ON THE COMPARISON OF CAUCHY MEAN VALUES

LÁSZLÓ LOSONCZI

Suppose that $x_1 \leq \cdots \leq x_n$ and $f^{(n-1)}, g^{(n-1)}$ exist, with $g^{(n-1)} \neq 0$, on $[x_1, x_n]$. Then there is a $t \in [x_1, x_n]$ (moreover $t \in (x_1, x_n)$ if $x_1 < x_n$) such that

$$\frac{[x_1, \ldots, x_n]_f}{[x_1, \ldots, x_n]_g} = \frac{f^{(n-1)}(t)}{g^{(n-1)}(t)}$$

where $[x_1, \ldots, x_n]_f$ denotes the divided difference of f at the points $x_1, \ldots, x_n$. This is the Cauchy Mean Value Theorem for divided differences (see e.g. E. Leach and M. Sholander, *Multi-variable extended mean values*, J. Math. Anal. Appl., **104** 1984, 390-407).

If the function $\dfrac{f^{(n-1)}}{g^{(n-1)}}$ is invertible then

$$t = \left(\frac{f^{(n-1)}}{g^{(n-1)}}\right)^{-1} \left(\frac{[x_1, \ldots, x_n]_f}{[x_1, \ldots, x_n]_g}\right)$$

is a mean value of $x_1, \ldots, x_n$. It is called the *Cauchy mean of the numbers* $x_1, \ldots, x_n$ and will be denoted by $D_{fg}(x_1, \ldots, x_n)$.

Here we completely solve the comparison problem of Cauchy means

$$D_{fg}(x_1, x_2, \ldots, x_n) \leq D_{FG}(x_1, x_2, \ldots, x_n)$$

where $x_1, x_2, \ldots, x_n \in I$, $n \geq 2$ is fixed, in the special cases $g = G$, $f = F$ and $\dfrac{f^{(n-1)}}{g^{(n-1)}} = \dfrac{F^{(n-1)}}{G^{(n-1)}}$. In the general case we find necessary conditions (which are not sufficient) and also sufficient conditions (which are not necessary).