Self-affine tiles and digit sets via the geometry of numbers

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Abstract: A self-affine tile in R^n is a compact set $T \subset R^n$ such that there are an expanding $n \times n$ real matrix M with $|\det(M)| = m$ integer and a finite set $D \subset R^n$ such that the set MT is tiled by the family $T + d_{d \in D}$. The latter set D is called a digit set. A compact set $\subset R^n$ is called lattice tiling in R^n if there is a point lattice $L \subset R^n$ such that R^n is tiled by the family $C + u_{u \in L}$. Given a point lattice $\Lambda \subset R^n$, the finite set $S \subset \Lambda$ is called a discrete lattice tiling in Λ, if there is a point lattice $L \subset \Lambda$ such that Λ is tiled by the family $S + u_{u \in L}$. The talk is devoted to the study of relations among the above three phenomena and their relation to the geometry of numbers. Our main tools are the new methods of papers [6]-[12] (based on a new "inequality approach"), where many refinements of basic results of geometry of numbers have been proved for any discrete subgroup L of R^n and any bounded set $A \subset R^n$. (As concern self-affine tiles and digit sets, see, e.g., [1]-[5].)

References

[1] Imre Kátai, "Generalized number systems and fractal geometry", Leaflets in Mathematics, University of Pécs, Pécs, Hungary, 1995, 1–40.

[2] Jeffrey C. Lagarias, Yang Wang, Integral self-affine tiles in $R\hat{}n$ I. Standard and nonstandard digit sets, *J. London Math. Soc.*, **54**(1996), 161–179.

[3] Jeffrey C. Lagarias, Yang Wang, Self-affine tiles in R^n, *Adv. Math.*, **121** (1996), 21–49.

[4] Andrew Vince, Replicating tessellations, *SIAM J. Discr. Math.*, **6** (1993), 501–521.

[5] Andrew Vince, Rep-tiling Euclidean space,*Aequationes Math.*, **50**(1995), 191–213.

[6] B. Uhrin, Some useful estimations in geometry of numbers, *Period. Math. Hungar.*, **11** (1980), 95–103

[7] B. Uhrin, On a generalization of Minkowski convex body theorem, *J. of Number Th.*, **13** (1981), 192–209

[8] G.Freiman, A.Heppes, B.Uhrin, A lower estimation for the cardinality of finite difference sets in R^n, In: K.Győry,

G.Halász, Eds., *Number Theory*, Budapest, 1987, Coll.Math.Soc.J.Bolyai **51**, North-Holland, Amsterdam-New York, 1989, 125–139.

[9] B.Uhrin, A description of the segment [1,T], where T

is the meeting number of a set-lattice, *Period. Math. Hungar.*, **26**(1993), 139–156.

[10] B.Uhrin, The index of a point lattice in a set,*J. Number Th.,* **54**(1995), 232–247.

[11] B.Uhrin, New lower bounds for the number of lattice points in a difference set, *Acta Sci. Math.(Szeged)*, **62**(1996), 81–99.

[12] B.Uhrin, Inner aperiodicities and partitions of sets, *Linear Algebra and Appl.*, **241-243** (1996), 851–876.