RESEARCH GROUP IN MATHEMATICAL INEQUALITIES AND APPLICATIONS

PROBLEM CORNER

Problem 1, (2009), Solution No. 1

Slavko Simic

Mathematical Institute SANU
Kneza Mihaila 36
11000 Belgrade, Serbia

Email: ssimic@turing.mi.sanu.ac.rs

More generally, let

$$A(p,x) := \sum p_i x_i, \quad H(p,x) := \left(\sum p_i/x_i \right)^{-1},$$

denote the generalized arithmetic and harmonic means, respectively.

If $0 < m \le x_i \le M < \infty$, then

$$0 \le A(p,x) - H(p,x) \le \left(\sqrt{M} - \sqrt{m} \right)^2.$$

Proof. The proof follows easily from Theorem 2.1 of [1], applied to the function $f(x) = 1/x$ and then changing variables $x_i \to 1/x_i$. ☐

References

[1] S. SIMIC, On an upper bound for Jensen's inequality, *J. Inequal. Pure Appl. Math.*, **10**(2) (2009), Art. 60.