MONOTONICITY OF $\left(1+\frac{1}{s}\right)^s$ ONCE MORE

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Abstract. We offer a very elementary proof of the well known fact.

Consider the function $h : \mathbf{R}_{>0} \times \mathbf{R} \setminus \{0\}$ defined by

$$h(u,s) = \frac{u^s - 1}{s}.$$

It is increasing in u (elementary [1]) and in s (for fixed u u^s is convex, so its divided difference increases), $h(1,s) = 0$ and for $s > -1$ $\lim_{u\to\infty} h(u,s) > 1$, therefore there exists a unique $u = u(s)$ such that $h(u(s),s) = 1$.
Since for $-1 < t < s$

$$h(u(s),s) = 1 = h(u(t),t) < h(u(t),s)$$

we see that $u(s) < u(t)$, thus $u(s) = (1+s)^{1/s}$ decreases, so for positive s

$$u(1/s) = \left(1+\frac{1}{s}\right)^s$$

increases.

Note that for positive s we have $u(s) < u(-1/2) = 4$, so the function $u(1/s)$ is bounded.

References

[1] Sir Arthur Conan Doyle, The Crooked Man, http://sherlock-holmes.classic-literature.co.uk/the-crooked-man/ p. 1.

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