Cauchy-Schwarz Inequalities in Hilbert Modules

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The aim in this paper is to study various types of Cauchy-Schwarz inequalities and related topics in Hilbert modules $(E, [\cdot, \cdot])$ over $\mathbb{C}^*$-algebras A. The results are qualitatively improved if, in particular, A becomes a H^*-algebra. For example, a inequality of the form

$$[x, y] + [y, x] \leq [x, x]^{1/2}[y, y]^{1/2} + [y, y]^{1/2}[x, x]^{1/2}, \quad x, y \in E,$$

proposed by C.P. Niculescu, does not hold for arbitrary $\mathbb{C}^*$-algebras A, but is proved to be true if A is a H^*-algebra.