On some inequalities for polynomial functions

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Abstract

Let Π_n denote the class of algebraic polynomials of degree no bigger than n. Let $P(x) = \sum_{k=0}^{n} a_k x^k$ and $\|P\|_{d\sigma} = \left(\int_R |P(x)|^2 d\sigma(x) \right)^{1/2}$, where $d\sigma(x)$ is a non-negative measure on $I\!R$. G. Milovanovic determined best constants C_{nk} such that

$$|a_k| \leq C_{nk} \|P\|_{d\sigma}, \quad \text{for } k = 0, 1, ..., n.$$

In the present work, we will propose a new way of proofing the above inequality, which will lead us in finding the optimal constant C, such that

$$\|P\|_\infty \leq C \|P\|_{d\sigma},$$

where $\|\cdot\|_\infty$ denotes the uniform norm on $[0, 1]$.

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