

ON APPROXIMATELY CONVEX FUNCTIONS

ZSOLT PÁLES

A real valued function f defined on a real interval I is called (ε, δ) -convex if it satisfies

$$f(tx + (1 - t)y) \leq tf(x) + (1 - t)f(y) + \varepsilon t(1 - t)|x - y| + \delta$$

for $x, y \in I, t \in [0, 1]$.

The main results of the paper offer various characterizations for (ε, δ) -convexity. One of the main results states that f is (ε, δ) -convex for some positive ε and δ if and only if f can be decomposed into the sum of a convex function, a function with bounded supremum norm, and a function with bounded Lipschitz-modulus. In the special case $\varepsilon = 0$, the results reduce to that of Hyers, Ulam, and Green obtained in 1952 concerning the so called δ -convexity.

REFERENCES

- [1] J. W. Green, *Approximately convex functions*, Duke Math. J. **19** (1952), 499–504.
- [2] D. H. Hyers and S. M. Ulam, *Approximately convex functions*, Proc. Amer. Math. Soc. **3** (1952), 821–828.

INSTITUTE OF MATHEMATICS AND INFORMATICS, UNIVERSITY OF DEBRECEN,
H-4010 DEBRECEN, PF. 12, HUNGARY
E-mail address: pales@math.klte.hu

2000 *Mathematics Subject Classification.* Primary 26A51, 26B25.

Key words and phrases. Convexity, (ε, δ) -convexity, Stability of convexity, (ε, δ) -subgradient, (ε, δ) -subdifferentiability.