Schur Convexity and Schur-Geometrically Concavity of Seiffert's Mean

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Abstract. The Schur- concavity and Schur-geometrically convexity of the Seiffert's mean with two positive numbers a,b in R_{++}^2 are discussed. Besides, some new inequalities are obtained.

Keywords: Seiffert's mean, Schur- convexity, Schur-geometrically concavity， inequality

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§1 Introduction

Seiffert's mean [1, p. 43] of two positive numbers *a* and *b* is defined as follows

$$P = P(a,b) = \begin{cases} \dfrac{a-b}{4\arctan\sqrt{a/b}-\pi} & a \neq b \\ a & a = b \end{cases}$$

In recent years, some further generalizations and applications about Seiffert's mean have been obtained in [2-5] and the references therein.

In this paper, the Schur-concavity and Schur-geometrically convexity of the Seiffert's mean with two positive numbers a,b in $R_{++}^2 := (0,+\infty)\times(0,+\infty)$ are discussed. Besides, some new inequalities are obtained.

§ 2 Main Results

Theorem 1. $P(a,b)$ is Schur-concave with (a,b) in R_{++}^2.

Theorem 2. $P(a,b)$ is Schur- geometrically convex with (a,b) in R_{++}^2.

§ 3 Applications

Theorem 3. For $(a,b)\in R_{++}^2$, with $a \le b$, we have

$$G(a,b) \le P\left(a^{\frac{3}{4}}b^{\frac{1}{4}}, a^{\frac{1}{4}}b^{\frac{3}{4}}\right) \le P(a,b) \le P\left(\frac{3a+b}{4}, \frac{a+3b}{4}\right) \le A(a,b),$$

where $G(a,b)$ and $A(a,b)$ is the arithmetic-mean and the geometry respectively.

Theorem 4. Let $0<a<b,\ c\ge 0$. Then

$$\left(a+b+2c\right)\left(\arctan\sqrt{\frac{a+c}{b+c}}\right)-\left(a+b\right)\left(\arctan\sqrt{\frac{a}{b}}\right)\geq\frac{c\pi}{2}.$$

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