SOME FUNCTIONS PRESERVING THE ORDER OF POSITIVE OPERATORS

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ABSTRACT We show some functions preserving the order of positive operators based on the Mond-Pečarić method as follows: Let A and B be positive operators on a Hilbert space H satisfying $MI \geq B \geq mI > 0$. Let $f(t)$ be a continuous convex function on $[m, M]$. If $g(t)$ is a continuous increasing convex function on $[m, M] \cup \mathsf{Sp}(A)$, then for a given $\alpha > 0$

$$A \geq B \geq 0 \quad \text{implies} \quad \alpha g(A) + \beta I \geq f(B)$$

where $\beta = \max_{m \leq t \leq M}\{f(m) + [(f(M) - f(m))/(M - m)](t - m) - \alpha g(t)\}$. We extend Kantorovich type operator inequalities via Ky Fan-Furuta constant as applications. Among others, we show the following inequality: If $A \geq B > 0$ and $MI \geq B \geq mI > 0$, then

$$\frac{M^{p-1}}{m^{q-1}} A^q \geq \frac{(q-1)^{q-1}}{q^q} \frac{(M^p - m^p)^q}{(M - m)(mM^p - Mm^p)^{q-1}} A^q \geq B^p$$

holds for all $p > 1$ and $q > 1$.