Research Group in Mathematical Inequalities and Applications

$v(G) > \sum_{m \in G} v(m)$

The value of the Group is greater than the sum of the values of its members.

Problem Corner

Problem 2, (2010), Solution No. 1

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We shall use the following, well known properties of convex functions:

a) for fixed x the function $g(t) = f(x+t) + f(x-t)$ increases for $t > 0$.
b) the divided difference increases in both variables.

Fix $x \in (a, \frac{a+b}{2}]$. Then, by property a)

$$\max_{\frac{s+t}{2}=x} F(s,t) = F(a, 2x-a) = f(a) + f(2x-a) - 2f(x)$$

Let $h(x) = f(a) + f(2x-a) - 2f(x)$. We have

$$\frac{h(x) - h(y)}{x - y} = 2\left[\frac{f(2x-a) - f(2y-a)}{(2x-a)-(2y-a)} - \frac{f(x) - f(y)}{x - y}\right] \geq 0$$

The inequality follows from property b), since $2x - a > x$ and $2y - a > y$. This means that h increases and its maximal value is $h(\frac{a+b}{2}) = F(a,b)$.

The proof in case $x \in [\frac{a+b}{2}, b)$ is symmetric.