

Order among Furuta type inequalities

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ABSTRACT. The order between parametrized Furuta inequality and parametrized grand Furuta inequality is determined as follows: If $A \geq B > 0$ and $\delta \in [0, 1]$, then

$$\begin{aligned} A^u \#_{\frac{\delta-u}{\beta-u}} (A^t \natural_{\frac{\beta-t}{p-t}} B^p) &\leq A^u \#_{\frac{\delta-u}{p-u}} B^p \leq B^\delta \\ &\leq A^\delta \leq B^u \#_{\frac{\delta-u}{p-u}} A^p \leq B^u \#_{\frac{\delta-u}{\beta-u}} (B^t \natural_{\frac{\beta-t}{p-t}} A^p) \end{aligned}$$

for $t \in [0, 1]$, $0 \leq t < p \leq \beta$ and $u \leq 0$.

More generally, if $\delta \in [0, p]$ under the above conditions, then

$$A^u \#_{\frac{\delta-u}{\beta-u}} (A^t \natural_{\frac{\beta-t}{p-t}} B^p) \leq A^u \#_{\frac{\delta-u}{p-u}} B^p \leq B^\delta$$

and

$$B^u \#_{\frac{\delta-u}{\beta-u}} (B^t \natural_{\frac{\beta-t}{p-t}} A^p) \geq B^u \#_{\frac{\delta-u}{p-u}} A^p \geq A^\delta.$$

The case of $\delta = 1$ gives the order between the Furuta inequality and grand Furuta inequality.

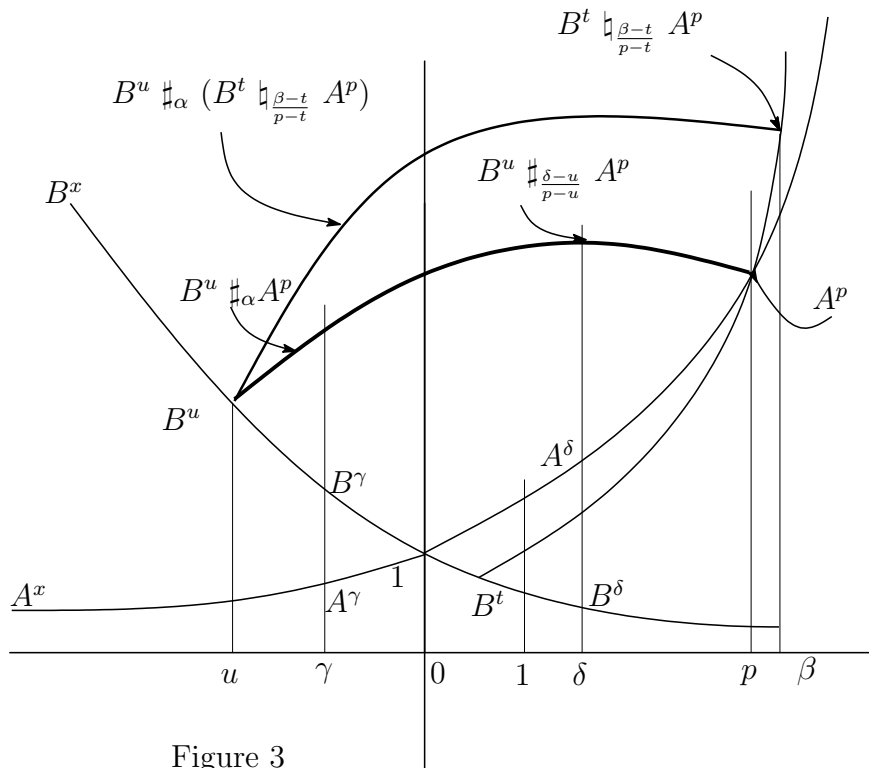


Figure 3