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SOME NEW HERMITE-HADAMARD INEQUALITIES ON $s-$CONVEX AND $s-$CONCAVE FUNCTIONS

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Abstract. In this paper, we obtain some new Hermite-Hadamard inequalities for $s-$convex and $s-$ concave functions. The analysis used in the proofs is fairly elementary and based on the use of Power-mean and Hölder inequalities.

1. INTRODUCTION

Throughout this paper, let $\|g\|_{\infty} = \sup_{t\in[a,b]} |g(t)|$.

The following inequality is well known in the literature as the Hermite-Hadamard integral inequality:

$$f\left(\frac{a+b}{2}\right) \leq \frac{1}{b-a}\int_a^b f(x)dx \leq \frac{f(a)+f(b)}{2} \tag{1.1}$$

where $f : I \subseteq \mathbb{R} \to \mathbb{R}$ is a convex function on the interval I of real numbers and $a, b \in I$ with $a < b$.

In [6], Hudzik and Maligranda considered among others the class of functions which are $s-$convex in the second sense. This class is defined in the following way: A function $f : \mathbb{R}^+ \to \mathbb{R}$, where $\mathbb{R}^+ = [0, \infty)$, is said to be $s-$convex in the second sense if

$$f(\alpha x + \beta y) \leq \alpha^s f(x) + \beta^s f(y)$$

for all $x, y \in [0, \infty)$, $\alpha, \beta \geq 0$ with $\alpha + \beta = 1$ and for some fixed $s \in (0, 1]$.

The class of $s-$convex functions in the second sense is usually denoted by K_s^2. It can be easily seen that for $s = 1$, $s-$convexity reduces to ordinary convexity of functions defined on $[0, \infty)$.

In [4], Dragomir and Fitzpatrick proved a variant of Hadamard's inequality which holds for $s-$convex functions in the second sense:

Theorem 1. *Suppose that $f : [0,\infty) \to [0,\infty)$ is an $s-$convex function in the second sense, where $s \in (0, 1)$ and let $a, b \in [0,\infty)$, $a < b$. If $f \in L^1[a, b]$, then the following inequalities hold:*

$$2^{s-1} f\left(\frac{a+b}{2}\right) \leq \frac{1}{b-a}\int_a^b f(x)dx \leq \frac{f(a)+f(b)}{s+1}. \tag{1.2}$$

The constant $k = \frac{1}{s+1}$ is the best possible in the second inequality in (1.2). The above inequalities are sharp.

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For recent results and generalizations concerning $s-$convex functions in the second sense see [1]-[6].

In [2], Kirmaci et al. established a new inequality which is given in the next theorem:

Theorem 2. *Let $f: I \rightarrow \mathbb{R}$, $I \subset [0,\infty)$, be a differentiable function on I° such that $f' \in L^1[a,b]$, where $a,b \in I$, $a<b$. If $|f'|^q$ is $s-$convex on $[a,b]$ for some fixed $s \in (0,1)$ and $q \geq 1$, then*

$$\left|\frac{f(a)+f(b)}{2}-\frac{1}{b-a}\int_a^b f(x)dx\right| \leq \frac{b-a}{2}\left(\frac{1}{2}\right)^{\frac{q-1}{q}}\left[\frac{s+\left(\frac{1}{2}\right)^s}{(s+1)(s+2)}\right]^{\frac{1}{q}}\left[|f'(a)|^q+|f'(b)|^q\right]^{\frac{1}{q}}. \tag{1.3}$$

In [7], K.L. Tseng et al. proved the following results:

Theorem 3. *Let $f: I^{\circ} \subseteq \mathbb{R} \rightarrow \mathbb{R}$ be a differentiable mapping on I°, $a,b \in I^{\circ}$ with $a<b, f' \in L[a,b]$, for $q \geq 1$, $|f'|^q$ is convex on $[a,b]$ and $g:[a,b] \rightarrow \mathbb{R}$ be a continuous mapping on $[a,b]$. Then, for all $x \in [a,b]$, we have the inequality:*

$$\begin{aligned}&\left|f(a)\int_a^x g(s)ds+f(b)\int_x^b g(s)ds-\int_a^b f(s)g(s)ds\right| \\ \leq\ & \|g\|_{\infty}\left[\frac{(x-a)^2+(b-x)^2}{2}\right]^{\frac{q-1}{q}} \\ &\times\left\{\frac{(x-a)^2(3b-x-2a)+(b-x)^3}{6(b-a)}|f'(a)|^q\right. \\ &\left.+\frac{(x-a)^3+(b-x)^2(2b+x-3a)}{6(b-a)}|f'(b)|^q\right\}^{\frac{1}{q}}.\end{aligned} \tag{1.4}$$

Corollary 1. *Let $g:[a,b] \rightarrow \mathbb{R}$ be symmetric to $\frac{a+b}{2}$ and $x=\frac{a+b}{2}$ in Theorem 3. Then, we have the inequality:*

$$\left|\frac{f(a)+f(b)}{2}\int_a^b g(s)ds-\int_a^b f(s)g(s)ds\right| \leq \|g\|_{\infty}\frac{(b-a)^2}{4}\left[\frac{|f'(a)|^q+|f'(b)|^q}{2}\right]^{\frac{1}{q}} \tag{1.5}$$

which is the "weighted trapezoid" inequality.

Theorem 4. *Let the assumptions of Theorem 3 hold. Then, for all $x \in [a,b]$, we have the inequality:*

$$
\begin{aligned}
&\left| f(x)\int_a^b g(s)ds - \int_a^b f(s)g(s)ds \right| \\
\leq\ & \|g\|_\infty \left[\frac{(x-a)^2 + (b-x)^2}{2} \right]^{\frac{q-1}{q}} \\
&\times \left\{ \frac{(x-a)^2 (3b-a-2x) + 2(b-x)^3}{6(b-a)} |f'(a)|^q \right. \\
&\left. + \frac{2(x-a)^3 + (b-x)^2 (b+2x-3a)}{6(b-a)} |f'(b)|^q \right\}^{\frac{1}{q}}.
\end{aligned} \tag{1.6}
$$

Corollary 2. *Let $x = \frac{a+b}{2}$ in Theorem 4. Then we have the inequality:*

$$
\begin{aligned}
&\left| f\left(\frac{a+b}{2}\right)\int_a^b g(s)ds - \int_a^b f(s)g(s)ds \right| \\
\leq\ & \|g\|_\infty \frac{(b-a)^2}{4} \left[\frac{|f'(a)|^q + |f'(b)|^q}{2} \right]^{\frac{1}{q}}
\end{aligned} \tag{1.7}
$$

which is the "weighted midpoint" inequality provided that $|f'|^q$ is convex on $[a,b]$.

In [8], Pearce and Pečarić proved the following theorem.

Theorem 5. *Let $f : I \to \mathbb{R}$, $I \subset \mathbb{R}$, be a differentiable mapping on I°, and $a, b \in I$, $a < b$. If $|f'|^q$ is convex on $[a,b]$ for some $q \geq 1$, then*

$$
\left| \frac{f(a)+f(b)}{2} - \frac{1}{b-a}\int_a^b f(x)dx \right| \leq \frac{b-a}{4} \left[\frac{|f'(a)|^q + |f'(b)|^q}{2} \right]^{\frac{1}{q}} \tag{1.8}
$$

and

$$
\left| f\left(\frac{a+b}{2}\right) - \frac{1}{b-a}\int_a^b f(x)dx \right| \leq \frac{b-a}{4} \left[\frac{|f'(a)|^q + |f'(b)|^q}{2} \right]^{\frac{1}{q}}. \tag{1.9}
$$

In [9], Fejér established the following inequality which is the weighted generalization of (1.1):

Theorem 6. *Let $f : I \subseteq \mathbb{R} \to \mathbb{R}$ be a convex mapping on an interval of real numbers I and let $a, b \in I$ with $a < b$. Then the inequality*

$$
f\left(\frac{a+b}{2}\right)\int_a^b g(x)dx \leq \int_a^b f(x)g(x)dx \leq \frac{f(a)+f(b)}{2}\int_a^b g(x)dx
$$

holds, where $g : [a,b] \to \mathbb{R}$ is nonnegative, integrable on $[a,b]$ and symmetric to $\frac{a+b}{2}$.

We use the Lemma 1 in order to prove our results (see [7]).

Lemma 1. *Let $f : I^{\circ} \subseteq \mathbb{R} \to \mathbb{R}$ be a differentiable mapping on I°, $a, b \in I^{\circ}$ with $a < b$ and let $g : [a, b] \to \mathbb{R}$. If $f', g \in L[a, b]$, then , for all $x \in [a, b]$, the following identity holds:*

$$f(a)\int_a^x g(s)ds+f(b)\int_x^b g(s)ds-\int_a^b f(s)g(s)ds=\int_a^b\left[\int_x^t g(s)ds\right]f'(t)dt. \tag{1.10}$$

2. MAIN RESULTS

Our main results are given in the following theorems.

Theorem 7. *Let $f : I \subseteq [0, \infty) \to \mathbb{R}$ be a differentiable mapping on I°, $a, b \in I^{\circ}$ with $a < b$, $f' \in L[a, b]$, for $q \geq 1$, $|f'|^q$ is $s-$convex on $[a, b]$ and let $g : [a, b] \to \mathbb{R}$ be a continuous mapping on $[a, b]$. Then, for all $x \in [a, b]$, we have the inequality:*

$$\begin{aligned}
&\left|f(a)\int_a^x g(u)du+f(b)\int_x^b g(u)du-\int_a^b f(u)g(u)du\right| \\
\leq\ & \|g\|_\infty\left[\frac{(x-a)^2+(b-x)^2}{2}\right]^{\frac{q-1}{q}}\left(\frac{1}{b-a}\right)^{\frac{s}{q}} \\
&\times\left\{\frac{(b-a)^{s+1}\left[x(s+2)-b-a(s+1)\right]+2\,(b-x)^{s+2}}{(s+1)(s+2)}\left|f'(a)\right|^q\right. \\
&\left.+\frac{(b-a)^{s+1}\left[a+b(s+1)-x(s+2)\right]+2\,(x-a)^{s+2}}{(s+1)(s+2)}\left|f'(b)\right|^q\right\}^{\frac{1}{q}}.
\end{aligned}$$

Proof. Let $x \in [a,b]$. Using Lemma 1, Power-mean integral inequality and the $s-$convexity of $|f'|^q$, it follows that

$$\begin{aligned}
&\left| f(a)\int_a^x g(u)du + f(b)\int_x^b g(u)du - \int_a^b f(u)g(u)du \right| \\
=& \left| \int_a^b \left[\int_x^t g(u)du \right] f'(t)dt \right| \\
\leq& \int_a^b \left| \int_x^t g(u)du f'(t) \right| dt \\
\leq& \left[\int_a^b \left| \int_x^t g(u)du \right| dt \right]^{\frac{q-1}{q}} \left[\int_a^b \left| \int_x^t g(u)du \right| |f'(t)|^q \, dt \right]^{\frac{1}{q}} \\
=& \|g\|_\infty \left[\frac{(x-a)^2+(b-x)^2}{2} \right]^{\frac{q-1}{q}} \left[\int_a^b |t-x| \, |f'(t)|^q \, dt \right]^{\frac{1}{q}} \\
=& \|g\|_\infty \left[\frac{(x-a)^2+(b-x)^2}{2} \right]^{\frac{q-1}{q}} \left[\int_a^b |t-x| \left| f'\left(\frac{b-t}{b-a}a + \frac{t-a}{b-a}b \right) \right|^q dt \right]^{\frac{1}{q}} \\
\leq& \|g\|_\infty \left[\frac{(x-a)^2+(b-x)^2}{2} \right]^{\frac{q-1}{q}} \left(\frac{1}{b-a} \right)^{\frac{s}{q}} \\
&\times \left[\int_a^b |t-x| \, (b-t)^s \, |f'(a)|^q \, dt + \int_a^b |t-x| \, (t-a)^s \, |f'(b)|^q \, dt \right]^{\frac{1}{q}}
\end{aligned}$$

where

$$\int_a^x (x-t)\,(b-t)^s \, dt = \frac{(b-a)^{s+1}\left[x(s+2) - b - a(s+1)\right] + (b-x)^{s+2}}{(s+1)(s+2)},$$

$$\int_x^b (t-x)\,(b-t)^s \, dt = \frac{(b-x)^{s+2}}{(s+1)(s+2)},$$

$$\int_a^x (x-t)\,(t-a)^s \, dt = \frac{(x-a)^{s+2}}{(s+1)(s+2)}$$

and

$$\int_x^b (t-x)\,(t-a)^s \, dt = \frac{(b-a)^{s+1}\left[a + b(s+1) - x(s+2)\right] + (b-x)^{s+2}}{(s+1)(s+2)}.$$

The proof is completed. □

Corollary 3. *Let $g : [a,b] \to \mathbb{R}$ symmetric to $\frac{a+b}{2}$ and $x = \frac{a+b}{2}$ in Theorem 7. Then we have the inequality*

$$\begin{aligned}
&\left| \frac{f(a)+f(b)}{2} \int_a^b g(u)du - \int_a^b f(u)g(u)du \right| \\
\leq& \|g\|_\infty \left[\frac{\frac{s}{2} + \frac{1}{2^{s+1}}}{(s+1)(s+2)} \right]^{\frac{1}{q}} \frac{(b-a)^2}{4^{1-\frac{1}{q}}} \left[|f'(a)|^q + |f'(b)|^q \right]^{\frac{1}{q}}.
\end{aligned}$$

Remark 1. *In Corollary 3, if we choose $s = 1$, we have the inequality in (1.5).*

Remark 2. *In Corollary 3, if we choose $g(u)=1$, we have the inequality in (1.3).*

Remark 3. *In Theorem 7, if we choose $s=1$, we have the inequality in (1.4).*

Theorem 8. *Let the assumptions of Theorem 7 hold. Then, for all $x\in[a,b]$, we have the inequality:*

$$\begin{aligned}
&\left|f(x)\int_a^b g(u)du-\int_a^b f(u)g(u)du\right| \\
\leq\ & \|g\|_\infty\left[\frac{(x-a)^2+(b-x)^2}{2}\right]^{\frac{q-1}{q}}\left(\frac{1}{b-a}\right)^{\frac{s}{q}} \\
&\times\left\{\frac{(b-a)^{s+2}+(b-x)^{s+1}\left[a(s+2)+bs-x(2s+2)\right]}{(s+1)(s+2)}\left|f'(a)\right|^q\right. \\
&\left.+\frac{(b-a)^{s+2}+(x-a)^{s+1}\left[x(2s+2)-b(s+2)-as\right]}{(s+1)(s+2)}\left|f'(b)\right|^q\right\}^{\frac{1}{q}}.
\end{aligned}$$

Proof. Let $x\in[a,b]$. By integration by parts, we have the following identities (see [7]):

$$f(x)\int_a^b g(u)du-\int_a^b f(u)g(u)du=\int_a^b S_g(t)f'(t)dt \tag{2.1}$$

where

$$S_g(t)=\begin{cases}\int_a^t g(u)du, & t\in[a,x),\\ -\int_t^b g(u)du, & t\in[x,b].\end{cases}$$

and

$$S(t)=\begin{cases}t-a, & t\in[a,x),\\ b-t, & t\in[x,b].\end{cases}.$$

Then

$$|S_g(t)|\leq\|g\|_\infty S(t),\quad t\in[a,b].$$

Using the identity(2.1), Power-mean integral inequality and the $s-$convexity of $|f'|^q$, it follows that

$$\begin{aligned}
&\left|f(x)\int_a^b g(u)du-\int_a^b f(u)g(u)du\right| \\
\leq\;& \int_a^b |S_g(t)|\,|f'(t)|\,dt \\
\leq\;& \|g\|_\infty \int_a^b S(t)\,|f'(t)|\,dt \\
\leq\;& \|g\|_\infty \left[\int_a^b S(t)dt\right]^{\frac{q-1}{q}} \\
&\times\left[\int_a^x (t-a)\left|f'\left(\frac{b-t}{b-a}a+\frac{t-a}{b-a}b\right)\right|^q dt+\int_x^b (b-t)\left|f'\left(\frac{b-t}{b-a}a+\frac{t-a}{b-a}b\right)\right|^q dt\right]^{\frac{1}{q}} \\
\leq\;& \|g\|_\infty \left[\int_a^b S(t)dt\right]^{\frac{q-1}{q}} \left(\frac{1}{b-a}\right)^{\frac{s}{q}} \\
&\times\left\{\int_a^x (t-a)\,(b-t)^s\,|f'(a)|^q\,dt+\int_x^b (b-t)^{s+1}\,|f'\,(a)|^q\,dt\right. \\
&\left.+\int_a^x (t-a)^{s+1}\,|f'(b)|^q\,dt+\int_x^b (b-t)\,(t-a)^s\,|f'\,(b)|^q\,dt\right\}^{\frac{1}{q}}
\end{aligned}$$

where

$$\int_a^b S(t)dt=\frac{(x-a)^2+(b-x)^2}{2},$$

$$\int_a^x (t-a)\,(b-t)^s dt=\frac{(b-a)^{s+2}+(b-x)^{s+1}\,[a(s+2)-b-x(s+1)]}{(s+1)(s+2)},$$

$$\int_x^b (b-t)^{s+1}dt=\frac{(b-x)^{s+2}}{s+2},$$

$$\int_x^b (b-t)\,(t-a)^s dt=\frac{(b-a)^{s+2}+(x-a)^{s+1}\,[x(s+1)-b(s+2)+a]}{(s+1)(s+2)}$$

and

$$\int_a^x (t-a)^{s+1}\,dt=\frac{(x-a)^{s+2}}{s+2}.$$

The proof is completed. □

Corollary 4. *Let $x=\frac{a+b}{2}$ in Theorem 8. Then we have the following inequality:*

$$\begin{aligned}
&\left|f\left(\frac{a+b}{2}\right)\int_a^b g(u)du-\int_a^b f(u)g(u)du\right| \\
\leq\;& \|g\|_\infty \frac{(b-a)^2}{4}\left[\frac{2^{s+2}-2}{(s+1)(s+2)}\right]^{\frac{1}{q}}\left[\frac{|f'(a)|^q+|f'(b)|^q}{2^s}\right]^{\frac{1}{q}}.
\end{aligned}$$

Remark 4. *In Corollary 4, if we choose $s=1$, we have the inequality in (1.7).*

Corollary 5. *In Corollary 4, if we choose $g(u) = 1$, we have the following inequality:*

$$
\begin{aligned}
&\left| f\left(\frac{a+b}{2}\right) - \frac{1}{b-a}\int_a^b f(u)du \right| \\
\leq\ & \frac{b-a}{4}\left[\frac{2^{s+2}-2}{(s+1)(s+2)}\right]^{\frac{1}{q}}\left[\frac{|f'(a)|^q+|f'(b)|^q}{2^s}\right]^{\frac{1}{q}}.
\end{aligned}
$$

Remark 5. *In Corollary 5, if we choose $s = 1$, we have the inequality in (1.9)*

Remark 6. *In Theorem 4, if we choose $s = 1$, we have the inequality in (1.6).*

Theorem 9. *Let $f : I \subseteq [0, \infty) \to \mathbb{R}$ be a differentiable mapping on I° such that $f' \in L[a, b]$ where $a, b \in I^\circ$ with $a < b$. If $|f'|^q$ is $s-$concave on $[a, b]$ for some fixed $s \in (0, 1)$ and $q > 1$, and $g : [a, b] \to \mathbb{R}$ be a continuous mapping on $[a, b]$. Then, for all $x \in [a, b]$, we have the inequalities:*

$$
\begin{aligned}
&\left| f(a)\int_a^x g(u)du + f(b)\int_x^b g(u)du - \int_a^b f(u)g(u)du \right| \\
\leq\ & \|g\|_\infty \left[\frac{(x-a)^{p+1}+(b-x)^{p+1}}{p+1}\right]^{\frac{1}{p}}\left[2^{s-1}\left|f'\left(\frac{a+b}{2}\right)\right|^q (b-a)\right]^{\frac{1}{q}}
\end{aligned}
$$

and

$$
\begin{aligned}
&\left| f(x)\int_a^b g(u)du - \int_a^b f(u)g(u)du \right| \\
\leq\ & \|g\|_\infty \left[\frac{(x-a)^{p+1}+(b-x)^{p+1}}{p+1}\right]^{\frac{1}{p}}\left[2^{s-1}\left|f'\left(\frac{a+b}{2}\right)\right|^q (b-a)\right]^{\frac{1}{q}}.
\end{aligned}
$$

Proof. Let $x \in [a, b]$ and we observe that $|f'|^q$ is $s-$concave on $[a, b]$. Using the Lemma 1, concavity of $|f'|^q$ and Hölder's integral inequality, we have

$$
\begin{aligned}
&\left| f(a)\int_a^x g(u)du + f(b)\int_x^b g(u)du - \int_a^b f(u)g(u)du \right| \\
\leq\ & \int_a^b \left|\int_x^t g(u)du\right| |f'(t)|\, dt \\
\leq\ & \|g\|_\infty \int_a^b |t-x|\, |f'(t)|\, dt \\
\leq\ & \|g\|_\infty \left(\int_a^b |t-x|^p\, dt\right)^{\frac{1}{p}} \left(\int_a^b |f'(t)|^q\, dt\right)^{\frac{1}{q}} \\
\leq\ & \|g\|_\infty \left[\frac{(x-a)^{p+1}+(b-x)^{p+1}}{p+1}\right]^{\frac{1}{p}}\left[2^{s-1}\left|f'\left(\frac{a+b}{2}\right)\right|^q (b-a)\right]^{\frac{1}{q}}.
\end{aligned}
$$

For the proof of second inequality we use (2.1), concavity of $|f'|^q$ and Hölder's integral inequality.

$$\begin{aligned}
&\left| f(x)\int_a^b g(u)du - \int_a^b f(u)g(u)du \right| \\
\leq\ & \|g\|_\infty \int_a^b S(t)\left|f'(t)\right| dt \\
\leq\ & \|g\|_\infty \left(\int_a^b [S(t)]^p\, dt\right)^{\frac{1}{p}} \left(\int_a^b \left|f'(t)\right|^q dt\right)^{\frac{1}{q}} \\
\leq\ & \|g\|_\infty \left[\frac{(x-a)^{p+1}+(b-x)^{p+1}}{p+1}\right]^{\frac{1}{p}} \left[2^{s-1}\left|f'\left(\frac{a+b}{2}\right)\right|^q (b-a)\right]^{\frac{1}{q}}
\end{aligned}$$

where

$$\int_a^b |t-x|^p\, dt = \int_a^b [S(t)]^p\, dt = \frac{(x-a)^{p+1}+(b-x)^{p+1}}{p+1}.$$

The proof is completed. □

Corollary 6. *For the first inequality in Theorem 9, let $g:[a,b]\to\mathbb{R}$ symmetric to $\frac{a+b}{2}$ and $x=\frac{a+b}{2}$. Then, we have the following inequality:*

$$\begin{aligned}
&\left| \frac{f(a)+f(b)}{2}\int_a^b g(u)du - \int_a^b f(u)g(u)du \right| \\
\leq\ & \|g\|_\infty (b-a)^2 \left(\frac{1}{2^p(p+1)}\right)^{\frac{1}{p}} \left[2^{s-1}\left|f'\left(\frac{a+b}{2}\right)\right|^q\right]^{\frac{1}{q}}.
\end{aligned}$$

And let $x=\frac{a+b}{2}$ in Theorem 9 for the second inequality. Then, we have the following inequality:

$$\begin{aligned}
&\left| f\left(\frac{a+b}{2}\right)\int_a^b g(u)du - \int_a^b f(u)g(u)du \right| \\
\leq\ & \|g\|_\infty (b-a)^2 \left(\frac{1}{2^p(p+1)}\right)^{\frac{1}{p}} \left[2^{s-1}\left|f'\left(\frac{a+b}{2}\right)\right|^q\right]^{\frac{1}{q}}.
\end{aligned}$$

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