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HERMITE-HADAMARD TYPE INEQUALITIES FOR $(h-(\alpha,m))$-CONVEX FUNCTIONS

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ABSTRACT. In this paper, we define $(h-(\alpha,m))$-convex functions that is a new class of convex functions as a generalization of convexity. We also prove some Hadamard's type inequalities.

1. INTRODUCTION

Let $f: I \subset \mathbb{R} \to \mathbb{R}$ be a convex function on the interval I of real numbers and $a, b \in I$ with $a < b$. The inequality

$$f\left(\frac{a+b}{2}\right) \leq \frac{1}{b-a}\int_a^b f(x)dx \leq \frac{f(a)+f(b)}{2} \tag{1.1}$$

is known as Hermite-Hadamard's inequality for convex functions, [2].

In [4], G.Toader defined the concept of m-convexity as the following:

Definition 1. *The function $f:[0,b] \to \mathbb{R}$ is said to be $m-$convex, where $m \in [0,1]$, if for every $x, y \in [0,b]$ and $t \in [0,1]$ we have:*

$$f(tx + m(1-t)y) \leq tf(x) + m(1-t)f(y).$$

Denote by $K_m(b)$ the set of the $m-$convex functions on $[0,b]$ for which $f(0) \leq 0$.

Some interesting and important inequalities for m-convex functions can be found in [5], [6], [7] and [8].

In [3], S. Varošanec defined the following class:

Definition 2. *Let $h: J \subseteq \mathbb{R} \to \mathbb{R}$ be a non-negative function, $h \neq 0$. We say that $f: I \to \mathbb{R}$ is an $h-$convex function, or that f belongs to the class $SX(h,I)$, if f is non-negative and for all $x, y \in I$,$\alpha \in (0,1)$ we have*

$$f(\alpha x + (1-\alpha)y) \leq h(\alpha)f(x) + h(1-\alpha)f(y) \tag{1.2}$$

If inequality (1.2) is reversed, then f is said to be $h-$concave, i.e. $f \in SV(h,I)$.

Obviously, if $h(\alpha) = \alpha$, then all non-negative convex functions belong to $SX(h,I)$ and all non-negative concave functions belong to $SV(h,I)$; if $h(\alpha) = \frac{1}{\alpha}$, then $SX(h,I) = Q(I)$; if $h(\alpha) = 1$, then $SX(h,I) \supseteq P(I)$; and if $h(\alpha) = \alpha^s$, where $s \in (0,1)$, then $SX(h,I) \supseteq K_s^2$.

In [1], Pachpatte established the new following Hadamard-type inequality for products of convex functions.

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Theorem 1. *Let $f, g : [a,b] \to [0,\infty)$ be convex functions on $[a,b] \subset \mathbb{R}$, $a < b$. Then*

$$\frac{1}{b-a}\int_a^b f(x)g(x)dx \le \frac{1}{3}M(a,b) + \frac{1}{6}N(a,b) \tag{1.3}$$

where $M(a,b) = f(a)g(a) + f(b)g(b)$ and $N(a,b) = f(a)g(b) + f(b)g(a)$.

2. DEFINITION AND MAIN RESULTS

Definition 3. *Let $h : J \subset \mathbb{R} \to \mathbb{R}$ be a non-negative function, $h \neq 0$. We say that $f : [0,b] \to \mathbb{R}$ is an $(h-(\alpha,m))-$ convex function, or that f belongs to the class $SX((h-(\alpha,m)),b)$, if f is non-negative and for all $x, y \in [0,b]$,$(\alpha,m) \in [0,1]^2$, $t \in [0,1]$ we have*

$$f(tx + m(1-t)y) \le h^{\alpha}(t)f(x) + m(1-h^{\alpha}(t))f(y) \tag{2.1}$$

If inequality (2.1) is reversed, then f is said to be $(h-(\alpha,m),b)-$ concave, i.e., $f \in SV(h-(\alpha,m),b)$.

Obviously, if $h(t) = t$, all non-negative $(\alpha,m)-$*convex* functions belong to $SX((h,(\alpha,m)),b)$ and all non-negative concave functions belong to $SV((h-(\alpha,m)),b)$.

The following theorem was obtained by using the $(h-(\alpha,m))-$*convex* function.

Theorem 2. *Let $h : J \subseteq \mathbb{R} \to \mathbb{R}$ be a non-negative function, $h \neq 0$. We say that $f : I = [0,b] \subset [0,\infty) \to \mathbb{R}$ is an $(h-(\alpha,m))-$ convex function, or that f belongs to the class $SX((h-(\alpha,m)),b)$, if f is non-negative and for all $0 \le ma \le a \le mb < b < \infty$, $(\alpha,m) \in [0,1] \times (0,1]$, $t \in [0,1]$. If $f \in L_1[ma,b]$, $h \in L_1[0,1]$,we have the following inequalities:*

(2.2)

$$\begin{aligned}\frac{1}{b-a}\int_a^b f(x)dx \le \min\Bigg\{ & f(a)\int_0^1 h^{\alpha}(t)dt + mf\left(\frac{b}{m}\right)\int_0^1 (1-h^{\alpha}(t))\,dt, \\ & f(b)\int_0^1 h^{\alpha}(t)dt + mf\left(\frac{a}{m}\right)\int_0^1 (1-h^{\alpha}(t))\,dt\Bigg\}\end{aligned}$$

and

(2.3)

$$\begin{aligned} f\left(\frac{a+b}{2}\right) &\le \frac{1}{b-a}\left[h^{\alpha}\left(\frac{1}{2}\right)\int_a^b f(x)dx + m\left(1-h^{\alpha}\left(\frac{1}{2}\right)\right)\int_a^b f\left(\frac{x}{m}\right)dx\right] \\ &\le h^{\alpha}\left(\frac{1}{2}\right)\left[f(a)\int_0^1 h^{\alpha}(t)\,dt + mf\left(\frac{b}{m}\right)\int_0^1 (1-h^{\alpha}(t))\,dt\right] \\ &\quad + \left(1-h^{\alpha}\left(\frac{1}{2}\right)\right)\left[mf\left(\frac{a}{m}\right)\int_0^1 (1-h^{\alpha}(t))\,dt + m^2 f\left(\frac{b}{m^2}\right)\int_0^1 h^{\alpha}(t)\,dt\right].\end{aligned}$$

Proof. By the properties of $(h-(\alpha,m))-$*convex* mappings, for any $t \in [0,1]$ and $(\alpha,m) \in [0,1] \times (0,1]$, we obtain the following inequality for $x, y \in I$

$$f(ta + (1-t)b) \le h^{\alpha}(t)f(a) + m(1-h^{\alpha}(t))f\left(\frac{b}{m}\right). \tag{2.4}$$

Integrating both side of (2.4) on $[0,1]$, we have

$$\begin{aligned} \int_0^1 f(ta+(1-t)b)dt &\leq \int_0^1 \left[h^\alpha(t)f(a)+m(1-h^\alpha(t))f\left(\frac{b}{m}\right)\right]dt \\ \frac{1}{b-a}\int_a^b f(x)dx &\leq f(a)\int_0^1 h^\alpha(t)dt+mf\left(\frac{b}{m}\right)\int_0^1 (1-h^\alpha(t))\,dt. \end{aligned} \tag{2.5}$$

Analogously, by using the $f(tb+(1-t)a)\leq h^\alpha(t)f(b)+m(1-h^\alpha(t))f\left(\frac{a}{m}\right)$, we have

$$\begin{aligned} \int_0^1 f(tb+(1-t)a)dt &\leq \int_0^1 \left[h^\alpha(t)f(b)+m(1-h^\alpha(t))f\left(\frac{a}{m}\right)\right]dt \\ \frac{1}{b-a}\int_a^b f(x)dx &\leq f(b)\int_0^1 h^\alpha(t)dt+mf\left(\frac{a}{m}\right)\int_0^1 (1-h^\alpha(t))\,dt. \end{aligned} \tag{2.6}$$

Combining the inequalities in (2.5) and (2.6), we obtain the inequality in (2.2).

To prove the first inequality in (2.3), by the $(h-(\alpha,m))-$*convexity* of f we have that

$$f\left(\frac{x+y}{2}\right)\leq h^\alpha\left(\frac{1}{2}\right)f(x)+m\left(1-h^\alpha\left(\frac{1}{2}\right)\right)f\left(\frac{y}{m}\right).$$

If we choose $x=ta+(1-t)b$, $y=tb+(1-t)a$, we get

$$f\left(\frac{a+b}{2}\right)\leq h^\alpha\left(\frac{1}{2}\right)f(ta+(1-t)b)+m\left(1-h^\alpha\left(\frac{1}{2}\right)\right)f\left(\frac{tb+(1-t)a}{m}\right) \tag{2.7}$$

for all $t\in[0,1]$. Then, integrating both side of (2.7) on $[0,1]$, we have

$$f\left(\frac{a+b}{2}\right)\leq \int_0^1 \left[h^\alpha\left(\frac{1}{2}\right)f(ta+(1-t)b)+m\left(1-h^\alpha\left(\frac{1}{2}\right)\right)f\left(\frac{tb+(1-t)a}{m}\right)\right]dt$$

Use of the changing variable, we have

$$f\left(\frac{a+b}{2}\right)\leq \frac{1}{b-a}\left[h^\alpha\left(\frac{1}{2}\right)\int_a^b f(x)dx+m\left(1-h^\alpha\left(\frac{1}{2}\right)\right)\int_a^b f\left(\frac{x}{m}\right)dx\right]$$

which is the first inequality in (2.3).

To prove the second inequality in (2.3), we use the right side of (2.7) and using $(h-(\alpha,m))-$*convexity* of f, we have

$$\begin{aligned} & h^\alpha\left(\frac{1}{2}\right)f(ta+(1-t)b)+m\left(1-h^\alpha\left(\frac{1}{2}\right)\right)f\left(\frac{tb+(1-t)a}{m}\right) \\ \leq\ & h^\alpha\left(\frac{1}{2}\right)\left[h^\alpha(t)f(a)+m(1-h^\alpha(t))f\left(\frac{b}{m}\right)\right] \\ & +m\left(1-h^\alpha\left(\frac{1}{2}\right)\right)\left[(1-h^\alpha(t))f\left(\frac{a}{m}\right)+mh^\alpha(t)f\left(\frac{b}{m^2}\right)\right] \end{aligned}$$

Integrating the both side of the above inequality, we have

$$\begin{aligned}
& h^{\alpha}\left(\frac{1}{2}\right) f\left(ta+(1-t)b\right)+m\left(1-h^{\alpha}\left(\frac{1}{2}\right)\right) f\left(\frac{tb+(1-t)a}{m}\right) \\
\leq\ & h^{\alpha}\left(\frac{1}{2}\right)\left[f(a)\int_0^1 h^{\alpha}(t)\,dt+mf\left(\frac{b}{m}\right)\int_0^1 (1-h(t))\,dt\right] \\
& +\left(1-h^{\alpha}\left(\frac{1}{2}\right)\right)\left[mf\left(\frac{a}{m}\right)\int_0^1 (1-h(t))\,dt+m^2 f\left(\frac{b}{m^2}\right)\int_0^1 h^{\alpha}(t)\,dt\right].
\end{aligned}$$

□

Corollary 1. *In the inequality in (2.2);*

- If we choose $m=1$, we have

$$\begin{aligned}
\frac{1}{b-a}\int_a^b f(x)dx \leq\ & \min\left\{f(a)\int_0^1 h^{\alpha}(t)dt+f(b)\int_0^1 \left(1-h^{\alpha}(t)\right)dt,\right. \\
& \left. f(b)\int_0^1 h^{\alpha}(t)dt+f(a)\int_0^1 \left(1-h^{\alpha}(t)\right)dt\right\}.
\end{aligned}$$

- If we choose $m=\alpha=1$, we have

$$\begin{aligned}
\frac{1}{b-a}\int_a^b f(x)dx \leq\ & \min\left\{f(a)\int_0^1 h(t)dt+f(b)\int_0^1 (1-h(t))\,dt,\right. \\
& \left. f(b)\int_0^1 h(t)dt+f(a)\int_0^1 (1-h(t))\,dt\right\}.
\end{aligned}$$

Remark 1. *In the inequality in (2.2); if we choose $m=\alpha=1$ and $h(t)=t$, we have*

$$\frac{1}{b-a}\int_a^b f(x)dx \leq \frac{f(a)+f(b)}{2}$$

*which is the right hand side of the H-H inequality in (*1.1).

Corollary 2. *In the inequality in (2.3), if we choose, $h(t)=t^{\frac{1}{\alpha}}$*

$$\begin{aligned}
f\left(\frac{a+b}{2}\right) \leq\ & \frac{1}{2(b-a)}\left[\int_a^b f(x)\,dx+m\int_a^b f\left(\frac{x}{m}\right)dx\right] \\
\leq\ & \frac{\left[f(a)+mf\left(\frac{b}{m}\right)\right]+\left[mf\left(\frac{a}{m}\right)+m^2 f\left(\frac{b}{m^2}\right)\right]}{4}.
\end{aligned}$$

Corollary 3. *In the inequality in (2.3), we choose $m=\alpha=1$, we obtain*

$$\begin{aligned}
& f\left(\frac{a+b}{2}\right) \\
\leq\ & h\left(\frac{1}{2}\right)\left[f(a)\int_0^1 h(t)\,dt+f(b)\int_0^1 (1-h(t))\,dt\right] \\
& +\left(1-h\left(\frac{1}{2}\right)\right)\left[f(a)\int_0^1 (1-h(t))\,dt+f(b)\int_0^1 h(t)\,dt\right].
\end{aligned}$$

Remark 2. *In the inequality in (2.3), we choose $m=\alpha=1$ and $h(t)=t$, we obtain*

$$f\left(\frac{a+b}{2}\right) \leq \frac{1}{b-a}\int_a^b f(x)dx \leq \frac{f(a)+f(b)}{2}$$

which is the H-H inequality in (1.1).

Theorem 3. *Let* $f \in SX(h_1-(\alpha,m))$, $g \in SX(h_2-(\alpha,m))$, $0 \leq ma \leq a \leq mb < b < \infty$, $(\alpha,m) \in [0,1]\times(0,1]$, $t \in [0,1]$, *be functions such that* $fg \in L_1[ma,b]$, $h_1h_2 \in L_1[0,1]$, *then*

$$
\begin{aligned}
(2.8)\qquad & \frac{1}{b-a}\int_0^1 f(x)\,g(x)\,dx \\
\leq\ & f(a)g(a)\int_0^1 h_1^\alpha(t)h_2^\alpha(t)dt + mf(a)g\left(\frac{b}{m}\right)\int_0^1 h_1^\alpha(t)\left(1-h_2^\alpha(t)\right)dt \\
& + mf\left(\frac{b}{m}\right)g(a)\int_0^1 h_2^\alpha(t)\left(1-h_1^\alpha(t)\right)dt \\
& + m^2 f\left(\frac{b}{m}\right)g\left(\frac{b}{m}\right)\int_0^1\left(1-h_1^\alpha(t)\right)\left(1-h_2^\alpha(t)\right)dt.
\end{aligned}
$$

Proof. Since $f \in SX(h_1-(\alpha,m))$, $g \in SX(h_2-(\alpha,m))$, we have

$$
\begin{aligned}
f(ta+(1-t)b) &\leq h_1^\alpha(t)f(a)+m(1-h_1^\alpha(t))f\left(\frac{b}{m}\right)\\
g(ta+(1-t)b) &\leq h_2^\alpha(t)g(a)+m(1-h_2^\alpha(t))g\left(\frac{b}{m}\right)
\end{aligned}
$$

for all $t \in [0,1]$. Since f and g are non-negative,

$$
\begin{aligned}
& f(ta+(1-t)b)g(ta+(1-t)b) \\
\leq\ & h_1^\alpha(t)h_2^\alpha(t)f(a)g(a)+mf(a)g\left(\frac{b}{m}\right)h_1^\alpha(t)\left(1-h_2^\alpha(t)\right) \\
& + mf\left(\frac{b}{m}\right)g(a)\,h_2^\alpha(t)\left(1-h_1^\alpha(t)\right)+m^2 f\left(\frac{b}{m}\right)g\left(\frac{b}{m}\right)\left(1-h_1^\alpha(t)\right)\left(1-h_2^\alpha(t)\right).
\end{aligned}
$$

Then if we integrate the both side of the above inequality on $[0,1]$, we have the inequality in (2.8). □

Corollary 4. *If we choose* $h_1(t)=h_2(t)=t$ *the inequality in (2.8), we get*

$$
\begin{aligned}
\frac{1}{b-a}\int_0^1 f(x)\,g(x)\,dx \leq\ & f(a)g(a)\frac{1}{1+2\alpha}+\left[f(a)g\left(\frac{b}{m}\right)+f\left(\frac{b}{m}\right)g(a)\right]\frac{m\alpha}{(\alpha+1)(2\alpha+1)} \\
& +\frac{2m^2\alpha^2}{(\alpha+1)(2\alpha+1)}g\left(\frac{b}{m}\right)f\left(\frac{b}{m}\right).
\end{aligned}
$$

Remark 3. *In Corollary 4,* $\alpha=m=1$, *we have*

$$
\frac{1}{b-a}\int_0^1 f(x)\,g(x)\,dx \leq \frac{1}{3}\left[f(a)g(a)+f(b)g(b)\right]+\frac{1}{6}\left[f(a)g(b)+f(b)g(a)\right]
$$

which is the inequality in (1.3).

Theorem 4. *Let* $h : J \subseteq \mathbb{R} \to \mathbb{R}$ *be a non-negative function,* $h \neq 0$. *We say that* $f : I=[0,b] \subset [0,\infty) \to \mathbb{R}$ *is an* $(h-(\alpha,m))-$*convex function, or that* f *belongs to the class* $SX((h-(\alpha,m)),b)$, *if* f *is non-negative and for all* $0 \leq ma \leq a \leq$

$mb < b < \infty,$ $(\alpha, m) \in [0,1] \times (0,1]$, $t \in [0,1]$. *If* $f \in L_1[ma,b]$, $h \in L_1[0,1]$, *we have*

$$\frac{1}{2\int_0^1 h^\alpha(t)\,dt + m}\left[\frac{1}{mb-a}\int_a^{mb} f(x)\,dx + \frac{1}{b-ma}\int_{ma}^b f(x)\,dx\right]$$
$$\leq \frac{f(a)+f(b)}{2}.$$

Proof. By the definition $(h-(\alpha,m))$ convexity of f, we can write;

$$\begin{aligned} f(ta+m(1-t)b) &\leq h^\alpha(t)f(a) + m(1-h^\alpha(t))f(b) \\ f(tb+m(1-t)a) &\leq h^\alpha(t)f(b) + m(1-h^\alpha(t))f(a) \\ f((1-t)a+mtb) &\leq h^\alpha(1-t)f(a) + mh^\alpha(t))f(b) \end{aligned}$$

and

$$f((1-t)b+mta) \leq h^\alpha(1-t)f(b) + mh^\alpha(t))f(a)$$

for all $t \in [0,1]$ and $(\alpha, m) \in [0,1] \times (0,1]$.

If we add the above inequalities, we get

$$\begin{aligned} & f(ta+m(1-t)b) + f(tb+m(1-t)a) + f((1-t)a+mtb) + f((1-t)b+mta) \\ \leq\ & h^\alpha(t)f(a) + m(1-h^\alpha(t))f(b) + h^\alpha(t)f(b) + m(1-h^\alpha(t))f(a) \\ & +h^\alpha(1-t)f(a) + mh^\alpha(t))f(b) + h^\alpha(1-t)f(b) + mh^\alpha(t))f(a) \\ =\ & h^\alpha(t)[f(a)+f(b)] + h^\alpha(1-t)[f(a)+f(b)] + m[f(a)+f(b)] \\ =\ & [f(a)+f(b)][h^\alpha(t)+h^\alpha(1-t)+m]. \end{aligned}$$

Integrating over $t \in [0,1]$, we obtain

$$\begin{aligned} & \frac{2}{mb-a}\int_a^{mb} f(x)dx + \frac{2}{b-ma}\int_{ma}^b f(x)dx \\ \leq\ & [f(a)+f(b)]\left[\int_0^1 h^\alpha(t)\,dt + \int_0^1 h^\alpha(1-t)\,dt + m\right]. \end{aligned}$$

If all corrections are made, we have

$$\frac{1}{\int_0^1 h^\alpha(t)\,dt + m + \int_0^1 h^\alpha(1-t)\,dt}\left[\frac{1}{mb-a}\int_a^{mb} f(x)\,dx + \frac{1}{b-ma}\int_{ma}^b f(x)\,dx\right]$$
$$\leq \frac{f(a)+f(b)}{2}.$$

Since $\int_0^1 h^\alpha(t)\,dt = \int_0^1 h^\alpha(1-t)\,dt$, we have the required inequality. □

Theorem 5. *Let* $h : J \subseteq \mathbb{R} \to \mathbb{R}$ *be a non-negative function,* $h \neq 0$. *We say that* $f : I = [0,b] \subset [0,\infty) \to \mathbb{R}$ *is an* $(h-(\alpha,m))-$ *convex function, or that* f *belongs to the class* $SX((h-(\alpha,m)),b)$, *if* f *is non-negative and for all* $0 \leq ma \leq a \leq mb < b < \infty$, $(\alpha,m) \in [0,1] \times (0,1]$, $t \in [0,1]$. *If* $f \in L_1[ma,b]$, $h \in L_1[0,1]$ *we have*

$$\frac{1}{b-a}\int_a^b f(u)\,du$$
$$\leq \frac{1}{2}\left[[f(a)+f(b)]\int_0^1 h^\alpha(t)\,dt + 2mf\left(\frac{a+b}{2m}\right)\int_0^1 [1-h^\alpha(t)]\,dt\right].$$

Proof. By the $(h-(\alpha,m))-convexity$ of f we have that

$$
\begin{aligned}
f\left(ta+(1-t)\frac{a+b}{2}\right) &\leq h^{\alpha}(t)f(a)+m\left[1-h^{\alpha}(1-t)\right]f\left(\frac{a+b}{2m}\right)\\
f\left(1-t)b+t\left(\frac{a+b}{2}\right)\right) &\leq h^{\alpha}(1-t)f(b)+m\left[1-h^{\alpha}(t)\right]f\left(\frac{a+b}{2m}\right).
\end{aligned}
$$

If we integrate the both side of the above inequalities over $[0,1]$, we have

$$\frac{2}{b-a}\int_{a}^{\frac{a+b}{2}}f(u)\,du\leq f(a)\int_{0}^{1}h^{\alpha}(t)\,dt+mf\left(\frac{a+b}{2m}\right)\int_{0}^{1}\left[1-h^{\alpha}(1-t)\right]dt$$

and

$$\frac{2}{b-a}\int_{\frac{a+b}{2}}^{b}f(u)\,du\leq f(b)\int_{0}^{1}h^{\alpha}(1-t)\,dt+mf\left(\frac{a+b}{2m}\right)\int_{0}^{1}\left[1-h^{\alpha}(t)\right]dt.$$

Then if we add the above inequalities we get

$$\frac{2}{b-a}\int_{a}^{b}f(u)\,du\leq f(a)\int_{0}^{1}h^{\alpha}(t)\,dt+2mf\left(\frac{a+b}{2m}\right)\int_{0}^{1}\left[1-h^{\alpha}(t)\right]dt+f(b)\int_{0}^{1}h^{\alpha}(1-t)\,dt.$$

Since $\int_0^1 h^{\alpha}(t)\,dt=\int_0^1 h^{\alpha}(1-t)\,dt$, we have

$$
\begin{aligned}
&\frac{1}{b-a}\int_{a}^{b}f(u)\,du\\
\leq\ &\frac{1}{2}\left[\left[f(a)+f(b)\right]\int_{0}^{1}h^{\alpha}(t)\,dt+2mf\left(\frac{a+b}{2m}\right)\int_{0}^{1}\left[1-h^{\alpha}(t)\right]dt\right].
\end{aligned}
$$

The proof is completed. □

Remark 4. *If in Theorem 5, we use $h(t)=1$ we have the right hand side of H-H inequality in (1.1).*

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