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A WEIGHTED COMPANION FOR THE OSTROWSKI AND THE GENERALIZED TRAPEZOID INEQUALITIES FOR MAPPINGS OF BOUNDED VARIATION

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Abstract. A new generalization of weighted companion for the Ostrowski and the generalized trapezoid inequalities for mappings of bounded variation are established.

1. Introduction

In 1938, A. Ostrowski [1], proved the following inequality for differentiable mappings with bounded derivatives:

Theorem 1. *Let $f : I \subset \mathbb{R} \to \mathbb{R}$ be a differentiable mapping on I°, the interior of the interval I, such that $f' \in L[a,b]$, where $a, b \in I$ with $a < b$. If $|f'(x)| \le M$, then the following inequality,*

$$\left| f(x) - \frac{1}{b-a}\int_a^b f(u)\,du \right| \le M(b-a)\left[\frac{1}{4} + \frac{\left(x-\frac{a+b}{2}\right)^2}{(b-a)^2}\right] \tag{1.1}$$

holds for all $x \in [a,b]$. The constant $\frac{1}{4}$ is the best possible in the sense that it cannot be replaced by a smaller constant.

In [2], Dragomir proved the following Ostrowski's inequality for mappings of bounded variation:

Theorem 2. *Let $f : [a,b] \to \mathbb{R}$ be a mapping of bounded variation on $[a,b]$. Then we have the inequalities:*

$$\left| f(x) - \frac{1}{b-a}\int_a^b f(t)\,dt \right| \le \left[\frac{1}{2} + \left|\frac{x-\frac{a+b}{2}}{b-a}\right|\right] \cdot \bigvee_a^b (f), \tag{1.2}$$

for any $x \in [a,b]$, where $\bigvee_a^b(f)$ denotes the total variation of f on $[a,b]$. The constant $\frac{1}{2}$ is best possible.

In [3], Tseng et al. have proved the following weighted Ostrowski inequality for mappings of bounded variation, as follows:

Theorem 3. *Let $0 \le \alpha \le 1$, $g : [a,b] \to [0,\infty)$ continuous and positive on (a,b) and let $h : [a,b] \to \mathbb{R}$ be differentiable such that $h'(t) = g(t)$ on $[a,b]$. Let $c =$*

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$h^{-1}\left(\left(1-\frac{\alpha}{2}\right)h(a)+\frac{\alpha}{2}h(b)\right)$ *and* $d=h^{-1}\left(\frac{\alpha}{2}h(a)+\left(1-\frac{\alpha}{2}\right)h(b)\right)$. *Suppose that* f *is of bounded variation on* $[a,b]$, *then for all* $x\in[c,d]$, *we have*

$$\left|\int_a^b f(t)g(t)\,dt-\left[(1-\alpha)f(x)+\alpha\frac{f(a)+f(b)}{2}\right]\int_a^b g(t)\,dt\right|\le K\cdot\bigvee_a^b(f) \tag{1.3}$$

where,

$$K=\begin{cases}\frac{1-\alpha}{2}\int_a^b g(t)\,dt+\left|h(x)+\frac{h(a)+h(b)}{2}\right|, & 0\le\alpha\le\frac{1}{2}\\ \max\left\{\frac{1-\alpha}{2}\int_a^b g(t)\,dt+\left|h(x)+\frac{h(a)+h(b)}{2}\right|,\frac{\alpha}{2}\int_a^b g(t)\,dt\right\}, & \frac{1}{2}<\alpha<\frac{2}{3}\\ \frac{\alpha}{2}\int_a^b g(t)\,dt, & \frac{2}{3}\le\alpha\le1\end{cases}$$

and $\bigvee_a^b(f)$ *is the total variation of* f *over* $[a,b]$. *The constant* $\frac{1-\alpha}{2}$ *for* $0\le\alpha\le\frac{1}{2}$ *and the constant* $\frac{\alpha}{2}$ *for* $\frac{2}{3}\le\alpha\le1$ *are the best possible.*

Motivated by [4], S.S. Dragomir in [5] has proved the following companion of the Ostrowski inequality for mappings of bounded variation:

Theorem 4. *Let* $f:[a,b]\to\mathbb{R}$ *be a mapping of bounded variation on* $[a,b]$. *Then we have the inequalities:*

$$\left|\frac{f(x)+f(a+b-x)}{2}-\frac{1}{b-a}\int_a^b f(t)\,dt\right|\le\left[\frac{1}{4}+\left|\frac{x-\frac{3a+b}{4}}{b-a}\right|\right]\cdot\bigvee_a^b(f), \tag{1.4}$$

for any $x\in\left[a,\frac{a+b}{2}\right]$, *where* $\bigvee_a^b(f)$ *denotes the total variation of* f *on* $[a,b]$. *The constant* $\frac{1}{4}$ *is best possible.*

In the recent work [6], Z. Liu, proved another generalization of weighted Ostrowski type inequality for mappings of bounded variation, as follows:

Theorem 5. *Let* $f:[a,b]\to\mathbb{R}$ *be a mapping of bounded variation,* $g:[a,b]\to[0,\infty)$ *continuous and positive on* (a,b). *Then for any* $x\in[a,b]$ *and* $\alpha\in[0,1]$, *we have*

$$\left|\int_a^b f(t)g(t)\,dt-\left[(1-\alpha)f(x)\int_a^b g(t)\,dt+\alpha\left(f(a)\int_a^x g(t)\,dt+f(b)\int_x^b g(t)\,dt\right)\right]\right| \le\left[\frac{1}{2}+\left|\frac{1}{2}-\alpha\right|\right]\left[\frac{1}{2}\int_a^b g(t)\,dt+\left|\int_a^x g(t)\,dt-\frac{1}{2}\int_a^b g(t)\,dt\right|\right]\cdot\bigvee_a^b(f) \tag{1.5}$$

where, $\bigvee_a^b(f)$ *denotes to the total variation of* f *over* $[a,b]$. *The constant* $\left[\frac{1}{2}+\left|\frac{1}{2}-\alpha\right|\right]$ *is the best possible.*

Indeed, W.J. Liu [7] has proved the above inequality (1.5) (essentially same). However, Z. Liu [6] proved the sharpness of the (1.5), and thus he improved the constant in W.J. Liu result. For details we recommend the reader to read the papers [6] and [7].

In this paper, a new generalization of weighted companion for the Ostrowski and the generalized trapezoid inequalities are proved. Therefore, several weighted inequalities are deduced.

2. A Weighted Companion for the Ostrowski–Trapezoid Inequality

Theorem 6. *Let $f : [a,b] \to \mathbb{R}$ be a mapping of bounded variation, $g : [a,b] \to [0,\infty)$ continuous and positive on (a,b). Then for any $x \in [a, \frac{a+b}{2}]$ and $\alpha \in [0,1]$, we have*

(2.1)
$$\left|\alpha\left[f(b)\int_x^b g(s)\,ds + f(a)\int_a^x g(s)\,ds\right] + (1-\alpha)\left[f(x)\int_a^{\frac{a+b}{2}} g(s)\,ds + f(a+b-x)\int_{\frac{a+b}{2}}^b g(s)\,ds\right] - \int_a^b f(t)\,g(t)\,dt\right|$$
$$\leq \left[\frac{1}{2} + \left|\frac{1}{2}-\alpha\right|\right]\cdot \max\left\{\int_a^x g(s)\,ds, \int_x^{a+b-x} g(s)\,ds, \int_{a+b-x}^b g(s)\,ds\right\}\cdot \bigvee_a^b (f)$$
$$\leq \left[\frac{1}{2} + \left|\frac{1}{2}-\alpha\right|\right]\cdot \int_a^b g(s)\,ds \cdot \bigvee_a^b (f)$$

where, $\bigvee_a^b (f)$ denotes the total variation of f on $[a,b]$.

Proof. Let $x \in [a, \frac{a+b}{2}]$. Define the mapping

$$S_g(t) = \begin{cases} (1-\alpha)\int_a^t g(s)\,ds + \alpha\int_x^t g(s)\,ds, & t \in [a,x] \\ (1-\alpha)\int_{\frac{a+b}{2}}^t g(s)\,ds + \alpha\int_x^t g(s)\,ds, & t \in (x, a+b-x] \\ (1-\alpha)\int_b^t g(s)\,ds + \alpha\int_x^t g(s)\,ds, & t \in (a+b-x, b] \end{cases}$$

for all $\alpha \in [0,1]$.

Using integration by parts, we have the following identity:

$$\int_a^b S_g(t)\,df(t) = \alpha\left[f(b)\int_x^b g(s)\,ds + f(a)\int_a^x g(s)\,ds\right] + (1-\alpha)\left[f(x)\int_a^{\frac{a+b}{2}} g(s)\,ds + f(a+b-x)\int_{\frac{a+b}{2}}^b g(s)\,ds\right] - \int_a^b f(t)\,g(t)\,dt$$

Now, we use the fact that for a continuous function $p : [a,b] \to \mathbb{R}$ and a function $\nu : [a,b] \to \mathbb{R}$ of bounded variation, one has the inequality

$$\left|\int_a^b p(t)\,d\nu(t)\right| \leq \sup_{t\in[a,b]} |p(t)| \bigvee_a^b (\nu). \tag{2.2}$$

Applying the inequality (2.2) for $p(t) = S_g(t)$, as above and $\nu(t) = f(t)$, $t \in [a,b]$, we get

$$\left|\alpha\left[f(b)\int_x^b g(s)\,ds + f(a)\int_a^x g(s)\,ds\right] + (1-\alpha)\left[f(x)\int_a^{\frac{a+b}{2}} g(s)\,ds + f(a+b-x)\int_{\frac{a+b}{2}}^b g(s)\,ds\right] - \int_a^b f(t)\,g(t)\,dt\right| \le \sup_{t\in[a,b]}|S_g(t)| \cdot \bigvee_a^b(f). \tag{2.3}$$

Putting

$$p_1(t) = (1-\alpha)\int_a^t g(s)\,ds + \alpha\int_x^t g(s)\,ds, \qquad t \in [a,x],$$
$$p_2(t) = (1-\alpha)\int_{\frac{a+b}{2}}^t g(s)\,ds + \alpha\int_x^t g(s)\,ds, \quad t \in (x, a+b-x],$$
$$p_3(t) = (1-\alpha)\int_b^t g(s)\,ds + \alpha\int_x^t g(s)\,ds, \qquad t \in (a+b-x, b].$$

Therefore, it is easy to check that $p_1(t)$ is increasing on the interval $[a,x)$, $p_2(t)$ is increasing on the interval $(x, a+b-x]$ and $p_3(t)$ is increasing on the interval $(a+b-x, b]$. Moreover, $p_1'(t) = p_2'(t) = p_3'(t) = g(t) > 0$. Thus,

$$\begin{aligned}\sup_{t\in[a,x]}|S_g(t)| &= p_1(x) - p_1(a)\\ &= \max\left\{(1-\alpha)\int_a^x g(s)\,ds, \alpha\int_a^x g(s)\,ds\right\}\\ &= \left[\frac{1}{2} + \left|\frac{1}{2} - \alpha\right|\right]\cdot\int_a^x g(s)\,ds,\end{aligned}$$

$$\begin{aligned}\sup_{t\in(x,a+b-x]}|S_g(t)| &= p_2(a+b-x) - p_2(x)\\ &= \max\left\{(1-\alpha)\int_x^{a+b-x} g(s)\,ds, \alpha\int_x^{a+b-x} g(s)\,ds\right\}\\ &= \left[\frac{1}{2} + \left|\frac{1}{2} - \alpha\right|\right]\cdot\int_x^{a+b-x} g(s)\,ds,\end{aligned}$$

and

$$\begin{aligned}\sup_{t\in(a+b-x,b]}|S_g(t)| &= p_3(b) - p_3(a+b-x)\\ &= \max\left\{(1-\alpha)\int_{a+b-x}^b g(s)\,ds, \alpha\int_{a+b-x}^b g(s)\,ds\right\}\\ &= \left[\frac{1}{2} + \left|\frac{1}{2} - \alpha\right|\right]\cdot\int_{a+b-x}^b g(s)\,ds,\end{aligned}$$

which follows that

$$\sup_{t\in(a,b]}|S_g(t)| = \max\left\{\sup_{t\in[a,x]}|S_g(t)|, \sup_{t\in(x,a+b-x]}|S_g(t)|, \sup_{t\in(a+b-x,b]}|S_g(t)|\right\}$$

$$= \left[\frac{1}{2}+\left|\frac{1}{2}-\alpha\right|\right]\cdot\max\left\{\int_a^x g(s)\,ds, \int_x^{a+b-x} g(s)\,ds, \int_{a+b-x}^b g(s)\,ds\right\} \tag{2.4}$$

By (2.3) and (2.4), we obtain the first inequality in (2.1).

To obtain the second inequality, since

$$\sup_{t\in(a,b]}|S_g(t)| \le \sup_{t\in[a,x]}|S_g(t)| + \sup_{t\in(x,a+b-x]}|S_g(t)| + \sup_{t\in(a+b-x,b]}|S_g(t)|$$

$$= \left[\frac{1}{2}+\left|\frac{1}{2}-\alpha\right|\right]\cdot\int_a^b g(s)\,ds. \tag{2.5}$$

By (2.3) and (2.5), we obtain the second inequality in (2.1).

which completes the proof. □

We remark that, Cerone, Dragomir and Pearce [8], have proved the following generalized trapezoid inequality for mappings of bounded variation:

$$\left|(b-x)f(b)+(x-a)f(a)-\int_a^b f(t)\,dt\right| \le \left[\frac{b-a}{2}+\left|x-\frac{a+b}{2}\right|\right]\cdot\bigvee_a^b(f)$$

for all $x\in[a,b]$. The constant $\frac{1}{2}$ is the best possible.

In what follows, we deduce another bound for the generalized trapezoid inequality for mappings of bounded variation

Theorem 7. *Let $f:[a,b]\to\mathbb{R}$ be a mapping of bounded variation. Then for any $x\in[a,\frac{a+b}{2}]$, we have*

$$\left|(b-x)f(b)+(x-a)f(a)-\int_a^b f(t)\,dt\right| \le \left[\frac{(b-x)}{2}+\left|(x-a)-\frac{b-x}{2}\right|\right]\cdot\bigvee_a^b(f), \tag{2.6}$$

for all $x\in[a,\frac{a+b}{2}]$, and the constant $\frac{1}{2}$ is the best possible.

Proof. Taking $g(t)=1$ on $[a,b]$ in (2.1) and choose $\alpha=1$, we get the desired result. To prove the sharpness of (2.6). Assume that (2.6) holds with constant $C_1>0$, i.e.,

$$\left|(b-x)f(b)+(x-a)f(a)-\int_a^b f(t)\,dt\right| \le \left[C_1(b-x)+\left|(x-a)-\frac{b-x}{2}\right|\right]\cdot\bigvee_a^b(f) \tag{2.7}$$

Define the mapping $f : [a, b] \to \mathbb{R}$ given by

$$f(t) = \begin{cases} 0, & t \in (a, b) \\ \\ 1, & t = a, b \end{cases}$$

which follows that $\int_a^b f(t)\,dt = 0$ and $\bigvee_a^b (f) = 2$, making of use (2.7) with $x = \frac{a+b}{2}$, we get

$$(b - a) \le (b - a)\left[\frac{C_1}{2} + \frac{1}{4}\right] \cdot 2$$

which gives that

$$\frac{1}{2} \le \frac{C_1}{2} + \frac{1}{4}$$

and therefore, $C_1 \ge \frac{1}{2}$, which proves the sharpness of (2.6). □

Remark 1. *In (2.1), if one chooses*

(1) $\alpha = 0$, *then we get*

$$\left| f(x) \int_a^{\frac{a+b}{2}} g(s)\,ds + f(a + b - x) \int_{\frac{a+b}{2}}^b g(s)\,ds - \int_a^b f(t)\,g(t)\,dt \right|$$
$$\le \max\left\{ \int_a^x g(s)\,ds, \int_x^{a+b-x} g(s)\,ds, \int_{a+b-x}^b g(s)\,ds \right\} \cdot \bigvee_a^b (f)$$
$$\le \int_a^b g(s)\,ds \cdot \bigvee_a^b (f)$$

which is the "weighted companion of Ostrowski inequality".

(2) $\alpha = 1$, *then we get*

$$\left| f(b) \int_x^b g(s)\,ds + f(a) \int_a^x g(s)\,ds - \int_a^b f(t)\,g(t)\,dt \right|$$
$$\le \max\left\{ \int_a^x g(s)\,ds, \int_x^{a+b-x} g(s)\,ds, \int_{a+b-x}^b g(s)\,ds \right\} \cdot \bigvee_a^b (f)$$
$$\le \int_a^b g(s)\,ds \cdot \bigvee_a^b (f)$$

which is the "generalized weighted trapezoid inequality".

Corollary 1. *If we take $g(t) = 1$ on $[a, b]$ in (2.1) then we get the following inequality*

(2.8)
$$\left| (b - a)\left[\alpha \cdot \frac{(b - x) f(b) + (x - a) f(a)}{b - a} + (1 - \alpha) \cdot \frac{f(x) + f(a + b - x)}{2}\right] - \int_a^b f(t)\,dt \right|$$
$$\le \left[\frac{1}{2} + \left|\frac{1}{2} - \alpha\right|\right] \cdot \left[\frac{(b - x)}{2} + \left|(x - a) - \frac{b - x}{2}\right|\right] \cdot \bigvee_a^b (f),$$

which is the "generalized companion of trapezoid–Ostrowski inequality". Moreover, if we choose $x = \frac{a+b}{2}$, then we get

$$\left|(b-a)\left[\alpha\frac{f(a)+f(b)}{2}+(1-\alpha)f\left(\frac{a+b}{2}\right)\right]-\int_a^b f(t)\,dt\right| \leq \left[\frac{1}{2}+\left|\frac{1}{2}-\alpha\right|\right]\cdot\frac{(b-a)}{2}\cdot\bigvee_a^b(f). \tag{2.9}$$

Its clear that $\alpha = \frac{1}{2}$ is best possible in (2.8), which gives the average trapezoid-midpoint inequality, as follows:

$$\left|\frac{(b-a)}{2}\left[\frac{f(a)+f(b)}{2}+f\left(\frac{a+b}{2}\right)\right]-\int_a^b f(t)\,dt\right| \leq \frac{(b-a)}{4}\bigvee_a^b(f). \tag{2.10}$$

The constant $\frac{1}{4}$ is the best possible.

Proof. We show that $\frac{1}{4}$ is the best possible. Assume that (2.10) holds with constant $C_2 > 0$, i.e.,

$$\left|\frac{(b-a)}{2}\left[\frac{f(a)+f(b)}{2}+f\left(\frac{a+b}{2}\right)\right]-\int_a^b f(t)\,dt\right| \leq C_2(b-a)\bigvee_a^b(f). \tag{2.11}$$

Define the mapping $f : [a,b] \to \mathbb{R}$ given by

$$f(t) = \begin{cases} 0, & t \in (a,b) \\ \frac{1}{2}, & t = a,b \end{cases}$$

which follows that $\int_a^b f(t)\,dt = 0$ and $\bigvee_a^b(f) = 1$, making of use (2.11), we get $\frac{(b-a)}{4} \leq C_2(b-a)$ and therefore $\frac{1}{4} \leq C_2$, which proves the sharpness of (2.10). □

Corollary 2. *Let $0 \leq \alpha \leq 1$. Let $f \in C^{(1)}[a,b]$. Then we have the inequality*

$$\begin{aligned} &\left|\alpha\left[f(b)\int_x^b g(s)\,ds + f(a)\int_a^x g(s)\,ds\right]\right. \\ &\qquad \left. + (1-\alpha)\left[f(x)\int_a^{\frac{a+b}{2}} g(s)\,ds + f(a+b-x)\int_{\frac{a+b}{2}}^b g(s)\,ds\right] - \int_a^b f(t)\,g(t)\,dt\right| \\ &\leq \left[\frac{1}{2}+\left|\frac{1}{2}-\alpha\right|\right]\cdot\max\left\{\int_a^x g(s)\,ds, \int_x^{a+b-x} g(s)\,ds, \int_{a+b-x}^b g(s)\,ds\right\}\cdot\|f'\|_1 \\ &\leq \left[\frac{1}{2}+\left|\frac{1}{2}-\alpha\right|\right]\cdot\int_a^b g(s)\,ds\cdot\|f'\|_1 \end{aligned} \tag{2.12}$$

for all $x \in [a, \frac{a+b}{2}]$, where $\|\cdot\|_1$ is the L_1 norm, namely $\|f'\|_1 := \int_a^b |f'(t)|\,dt$.

Corollary 3. *Let $0 \le \alpha \le 1$. Let $f : [a,b] \to \mathbb{R}$ be a Lipschitzian mapping with the constant $L > 0$. Then we have the inequality*

$$\begin{aligned}
&\left|\alpha\left[f(b)\int_x^b g(s)\,ds + f(a)\int_a^x g(s)\,ds\right]\right. \\
&\qquad\left. + (1-\alpha)\left[f(x)\int_a^{\frac{a+b}{2}} g(s)\,ds + f(a+b-x)\int_{\frac{a+b}{2}}^b g(s)\,ds\right] - \int_a^b f(t)\,g(t)\,dt\right| \\
&\le L\left[\frac{1}{2} + \left|\frac{1}{2}-\alpha\right|\right]\cdot(b-a)\cdot\max\left\{\int_a^x g(s)\,ds, \int_x^{a+b-x} g(s)\,ds, \int_{a+b-x}^b g(s)\,ds\right\} \\
&\le L\left[\frac{1}{2} + \left|\frac{1}{2}-\alpha\right|\right]\cdot(b-a)\cdot\int_a^b g(s)\,ds
\end{aligned} \tag{2.13}$$

for all $x \in [a, \frac{a+b}{2}]$.

Corollary 4. *Let $0 \le \alpha \le 1$. Let $f : [a,b] \to \mathbb{R}$ be a monotonic mapping. Then we have the inequality*

$$\begin{aligned}
&\left|\alpha\left[f(b)\int_x^b g(s)\,ds + f(a)\int_a^x g(s)\,ds\right]\right. \\
&\qquad\left. + (1-\alpha)\left[f(x)\int_a^{\frac{a+b}{2}} g(s)\,ds + f(a+b-x)\int_{\frac{a+b}{2}}^b g(s)\,ds\right] - \int_a^b f(t)\,g(t)\,dt\right| \\
&\le \left[\frac{1}{2} + \left|\frac{1}{2}-\alpha\right|\right]\cdot\max\left\{\int_a^x g(s)\,ds, \int_x^{a+b-x} g(s)\,ds, \int_{a+b-x}^b g(s)\,ds\right\}\cdot|f(b)-f(a)| \\
&\le \left[\frac{1}{2} + \left|\frac{1}{2}-\alpha\right|\right]\cdot\int_a^b g(s)\,ds\cdot|f(b)-f(a)|
\end{aligned} \tag{2.14}$$

for all $x \in [a, \frac{a+b}{2}]$.

Remark 2. *Let the assumptions of Theorem 6 hold, for a function f if one replace the condition of bounded variation to be L-Lipschitzian on $[a,b]$. Then for any*

$x \in [a, \frac{a+b}{2}]$ and $\alpha \in [0,1]$, we have

$$\begin{aligned} &\left|\alpha\left[f(b)\int_x^b g(s)\,ds + f(a)\int_a^x g(s)\,ds\right]\right. \\ &\left. +(1-\alpha)\left[f(x)\int_a^{\frac{a+b}{2}} g(s)\,ds + f(a+b-x)\int_{\frac{a+b}{2}}^b g(s)\,ds\right] - \int_a^b f(t)\,g(t)\,dt\right| \\ &\le L\left[(1-\alpha)\left(\left\|\int_a^t g(s)\,ds\right\|_{1,[a,x]} + \left\|\int_{\frac{a+b}{2}}^t g(s)\,ds\right\|_{1,[x,a+b-x]} + \left\|\int_b^t g(s)\,ds\right\|_{1,[a+b-x,b]}\right)\right. \\ &\left. +\alpha\left\|\int_x^t g(s)\,ds\right\|_{1,[a,b]}\right]. \end{aligned} \tag{2.15}$$

The proof of the above inequality may be done in similar manner of proof Theorem 6, by using the well known fact, for a Riemann integrable function $p : [c,d] \to \mathbb{R}$ and L-Lipschitzian function $\nu : [c,d] \to \mathbb{R}$, one has the inequality

$$\left|\int_c^d p(t)\,d\nu(t)\right| \le L\int_c^d |p(t)|\,dt, \tag{2.16}$$

and we shall omit the details. Moreover, if we take $g(t) = 1$ on $[a,b]$ in (2.15) then we get the following inequality

$$\begin{aligned} &\left|(b-a)\left[\alpha\cdot\frac{(b-x)f(b)+(x-a)f(a)}{b-a} + (1-\alpha)\cdot\frac{f(x)+f(a+b-x)}{2}\right] - \int_a^b f(t)\,dt\right| \\ &\quad \le L\left[(1-\alpha)\left(\frac{1}{8}(b-a)^2 + 2\left(x-\frac{3a+b}{4}\right)^2\right) + \alpha\frac{(x-a)^2+(b-x)^2}{2}\right], \end{aligned} \tag{2.17}$$

which is the "generalized companion of trapezoid–Ostrowski inequality". Moreover, if we choose $x = \frac{a+b}{2}$, then we get

$$\left|(b-a)\left[\alpha\frac{f(a)+f(b)}{2} + (1-\alpha)f\left(\frac{a+b}{2}\right)\right] - \int_a^b f(t)\,dt\right| \le \frac{L}{4}(b-a)^2. \tag{2.18}$$

Finally, we note that, several special cases may be deduced from (2.15) and we shall left the details to the interested reader.

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