \Rightarrow \{31, 1441, 3361, 74431, 2307361\}$.

233. $\{720, 1728, 1872\} \Rightarrow \{12, 3457, 3743, 43200, 518400\}$.

234. $\{720, 756, 1044\} \Rightarrow \{18, 1441, 2087, 31752, 571536\}$.

235. $\{726, 968, 1210\} \Rightarrow \{22, 1453, 2419, 42592, 937024\}$.

236. $\{728, 1254, 1450\} \Rightarrow \{14, 2509, 2899, 37856, 529984\}$.

237. $\{731, 780, 1069\} \Rightarrow \{17, 1561, 2137, 31433, 534361\}$.

238. $\{735, 1088, 1313\} \Rightarrow \{15, 2177, 2625, 36015, 540225\}$.

239. $\{736, 930, 1186\} \Rightarrow \{16, 1861, 2371, 33856, 541696\}$.

240. $\{738, 1600, 1762\} \Rightarrow \{32, 1477, 3523, 80000, 2560000\}$.

241. $\{74, 1368, 1370\} \Rightarrow \{36, 149, 2739, 51984, 1871424\}$.

242. $\{740, 1269, 1469\} \Rightarrow \{27, 1481, 2937, 59643, 1610361\}$.

243. $\{741, 1540, 1709\} \Rightarrow \{13, 3081, 3417, 42237, 549081\}$.

244. $\{744, 1850, 1994\} \Rightarrow \{12, 3701, 3987, 46128, 553536\}$.

245. $\{744, 817, 1105\} \Rightarrow \{19, 1489, 2209, 35131, 667489\}$.

246. $\{748, 1035, 1277\} \Rightarrow \{23, 1497, 2553, 46575, 1071225\}$.

247. $\{756, 1360, 1556\} \Rightarrow \{14, 2721, 3111, 40824, 571536\}$.

248. $\{756, 1683, 1845\} \Rightarrow \{33, 1513, 3689, 85833, 2832489\}$.

249. $\{76, 1443, 1445\} \Rightarrow \{37, 153, 2889, 56277, 2082249\}$.

250. $\{760, 1344, 1544\} \Rightarrow \{28, 1521, 3087, 64512, 1806336\}$.

251. $\{765, 1188, 1413\} \Rightarrow \{15, 2377, 2825, 39015, 585225\}$.

252. $\{765, 868, 1157\} \Rightarrow \{17, 1737, 2313, 34425, 585225\}$.

253. $\{767, 1656, 1825\} \Rightarrow \{13, 3313, 3649, 45253, 588289\}$.

254. $\{768, 1024, 1280\} \Rightarrow \{16, 2049, 2559, 36864, 589824\}$.

255. $\{768, 880, 1168\} \Rightarrow \{20, 1537, 2335, 38720, 774400\}$.

256. $\{770, 1104, 1346\} \Rightarrow \{24, 1541, 2691, 50784, 1218816\}$.

257. $\{774, 1768, 1930\} \Rightarrow \{34, 1549, 3859, 91936, 3125824\}$.

258. $\{780, 1421, 1621\} \Rightarrow \{29, 1561, 3241, 69629, 2019241\}$.

259. $\{784, 1470, 1666\} \Rightarrow \{14, 2941, 3331, 43904, 614656\}$.

260. $\{792, 1175, 1417\} \Rightarrow \{25, 1585, 2833, 55225, 1380625\}$.

261. $\{792, 806, 1130\} \Rightarrow \{18, 1613, 2259, 34848, 627264\}$.

262. $\{792, 945, 1233\} \Rightarrow \{21, 1585, 2465, 42525, 893025\}$.

263. $\{793, 1776, 1945\} \Rightarrow \{13, 3553, 3889, 48373, 628849\}$.

264. $\{795, 1292, 1517\} \Rightarrow \{15, 2585, 3033, 42135, 632025\}$.

265. $\{799, 960, 1249\} \Rightarrow \{17, 1921, 2497, 37553, 638401\}$.

266. $\{800, 1122, 1378\} \Rightarrow \{16, 2245, 2755, 40000, 640000\}$.

267. $\{800, 1500, 1700\} \Rightarrow \{30, 1601, 3399, 75000, 2250000\}$.

268. $\{812, 1584, 1780\} \Rightarrow \{14, 3169, 3559, 47096, 659344\}$.

269. $\{814, 1248, 1490\} \Rightarrow \{26, 1629, 2979, 59904, 1557504\}$.

270. $\{816, 1012, 1300\} \Rightarrow \{22, 1633, 2599, 46552, 1024144\}$.

271. $\{820, 1581, 1781\} \Rightarrow \{31, 1641, 3561, 80631, 2499561\}$.

272. $\{825, 1400, 1625\} \Rightarrow \{15, 2801, 3249, 45375, 680625\}$.

273. $\{828, 896, 1220\} \Rightarrow \{18, 1793, 2439, 38088, 685584\}$.

274. $\{832, 1224, 1480\} \Rightarrow \{16, 2449, 2959, 43264, 692224\}$.

275. $\{832, 855, 1193\} \Rightarrow \{19, 1665, 2385, 38475, 731025\}$.

276. $\{833, 1056, 1345\} \Rightarrow \{17, 2113, 2689, 40817, 693889\}$.

277. $\{836, 1323, 1565\} \Rightarrow \{27, 1673, 3129, 64827, 1750329\}$.

278. $\{840, 1081, 1369\} \Rightarrow \{23, 1681, 2737, 50807, 1168561\}$.

279. $\{840, 1664, 1864\} \Rightarrow \{32, 1681, 3727, 86528, 2768896\}$.

280. $\{840, 1702, 1898\} \Rightarrow \{14, 3405, 3795, 50400, 705600\}$.

281. $\{855, 1512, 1737\} \Rightarrow \{15, 3025, 3473, 48735, 731025\}$.

282. $\{858, 1400, 1642\} \Rightarrow \{28, 1717, 3283, 70000, 1960000\}$.

283. $\{858, 920, 1258\} \Rightarrow \{20, 1717, 2515, 42320, 846400\}$.

284. $\{86, 1848, 1850\} \Rightarrow \{42, 173, 3699, 81312, 3415104\}$.

285. $\{860, 1749, 1949\} \Rightarrow \{33, 1721, 3897, 92697, 3059001\}$.

286. $\{864, 1152, 1440\} \Rightarrow \{24, 1729, 2879, 55296, 1327104\}$.

287. $\{864, 1330, 1586\} \Rightarrow \{16, 2661, 3171, 46656, 746496\}$.

288. $\{864, 990, 1314\} \Rightarrow \{18, 1981, 2627, 41472, 746496\}$.

289. $\{867, 1156, 1445\} \Rightarrow \{17, 2313, 2889, 44217, 751689\}$.

290. $\{880, 1479, 1721\} \Rightarrow \{29, 1761, 3441, 75429, 2187441\}$.

291. $\{884, 987, 1325\} \Rightarrow \{21, 1769, 2649, 46389, 974169\}$.

292. $\{885, 1628, 1853\} \Rightarrow \{15, 3257, 3705, 52215, 783225\}$.

293. $\{888, 1225, 1513\} \Rightarrow \{25, 1777, 3025, 60025, 1500625\}$.

294. $\{893, 924, 1285\} \Rightarrow \{19, 1849, 2569, 41971, 797449\}$.

295. $\{896, 1440, 1696\} \Rightarrow \{16, 2881, 3391, 50176, 802816\}$.

296. $\{900, 1088, 1412\} \Rightarrow \{18, 2177, 2823, 45000, 810000\}$.

297. $\{901, 1260, 1549\} \Rightarrow \{17, 2521, 3097, 47753, 811801\}$.

298. $\{902, 1560, 1802\} \Rightarrow \{30, 1805, 3603, 81120, 2433600\}$.

299. $\{910, 1056, 1394\} \Rightarrow \{22, 1821, 2787, 50688, 1115136\}$.

300. $\{912, 1300, 1588\} \Rightarrow \{26, 1825, 3175, 65000, 1690000\}$.

301. $\{915, 1748, 1973\} \Rightarrow \{15, 3497, 3945, 55815, 837225\}$.

302. $\{924, 1643, 1885\} \Rightarrow \{31, 1849, 3769, 87079, 2699449\}$.

303. $\{928, 1554, 1810\} \Rightarrow \{16, 3109, 3619, 53824, 861184\}$.

304. $\{931, 1020, 1381\} \Rightarrow \{19, 2041, 2761, 45619, 866761\}$.

305. $\{935, 1368, 1657\} \Rightarrow \{17, 2737, 3313, 51425, 874225\}$.

306. $\{936, 1127, 1465\} \Rightarrow \{23, 1873, 2929, 55223, 1270129\}$.

307. $\{936, 1190, 1514\} \Rightarrow \{18, 2381, 3027, 48672, 876096\}$.

308. $\{936, 1377, 1665\} \Rightarrow \{27, 1873, 3329, 70227, 1896129\}$.

309. $\{946, 1728, 1970\} \Rightarrow \{32, 1893, 3939, 93312, 2985984\}$.

310. $\{952, 960, 1352\} \Rightarrow \{20, 1905, 2703, 46080, 921600\}$.

311. $\{960, 1456, 1744\} \Rightarrow \{28, 1921, 3487, 75712, 2119936\}$.

312. $\{960, 1672, 1928\} \Rightarrow \{16, 3345, 3855, 57600, 921600\}$.

313. $\{962, 1200, 1538\} \Rightarrow \{24, 1925, 3075, 60000, 1440000\}$.

314. $\{969, 1120, 1481\} \Rightarrow \{19, 2241, 2961, 49419, 938961\}$.

315. $\{969, 1480, 1769\} \Rightarrow \{17, 2961, 3537, 55233, 938961\}$.

316. $\{972, 1296, 1620\} \Rightarrow \{18, 2593, 3239, 52488, 944784\}$.

317. $\{980, 1029, 1421\} \Rightarrow \{21, 1961, 2841, 50421, 1058841\}$.

318. $\{984, 1537, 1825\} \Rightarrow \{29, 1969, 3649, 81461, 2362369\}$.

319. $\{988, 1275, 1613\} \Rightarrow \{25, 1977, 3225, 65025, 1625625\}$.

320. $\{1000, 1050, 1450\} \Rightarrow \{20, 2101, 2899, 50000, 1000000\}$.

321. $\{1003, 1596, 1885\} \Rightarrow \{17, 3193, 3769, 59177, 1006009\}$.

322. $\{1007, 1224, 1585\} \Rightarrow \{19, 2449, 3169, 53371, 1014049\}$.

323. $\{1008, 1100, 1492\} \Rightarrow \{22, 2017, 2983, 55000, 1210000\}$.

324. $\{1008, 1406, 1730\} \Rightarrow \{18, 2813, 3459, 56448, 1016064\}$.

325. $\{1008, 1620, 1908\} \Rightarrow \{30, 2017, 3815, 87480, 2624400\}$.

326. $\{1014, 1352, 1690\} \Rightarrow \{26, 2029, 3379, 70304, 1827904\}$.

327. $\{1032, 1705, 1993\} \Rightarrow \{31, 2065, 3985, 93775, 2907025\}$.

328. $\{1036, 1173, 1565\} \Rightarrow \{23, 2073, 3129, 59823, 1375929\}$.

329. $\{1040, 1152, 1552\} \Rightarrow \{20, 2305, 3103, 54080, 1081600\}$.

330. $\{1040, 1431, 1769\} \Rightarrow \{27, 2081, 3537, 75843, 2047761\}$.

331. $\{1044, 1520, 1844\} \Rightarrow \{18, 3041, 3687, 60552, 1089936\}$.

332. $\{1045, 1332, 1693\} \Rightarrow \{19, 2665, 3385, 57475, 1092025\}$.

333. $\{1064, 1248, 1640\} \Rightarrow \{24, 2129, 3279, 64896, 1557504\}$.

334. $\{1066, 1512, 1850\} \Rightarrow \{28, 2133, 3699, 81648, 2286144\}$.

335. $\{1071, 1080, 1521\} \Rightarrow \{21, 2161, 3041, 54621, 1147041\}$.

336. $\{1080, 1258, 1658\} \Rightarrow \{20, 2517, 3315, 58320, 1166400\}$.

337. $\{1080, 1638, 1962\} \Rightarrow \{18, 3277, 3923, 64800, 1166400\}$.

338. $\{1083, 1444, 1805\} \Rightarrow \{19, 2889, 3609, 61731, 1172889\}$.

339. $\{1092, 1325, 1717\} \Rightarrow \{25, 2185, 3433, 70225, 1755625\}$.

340. $\{1092, 1595, 1933\} \Rightarrow \{29, 2185, 3865, 87725, 2544025\}$.

341. $\{1110, 1144, 1594\} \Rightarrow \{22, 2221, 3187, 59488, 1308736\}$.

342. $\{1113, 1184, 1625\} \Rightarrow \{21, 2369, 3249, 58989, 1238769\}$.

343. $\{1120, 1368, 1768\} \Rightarrow \{20, 2737, 3535, 62720, 1254400\}$.

344. $\{1120, 1404, 1796\} \Rightarrow \{26, 2241, 3591, 75816, 1971216\}$.

345. $\{1121, 1560, 1921\} \Rightarrow \{19, 3121, 3841, 66139, 1256641\}$.

346. $\{1140, 1219, 1669\} \Rightarrow \{23, 2281, 3337, 64607, 1485961\}$.

347. $\{1148, 1485, 1877\} \Rightarrow \{27, 2297, 3753, 81675, 2205225\}$.

348. $\{1155, 1292, 1733\} \Rightarrow \{21, 2585, 3465, 63525, 1334025\}$.

349. $\{1160, 1482, 1882\} \Rightarrow \{20, 2965, 3763, 67280, 1345600\}$.

350. $\{1170, 1296, 1746\} \Rightarrow \{24, 2341, 3491, 69984, 1679616\}$.

351. $\{1176, 1568, 1960\} \Rightarrow \{28, 2353, 3919, 87808, 2458624\}$.

352. $\{1188, 1216, 1700\} \Rightarrow \{22, 2433, 3399, 64152, 1411344\}$.

353. $\{1197, 1404, 1845\} \Rightarrow \{21, 2809, 3689, 68229, 1432809\}$.

354. $\{1200, 1375, 1825\} \Rightarrow \{25, 2401, 3649, 75625, 1890625\}$.

355. $\{1200, 1600, 2000\} \Rightarrow \{20, 3201, 3999, 72000, 1440000\}$.

356. $\{1230, 1456, 1906\} \Rightarrow \{26, 2461, 3811, 81536, 2119936\}$.

357. $\{1232, 1326, 1810\} \Rightarrow \{22, 2653, 3619, 68992, 1517824\}$.

358. $\{1239, 1520, 1961\} \Rightarrow \{21, 3041, 3921, 73101, 1535121\}$.

359. $\{1248, 1265, 1777\} \Rightarrow \{23, 2497, 3553, 69575, 1600225\}$.

360. $\{1260, 1539, 1989\} \Rightarrow \{27, 2521, 3977, 87723, 2368521\}$.

361. $\{1276, 1440, 1924\} \Rightarrow \{22, 2881, 3847, 74008, 1628176\}$.

362. $\{1280, 1344, 1856\} \Rightarrow \{24, 2561, 3711, 75264, 1806336\}$.

363. $\{1311, 1360, 1889\} \Rightarrow \{23, 2721, 3777, 74727, 1718721\}$.

364. $\{1312, 1425, 1937\} \Rightarrow \{25, 2625, 3873, 81225, 2030625\}$.

365. $\{1392, 1394, 1970\} \Rightarrow \{24, 2789, 3939, 80736, 1937664\}$.

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