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A NEW SHARPENING OF THE ERDÖS-MORDELL INEQUALITY AND RELATED INEQUALITIES

JIAN LIU

Abstract. In this paper we point out a new sharpening of the Erdös-Mordell inequality. We also prove the weighted generalization of this result and its analogous. Some related conjectures for the polygon are put forward.

1. Introduction

The famous Erdös-Mordell inequality in geometric inequalities states the following: Let P be an interior point of a triangle ABC. Let R_1, R_2, R_3 be the distances from P to the vertices A, B, C, and let r_1, r_2, r_3 be the distance from P to the sides BC, CA, AB, respectively. Then

$$R_1 + R_2 + R_3 \geq 2(r_1 + r_2 + r_3), \tag{1.1}$$

with equality if and only if $\triangle ABC$ is equilateral and P is its center. This equality was conjectured by Erdös [1] in 1935 and was first proved by Mordell [2] in the same year. Since then, a lot of proofs have been given, various generalizations, refinements and variations were also studied (see [3]-[29]). We recall here some related results.

In [2], Barrow proved the stronger inequality:

$$R_1 + R_2 + R_3 \geq 2(w_1 + w_2 + w_3), \tag{1.2}$$

where w_1, w_2, w_3 are the internal angle-bisectors of $\angle BPC, \angle CPA, \angle APB$ respectively.

The authors of the monograph [13] gave the following weighted generalization:

$$x^2R_1 + y^2R_2 + z^2R_3 \geq 2(yzr_1 + zxr_2 + xyr_3), \tag{1.3}$$

with equality if and only if P is the circumcenter of $\triangle ABC$ and $x : y : z = \sin A : \sin B : \sin C$, where A, B, C denote the angles of $\triangle ABC$.

In a recent paper [31], the author pointed out the following refinements:

$$\begin{aligned}
&R_1 + R_2 + R_3 \geq \frac{1}{2}\left(\sqrt{a^2 + 4r_1^2} + \sqrt{b^2 + 4r_2^2} + \sqrt{c^2 + 4r_3^2}\right) \\
&\geq \left(\frac{c}{b} + \frac{b}{c}\right) r_1 + \left(\frac{a}{c} + \frac{c}{a}\right) r_2 + \left(\frac{b}{a} + \frac{a}{b}\right) r_3 \\
&\geq 2\sqrt{h_a r_1 + h_b r_2 + h_c r_3} \\
&\geq 2(r_1 + r_2 + r_3),
\end{aligned} \tag{1.4}$$

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where a, b, c denote the sides BC, CA, AB respectively and h_a, h_b, h_c the corresponding altitudes of $\triangle ABC$.

The purpose of this paper is to point out a new sharpening of the Erdös-Mordell inequality. In fact, we will establish a more general weighted inequality and prove another similar result.

2. Main results and their proofs

For Erdös-Mordell inequality (1.1), we have the following sharpening:

Theorem 2.1. *For any interior point P of $\triangle ABC$, we have*

$$\sqrt{(r_2+r_3)R_1}+\sqrt{(r_3+r_1)R_2}+\sqrt{(r_1+r_2)R_3} \geq 2(r_1+r_2+r_3), \tag{2.1}$$

with equality if and only if $\triangle ABC$ is equilateral and P is its center.

Note that $R_1+r_2+r_3 \geq 2\sqrt{R_1(r_2+r_3)}$ and its two analogues, we know that

$$\frac{1}{2}(R_1+R_2+R_3)+r_1+r_2+r_3 \geq \sqrt{(r_2+r_3)R_1}+\sqrt{(r_3+r_1)R_2}+\sqrt{(r_1+r_2)R_3}.$$

From this and (2.1), Erdös-Mordell inequality (1.1) follows at once. Also, according to (1.1) and above inequality we see that

$$R_1+R_2+R_3 \geq \sqrt{(r_2+r_3)R_1}+\sqrt{(r_3+r_1)R_2}+\sqrt{(r_1+r_2)R_3}, \tag{2.2}$$

which shows that (2.1) is a sharpening of the Erdoös-Mordell inequality.

Actually, we have the following general result:

Theorem 2.2. *Let x, y, z be arbitrary real numbers and let u, v, w be positive numbers. Then for any interior point P of $\triangle ABC$ holds:*

$$\begin{aligned} &x^2\sqrt{(v+w)R_1}+y^2\sqrt{(w+u)R_2}+z^2\sqrt{(u+v)R_3} \\ &\geq 2\left(yz\sqrt{ur_1}+zx\sqrt{vr_2}+xy\sqrt{wr_3}\right), \end{aligned} \tag{2.3}$$

with equality if and only if $x=y=z, u=v=w$, $\triangle ABC$ is equilateral and P is its center.

In (2.3), put $u=r_1, v=r_2, w=r_3$, we get

$$x^2\sqrt{(r_2+r_3)R_1}+y^2\sqrt{(r_3+r_1)R_2}+z^2\sqrt{(r_1+r_2)R_3} \geq 2(yzr_1+zxr_2+xyr_3), \tag{2.4}$$

which is the weighted generalization of (2.1).

In (2.3) put $x=y=z=1$, then we have the following weighted inequality with three positive real numbers u, v, w:

$$\sqrt{R_1(v+w)}+\sqrt{R_2(w+u)}+\sqrt{R_3(u+v)} \geq 2(\sqrt{ur_1}+\sqrt{vr_2}+\sqrt{wr_3}). \tag{2.5}$$

Clearly, inequality (2.1) is a special case of this inequality.

When the author obtained inequality (2.3), the following analogues was also found.

Theorem 2.3. *Let x, y, z be arbitrary real numbers and let u, v, w be positive numbers. Then for any interior point P of $\triangle ABC$ holds:*

$$x^2\sqrt{\frac{v+w}{u}R_1}+y^2\sqrt{\frac{w+u}{v}R_2}+z^2\sqrt{\frac{u+v}{w}R_3} \geq 2\left(yz\sqrt{r_1}+zx\sqrt{r_2}+xy\sqrt{r_3}\right). \tag{2.6}$$

with equality if and only if $x = y = z, u = v = w$, $\triangle ABC$ *is equilateral and* P *is its center.*

In particular, we have

$$\sqrt{\frac{v+w}{u}R_1} + \sqrt{\frac{w+u}{v}R_2} + \sqrt{\frac{u+v}{w}R_3} \geq 2\left(\sqrt{r_1} + \sqrt{r_2} + \sqrt{r_3}\right). \tag{2.7}$$

In (2.6) put $u = R_1, v = R_2, w = R_3$, then

$$x^2\sqrt{R_2+R_3} + y^2\sqrt{R_3+R_1} + z^2\sqrt{R_1+R_2} \geq 2\left(yz\sqrt{r_1} + zx\sqrt{r_2} + xy\sqrt{r_3}\right). \tag{2.8}$$

which is posed as a conjecture by the author in [29].

To prove our theorems, we will use the weighted Erdös-Mordell inequality (1.3) and the following useful lemma:

Lemma 2.1. *Let* $\alpha_1, \beta_1, \gamma_1, \lambda_1, \mu_1, \nu_1$, $\alpha_2, \beta_2, \gamma_2, \lambda_2, \mu_2, \nu_2$ *be positive real numbers. If the following inequality:*

$$\alpha_1 x^2 + \beta_1 y^2 + \gamma_1 z^2 \geqslant \lambda_1 yz + \mu_1 zx + \nu_1 xy, \tag{2.9}$$

$$\alpha_2 x^2 + \beta_2 y^2 + \gamma_2 z^2 \geqslant \lambda_2 yz + \mu_2 zx + \nu_2 xy, \tag{2.10}$$

hold for any real numbers x, y, z. *Then*

$$\sqrt{\alpha_1\alpha_2}x^2 + \sqrt{\beta_1\beta_2}y^2 + \sqrt{\gamma_1\gamma_2}z^2 \geqslant \sqrt{\lambda_1\lambda_2}yz + \sqrt{\mu_1\mu_2}zx + \sqrt{\nu_1\nu_2}xy. \tag{2.11}$$

In [32], the author established " The Composite Theorem " of the ternary quadratic inequality by using the classical Hölder inequality. The above lemma is only a consequence of it.

Next, we prove the previous theorems at the same time.

Proof. We first prove Theorem 2.2. For arbitrary real numbers x, y, z and positive real numbers u, v, w, we have

$$u(y-z)^2 + v(z-x)^2 + w(x-y)^2 \geq 0,$$

which implies that

$$(v+w)x^2 + (w+u)y^2 + (u+v)z^2 \geq 2(yzu + zxv + xyw), \tag{2.12}$$

with equality if and only if $x = y = z$.

Now, applying Lemma 2.1 to inequality (1.3) and (2.12), we obtain the inequality of Theorem 2.2 at once. From [32], it is easy to know that the equality in (2.11) holds if and only if both (2.9) and (2.10) become equality and $\alpha_1 : \alpha_2 = \beta_1 : \beta_2 : \gamma_1 : \gamma_2$. According to this conclusion and the equality conditions of (1.3) and (2.12), we conclude easily that the equality in (2.3) occurs if and only if $x = y = z, u = v = w$, $\triangle ABC$ is equilateral and P is its center. This completes the proof of Theorem 2.2.

Because Theorem 2.1 is an evident corollary of Theorem 2.2, hence Theorem 2.1 is also proved.

By replacing $u \to vw, v \to wu, w \to uv$ in (2.12) and then dividing both sides by uvw, we get again:

$$\frac{v+w}{u}x^2 + \frac{w+u}{v}y^2 + \frac{u+v}{w}z^2 \geq 2(yz + zx + xy), \tag{2.13}$$

with equality if and only if $x : y : z = u : v : w$.

Applying Lemma 2.1 to (1.3) and (2.13), we immediately obtain inequality (2.6), and we easily know equality in (2.6) is just the same as in (2.3). The proof of the Theorem 2.2 is completed. □

The author has failed to prove Theorem 2.1 directly for many times. It keeps to be an interesting problem.

Remark 2.1. *According to the generalization of Lemma 2.1 (see* [32]*), we can give the more general result of Theorem 2.2.*

Remark 2.2. *Since we have the known quadratic inequality:*

$$x^2R_1 + y^2R_2 + z^2R_3 \geq 2(yzw_1 + zxw_2 + xyw_3), \tag{2.14}$$

which similar to (1.3). By the same way to prove (2.3), we also have the following inequality:

$$\begin{aligned} &x^2\sqrt{(v+w)R_1} + y^2\sqrt{(w+u)R_2} + z^2\sqrt{(u+v)R_3} \\ &\geq 2\left(yz\sqrt{uw_1} + zx\sqrt{vw_2} + xy\sqrt{ww_3}\right), \end{aligned} \tag{2.15}$$

which is similar to Theorem 2.2. Particularly, we have the following stronger version of Barrow's inequality:

$$\sqrt{(w_2+w_3)R_1} + \sqrt{(w_3+w_1)R_2} + \sqrt{(w_1+w_2)R_3} \geq 2(w_1+w_2+w_3). \tag{2.16}$$

Remark 2.3. *From the proof of Theorem 2.3, we can see that the following interesting conclusion is valid:*

If $p_1, p_2, p_3, q_1, q_2, q_3$ are positive and the following inequality:

$$p_1x^2 + p_2y^2 + p_3z^2 \geq q_1yz + q_2zx + q_3xy. \tag{2.17}$$

holds for any real numbers x, y, z. Then holds:

$$x^2\sqrt{p_2+p_3} + y^2\sqrt{p_3+p_1} + z^2\sqrt{p_1+p_2} \geq 2(\sqrt{q_1}yz + \sqrt{q_2}zx + \sqrt{q_3}xy). \tag{2.18}$$

Form this and known inequality (see [33]*)*

$$x^2R_a + y^2R_a + z^2R_c \geq 2(yzr_1 + zxr_2 + xyr_3) \tag{2.19}$$

where R_a, R_b, R_c are the circumradius of $\triangle PBC, \triangle PCA, \triangle PAB$. We know again that

$$x^2\sqrt{R_b+R_c} + y^2\sqrt{R_c+R_a} + z^2\sqrt{R_a+R_b} \geq 2\left(yz\sqrt{r_1} + zx\sqrt{r_2} + xy\sqrt{r_3}\right), \tag{2.20}$$

which was posed by the author in [29].

Remark 2.4. *If we apply geometric transformations (see e.g.* [10],[13]*) to our theorems, then we can get some new results. For example, make transformation by Theorem 2.2 we can get*

$$\begin{aligned} &x^2\sqrt{(v+w)R_a} + y^2\sqrt{(w+u)R_b} + z^2\sqrt{(u+v)R_c} \\ &\geq \sqrt{2}\left(yz\sqrt{uR_1} + zx\sqrt{vR_2} + xy\sqrt{wR_3}\right). \end{aligned} \tag{2.21}$$

3. Some conjectures

In this section, we will propose some conjecture inequalities for the polygon.

In 1953, L.Fejes Tóth [4] conjectured Erdös-Mordell inequality can be generalized to the polygon:

$$\sum_{i=1}^{n} R_i \geq \left(\sec\frac{\pi}{n}\right)\sum_{i=1}^{n} r_i. \tag{3.1}$$

where R_i are the distances from the interior point P to the vertices A_i ($i = 1, 2, \cdots, n$) and r_i the distances from P to the sides A_iA_{i+1} ($i = 1, 2, \cdots, n.A_{n+1} = A_1$). This generalization has first proved by N.Ozeki [7] in 1957. Here, we give a stronger conjecture which generalizes Theorem 2.1. Namely,

Conjecture 3.1. *Let P be an arbitrary interior point of the polygon $A_1A_2\cdots A_n$, then we have*

$$\sum_{i=1}^{n}\sqrt{(r_i + r_{i+1})R_{i+1}} \geq \sqrt{2\sec\frac{\pi}{n}}\sum_{i=1}^{n} r_i, \tag{3.2}$$

where $A_{n+1} = A_1, r_{n+1} = r_1, R_{n+1} = R_1$.

Clearly, if the above conjecture holds true, then it would be showed easily that (3.1) follows from (3.2) by using AM-GM inequality.

Inequality (2.1) and the refinements (1.4) of the Erdös-Mordell inequality prompt the author to conjecture that

$$\begin{aligned}&\sqrt{a^2 + 4r_1^2} + \sqrt{b^2 + 4r_2^2} + \sqrt{c^2 + 4r_3^2}\\ &\geq 2\left[\sqrt{(r_2 + r_3)R_1} + \sqrt{(r_3 + r_1)R_2} + \sqrt{(r_1 + r_2)R_3}\right].\end{aligned} \tag{3.3}$$

Further, we conjecture that the stronger inequality holds:

$$\sqrt{a^2 + 4r_1^2} + \sqrt{b^2 + 4r_2^2} + \sqrt{c^2 + 4r_3^2} \geq R_1 + R_2 + R_3 + 2(r_1 + r_2 + r_3). \tag{3.4}$$

Generally, we propose the following conjecture for the polygon.

Conjecture 3.2. *For any interior point P of the polygon $A_1A_2\cdots A_n$, we have*

$$\sum_{i=1}^{n}\sqrt{a_i^2 + 4r_i^2} \geq \sum_{i=1}^{n} R_i + \left(\sec\frac{\pi}{n}\right)\sum_{i=1}^{n} r_i, \tag{3.5}$$

where a_i denote the lengths of sides $A_iA_{i+1}(i = 1, 2, \cdots, n)$.

Remark 3.1. *It is easy to prove that*

$$2\sum_{i=1}^{n} R_i \geq \sum_{i=1}^{n}\sqrt{a_i^2 + 4r_i^2}. \tag{3.6}$$

Therefore, inequality (3.5 is stronger than Erdös-Mordell inequality (3.1) of the polygon. The weaker inequality of (3.5):

$$\sum_{i=1}^{n}\sqrt{a_i^2 + 4r_i^2} \geq 2\left(\sec\frac{\pi}{n}\right)\sum_{i=1}^{n} r_i \tag{3.7}$$

has not been yet proven.

Considering the generalization of the following inequality:

$$\sqrt{R_2+R_3}+\sqrt{R_3+R_1}+\sqrt{R_1+R_2}\geq 2(\sqrt{r_1}+\sqrt{r_2}+\sqrt{r_3}), \tag{3.8}$$

which is the special case of Theorem 2.2. We put forward

Conjecture 3.3. *Let $0<k<1$. For any interior point P of the polygon $A_1A_2\cdots A_n$, we have*

$$\sum_{i=1}^{n}(R_i+R_{i+1})^k\geq\left(2\sec\frac{\pi}{n}\right)^k\sum r_i^k. \tag{3.9}$$

The inequality is reversed when $-1\leq k<0$.

By inequality (1.3), we have

$$y^2R_1+z^2R_2+x^2R_3\geq 2(zxr_1+xyr_2+yzr_3),$$
$$z^2R_1+x^2R_2+y^2R_3\geq 2(xyr_1+yzr_2+zxr_3).$$

Adding up them gives

$$x^2(R_2+R_3)+y^2(R_3+R_1)+z^2(R_1+R_2)\geq 2\left[yz(r_2+r_3)+zx(r_3+r_1)+xy(r_1+r_2)\right]. \tag{3.10}$$

Applying Lemma 2.1 to inequality (2.12) and (3.10), one obtains

$$\begin{aligned}&x^2\sqrt{(v+w)(R_2+R_3)}+y^2\sqrt{(w+u)(R_3+R_1)}+z^2\sqrt{(u+v)(R_1+R_2)}\\&\geq\sqrt{2}\left[yz\sqrt{u(r_2+r_3)}+zx\sqrt{v(r_3+r_1)}+xy\sqrt{w(r_1+r_2)}\right],\end{aligned} \tag{3.11}$$

which is similar to the inequality of Theorem 2.2. In particular, we have

$$\sqrt{R_2+R_3}+\sqrt{R_3+R_1}+\sqrt{R_1+R_2}\geq\sqrt{2}\left(\sqrt{r_2+r_3}+\sqrt{r_3+r_1}+\sqrt{r_1+r_2}\right). \tag{3.12}$$

For this inequality, we pose the following generalization:

Conjecture 3.4. *Let $0<k<1$. For any interior point P of the polygon $A_1A_2\cdots A_n$, we have*

$$\sum_{i=1}^{n}(R_i+R_{i+1})^k\geq\left(2\sec\frac{\pi}{n}\right)^k\sum_{i=1}^{n}(r_i+r_{i+1})^k. \tag{3.13}$$

The inequality is reversed when $-1\leq k<0$.

Remark 3.2. *Let w_i denote the internal angle-bisectors of $\angle A_iPA_{i+1}$ ($i=1,2,\cdots,n$ and $A_{n+1}=A_1$). We also suppose that the inequalities of the previous four conjectures still hold after replacing r_i by w_i.*

In (2.21), put $x=y=z=1, u=R_1, v=R_2, w=R_3$, we get

$$\sqrt{R_a(R_2+R_3)}+\sqrt{R_b(R_3+R_1)}+\sqrt{R_c(R_1+R_2)}\geq\sqrt{2}(R_1++R_2+R_3). \tag{3.14}$$

For this inequality, we put forward

Conjecture 3.5. *For any interior point P of the polygon $A_1A_2\cdots A_n$, we have*

$$\sum_{i=1}^{n}\sqrt{(R_i+R_{i+1})\rho_i}\geq\sqrt{\sec\frac{\pi}{n}}\sum_{i=1}^{n}R_i. \tag{3.15}$$

where ρ_i denote the circumcenters of $\triangle A_iPA_{i+1}$ ($i=1,2,\cdots n$ and $A_{n+1}=A_1$).

Inequality (3.15) even holds for any point P in the plane. But we can not prove the weaker linear inequality now:

$$\sum_{i=1}^{n} \rho_i \geq \frac{1}{2}\left(\sec\frac{\pi}{n}\right)\sum_{i=1}^{n} R_i. \tag{3.16}$$

We also think this inequality has the following exponential generalization:

Conjecture 3.6. *Let $0 < k \leq 1$. For any interior point P of the polygon $A_1A_2\cdots A_n$, we have*

$$\sum_{i=1}^{n} \rho_i^k \geq \left(\frac{1}{2}\sec\frac{\pi}{n}\right)^k \sum_{i=1}^{n} R_i^k. \tag{3.17}$$

The inequality is reversed when $-1 \leq k < 0$.

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East China Jiaotong University, Jiangxi province Nanchang City, 330013, China
E-mail address: China99jian@163.com