SOME OSTROWSKI TYPE INEQUALITIES FOR QUASI-CONVEX FUNCTIONS WITH APPLICATIONS TO SPECIAL MEANS

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Abstract. Some inequalities of Ostrowski's type for quasi-convex functions are introduced. An improvements for some Midpoint type inequalities are given. Some applications to special means are also obtained.

1. Introduction

Let $f : I \subset [0, \infty) \to \mathbb{R}$ be a differentiable mapping on I°, the interior of the interval I, such that $f' \in L[a, b]$, where $a, b \in I$ with $a < b$. If $|f'(x)| \le M$, then the following inequality,

$$\left| f(x) - \frac{1}{b-a} \int_a^b f(u)\, du \right| \le \frac{M}{b-a} \cdot \left[\frac{(x-a)^2 + (b-x)^2}{2} \right] \tag{1.1}$$

holds. This result is known in the literature as the *Ostrowski inequality*. For recent results and generalizations concerning Ostrowski's inequality see [4]–[10] and the references therein.

The notion of *quasi-convex functions* generalizes the notion of convex functions. More precisely, a function $f : [a, b] \to \mathbb{R}$ is said quasi-convex on $[a, b]$ if

$$f(\lambda x + (1-\lambda) y) \le \max\{f(x), f(y)\},$$

for any $x, y \in [a, b]$ and $\lambda \in [0, 1]$. Clearly, any convex function is a quasi-convex function. Furthermore, there exist quasi-convex functions which are not convex (see [10]). For refinements inequalities concerning quasi-convex functions, see [2], [3], [9] and [10].

Recently, Alomari et al. [2] established several inequalities for functions whose first derivatives in absolute value are quasi-convex. Namely, the authors obtained the following results:

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Theorem 1. *Let $f : I^{\circ} \subset \mathbb{R} \to \mathbb{R}$ be a differentiable mapping on I°, $a, b \in I^{\circ}$ with $a < b$. If $|f'|^q$ is quasi-convex on $[a, b]$, $q \geq 1$, then the following inequality holds:*

$$\left| \frac{1}{b-a} \int_a^b f(x)\,dx - f\left(\frac{a+b}{2}\right) \right| \leq \frac{b-a}{8} \left[\left(\max\left\{ \left| f'\left(\frac{a+b}{2}\right) \right|^q, |f'(b)|^q \right\} \right)^{\frac{1}{q}} + \left(\max\left\{ \left| f'\left(\frac{a+b}{2}\right) \right|^q, |f'(a)|^q \right\} \right)^{\frac{1}{q}} \right]. \tag{1.2}$$

Corollary 1. *Let f be as in Theorem 1. Additionally, if*

(1) *$|f'|$ is increasing, then we have*

$$\left| \frac{1}{b-a} \int_a^b f(x)\,dx - f\left(\frac{a+b}{2}\right) \right| \leq \frac{b-a}{8} \left[|f'(b)| + \left| f'\left(\frac{a+b}{2}\right) \right| \right], \tag{1.3}$$

(2) *$|f'|$ is decreasing, then we have*

$$\left| \frac{1}{b-a} \int_a^b f(x)\,dx - f\left(\frac{a+b}{2}\right) \right| \leq \frac{b-a}{8} \left[|f'(a)| + \left| f'\left(\frac{a+b}{2}\right) \right| \right], \tag{1.4}$$

(3) *$f'(a) = f'(b) = 0$, then we have*

$$\left| \frac{1}{b-a} \int_a^b f(x)\,dx - f\left(\frac{a+b}{2}\right) \right| \leq \frac{b-a}{4} \left| f'\left(\frac{a+b}{2}\right) \right|. \tag{1.5}$$

The aim of this paper is to establish some Ostrowski type inequalities for the class of functions whose derivatives in absolute value are quasi-convex functions.

2. Ostrowski's Type Inequalities

In order to prove our main theorems, we need the following lemma (see [1]):

Lemma 1. *Let $f : I \subset \mathbb{R} \to \mathbb{R}$ be a differentiable mapping mapping on I° where $a, b \in I$ with $a < b$. If $f' \in L[a, b]$, then the following equality holds:*

$$f(x) - \frac{1}{b-a} \int_a^b f(u)\,du = (b-a) \int_0^1 p(t)\, f'(ta + (1-t)b)\,dt \tag{2.1}$$

for each $t \in [0, 1]$, where

$$p(t) = \begin{cases} t, & t \in \left[0, \frac{b-x}{b-a}\right] \\ t-1, & t \in \left(\frac{b-x}{b-a}, 1\right] \end{cases},$$

for all $x \in [a, b]$.

The following result may be stated:

Theorem 2. *Let $f : I \subset [0, \infty) \to \mathbb{R}$ be a differentiable mapping on I° such that $f' \in L[a,b]$, where $a, b \in I$ with $a < b$. If $|f'|$ is quasi-convex on $[a,b]$, then the following inequality holds:*

$$\left| f(x) - \frac{1}{b-a} \int_a^b f(u)\, du \right|$$
$$\leq \frac{(b-x)^2}{2(b-a)} \max\left\{ |f'(x)|, |f'(b)| \right\} + \frac{(x-a)^2}{2(b-a)} \max\left\{ |f'(x)|, |f'(a)| \right\}, \tag{2.2}$$

for each $x \in [a,b]$.

Proof. By Lemma 1 and since $|f'|$ is quasi-convex, then we have

$$\begin{aligned} &\left| f(x) - \frac{1}{b-a} \int_a^b f(u)\, du \right| \\ &\leq (b-a) \int_0^{\frac{b-x}{b-a}} t \cdot \max\left\{ |f'(x)|, |f'(b)| \right\} dt \\ &\qquad + \int_{\frac{b-x}{b-a}}^1 (1-t) \cdot \max\left\{ |f'(x)|, |f'(a)| \right\} dt \\ &= (b-a) \max\left\{ |f'(x)|, |f'(b)| \right\} \int_0^{\frac{b-x}{b-a}} t dt \\ &\qquad + (b-a) \max\left\{ |f'(x)|, |f'(a)| \right\} \int_{\frac{b-x}{b-a}}^1 (1-t)\, dt \\ &= \frac{(b-x)^2}{2(b-a)} \max\left\{ |f'(x)|, |f'(b)| \right\} + \frac{(x-a)^2}{2(b-a)} \max\left\{ |f'(x)|, |f'(a)| \right\}. \end{aligned}$$

This completes the proof. ∎

Corollary 2. *In Theorem 2. Additionally, if $|f'(x)| \leq M$, $M > 0$, then inequality (1.1) holds.*

Corollary 3. *In Theorem 2, Additionally, if*

(1) $|f'|$ *is increasing, then we have*

$$\left| f(x) - \frac{1}{b-a} \int_a^b f(u)\, du \right| \leq \frac{(b-x)^2}{2(b-a)} |f'(b)| + \frac{(x-a)^2}{2(b-a)} |f'(x)|. \tag{2.3}$$

(2) $|f'|$ *is decreasing, then we have*

$$\left| f(x) - \frac{1}{b-a} \int_a^b f(u)\, du \right| \leq \frac{(b-x)^2}{2(b-a)} |f'(x)| + \frac{(x-a)^2}{2(b-a)} |f'(a)|. \tag{2.4}$$

Corollary 4. *In Theorem 2, choose $x = \frac{a+b}{2}$, then*

$$\left| f\left(\frac{a+b}{2}\right) - \frac{1}{b-a} \int_a^b f(u)\, du \right|$$
$$\leq \frac{(b-a)}{8} \left[\max\left\{ \left| f'\left(\frac{a+b}{2}\right) \right|, |f'(b)| \right\} + \max\left\{ \left| f'\left(\frac{a+b}{2}\right) \right|, |f'(a)| \right\} \right]. \tag{2.5}$$

Corollary 5. *In Corollary 4. Choosing* $x = \frac{a+b}{2}$,

(1) *If* $|f'|$ *is increasing, then we have*

$$\left|f\left(\frac{a+b}{2}\right)-\frac{1}{b-a}\int_a^b f(u)\,du\right| \le \frac{b-a}{8}\left[\left|f'(b)\right|+\left|f'\left(\frac{a+b}{2}\right)\right|\right]. \tag{2.6}$$

(2) *If* $|f'|$ *is decreasing, then we have*

$$\left|f\left(\frac{a+b}{2}\right)-\frac{1}{b-a}\int_a^b f(u)\,du\right| \le \frac{b-a}{8}\left[\left|f'(a)\right|+\left|f'\left(\frac{a+b}{2}\right)\right|\right]. \tag{2.7}$$

(3) *If* $f'(a) = f'(b) = 0$, *then we have*

$$\left|f\left(\frac{a+b}{2}\right)-\frac{1}{b-a}\int_a^b f(u)\,du\right| \le \frac{b-a}{4}\left|f'\left(\frac{a+b}{2}\right)\right| \tag{2.8}$$

Remark 1. *We note that the inequalities (2.6)–(2.8) improve the inequalities (1.3)–(1.5), respectively.*

The corresponding version for powers of the absolute value of the first derivative is incorporated in the following result:

Theorem 3. *Let* $f : I \subset [0,\infty) \to \mathbb{R}$ *be a differentiable mapping on* I° *such that* $f' \in L[a,b]$, *where* $a, b \in I$ *with* $a < b$. *If* $|f'|$ *is quasi-convex on* $[a,b]$, *then the following inequality holds:*

$$\begin{aligned} &\left|f(x)-\frac{1}{b-a}\int_a^b f(u)\,du\right| \\ &\le \left(\frac{(b-x)^{p+1}}{(b-a)(p+1)}\right)^{\frac{1}{p}}\left[\max\left\{\left|f'(x)\right|^q,\left|f'(b)\right|^q\right\}\right]^{\frac{1}{q}} \\ &\quad + \left(\frac{(x-a)^{p+1}}{(b-a)(p+1)}\right)^{\frac{1}{p}}\left[\max\left\{\left|f'(x)\right|^q,\left|f'(a)\right|^q\right\}\right]^{\frac{1}{q}}, \end{aligned} \tag{2.9}$$

for each $x \in [a,b]$, *where* $\frac{1}{p}+\frac{1}{q} = 1$.

Proof. Suppose that $p>1$. From Lemma 1 and using the Hölder inequality, we have

$$\begin{aligned}
&\left| f(x) - \frac{1}{b-a}\int_a^b f(u)\,du \right| \\
&\leq (b-a)\int_0^{\frac{b-x}{b-a}} t\left|f'(ta+(1-t)b)\right| dt \\
&\qquad + (b-a)\int_{\frac{b-x}{b-a}}^1 |t-1|\left|f'(ta+(1-t)b)\right| dt \\
&\leq (b-a)\left(\int_0^{\frac{b-x}{b-a}} t^p dt\right)^{1/p}\left(\int_0^{\frac{b-x}{b-a}} \left|f'(ta+(1-t)b)\right|^q dt\right)^{1/q} \\
&\qquad + (b-a)\left(\int_{\frac{b-x}{b-a}}^1 (1-t)^p\,dt\right)^{1/p}\left(\int_{\frac{b-x}{b-a}}^1 \left|f'(ta+(1-t)b)\right|^q dt\right)^{1/q} \\
&= \frac{(b-x)^{\frac{p+1}{p}}}{(b-a)^{\frac{1}{p}}(p+1)^{\frac{1}{p}}}\left[\max\left\{|f'(x)|^q, |f'(b)|^q\right\}\right]^{1/q} \\
&\qquad + \frac{(x-a)^{\frac{p+1}{p}}}{(b-a)^{\frac{1}{p}}(p+1)^{\frac{1}{p}}}\left[\max\left\{|f'(x)|^q, |f'(a)|^q\right\}\right]^{1/q}.
\end{aligned}$$

This completes the proof. ∎

Corollary 6. *In Theorem 3. Additionally, if* $|f'(x)| \leq M$, $M>0$, *then inequality (1.1) holds.*

Corollary 7. *In Theorem 3, Additionally, if*

(1) $|f'|$ *is increasing, then we have*

$$\left| f(x) - \frac{1}{b-a}\int_a^b f(u)\,du \right| \leq \frac{1}{(b-a)^{\frac{1}{p}}(p+1)^{\frac{1}{p}}}\left[(b-x)^{\frac{p+1}{p}}|f'(b)| + (x-a)^{\frac{p+1}{p}}|f'(x)|\right] \tag{2.10}$$

(2) $|f'|$ *is decreasing, then we have*

$$\left| f(x) - \frac{1}{b-a}\int_a^b f(u)\,du \right| \leq \frac{1}{(b-a)^{\frac{1}{p}}(p+1)^{\frac{1}{p}}}\left[(b-x)^{\frac{p+1}{p}}|f'(x)| + (x-a)^{\frac{p+1}{p}}|f'(a)|\right]. \tag{2.11}$$

Corollary 8. *In Theorem 3, choose* $x = \frac{a+b}{2}$*, then*

$$\left|f\left(\frac{a+b}{2}\right)-\frac{1}{b-a}\int_a^b f(u)\,du\right| \leq \frac{(b-a)}{2^{1/p}(p+1)^{1/p}}\left[\max\left\{\left|f'\left(\frac{a+b}{2}\right)\right|^q,|f'(b)|^q\right\}^{\frac{1}{q}} + \max\left\{\left|f'\left(\frac{a+b}{2}\right)\right|^q,|f'(a)|^q\right\}\right]^{\frac{1}{q}}. \tag{2.12}$$

Corollary 9. *In Corollary 8. Additionally, if*

(1) $|f'|$ *is increasing, then we have*

$$\left|f\left(\frac{a+b}{2}\right)-\frac{1}{b-a}\int_a^b f(x)\,dx\right| \leq \frac{(b-a)}{2^{1/p}(p+1)^{1/p}}\left[|f'(b)|+\left|f'\left(\frac{a+b}{2}\right)\right|\right]. \tag{2.13}$$

(2) $|f'|$ *is decreasing, then we have*

$$\left|f\left(\frac{a+b}{2}\right)-\frac{1}{b-a}\int_a^b f(x)\,dx\right| \leq \frac{(b-a)}{2^{1/p}(p+1)^{1/p}}\left[|f'(a)|+\left|f'\left(\frac{a+b}{2}\right)\right|\right]. \tag{2.14}$$

(3) *If* $f'(a) = f'(b) = 0$*, then we have*

$$\left|f\left(\frac{a+b}{2}\right)-\frac{1}{b-a}\int_a^b f(u)\,du\right| \leq \frac{2^{1-\frac{1}{p}}}{(p+1)^{1/p}}(b-a)\left|f'\left(\frac{a+b}{2}\right)\right| \tag{2.15}$$

A different approach leads to the following result:

Theorem 4. *Let* $f : I \subset [0,\infty) \to \mathbb{R}$ *be a differentiable mapping on* I° *such that* $f' \in L[a,b]$*, where* $a, b \in I$ *with* $a < b$*. If* $|f'|^q$ *is quasi-convex on* $[a,b]$*,* $q \geq 1$*, and* $|f'(x)| \leq M$*,* $x \in [a,b]$*, then the following inequality holds:*

$$\left|f(x)-\frac{1}{b-a}\int_a^b f(u)\,du\right| \leq \frac{(x-a)^2}{2(b-a)}\left(\max\left\{|f'(x)|^q,|f'(a)|^q\right\}\right)^{\frac{1}{q}} + \frac{(b-x)^2}{2(b-a)}\left(\max\left\{|f'(x)|^q,|f'(b)|^q\right\}\right)^{\frac{1}{q}} \tag{2.16}$$

for each $x \in [a,b]$.

Proof. Suppose that $q \geq 1$. From Lemma 1 and using the well known power mean inequality, we have

$$\begin{aligned}
&\left| f(x) - \frac{1}{b-a} \int_a^b f(u)\,du \right| \\
&\leq (b-a) \int_0^{\frac{b-x}{b-a}} t \left| f'(ta + (1-t)b) \right| dt \\
&\qquad + (b-a) \int_{\frac{b-x}{b-a}}^1 |t-1| \left| f'(ta + (1-t)b) \right| dt \\
&\leq (b-a) \left(\int_0^{\frac{b-x}{b-a}} t dt \right)^{1-1/q} \left(\int_0^{\frac{b-x}{b-a}} t \left| f'(ta + (1-t)b) \right|^q dt \right)^{1/q} \\
&\qquad + (b-a) \left(\int_{\frac{b-x}{b-a}}^1 (1-t)\,dt \right)^{1-1/q} \left(\int_{\frac{b-x}{b-a}}^1 (1-t) \left| f'(ta + (1-t)b) \right|^q dt \right)^{1/q}.
\end{aligned}$$

Since $|f'|^q$ is quasi-convex, we have

$$\begin{aligned}
\int_0^{\frac{b-x}{b-a}} t \left| f'(ta + (1-t)b) \right|^q dt &\leq \int_0^{\frac{b-x}{b-a}} t \cdot \max\left\{ |f'(x)|^q, |f'(b)|^q \right\} dt \\
&= \frac{(b-x)^2}{2(b-a)^2} \cdot \max\left\{ |f'(x)|^q, |f'(b)|^q \right\}
\end{aligned}$$

and

$$\begin{aligned}
\int_{\frac{b-x}{b-a}}^1 (1-t) \left| f'(ta + (1-t)b) \right|^q dt &\leq \int_{\frac{b-x}{b-a}}^1 (1-t) \cdot \max\left\{ |f'(a)|^q, |f'(x)|^q \right\} dt \\
&= \frac{(x-a)^2}{2(b-a)^2} \cdot \max\left\{ |f'(a)|^q, |f'(x)|^q \right\}
\end{aligned}$$

Therefore, we have

$$\begin{aligned}
\left| f(x) - \frac{1}{b-a} \int_a^b f(u)\,du \right| &\leq \frac{(x-a)^2}{2(b-a)} \left(\max\left\{ |f'(x)|^q, |f'(a)|^q \right\} \right)^{\frac{1}{q}} \\
&\qquad + \frac{(b-x)^2}{2(b-a)} \left(\max\left\{ |f'(x)|^q, |f'(b)|^q \right\} \right)^{\frac{1}{q}},
\end{aligned}$$

which is required. ∎

Corollary 10. *In Theorem 4. Additionally, if* $|f'(x)| \leq M$, $M > 0$, *then inequality (1.1) holds.*

Corollary 11. *In Theorem 4, Additionally, if*

(1) $|f'|$ *is increasing, then (2.3) holds.*
(2) $|f'|$ *is decreasing, then (2.4) holds.*

Corollary 12. *In Theorem 4, choose* $x = \frac{a+b}{2}$*, then*

$$\left| f\left(\frac{a+b}{2}\right) - \frac{1}{b-a}\int_a^b f(u)\,du \right| \le \frac{(b-a)}{8}\left[\left(\max\left\{\left|f'\left(\frac{a+b}{2}\right)\right|^q, |f'(b)|^q\right\}\right)^{1/q} + \left(\max\left\{\left|f'\left(\frac{a+b}{2}\right)\right|^q, |f'(a)|^q\right\}\right)^{1/q}\right]. \tag{2.17}$$

Corollary 13. *In Corollary 12. Choosing* $x = \frac{a+b}{2}$*,*

1. *If* $|f'|$ *is increasing, then (2.6) holds.*
2. *If* $|f'|$ *is decreasing, then (2.7) holds.*
3. *If* $f'(a) = f'(b) = 0$*, then (2.8) holds.*

Remark 2. *We note that the inequality (2.17) improve the inequality (1.2).*

3. Applications to Special Means

We shall consider the means for arbitrary real numbers α, β $(\alpha \neq \beta)$. We take

1. Arithmetic mean :
$$A(\alpha,\beta) = \frac{\alpha+\beta}{2}, \quad \alpha,\beta \in \mathbb{R}.$$
2. Logarithmic mean :
$$L(\alpha,\beta) = \frac{\alpha-\beta}{\ln|\alpha| - \ln|\beta|}, \ |\alpha| \neq |\beta|, \ \alpha,\beta \neq 0, \ \alpha,\beta \in \mathbb{R}.$$
3. Generalized log-mean:
$$L_n(\alpha,\beta) = \left[\frac{\beta^{n+1} - \alpha^{n+1}}{(n+1)(\beta-\alpha)}\right]^{\frac{1}{n}}, \ n \in \mathbb{Z}, \ \alpha,\beta \in \mathbb{R}, \ \alpha \neq \beta.$$

Now, using the results of Section 2, we give some applications to special means of real numbers.

Proposition 1. *Let* $a, b \in \boldsymbol{R}$*,* $a < b$ *and* $0 \notin [a,b]$*. Then, for all* $p > 1$*, we have*

$$\left|A^{-1}(a,b) - L^{-1}(a,b)\right| \le \frac{(b-a)}{2^{1/p}(p+1)^{1/p}}\left\{\left[\sup\left(\left|\frac{a+b}{2}\right|^{-\frac{2p}{p-1}}, |a|^{-\frac{2p}{p-1}}\right)\right]^{\frac{p-1}{p}} + \left[\sup\left(\left|\frac{a+b}{2}\right|^{-\frac{2p}{p-1}}, |b|^{-\frac{2p}{p-1}}\right)\right]^{\frac{p-1}{p}}\right\}.$$

Proof. The assertion follows from Corollary 8 applied to the quasi-convex mapping $f(x) = 1/x$, $x \in [a,b]$. ∎

Proposition 2. *Let $a, b \in \boldsymbol{R}$, $a < b$ and $n \in \boldsymbol{N}$, $n \geq 2$. Then, for all $q \geq 1$, we have*

$$\begin{aligned} \left|A^{n}(a,b) - L_{n}^{n}(a,b)\right| &\leq n\left(\frac{b-a}{8}\right)\left\{\left[\sup\left(\left|\frac{a+b}{2}\right|^{(n-1)q}, |a|^{(n-1)q}\right)\right]^{1/q}\right. \\ &\quad \left. + \left[\sup\left(\left|\frac{a+b}{2}\right|^{(n-1)q}, |b|^{(n-1)q}\right)\right]^{1/q}\right\}. \end{aligned}$$

Proof. The assertion follows from Corollary 12 applied to the quasi-convex mapping $f(x) = x^n$, $x \in \mathbf{R}$. ∎

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