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A Three- Step Iterative Scheme for Generalized Φ-Hemicontractive operators in Banach spaces [*†]

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Abstract We prove some convergence results for a new three- step iteration scheme for generalized Φ-hemicontractive operators defined on Banach spaces. The results are generalizations of the work of several authors. In particular, they generalize the recent results of Fan and Xue (2009) which is in turn a correction of Rafiq (2006).

1 Introduction

We denote by J the normalized duality mapping from X into 2^{X^*} by

$$J(x) = \{f \in \mathrm{X}^* : \langle x, f\rangle = \|x\|^2 = \|f\|^2\}$$

where X^* denotes the dual space of X and $\langle ., .\rangle$ denotes the generalized duality pairing.

Definition 1.1 [25]. A mapping $T : X \to X$ is called strongly pseudocontractive if for all $x, y \in X$, there exist $j(x - y) \in J(x - y)$ and a constant $k \in (0, 1)$ such that

$$\langle Tx - Ty, j(x - y)\rangle \leq (1 - k)\|x - y\|^2.$$

Definition 1.2 [25]. A mapping T is called strongly ϕ-pseudocontractive if for all $x, y \in X$, there exist $j(x - y) \in J(x - y)$ and a strictly increasing function $\phi : [0, \infty) \to [0, \infty)$ with $\phi(0) = 0$ such that

$$\langle Tx - Ty, j(x - y)\rangle \leq \|x - y\|^2 - \phi(\|x - y\|)\|x - y\|.$$

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Definition 1.3 [25]. A mapping T is called generalized strongly Φ-pseudocontractive if for all $x, y \in X$, there exist $j(x-y) \in J(x-y)$ and a strictly increasing function $\Phi : [0, \infty) \to [0, \infty)$ with $\Phi(0) = 0$ such that

$$\langle Tx - Ty, j(x-y)\rangle \leq \|x-y\|^2 - \Phi(\|x-y\|).$$

It is clear from above definitions that every strongly ϕ- pseudocontractive operator is a generalized strongly Φ-pseudocontractive operator with $\Phi : [0,\infty) \to [0, \infty)$ defined by $\Phi(s) = \phi(s)s$, and every strongly pseudocontractive operator is strongly ϕ- pseudocontractive operator where ϕ is defined by $\phi(s) = ks$ for $k \in (0, 1)$ while the converses need not be true. An example by Hirano and Huang [5] showed that a strongly ϕ-pseudocontractive operator T is not always a strongly pseudocontractive operator.
Definition 1.4 [20]. A mapping $T : X \to X$ is called uniformly continuous generalized Φ- hemicontractive mapping if there exist $\rho \in F(T)$ $F(T) = (x \in K : Tx = x) \neq \phi$, and a strictly increasing function $\Phi : [0, \infty) \to [0, \infty)$, $\Phi(0) = 0$ such that for all $x \in K$, there exists $j(x - \rho) \in J(x - \rho)$ such that

$$\langle Tx - \rho, j(x-\rho)\rangle \leq \|x-\rho\|^2 - \Phi(\|x-\rho\|).$$

where K is a closed convex subset of X.
The class of generalized Φ- hemicontractive mappings is the most general among those defined above for which T has a unique fixed point. These classes of operators have been studied by several authors (see, for example [2-3],[5-25]).
The Mann iteration scheme [12], introduced in 1953, was used to prove the convergence of the sequence to the fixed points of mappings of which the Banach principle is not applicable. In 1974, Ishikawa [9] devised a new iteration scheme to establish the convergence of a Lipschitzian pseudocontractive map when Mann iteration process failed to converge. Noor et al.[17], gave the following three-step iteration process for solving non-linear operator equations in real Banach spaces.

Let K be a nonempty closed convex subset of E and $T : K \to K$ be a mapping. For an arbitrary $x_0 \in K$, the sequence $\{x_n\}_{n=0}^{\infty} \subset K$, defined by

$$\begin{aligned} x_{n+1} &= (1-\alpha_n)x_n + \alpha_n T y_n \\ y_n &= (1-\beta_n)x_n + \beta_n T z_n \\ z_n &= (1-\gamma_n)x_n + \gamma_n T x_n, \quad n \geq 0, \end{aligned} \tag{1.1}$$

where $\{\alpha_n\}_{n=0}^{\infty}$,$\{\beta_n\}_{n=0}^{\infty}$ and $\{\gamma_n\}_{n=0}^{\infty}$ are three sequences satisfying $\alpha_n, \beta_n, \gamma_n \in [0, 1]$ for each n, is called the three-step iteration (or the Noor iteration). When $\gamma_n = 0$, then the three-step iteration reduces to the Ishikawa iterative sequence $\{x_n\}_{n=0}^{\infty} \subset K$ defined by

$$\begin{aligned} x_{n+1} &= (1-\alpha_n)x_n + \alpha_n T y_n \\ y_n &= (1-\beta_n)x_n + \beta_n T x_n, \quad n \geq 0. \end{aligned} \tag{1.2}$$

If $\beta_n = \gamma_n = 0$, then (1.1) becomes the Mann iteration. It is the sequence $\{x_n\}_{n=0}^{\infty} \subset K$ defined by

$$x_{n+1} = (1-\alpha_n)x_n + \alpha_n T x_n, \quad n \geq 0. \tag{1.3}$$

Glowinski and Le Tallec [4] used a three-step iterative scheme to solve elastoviscoplasticity, liquid crystal and eigen-value problems. They have shown that the three-step approximation scheme performs better than the two-step and one-step iterative methods.

Haubruge et al.[6] studied the convergence analysis of three-step iterative schemes of Glowinski and Le Tallec [4] and applied these three-step iteration to obtain new splitting type algorithms for solving variational inequalities, separable convex programming and minimization of a sum of convex functions. They also proved that three-step iterations also lead to highly parallelized algorithms under certain conditions. Thus, it is clear that three-step schemes play an important part in solving various problems, which arise in pure and applied sciences.

Rafiq [18], recently introduced the following new type of iteration- the modified three-step iteration process, to approximate the unique common fixed points of a three strongly pseudocontractive mappings in Banach spaces.

Let $T_1, T_2, T_3 : K \to K$ be three mappings. For any given $x_0 \in K$, the modified three-step iteration $\{x_n\}_{n=0}^{\infty} \subset K$ is defined by

$$\begin{aligned} x_{n+1} &= (1-\alpha_n)x_n + \alpha_n T_1 y_n \\ y_n &= (1-\beta_n)x_n + \beta_n T_2 z_n \\ z_n &= (1-\gamma_n)x_n + \gamma_n T_3 x_n, \quad n \geq 0 \end{aligned} \tag{1.4}$$

where $\{\alpha_n\}_{n=0}^{\infty}$,$\{\beta_n\}_{n=0}^{\infty}$ and $\{\gamma_n\}_{n=0}^{\infty}$ are three real sequences satisfying some conditions. It is clear that the iteration schemes (1.1)-(1.3) are special cases of (1.4)

It is worth mentioning that, several authors, for example, Xue and Fan [21] recently used the iteration in equation (1.4) to approximate the common fixed points of three pseudocontractive operators in Banach spaces. Infact,they stated and proved the corrected version of Rafiq's result [18] thus :

Theorem XF. Let X be a real Banach Space and K be a nonempty closed convex subset of X. Let T_1, T_2 and T_3 be strongly pseudocontractive self maps of K with $T_1(K)$ bounded and T_1, T_2 and T_3 uniformly continuous. Let $\{x_n\}_{n=0}^{\infty}$ be defined by (1.4),where $\{a_n\}_{n=0}^{\infty}$, $\{b_n\}_{n=0}^{\infty}$ and $\{c_n\}_{n=0}^{\infty}$ are three real sequences in [0,1] such that: (i) $a_n, b_n \to 0$ as $n \to \infty$ and (ii)$\sum_{n=0}^{\infty} a_n = \infty$. If $F(T_1) \cap F(T_2) \cap F(T_3) \neq \phi$, then the sequence $\{x_n\}_{n=0}^{\infty}$ converges strongly to the common fixed point of T_1, T_2 and T_3.

This result itself is a generalization of many previous results (see[18] and the references there in).

For three mappings, it is desirable to devise a general iteration scheme which extend the Mann iteration , the Ishikawa iteration , the Noor iteration and the modified Noor iteration . To achieve this goal, we introduce a new iteration process for three generalized Φ- hemicontractive operators as follows:

Let K be a non-empty closed convex subset of a Banach psace X. Suppose that $\{\alpha_n\}_{n=0}^{\infty}$,$\{\alpha'_n\}_{n=0}^{\infty}$, $\{\beta_n\}_{n=0}^{\infty}$,$\{\gamma_n\}_{n=0}^{\infty}$ are real sequences in [0,1] satisfying some conditions.
Let $T_1, T_2, T_3 : K \to K$ be three mappings. The iteration scheme we introduce is defined as follows:
For any given $x_0 \in K$,

$$\begin{aligned} x_{n+1} &= (1-\alpha_n-\alpha'_n)x_n + \alpha_n T_1 y_n + \alpha'_n T_1 x_n \\ y_n &= (1-\beta_n)x_n + \beta_n T_2 z_n \\ z_n &= (1-\gamma_n)x_n + \gamma_n T_3 x_n \quad n \geq 0 \end{aligned} \tag{1.5}$$

When $T_1 = T_2 = T_3 = T$ in equation (1.5), we obtain a version given by

$$\begin{aligned} x_{n+1} &= (1-\alpha_n-\alpha'_n)x_n + \alpha_n T y_n + \alpha'_n T x_n \\ y_n &= (1-\beta_n)x_n + \beta_n T z_n \\ z_n &= (1-\gamma_n)x_n + \gamma_n T x_n \quad n \geq 0. \end{aligned} \tag{1.6}$$

Since T_1 is self mapping and K is convex , then we can find an $u_n \in K$ such that $T_1 x_n = u_n$. In this case,(1.5) reduces to:

$$\begin{aligned} x_{n+1} &= (1-\alpha_n-\alpha'_n)x_n + \alpha_n T_1 y_n + \alpha'_n u_n \\ y_n &= (1-\beta_n)x_n + \beta_n T_2 z_n \\ z_n &= (1-\gamma_n)x_n + \gamma_n T_3 x_n \quad n \geq 0. \end{aligned} \tag{1.7}$$

Obviously, when $\alpha'_n = 0$ in equation (1.5) we obtaine (1.4). We observed that the iteration (1.5), (1.6) and (1.7) are well defined and are generalization of (1.1)- (1.4).

In this paper, we use our newly introduced iteration process (1.5) and prove that it converges strongly to a unique common fixed point of a three generalized Φ- hemicontractive mappings in Banach spaces. Our result extends the recent results of Xue and Fan [21] to a three generalized Φ- hemicontractive mappings, which itself is a generalization of many of the previous results.

In order to obtain the main results, the following lemmas are needed.
Lemma 1.2[2]. Let E be real Banach Space and $J : E \to 2^{E^*}$ be the normalized duality mapping.Then,for any $x, y \in E$

$$\|x+y\|^2 \leq \|x\|^2 + 2 < y, j(x+y) >, \forall j(x+y) \in J(x+y)$$

Lemma 1.3[14]. Let $\Phi : [0, \infty) \to [0, \infty)$ be an increasing function with $\Phi(x) = 0 \Leftrightarrow x = 0$ and let $\{b_n\}_{n=0}^{\infty}$ be a positive real sequence satisfying

$$\sum_{n=0}^{\infty} b_n = +\infty \quad \text{and} \quad \lim_{n\to\infty} b_n = 0.$$

Suppose that $\{a_n\}_{n=0}^{\infty}$ is a nonnegative real sequence. If there exists an integer $N_0 > 0$ satisfying

$$a_{n+1}^2 < a_n^2 + o(b_n) - b_n \Phi(a_{n+1}), \quad \forall n \geq N_0$$

where $\lim_{n\to\infty} \frac{o(b_n)}{b_n} = 0$, then $\lim_{n\to\infty} a_n = 0$.

2.Main results

Theorem 2.1. *Let X be a real Banach space , K a nonempty closed and convex subset of X and $T_1, T_2, T_3 : K \to K$ be uniformly continuous and generalized Φ-hemicontractive mappings such that $T_1(K)$ is bounded with $p \in F(T_1) \cap F(T_2) \cap F(T_3)$. Let $\{x_n\}$ be a sequence defined by (1.7) where $\{\alpha_n\}_{n=0}^{\infty}$,$\{\alpha'_n\}_{n=0}^{\infty}$, $\{\beta_n\}_{n=0}^{\infty}$ and $\{\gamma_n\}_{n=0}^{\infty}$ are three sequences in [0,1] satisfying*

(i) $\lim_{n\to\infty} \alpha_n = \lim_{n\to\infty} \alpha'_n = \lim_{n\to\infty} \beta_n = 0$
and

(ii) $\sum_{n=1}^{\infty} (\alpha_n + \alpha'_n) = \infty$.

(iii) $\lim_{n\to\infty} \frac{\alpha'_n}{\delta_n} = 0$, *where* $\delta_n = \alpha_n + \alpha'_n$.

If $F(T_1) \cap F(T_2) \cap F(T_3) \neq \emptyset$, then the sequence $\{x_n\}_{n=0}^{\infty}$ converges strongly to the unique common fixed point of T_1, T_2 and T_3.

Proof. The uniqueness of the fixed point comes from the definition of Φ-hemicontractive mapping. By assumption, we have $p \in F(T_1) \cap F(T_2) \cap F(T_3)$.

Let $D_1 = \|x_0 - p\| + \sup_{n\geq 0} \|T_1 y_n - p\| + \sup_{n\geq} \|u_n - \rho\|$. We prove by induction that $\|x_n - p\| \leq D_1$ for all n.

It is clear that, $\|x_0 - p\| \leq D_1$. Assume that $\|x_n - p\| \leq D_1$ holds. We will prove that $\|x_{n+1} - p\| \leq D_1$. Indeed, from (1.7), we obtain

$$\begin{aligned} \|x_{n+1} - p\| &= \|(1 - \delta_n)(x_n - p) + \alpha_n(T_1 y_n - p) + \alpha'_n(u_n - \rho)\| \\ &\leq (1 - \delta_n)\|x_n - p\| + \alpha_n\|T_1 y_n - p\| + \alpha'_n\|u_n - p\| \\ &\leq (1 - \delta_n)D_1 + \alpha_n D_1 + \alpha'_n D_1. \end{aligned}$$

where $\delta_n = \alpha_n + \alpha'_n$. Hence the sequence $\{x_n\}$ is bounded.
Using the uniformly continuity of T_3, we have $\{T_3 x_n\}$ is bounded. Denote $D_2 = \max\{D_1, \sup\{\|T_3 x_n - p\|\}$, then

$$\begin{aligned} \|z_n - p\| &\leq (1 - \gamma_n)\|x_n - p\| + \gamma_n\|T_3 x_n - p\| \\ &\leq (1 - \gamma_n)D_1 + \gamma_n D_2 \\ &\leq (1 - \gamma_n)D_2 + \gamma_n D_2 = D_2 \end{aligned}$$

By the virtue of the uniform continuity of T_2, we have that $\{T_2 z_n\}$ is bounded. Set $D = \sup_{n\geq 0}\|T_2 z_n - p\| + D_2$. From equation (1.7) we have,in view of Definition 1.4, that

$$\begin{aligned}
\|x_{n+1}-p\|^2 &= \langle x_{n+1}-p, j(x_{n+1}-p)\rangle \\
&\leq (1-\delta_n)\|x_n-p\|\|x_{n+1}-p\| + \delta_n\langle T_1 y_n - p, j(x_{n+1}-p)\rangle \\
&\quad +\alpha''\langle u_n - T_1 y_n, j(x_{n+1}-\rho) \\
&= (1-\delta_n)\|x_n-p\|\|x_{n+1}-p\| \\
&\quad +\delta_n\langle T_1 x_{n+1}-p, j(x_{n+1}-p)\rangle \\
&\quad +\delta_n\langle T_1 y_n - T_1 x_{n+1}, j(x_{n+1}-p)\rangle + \alpha' D_1\|x_{n+1}-\rho\| \\
&\leq (1-\delta_n)\|x_n-p\|\|x_{n+1}-p\| + \delta_n\sigma_n\|x_{n+1}-p\| \\
&\quad +\delta_n(\|x_{n+1}-p\|^2 - \Phi(\|x_{n+1}-p\|)) \\
&\quad +\alpha'_n D_1\|x_{n+1}-\rho\| \\
&= (1-\delta_n)\|x_n-p\|\|x_{n+1}-p\| + (\delta_n\sigma_n + \alpha'_n D_1)\|x_{n+1}-\rho\| \\
&\quad +\delta_n(\|x_{n+1}-p\|^2 - \Phi(\|x_{n+1}-p\|)) \\
&\leq (1-\delta_n)\|x_n-p\|\|x_{n+1}-p\| + \delta_n r_n\|x_{n+1}-p\| \\
&\quad +\delta_n(\|x_{n+1}-p\|^2 - \Phi(\|x_{n+1}-p\|))
\end{aligned} \tag{2.1}$$

where $r_n = \sigma_n + \frac{\alpha'_n D_1}{\delta_n}$, $\sigma_n = \|T_1 y_n - T_1 x_{n+1}\|$. Observe that

$$\begin{aligned}
\|x_{n+1}-y_n\| &= \beta_n\|x_n - T_2 z_n\| + \alpha'_n\|u_n - T_1 y_n\| \\
&\quad +\delta_n\|T_1 y_n - x_n\| \\
&\leq 2D(\delta_n + \alpha'_n) + (D_1 + D)\beta_n
\end{aligned}$$

This implies that $\lim_{n\to\infty}\|x_{n+1}-y_n\| = 0$ by (i). Since T_1 is uniformly continuous, we have

$$\sigma_n = \|T_1 x_{n+1} - T_1 y_n\| \to 0, \quad (n\to\infty) \tag{2.2}$$

Observe from the fact that $2AB \leq A^2 + B^2$ that

$$(1-\delta_n)\|x_n-p\|.\|x_{n+1}-p\| \leq \frac{1}{2}((1-\delta_n)^2\|x_n-p\|^2 + \|x_{n+1}-p\|^2),$$

and

$$\|x_{n+1}-p\| \leq \frac{1}{2}(1 + \|x_{n+1}-p\|^2). \tag{2.3}$$

Substituting (2.2) and (2.3) into (2.1), we obtain

$$\begin{aligned}
\|x_{n+1}-p\|^2 &\leq \tfrac{1}{2}((1-\delta_n)^2\|x_n-p\|^2 + \|x_{n+1}-p\|^2) + \delta_n\|x_{n+1}-p\|^2 \\
&\quad -\delta_n\Phi(\|x_{n+1}-p\|) + \delta_n r_n.\tfrac{1}{2}(1 + \|x_{n+1}-p\|^2)
\end{aligned}$$

$$(1-2\delta_n-\delta_n r_n)\|x_{n+1}-p\|^2 \leq (1-\delta_n)^2\|x_n-p\|^2 - 2\delta_n\Phi(x_{n+1}-p) + \delta_n r_n. \tag{2.4}$$

Since $\lim_{n\to\infty}\delta_n = \lim_{n\to\infty}\delta_n r_n = 0$, there exists a natural number N_0 such that

$$\frac{1}{2} < 1 - 2\delta_n - \delta_n r_n < 1$$

for all $n > N_0$. Then, (2.4) implies that

$$\begin{aligned} \|x_{n+1} - p\|^2 &\leq \tfrac{(1-\delta_n)^2}{1-2\delta_n-\delta_n r_n}\|x_n - p\|^2 - \tfrac{2\delta_n}{1-2\delta_n-\delta_n r_n}\Phi(\|x_{n+1} - p\|) \\ &\quad + \tfrac{\delta_n r_n}{1-2\delta_n-\delta_n r_n} \\ &\leq \|x_n - p\|^2 + \delta_n \tfrac{(\delta_n + r_n)}{1-2\delta_n-\delta_n r_n}\|x_n - p\|^2 \\ &\quad - \tfrac{2\delta_n}{1-2\delta_n-\delta_n r_n}\Phi(\|x_{n+1} - p\|) + \tfrac{\delta_n r_n}{1-2\delta_n-\delta_n r_n} \end{aligned} \tag{2.5}$$

Since $\|x_n - p\| \leq D$, it follows from (2.5) that $\forall\ n \geq N_0$,

$$\begin{aligned} \|x_{n+1} - p\|^2 &\leq \|x_n - p\|^2 + 2\delta_n(\delta_n + r_n)D^2 - 2\delta_n\Phi(\|x_{n+1} - p\|) \\ &\quad +2\delta_n r_n \\ &= \|x_n - p\|^2 - 2\delta_n\Phi(\|x_{n+1} - p\|) + 2\delta_n((\delta_n + r_n)D^2 + r_n) \\ &\leq \|x_n - p\|^2 - 2\delta_n\Phi(\|x_{n+1} - p\|) \\ &\quad +2\delta_n((\delta_n + r_n)D^2 + r_n), \quad \forall n \geq N_0 \end{aligned} \tag{2.6}$$

Taking $b_n = 2\delta_n$ and observing that

$$\lim_{n\to\infty} \frac{2\delta_n((\delta_n + r_n)D^2 + r_n)}{2\delta_n} = \lim_{n\to\infty}((\delta_n + r_n)D^2 + r_n) = 0,$$

then (2.6) becomes

$$a_{n+1}^2 \leq a_n^2 - b_n\Phi(a_{n+1}) + o(b_n), \quad \forall n \geq N_0$$

This with Lemma 1.1 showed that $a_n \to 0$ as $n \to \infty$, that is ,

$$\lim_{n\to\infty} \|x_n - p\| = 0.$$

This completes the proof.

Remarks 1. Theorem 2.1 generalizes Theorem 2 of [21] which in turn is a correction of Theorem 2 of [18] in that the three strongly pseudocontractive maps in [18]and [21] are replaced by three generalized Φ-hemicontractive mappings.
(ii.) Theorem 2.1 also extend Theorem 2 of [18] and Theorem 2 of [21] to a more general iterative process.

Corollary 2.2. Let X be a Banach space and K a non-empty closed, convex subset of X. Let T be a uniformly continuous and generalized Φ-hemicontractive self map of K with $T(K)$ bounded. Let $\{x_n\}_{n=0}^{\infty}$ be a sequence define by (1.7) where $\{\alpha_n\}_{n=0}^{\infty}$,$\{\beta_n\}_{n=0}^{\infty}$ and $\{\gamma_n\}_{n=0}^{\infty}$ are three real sequences in [0,1] satisfying

(i) $\lim_{n\to\infty} \alpha_n = \lim_{n\to\infty} \alpha'_n = \lim_{n\to\infty} \beta_n = 0$

(ii) $\sum_{n=1}^{\infty}(\alpha_n + \alpha'_n) = \infty$

Then, the sequence $\{x_n\}_{n=0}^{\infty}$ converges strongly to the unique fixed point of T.
Proof. It follows from Theorem 2.1 with $T_1 = T_2 = T_3 = T$.

Theorem 2.3. *Let X be a real Banach space , K a nonempty closed and convex subset of X and $T_1, T_2, T_3 : K \to K$ be uniformly continuous Φ-hemicontractive mappings such that $T_1(K)$ is bounded with $p \in F(T_1) \cap F(T_2) \cap F(T_3)$. Let $\{x_n\}$ be a sequence defined by (1.5) where $\{\alpha_n\}_{n=0}^{\infty}$,$\{\alpha'_n\}_{n=0}^{\infty}$,$\{\beta_n\}_{n=0}^{\infty}$ and $\{\gamma_n\}_{n=0}^{\infty}$ are three sequences in [0,1] satisfying*

(i) $\lim_{n\to\infty} \alpha_n = \lim_{n\to\infty} \alpha'_n = \lim_{n\to\infty} \beta_n = 0$
and

(ii) $\sum_{n=1}^{\infty}(\alpha_n + \alpha'_n) = \infty$.

If $F(T_1) \cap F(T_2) \cap F(T_3) \neq \emptyset$, then the sequence $\{x_n\}_{n=0}^{\infty}$ converges strongly to the unique common fixed point of T_1, T_2 and T_3.
Proof. It follows from Theorem 2.1 with $u_n = T_1$

The following remark indicates some ways in which the main theorems of this paper can be applied to certain quasi-accretive maps.
Remarks 2. (i) The operator T is a generalized Φ- hemi-contractive if and only if $(I - T)$ is generalized Φ-quasi-accretive.
(ii) Let $T, S : X \to X$, and $f \in X$ be given. A fixed point for the map $Tx = f + (I - S)x$, for all $X \in X$, is a solution for $Sx = f$, and conversely
(iii) Consider iteration (1.7) with $T_i = f + (I - S_i)x$ to obtain a convergence result to the solution of $S_i x = f$.

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