A DOUBLE-WEIGHTED REFINEMENT OF JENSEN'S INEQUALITY WITH APPLICATIONS

J. ROOIN[1] AND A. MORASSAEI[2]

Abstract. Let $(X, \mathcal{A}, \mu)$ and $(Y, \mathcal{B}, \lambda)$ be two probability measure spaces, I an interval of the real line, $f \in L^1(\mu)$, $f(x) \in I$ for each $x \in X$, and φ a real-valued convex function on I. We show that, if ω_0 and ω_1 are two appropriate weight functions on $X \times Y$, then

$$\varphi\left(\int_X f d\mu\right) \leq \int_Y A\left(\varphi; F_0(y), F_1(y)\right) d\lambda(y) \leq \int_X (\varphi \circ f) d\mu,$$

where A denotes the arithmetic mean of φ on the closed interval with end points $F_0(y)$ and $F_1(y)$, and for λ-almost all $y \in Y$'s

$$F_k(y) = \int_X f(x)\omega_k(x, y) d\mu(x) \qquad (k = 0, 1).$$

Finally, we give nice applications in refining Information inequality and some important inequalities between means.

[1] Department of Mathematics, Institute for Advanced Studies in Basic Sciences, 45195-1159, Zanjan, Iran.

E-mail address: rooin@iasbs.ac.ir

[2] Department of Mathematics, Faculty of Sciences, University of Zanjan, 45195-313, and, Education Office of Zanjan, Zanjan, Iran.

E-mail address: morassae@iasbs.ac.ir

2000 *Mathematics Subject Classification.* 26D15, 26A51, 28A35.

Key words and phrases. Jensen's inequality, Weight function, Mean, Information inequality.