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NEW INEQUALITIES OF OSTROWSKI TYPE FOR MAPPINGS WHOSE DERIVATIVES ARE (α, m)-CONVEX VIA FRACTIONAL INTEGRALS

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ABSTRACT. New identity similar to an identity of [4] for fractional integrals have been defined. Then making use of this identity, some new Ostrowski type inequalities for Riemann-Liouville fractional integral have been developed. Our results have some relationships with the results of Özdemir et. al., proved in [4] [published in KYUNGPOOK Math. J. 50(2010), 371-378] and the analysis used in the proofs is simple.

1. INTRODUCTION

In 1938, A.M. Ostrowski proved the following interesting and useful integral inequality (see also [1], page 468):

Theorem 1. *Let $f: I \to \mathbb{R}$, where $I \subset \mathbb{R}$ is an interval, be a mapping differentiable in the interior I° of I, and let $a, b \in I^\circ$ with $a < b$. If $|f'(x)| \leq M$ for all $x \in [a, b]$, then the following inequality holds:*

$$\left| f(x) - \frac{1}{b-a} \int_a^b f(t)\,dt \right| \leq M(b-a) \left[\frac{1}{4} + \frac{\left(x - \frac{a+b}{2}\right)^2}{(b-a)^2} \right]$$

for all $x \in [a, b]$.

The constant $\frac{1}{4}$ is the best possible in the sense that it cannot be replaced by a smaller one.

This inequality gives an upper bound for the approximation of the integral average $\frac{1}{b-a} \int_a^b f(t)\,dt$ by the value $f(x)$ at point $x \in [a, b]$. In recent years, such inequalities were studied extensively by many researchers and numerious generalizations, extensions and variants of them appeared in a number of papers see ([4], [11], [12] and [13]).

In [14], V.G. Miheşan defined $(\alpha, m)-$convexity as the following:

Definition 1. *The function $f: [0, b] \to \mathbb{R}$, $b > 0$, is said to be $(\alpha, m)-$convex, where $(\alpha, m) \in [0, 1]^2$, if we have*

$$f(tx + m(1-t)y) \leq t^\alpha f(x) + m(1 - t^\alpha) f(y)$$

for all $x, y \in [0, b]$ and $t \in [0, 1]$.

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Denote by $K_m^{\alpha}(b)$ the class of all $(\alpha,m)-$convex functions on $[0,b]$ for which $f(0)\leq 0$.

It can be easily seen that for $(\alpha,m)=(1,m)$, (α,m) convexity reduces to $m-$convexity; $(\alpha,m)=(\alpha,1)$, $(\alpha,m)-$convexity reduces to $\alpha-$convexity and for $(\alpha,m)=(1,1)$, $(\alpha,m)-$ convexity reduces to the concept of usual convexity defined on $[0,b]$, $b>0$. For recent results and generalizations concerning $(\alpha,m)-$ convex functions, see ([2]-[4]).

In order to prove our results we need the following equality which was given in ([4], page 372) by Özdemir et al. :

$$mf(x)-\frac{1}{b-a}\int_{ma}^{mb}f(u)\,du = \frac{(x-ma)^2}{b-a}\int_0^1 tf'(tx+m(1-t)a)\,dt -\frac{(mb-x)^2}{b-a}\int_0^1 tf'(tx+m(1-t)b)\,dt \tag{1.1}$$

which is a special case of Lemma 1 in [15] with $ma\to a$ and $mb\to b$.

Using the inequality in (1.1), Özdemir et al. in [4] established the following results which holds for $(\alpha,m)-$convex functions.

Theorem 2. *Let I be an open interval such that $[0,\infty)\subset I$ and $f:I\to\mathbb{R}$ be a differentiable function on I such that $f'\in L[ma,mb]$ where $ma,mb\in I$ with $a<b$. If $|f'|^q$ is $(\alpha,m)-$convex on $[ma,mb]$ for $(\alpha,m)\in[0,1]^2$, $q>1$, $\frac{1}{p}+\frac{1}{q}=1$ and $|f'(x)|\leq M$, $x\in[ma,mb]$, then the following inequality holds:*

$$\left|mf(x)-\frac{1}{b-a}\int_{ma}^{mb}f(u)\,du\right| \leq M\left(\frac{\alpha m+1}{\alpha+1}\right)^{\frac{1}{q}}\left(\frac{1}{p+1}\right)^{\frac{1}{p}}\frac{(x-ma)^2+(mb-x)^2}{b-a}$$

for each $x\in[ma,mb]$.

Theorem 3. *Let I be an open interval such that $[0,\infty)\subset I$ and $f:I\to\mathbb{R}$ be a differentiable function on I such that $f'\in L[ma,mb]$ where $ma,mb\in I$ with $a<b$. If $|f'|^q$ is $(\alpha,m)-$convex on $[ma,mb]$ for $(\alpha,m)\in[0,1]^2$, $q\in[1,\infty)$ and $|f'(x)|\leq M$, then the following inequality holds:*

$$\left|mf(x)-\frac{1}{b-a}\int_{ma}^{mb}f(u)\,du\right| \leq M\left(\frac{2+\alpha m}{\alpha+2}\right)^{\frac{1}{q}}\frac{(x-ma)^{\alpha+1}+(mb-x)^{\alpha+1}}{(b-a)\,2}$$

for each $x\in[ma,mb]$.

We give some necessary definitions and mathematical preliminaries of fractional calculus theory which are used throughout this paper.

Definition 2. *Let $f\in L_1[a,b]$. The Riemann-Liouville integrals $J_{a^+}^{\alpha}(f)$ and $J_{b^-}^{\alpha}(f)$ of order $\alpha>0$ with $a\geq 0$ are defined by*

$$J_{a^+}^{\alpha}f(x)=\frac{1}{\Gamma(\alpha)}\int_a^x (x-t)^{\alpha-1}f(t)\,dt,\quad x>a$$

and

$$J_{b^-}^{\alpha}f(x)=\frac{1}{\Gamma(\alpha)}\int_x^b (t-x)^{\alpha-1}f(t)\,dt,\quad x<b$$

where $\Gamma(\alpha) = \int_0^\infty e^{-t} u^{\alpha-1} du$. *Here* $J_{a+}^0 f(x) = J_{b-}^0 f(x) = f(x)$.

In the case of $\alpha = 1$, the fractional integral reduces to the classical integral. Some recent results and properties concerning this operator can be found ([5]-[10] and [16]).

We establish new Ostrowski type inequalities for $(\alpha, m)-$convex functions via Riemann-Liouville fractional integral. An interesting feature of our results is that they provide new estimates on these types of inequalities for fractional integrals.

2. Ostrowski Type Inequalities for Fractional Integrals

In order to prove our main results we need the following identity:

Lemma 1. *Let I be an open real interval such that $[0, \infty) \subset I$ and $f : I \to \mathbb{R}$ be a differentiable function on I such that $f' \in L([ma, mb])$, where $[ma, mb] \in I$ with $a < b$ then for all $x \in (ma, mb)$ and $\alpha > 0$ we have:*

$$\begin{aligned} & \frac{(x-ma)^\alpha + (mb-x)^\alpha}{b-a} f(x) - \frac{\Gamma(\alpha+1)}{b-a} \left[J_{x-}^\alpha f(ma) + J_{x+}^\alpha f(mb) \right] \\ = & \frac{(x-ma)^{\alpha+1}}{b-a} \int_0^1 t^\alpha f'(tx + m(1-t)a)\, dt \\ & - \frac{(mb-x)^{\alpha+1}}{b-a} \int_0^1 t^\alpha f'(tx + m(1-t)b)\, dt \end{aligned}$$

where $\Gamma(\alpha) = \int_0^\infty e^{-t} u^{\alpha-1} du$.

Proof. Integration by parts we have

$$\begin{aligned} & \int_0^1 t^\alpha f'(tx + m(1-t)a)\, dt \\ = & \left. t^\alpha \frac{f(tx + m(1-t)a)}{x-ma} \right|_0^1 - \int_0^1 \alpha t^{\alpha-1} \frac{f(tx + m(1-t)a)}{x-ma} dt \\ = & \frac{f(x)}{x-ma} - \frac{\alpha}{x-ma} \int_{ma}^x \left(\frac{u-ma}{x-ma} \right)^{\alpha-1} f(u) \frac{du}{x-ma} \\ = & \frac{f(x)}{x-ma} - \frac{\Gamma(\alpha+1)}{(x-ma)^{\alpha+1}} \frac{1}{\Gamma(\alpha)} \int_{ma}^x (u-ma)^{\alpha-1} f(u)\, du \end{aligned}$$

$$= \frac{f(x)}{x-ma} - \frac{\Gamma(\alpha+1)}{(x-ma)^{\alpha+1}} J_{x-}^\alpha f(ma) \tag{2.1}$$

and similarly

$$\begin{aligned}
&\int_0^1 t^{\alpha} f'(tx+m(1-t)b)\,dt \\
&= t^{\alpha}\frac{f(tx+m(1-t)b)}{x-mb}\bigg|_0^1 - \int_0^1 \alpha t^{\alpha-1}\frac{f(tx+m(1-t)b)}{x-mb}dt \\
&= \frac{f(x)}{x-mb}+\frac{\alpha}{(mb-x)^2}\int_x^{mb}\left(\frac{mb-u}{mb-x}\right)^{\alpha-1} f(u)\,du \\
&= \frac{f(x)}{x-mb}+\frac{\Gamma(\alpha+1)}{(mb-x)^{\alpha+1}}\frac{1}{\Gamma(\alpha)}\int_x^{mb}(mb-u)^{\alpha-1} f(u)\,du \\
&= \frac{f(x)}{x-mb}+\frac{\Gamma(\alpha+1)}{(mb-x)^{\alpha+1}} J_{x^+}^{\alpha} f(mb) \qquad (2.2)
\end{aligned}$$

Multiplying the both sides of 2.1 and 2.2 by $\frac{(x-ma)^{\alpha+1}}{b-a}$ and $\frac{(mb-x)^{\alpha+1}}{b-a}$, respectively, we have

(2.3)
$$\frac{(x-ma)^{\alpha+1}}{b-a}\int_0^1 t^{\alpha} f'(tx+m(1-t)a)\,dt = \frac{(x-ma)^{\alpha}}{b-a} f(x) - \frac{\Gamma(\alpha+1)}{b-a} J_{x^-}^{\alpha} f(ma)$$

and

(2.4)
$$\frac{(mb-x)^{\alpha+1}}{b-a}\int_0^1 t^{\alpha} f'(tx+m(1-t)b)\,dt = -\frac{(mb-x)^{\alpha}}{b-a} f(x) + \frac{\Gamma(\alpha+1)}{b-a} J_{x^+}^{\alpha} f(mb).$$

If we add the inequalities in 2.3 and 2.4, we get the desired result. □

Using Lemma 1, we can obtain the following fractional integral inequalities:

Theorem 4. *Let I be an open interval such that $[0,\infty)\subset I$ and $f: I\to\mathbb{R}$ be a differentiable function on I such that $f'\in L[ma,mb]$ where $ma, mb\in I$ with $a<b$. If $|f'|$ is $(\alpha,m)-$convex on $[ma,mb]$ for $(\alpha,m)\in[0,1]^2$ and $|f'(x)|\le M$, then the following inequality holds:*

$$\begin{aligned}
&\left|\frac{(x-ma)^{\alpha}+(mb-x)^{\alpha}}{b-a} f(x) - \frac{\Gamma(\alpha+1)}{b-a}\left[J_{x^-}^{\alpha} f(ma) + J_{x^+}^{\alpha} f(mb)\right]\right| \\
\le\;& M\left(\frac{1+m\alpha}{2\alpha+1}\right)\frac{(x-ma)^{\alpha+1}+(mb-x)^{\alpha+1}}{b-a}
\end{aligned}$$

for all $x\in[ma,mb]$.

Proof. From Lemma 1 and using the $(\alpha,m)-$convexity of $|f'|$, we have

$$\begin{aligned}
&\left|\frac{(x-ma)^{\alpha}+(mb-x)^{\alpha}}{b-a}f(x)-\frac{\Gamma(\alpha+1)}{b-a}\left[J_{x-}^{\alpha}f(ma)+J_{x+}^{\alpha}f(mb)\right]\right| \\
\leq\;& \frac{(x-ma)^{\alpha+1}}{b-a}\int_0^1 t^{\alpha}\left|f'(tx+m(1-t)a)\right|dt+\frac{(mb-x)^{\alpha+1}}{b-a}\int_0^1 t^{\alpha}\left|f'(tx+m(1-t)b)\right|dt \\
\leq\;& \frac{(x-ma)^{\alpha+1}}{b-a}\int_0^1 t^{\alpha}\left[t^{\alpha}\left|f'(x)\right|+m(1-t^{\alpha})\left|f'(a)\right|\right]dt \\
&+\frac{(mb-x)^{\alpha+1}}{b-a}\int_0^1 t^{\alpha}\left[t^{\alpha}\left|f'(x)\right|+m(1-t^{\alpha})\left|f'(b)\right|\right]dt \\
\leq\;& M\frac{(x-ma)^{\alpha+1}}{b-a}\int_0^1\left[t^{2\alpha}+m\left(t^{\alpha}-t^{2\alpha}\right)\right]dt \\
&+M\frac{(mb-x)^{\alpha+1}}{b-a}\int_0^1\left[t^{2\alpha}+m\left(t^{\alpha}-t^{2\alpha}\right)\right]dt \\
\leq\;& M\left(\frac{1+m\alpha}{2\alpha+1}\right)\frac{(x-ma)^{\alpha+1}+(mb-x)^{\alpha+1}}{b-a}
\end{aligned}$$

where we have used the fact that

$$\int_0^1\left[t^{2\alpha}+m\left(t^{\alpha}-t^{2\alpha}\right)\right]dt=\frac{1+m\alpha}{2\alpha+1}.$$

The proof is completed. □

The corresponding version for powers of the absolute value of the first derivative is incorporated in the following results:

Theorem 5. *Let I be an open interval such that $[0,\infty)\subset I$ and $f:I\to\mathbb{R}$ be a differentiable function on I such that $f'\in L[ma,mb]$ where $ma,mb\in I$ with $a<b$. If $|f'|^q$ is $(\alpha,m)-$convex on $[ma,mb]$ for $(\alpha,m)\in[0,1]^2$, $q>1$, $\frac{1}{p}+\frac{1}{q}=1$ and $|f'(x)|\leq M$, then the following inequality holds:*

$$\begin{aligned}
&\left|\frac{(x-ma)^{\alpha}+(mb-x)^{\alpha}}{b-a}f(x)-\frac{\Gamma(\alpha+1)}{b-a}\left[J_{x-}^{\alpha}f(ma)+J_{x+}^{\alpha}f(mb)\right]\right| \\
\leq\;& M\left(\frac{1+m\alpha}{\alpha+1}\right)^{\frac{1}{q}}\left(\frac{1}{\alpha p+1}\right)^{\frac{1}{p}}\frac{(x-ma)^{\alpha+1}+(mb-x)^{\alpha+1}}{b-a}
\end{aligned}$$

for all $x\in[ma,mb]$.

Proof. Suppose that $p>1$. From Lemma 1 and using the Hölder inequality, we have

$$\begin{aligned}
&\left|\frac{(x-ma)^{\alpha}+(mb-x)^{\alpha}}{b-a}f(x)-\frac{\Gamma(\alpha+1)}{b-a}\left[J_{x-}^{\alpha}f(ma)+J_{x+}^{\alpha}f(mb)\right]\right| \\
\leq\;& \frac{(x-ma)^{\alpha+1}}{b-a}\int_0^1 t^{\alpha}\left|f'(tx+m(1-t)a)\right|dt+\frac{(mb-x)^{\alpha+1}}{b-a}\int_0^1 t^{\alpha}\left|f'(tx+m(1-t)b)\right|dt \\
\leq\;& \frac{(x-ma)^{\alpha+1}}{b-a}\left(\int_0^1 t^{\alpha p}dt\right)^{\frac{1}{p}}\left(\int_0^1\left|f'(tx+m(1-t)a)\right|^q dt\right)^{\frac{1}{q}} \\
&+\frac{(mb-x)^{\alpha+1}}{b-a}\left(\int_0^1 t^{\alpha p}dt\right)^{\frac{1}{p}}\left(\int_0^1\left|f'(tx+m(1-t)b)\right|^q dt\right)^{\frac{1}{q}}
\end{aligned}$$

Since $|f'|^q$ is $(\alpha,m)-$convex function and $|f'(x)|\leq M$, then we have

$$\begin{aligned}\left(\int_0^1 |f'(tx+m(1-t)a)|^q\,dt\right)^{\frac{1}{q}} &\leq \left(\int_0^1 \left[t^\alpha |f'(x)|^q + m(1-t^\alpha)|f'(a)|^q\right]dt\right)^{\frac{1}{q}}\\ &= M\left(\frac{1+\alpha m}{\alpha+1}\right)^{\frac{1}{q}}\end{aligned}$$

and similarly

$$\begin{aligned}\left(\int_0^1 |f'(tx+m(1-t)b)|^q\,dt\right)^{\frac{1}{q}} &\leq \left(\int_0^1 \left[t^\alpha |f'(x)|^q + m(1-t^\alpha)|f'(b)|^q\right]dt\right)^{\frac{1}{q}}\\ &= M\left(\frac{1+\alpha m}{\alpha+1}\right)^{\frac{1}{q}}.\end{aligned}$$

Therefore, we have

$$\begin{aligned}&\left|\frac{(x-ma)^\alpha+(mb-x)^\alpha}{b-a}f(x)-\frac{\Gamma(\alpha+1)}{b-a}\left[J_{x^-}^\alpha f(ma)+J_{x^+}^\alpha f(mb)\right]\right|\\ \leq\ & \frac{(x-ma)^{\alpha+1}}{b-a}\left(\frac{1}{\alpha p+1}\right)^{\frac{1}{p}}\left(M^q\frac{1+\alpha m}{\alpha+1}\right)^{\frac{1}{q}}\\ &+\frac{(mb-x)^{\alpha+1}}{b-a}\left(\frac{1}{\alpha p+1}\right)^{\frac{1}{p}}\left(M^q\frac{1+\alpha m}{\alpha+1}\right)^{\frac{1}{q}}\\ =\ & M\left(\frac{1}{\alpha p+1}\right)^{\frac{1}{p}}\left(\frac{1+\alpha m}{\alpha+1}\right)^{\frac{1}{q}}\frac{(x-ma)^{\alpha+1}+(mb-x)^{\alpha+1}}{b-a}.\end{aligned}$$

This completes the proof. □

Remark 1. *In Theorem 5, if we choose $\alpha=1$ we get the result in Theorem 2 with $\alpha=1$.*

Theorem 6. *Let I be an open interval such that $[0,\infty)\subset I$ and $f:I\to\mathbb{R}$ be a differentiable function on I such that $f'\in L[ma,mb]$ where $ma,mb\in I$ with $a<b$. If $|f'|^q$ is $(\alpha,m)-$convex on $[ma,mb]$ for $(\alpha,m)\in[0,1]^2$, $q\geq 1$ and $|f'(x)|\leq M$, then the following inequality holds:*

$$\begin{aligned}&\left|\frac{(x-ma)^\alpha+(mb-x)^\alpha}{b-a}f(x)-\frac{\Gamma(\alpha+1)}{b-a}\left[J_{x^-}^\alpha f(ma)+J_{x^+}^\alpha f(mb)\right]\right|\\ \leq\ & M\left(\frac{\alpha(m+1)+1}{2\alpha+1}\right)^{\frac{1}{q}}\frac{(x-ma)^{\alpha+1}+(mb-x)^{\alpha+1}}{(b-a)(\alpha+1)}\end{aligned}$$

for all $x\in[ma,mb]$.

Proof. From Lemma 1 and using the well known power mean inequality, we have

$$
\begin{aligned}
&\left|\frac{(x-ma)^{\alpha}+(mb-x)^{\alpha}}{b-a}f(x)-\frac{\Gamma(\alpha+1)}{b-a}\left[J_{x-}^{\alpha}f(ma)+J_{x+}^{\alpha}f(mb)\right]\right| \\
\leq\ & \frac{(x-ma)^{\alpha+1}}{b-a}\int_0^1 t^{\alpha}\left|f'(tx+m(1-t)a)\right|dt+\frac{(mb-x)^{\alpha+1}}{b-a}\int_0^1 t^{\alpha}\left|f'(tx+m(1-t)b)\right|dt \\
\leq\ & \frac{(x-ma)^{\alpha+1}}{b-a}\left(\int_0^1 t^{\alpha}dt\right)^{1-\frac{1}{q}}\left(\int_0^1 t^{\alpha}\left|f'(tx+m(1-t)a)\right|^q dt\right)^{\frac{1}{q}} \\
&+\frac{(mb-x)^{\alpha+1}}{b-a}\left(\int_0^1 t^{\alpha}dt\right)^{1-\frac{1}{q}}\left(\int_0^1 t^{\alpha}\left|f'(tx+m(1-t)b)\right|^q dt\right)^{\frac{1}{q}}
\end{aligned}
\tag{2.5}
$$

Since $|f'|^q$ is $(\alpha,m)-$convex function and $|f'(x)|\leq M$, then we have

$$
\int_0^1 t^{\alpha}\left|f'(tx+m(1-t)a)\right|^q dt = \int_0^1 t^{\alpha}\left|f'(tx+m(1-t)b)\right|dt\leq M^q\frac{\alpha(m+1)+1}{(2\alpha+1)(\alpha+1)}. \tag{2.6}
$$

Then using the inequality (2.6) in (2.5) and computing the integrals in (2.5), we get the desired result. □

Remark 2. *In Theorem 6, if we choose* $\alpha=1$ *we get the result in Theorem 3 with* $\alpha=1$.

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