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ON SOME NEW INEQUALITIES OF HERMITE-HADAMARD TYPE FOR FUNCTIONS WHOSE DERIVATIVES IN ABSOLUTE VALUE ARE s-CONVEX IN THE SECOND SENSE

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ABSTRACT. Several new inequalities for of Hermite-Hadamard type for functions whose derivatives in absolute value are s-convex in the second sense are established. Some applications to special means of positive real numbers are given as well.

1. INTRODUCTION

Let $f : I \subseteq \mathbb{R} \to \mathbb{R}$, $\emptyset \neq I \subseteq \mathbb{R}$ be a convex function on I and a, $b \in I$ with $a < b$, then the inequalities

$$f\left(\frac{a+b}{2}\right) \leq \frac{1}{b-a}\int_a^b f(x)\,dx \leq \frac{f(a)+f(b)}{2}, \tag{1.1}$$

hold and are known as Hermite-Hadamard inequalities. The inequalities in (1.1) hold in reversed order if f is a concave function.

In the paper [8], H. Hudzik and L. Maligranda considered, among others, the class of functions which are s-convex in the second sense. This class of functions is defined as follows:

A function $f : [0,\infty) \to \mathbb{R}$ is said to be s-convex in the second sense if

$$f(\lambda x + (1-\lambda)y) \leq \lambda^s f(x) + (1-\lambda)^s f(y),$$

holds for all x, $y \in [0,\infty)$, $\lambda \in [0,1]$ and for some fixed $s \in (0,1]$.

The class of s-convex functions in the second sense is usually denoted by K_s^2. It is to be noted that for $s = 1$, s-convexity is merely the usual convexity.

In [6], Dragomir and Fitzpatrick proved a variant of Hermite-Hadamard's inequality which holds for s-convex functions:

Theorem 1. [6] *Suppose $f : [0,\infty) \to [0,\infty)$ is an s-convex function in the second sense, where $s \in (0,1)$ and let a, $b \in [0,\infty)$, $a < b$. If $f \in L^1[a,b]$, then the following inequalities hold:*

$$2^{s-1} f\left(\frac{a+b}{2}\right) \leq \frac{1}{b-a}\int_a^b f(x)\,dx \leq \frac{f(a)+f(b)}{s+1}, \tag{1.2}$$

The constant $k = \frac{1}{s+1}$ is the best possible in the second inequality in (1.2). The inequalities in (1.2) hold in reversed order if f is s-concave.

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In recent years, many authors have established several inequalities of Hermite-Hadamard type for convex functions and s-convex functions in the second sense see for instance the works in [1]-[19] and the references therein.

The author established the following inequalities for functions whose derivatives in absolute value are convex:

Theorem 2. [14]*Let* $f : I \subseteq \mathbb{R} \to \mathbb{R}$ *be a differentiable function on* I° *such that* $f' \in L[a,b]$, *where* $a, b \in I$ *with* $a < b$. *If* $|f'|$ *is convex on* $[a,b]$, *then the following inequality holds:*

$$\left| f(x) + \frac{(b-x) f(b) + (x-a) f(a)}{b-a} - \frac{2}{b-a} \int_a^b f(u)\,du \right| \leq \frac{(x-a)^2}{b-a}\left[\frac{|f'(x)| + |f'(a)|}{4}\right] + \frac{(b-x)^2}{b-a}\left[\frac{|f'(x)| + |f'(b)|}{4}\right] \tag{1.3}$$

for all $x \in [a,b]$.

Corollary 1. [14]*In Theorem 2, if we choose* $x = \frac{a+b}{2}$ *and then using the convexity of* $|f'|$, *we get the following inequality:*

$$\left| f\left(\frac{a+b}{2}\right) + \frac{f(b) + f(a)}{2} - \frac{2}{b-a} \int_a^b f(u)\,du \right| \leq \left(\frac{b-a}{8}\right) \left[|f'(a)| + |f'(b)|\right]. \tag{1.4}$$

Theorem 3. [14]*Let* $f : I \subseteq \mathbb{R} \to \mathbb{R}$ *be a differentiable function on* I° *such that* $f' \in L[a,b]$, *where* $a, b \in I$ *with* $a < b$. *If* $|f'|^q$ *is convex on* $[a,b]$ *for some fixed* $q > 1$, *then the following inequality holds:*

$$\begin{aligned} &\left| f(x) + \frac{(b-x) f(b) + (x-a) f(a)}{b-a} - \frac{2}{b-a} \int_a^b f(u)\,du \right| \\ &\leq \left(\frac{1}{p+1}\right)^{\frac{1}{p}} \left(\frac{1}{2}\right)^{\frac{2}{q}+1} \left\{ \frac{(x-a)^2}{b-a}\left[3|f'(x)|^q + |f'(a)|^q\right]^{\frac{1}{q}} \right. \\ &\quad + \frac{(x-a)^2}{b-a}\left[|f'(x)|^q + 3|f'(a)|^q\right]^{\frac{1}{q}} \\ &\quad + \frac{(b-x)^2}{b-a}\left[3|f'(x)|^q + |f'(b)|^q\right]^{\frac{1}{q}} \\ &\quad \left. + \frac{(b-x)^2}{b-a}\left[|f'(x)|^q + 3|f'(b)|^q\right]^{\frac{1}{q}} \right\}, \end{aligned} \tag{1.5}$$

for all $x \in [a,b]$ *and* $\frac{1}{p} + \frac{1}{q} = 1$.

Corollary 2. [14]*In Theorem 3, if we choose* $x=\frac{a+b}{2}$ *and then using the convexity of* $|f'|^q$*, we get the following inequality:*

$$\begin{aligned}
&\left|f\left(\frac{a+b}{2}\right)+\frac{f(a)+f(b)}{2}-\frac{2}{b-a}\int_a^b f(u)\,du\right| \\
&\leq\left(\frac{1}{p+1}\right)^{\frac{1}{p}}\left(\frac{1}{2}\right)^{\frac{2}{q}+1}\left(\frac{b-a}{4}\right)\left\{\left[|f'(a)|^q+3\left|f'\left(\frac{a+b}{2}\right)\right|^q\right]^{\frac{1}{q}}\right. \\
&\qquad+\left[\left|f'\left(\frac{a+b}{2}\right)\right|^q+3|f'(a)|^q\right]^{\frac{1}{q}} \\
&\left.+\left[3\left|f'\left(\frac{a+b}{2}\right)\right|^q+|f'(b)|^q\right]^{\frac{1}{q}}+\left[\left|f'\left(\frac{a+b}{2}\right)\right|^q+3|f'(b)|^q\right]^{\frac{1}{q}}\right\} \\
&\leq\left(\frac{1}{p+1}\right)^{\frac{1}{p}}\left(\frac{1}{2}\right)^{\frac{3}{q}+1}\left[1+3^{\frac{1}{q}}+5^{\frac{1}{q}}+7^{\frac{1}{q}}\right]\left(\frac{b-a}{4}\right)\left[|f'(a)|+|f'(b)|\right]. \qquad (1.6)
\end{aligned}$$

Theorem 4. *Let* $f:I\subseteq\mathbb{R}\to\mathbb{R}$ *be a differentiable function on* I° *such that* $f'\in L[a,b]$*, where* $a,b\in I$ *with* $a<b$*. If* $|f'|^q$ *is convex on* $[a,b]$ *for some fixed* $q\geq 1$*, then the following inequality holds:*

$$\begin{aligned}
&\left|f(x)+\frac{(b-x)f(b)+(x-a)f(a)}{b-a}-\frac{2}{b-a}\int_a^b f(u)\,du\right| \\
&\leq\left(\frac{1}{4}\right)\left(\frac{1}{6}\right)^{\frac{1}{q}}\left\{\frac{(x-a)^2}{b-a}\left[\left(5|f'(x)|^q+|f'(a)|^q\right)^{\frac{1}{q}}+\left(|f'(x)|^q+5|f'(a)|^q\right)^{\frac{1}{q}}\right]\right. \\
&\qquad\left.+\frac{(b-x)^2}{b-a}\left[\left(5|f'(x)|^q+|f'(b)|^q\right)^{\frac{1}{q}}+\left(|f'(x)|^q+5|f'(b)|^q\right)^{\frac{1}{q}}\right]\right\}, \qquad (1.7)
\end{aligned}$$

for all $x\in[a,b]$.

Corollary 3. *In Theorem 4, if we choose* $x=\frac{a+b}{2}$ *and using similar arguments as in Corollary 2, we get the following inequality:*

$$\begin{aligned}
&\left|f\left(\frac{a+b}{2}\right)+\frac{f(a)+f(b)}{2}-\frac{2}{b-a}\int_a^b f(u)\,du\right| \\
&\leq\left(\frac{1}{3}\right)^{\frac{1}{q}}\left(\frac{1}{2}\right)^{\frac{2}{q}}\left[1+5^{\frac{1}{q}}+7^{\frac{1}{q}}+11^{\frac{1}{q}}\right]\left(\frac{b-a}{16}\right)\left[|f'(a)|+|f'(b)|\right]. \qquad (1.8)
\end{aligned}$$

The interesting features of the established results from [14] are that they give, in fact, the estimate of difference between the middle and sum of rightmost and leftmost terms connected with the Hermite Hadamard's inequalities (1.1).

The main purpose of this paper is to establish some entirely new inequalities of Hermite-Hadamard type for functions whose derivatives in absolute value are s-convex in the second sense. Our results give an estimate of the difference between the middle and sum of rightmost and leftmost terms connected with the Hermite Hadamard inequalties. given above in (1.1).

2. Main Results

In order to prove our results we need the following lemma:

Lemma 1. [14, Lemma 1, Page 2.]*Let $f : I \subseteq \mathbb{R} \to \mathbb{R}$ be a differentiable function on I° , the interior of I, where $a, b \in I$ with $a < b$. If $f' \in L[a, b]$, then the following equality holds:*

$$
\begin{aligned}
&f(x) + \frac{(b-x) f(b) + (x-a) f(a)}{b-a} - \frac{2}{b-a} \int_a^b f(u)\, du \\
&= \frac{(x-a)^2}{b-a} \int_0^1 \frac{t}{2} f'\left(\frac{1+t}{2}x + \frac{1-t}{2}a\right) dt - \frac{(x-a)^2}{b-a} \int_0^1 \frac{t}{2} f'\left(\frac{1-t}{2}x + \frac{1+t}{2}a\right) dt \\
&- \frac{(b-x)^2}{b-a} \int_0^1 \frac{t}{2} f'\left(\frac{1+t}{2}x + \frac{1-t}{2}b\right) dt + \frac{(b-x)^2}{b-a} \int_0^1 \frac{t}{2} f'\left(\frac{1-t}{2}x + \frac{1+t}{2}b\right) dt,
\end{aligned} \tag{2.1}
$$

for all $x \in [a, b]$.

Theorem 5. *Let $f : I \to \mathbb{R}$, $I \subset [0, \infty)$, be a differentiable function on I° such that $f' \in L[a, b]$, where $a, b \in I$ with $a < b$. If $|f'|$ is s-convex on $[a, b]$, $s \in (0, 1]$, then the following inequality holds:*

$$
\begin{aligned}
&\left| f(x) + \frac{(b-x) f(b) + (x-a) f(a)}{b-a} - \frac{2}{b-a} \int_a^b f(u)\, du \right| \\
&\leq \frac{s + 2^{-s}}{(s+1)(s+2)} \left\{ \frac{(x-a)^2}{b-a} \left[|f'(x)| + |f'(a)| \right] + \frac{(b-x)^2}{b-a} \left[|f'(x)| + |f'(b)| \right] \right\},
\end{aligned} \tag{2.2}
$$

for all $x \in [a, b]$.

Proof. From Lemma 1, we have

$$
\begin{aligned}
&\left| f(x) + \frac{(b-x) f(b) + (x-a) f(a)}{b-a} - \frac{2}{b-a} \int_a^b f(u)\, du \right| \\
&\leq \frac{(x-a)^2}{b-a} \left[\int_0^1 \frac{t}{2} \left| f'\left(\frac{1+t}{2}x + \frac{1-t}{2}a\right) \right| dt + \int_0^1 \frac{t}{2} \left| f'\left(\frac{1-t}{2}x + \frac{1+t}{2}a\right) \right| dt \right] \\
&+ \frac{(b-x)^2}{b-a} \left[\int_0^1 \frac{t}{2} \left| f'\left(\frac{1+t}{2}x + \frac{1-t}{2}b\right) \right| dt + \int_0^1 \frac{t}{2} \left| f'\left(\frac{1-t}{2}x + \frac{1+t}{2}b\right) \right| dt \right]
\end{aligned} \tag{2.3}
$$

Since $|f'|$ is s-convex on $[a, b]$, we have

$$
\begin{aligned}
&\int_0^1 \frac{t}{2} f'\left(\frac{1+t}{2}x + \frac{1-t}{2}a\right) dt + \int_0^1 \frac{t}{2} \left| f'\left(\frac{1-t}{2}x + \frac{1+t}{2}a\right) \right| dt \\
&\qquad \leq \left[\int_0^1 \frac{t}{2} \left(\frac{1-t}{2}\right)^s dt + \int_0^1 \frac{t}{2} \left(\frac{1+t}{2}\right)^s dt \right] \left[|f'(x)| + |f'(a)| \right]
\end{aligned} \tag{2.4}
$$

and

$$\int_0^1 \frac{t}{2} f'\left(\frac{1+t}{2}x+\frac{1-t}{2}a\right) dt + \int_0^1 \frac{t}{2}\left|f'\left(\frac{1-t}{2}x+\frac{1+t}{2}a\right)\right| dt$$
$$\leq \left[\int_0^1 \frac{t}{2}\left(\frac{1-t}{2}\right)^s dt + \int_0^1 \frac{t}{2}\left(\frac{1+t}{2}\right)^s dt\right]\left[\left|f'(x)\right| + \left|f'(b)\right|\right]. \quad (2.5)$$

By making use of the inequalites (2.4), (2.5) and the fact

$$\int_0^1 \frac{t}{2}\left(\frac{1-t}{2}\right)^s dt + \int_0^1 \frac{t}{2}\left(\frac{1+t}{2}\right)^s dt = \frac{s+2^{-s}}{(s+1)(s+2)}.$$

in the inequality (2.3), we get the inequality (2.2).
This completes the proof of the theorem. □

Corollary 4. *Under the assumptions of Theorem 5, if we take* $x = \frac{a+b}{2}$, *we get the following inequality:*

$$\left|f\left(\frac{a+b}{2}\right) + \frac{f(a)+f(b)}{2} - \frac{2}{b-a}\int_a^b f(u)\,du\right|$$
$$\leq \frac{(s+2^{-s})\left(2^{1-s}+1\right)}{(s+1)(s+2)}\left(\frac{b-a}{4}\right)\left[\left|f'(a)\right| + \left|f'(b)\right|\right]. \quad (2.6)$$

Proof. It follows from Theorem 5 and using the s-convexity of f. □

Remark 1. *If we take* $s = 1$ *in Theorem 5 and Corollary 4 we get the inequalities (1.3) and (1.4) respectively.*

The corresponding version of the inequality (2.2) for powers in terms of the first derivative is incorporated as follows:

Theorem 6. *Let* $f : I \subseteq \mathbb{R} \to \mathbb{R}$ *be a differentiable function on* I° *such that* $f' \in L[a,b]$, *where* $a, b \in I$ *with* $a < b$. *If* $|f'|^q$ *is s-convex on* $[a,b]$ *for some fixed* $q > 1$, $s \in (0,1]$, *then the following inequality holds:*

$$\left|f(x) + \frac{(b-x)f(b)+(x-a)f(a)}{b-a} - \frac{2}{b-a}\int_a^b f(u)\,du\right|$$
$$\leq \frac{1}{2}\left(\frac{1}{p+1}\right)^{\frac{1}{p}}\left(\frac{1}{s+1}\right)^{\frac{1}{q}}\left\{\frac{(x-a)^2}{b-a}\left[\left(\left(2-2^{-s}\right)\left|f'(x)\right|^q + 2^{-s}\left|f'(a)\right|^q\right)^{\frac{1}{q}}\right.\right.$$
$$\left. + \left(2^{-s}\left|f'(x)\right|^q + \left(2-2^{-s}\right)\left|f'(a)\right|^q\right)^{\frac{1}{q}}\right]$$
$$+ \frac{(b-x)^2}{b-a}\left[\left(\left(2-2^{-s}\right)\left|f'(x)\right|^q + 2^{-s}\left|f'(b)\right|^q\right)^{\frac{1}{q}}\right.$$
$$\left.\left. + \left(2^{-s}\left|f'(x)\right|^q + \left(2-2^{-s}\right)\left|f'(b)\right|^q\right)^{\frac{1}{q}}\right]\right\}, \quad (2.7)$$

for all $x \in [a,b]$ *and* $\frac{1}{p} + \frac{1}{q} = 1$.

Proof. From Lemma 1 and using the well-known Hölder integral inequality, we have

$$\left|f(x)+\frac{(b-x)f(b)+(x-a)f(a)}{b-a}-\frac{2}{b-a}\int_a^b f(u)\,du\right|$$
$$\leq \frac{(x-a)^2}{b-a}\left(\int_0^1\left(\frac{t}{2}\right)^p dt\right)^{\frac{1}{p}}\left(\int_0^1\left|f'\left(\frac{1+t}{2}x+\frac{1-t}{2}a\right)\right|^q dt\right)^{\frac{1}{q}}$$
$$+\frac{(x-a)^2}{b-a}\left(\int_0^1\left(\frac{t}{2}\right)^p dt\right)^{\frac{1}{p}}\left(\int_0^1\left|f'\left(\frac{1-t}{2}x+\frac{1+t}{2}a\right)\right|^q dt\right)^{\frac{1}{q}}$$
$$+\frac{(b-x)^2}{b-a}\left(\int_0^1\left(\frac{t}{2}\right)^p dt\right)^{\frac{1}{p}}\left(\int_0^1\left|f'\left(\frac{1+t}{2}x+\frac{1-t}{2}b\right)\right|^q dt\right)^{\frac{1}{q}}$$
$$+\frac{(b-x)^2}{b-a}\left(\int_0^1\left(\frac{t}{2}\right)^p dt\right)^{\frac{1}{p}}\left(\int_0^1\left|f'\left(\frac{1-t}{2}x+\frac{1+t}{2}b\right)\right|^q dt\right)^{\frac{1}{q}}, \quad (2.8)$$

for all $x\in[a,b]$.
Since $|f'|^q$ is s-convex on $[a,b]$, we have

$$\int_0^1\left|f'\left(\frac{1+t}{2}x+\frac{1-t}{2}a\right)\right|^q dt$$
$$\leq \int_0^1\left[\left(\frac{1+t}{2}\right)^s|f'(x)|^q+\left(\frac{1-t}{2}\right)^s|f'(a)|^q\right]dt$$
$$=\frac{2-2^{-s}}{s+1}|f'(x)|^q+\frac{2^{-s}}{s+1}|f'(a)|^q. \quad (2.9)$$

Similarly,

$$\int_0^1\left|f'\left(\frac{1-t}{2}x+\frac{1+t}{2}a\right)\right|^q dt\leq\frac{2^{-s}}{s+1}|f'(x)|^q+\frac{2-2^{-s}}{s+1}|f'(a)|^q, \quad (2.10)$$

$$\int_0^1\left|f'\left(\frac{1+t}{2}x+\frac{1-t}{2}b\right)\right|^q dt\leq\frac{2-2^{-s}}{s+1}|f'(x)|^q+\frac{2^{-s}}{s+1}|f'(b)|^q \quad (2.11)$$

and

$$\int_0^1\left|f'\left(\frac{1-t}{2}x+\frac{1+t}{2}b\right)\right|^q dt\leq\frac{2^{-s}}{s+1}|f'(x)|^q+\frac{2-2^{-s}}{s+1}|f'(b)|^q. \quad (2.12)$$

Using the inequalities (2.9)-(2.12) in (2.8) and the fact

$$\int_0^1\left(\frac{t}{2}\right)^p dt=\frac{1}{2^p}\frac{1}{p+1},$$

we get inequality (2.7).
This completes the proof of the theorem. □

Remark 2. *If in Theorem 6, we take $s=1$, we get the inequality (1.5).*

Corollary 5. *Under the assumptions of Theorem 6, if we choose* $x = \frac{a+b}{2}$ *. Then*

$$\left| f\left(\frac{a+b}{2}\right) + \frac{f(a)+f(b)}{2} - \frac{2}{b-a}\int_a^b f(u)\,du \right|$$
$$\leq \left(\frac{b-a}{8}\right)\left(\frac{1}{p+1}\right)^{\frac{1}{p}}\left(\frac{1}{s+1}\right)^{\frac{1}{q}}\left\{\left[\left(2-2^{-s}\right)\left|f'\left(\frac{a+b}{2}\right)\right|^q + 2^{-s}\left|f'(a)\right|^q\right]^{\frac{1}{q}}\right.$$
$$+\left[2^{-s}\left|f'\left(\frac{a+b}{2}\right)\right|^q + \left(2-2^{-s}\right)\left|f'(a)\right|^q\right]^{\frac{1}{q}} + \left[\left(2-2^{-s}\right)\left|f'\left(\frac{a+b}{2}\right)\right|^q + 2^{-s}\left|f'(b)\right|^q\right]^{\frac{1}{q}}$$
$$\left. + \left[2^{-s}\left|f'\left(\frac{a+b}{2}\right)\right|^q + \left(2-2^{-s}\right)\left|f'(b)\right|^q\right]^{\frac{1}{q}}\right\}$$
$$\leq \left(\frac{b-a}{8}\right)\left(\frac{1}{p+1}\right)^{\frac{1}{p}}\left(\frac{1}{s+1}\right)^{\frac{1}{q}}\left[2^{-\frac{2s}{q}} + \left(2^{1-s}-2^{-2s}\right)^{\frac{1}{q}}\right.$$
$$\left. + \left(2^{-2s}-2^{-s}+2\right)^{\frac{1}{q}} + \left(2^{1-s}-2^{-2s}+2^{-s}\right)^{\frac{1}{q}}\right]\left[\left|f'(a)\right| + \left|f'(b)\right|\right]. \quad (2.13)$$

Proof. It follows from Theorem 6. The second inequality is obtained by using the s-convexity of $|f'|^q$ and the fact that

$$\sum_{k=1}^{n}(u_k+v_k)^s \leq \sum_{k=1}^{n}(u_k)^s + \sum_{k=1}^{n}(v_k)^s,$$

for all $u_k, v_k \geq 0, 1 \leq k \leq n$ and $0 \leq s < 1$. □

Remark 3. *If in Corollary 5, we take* $s = 1$*, we get the inequality (1.6).*

Remark 4. *Since for* $p,\ q > 1$ *and* $s \in (0,1]$*,* $\frac{1}{2} \leq \left(\frac{1}{p+1}\right)^{\frac{1}{p}} \leq 1$*,* $\left(\frac{1}{s+1}\right)^{\frac{1}{q}} \leq 1$*, we have from (2.13) the following inequality:*

$$\left| f\left(\frac{a+b}{2}\right) + \frac{f(a)+f(b)}{2} - \frac{2}{b-a}\int_a^b f(u)\,du \right|$$
$$\leq \left(\frac{b-a}{8}\right)\left\{\left[\left(2-2^{-s}\right)\left|f'\left(\frac{a+b}{2}\right)\right|^q + 2^{-s}\left|f'(a)\right|^q\right]^{\frac{1}{q}}\right.$$
$$+\left[2^{-s}\left|f'\left(\frac{a+b}{2}\right)\right|^q + \left(2-2^{-s}\right)\left|f'(a)\right|^q\right]^{\frac{1}{q}} + \left[\left(2-2^{-s}\right)\left|f'\left(\frac{a+b}{2}\right)\right|^q + 2^{-s}\left|f'(b)\right|^q\right]^{\frac{1}{q}}$$
$$\left. + \left[2^{-s}\left|f'\left(\frac{a+b}{2}\right)\right|^q + \left(2-2^{-s}\right)\left|f'(b)\right|^q\right]^{\frac{1}{q}}\right\} \leq \left(\frac{b-a}{8}\right)\left[2^{-\frac{2s}{q}} + \left(2^{1-s}-2^{-2s}\right)^{\frac{1}{q}}\right.$$
$$\left. + \left(2^{-2s}-2^{-s}+2\right)^{\frac{1}{q}} + \left(2^{1-s}-2^{-2s}+2^{-s}\right)^{\frac{1}{q}}\right]\left[\left|f'(a)\right| + \left|f'(b)\right|\right]. \quad (2.14)$$

Theorem 7. *Let* $f : I \subseteq \mathbb{R} \to \mathbb{R}$ *be a differentiable function on* I° *such that* $f' \in L[a,b]$*, where* $a, b \in I$ *with* $a < b$*. If* $|f'|^q$ *is s-convex on* $[a,b]$ *for some fixed*

$q>1$, $s\in(0,1]$, then the following inequality holds:

$$\left|f(x)+\frac{(b-x)f(b)+(x-a)f(a)}{b-a}-\frac{2}{b-a}\int_a^b f(u)\,du\right|$$
$$\leq\frac{1}{2}\left(\frac{1}{p+1}\right)^{\frac{1}{p}}\left(\frac{1}{s+1}\right)^{\frac{1}{q}}\left\{\frac{(x-a)^2}{b-a}\left[|f'(x)|^q+\left|f'\left(\frac{x+a}{2}\right)\right|^q\right]^{\frac{1}{q}}\right.$$
$$+\frac{(x-a)^2}{b-a}\left[|f'(a)|^q+\left|f'\left(\frac{x+a}{2}\right)\right|^q\right]^{\frac{1}{q}}+\frac{(b-x)^2}{b-a}\left[|f'(x)|^q+\left|f'\left(\frac{x+b}{2}\right)\right|^q\right]^{\frac{1}{q}}$$
$$\left.+\frac{(b-x)^2}{b-a}\left[|f'(b)|^q+\left|f'\left(\frac{x+b}{2}\right)\right|^q\right]^{\frac{1}{q}}\right\},\quad(2.15)$$

for all $x\in[a,b]$ and $\frac{1}{p}+\frac{1}{q}=1$.

Proof. From Lemma 1 and using the well-known Hölder integral inequality, we have

$$\left|f(x)+\frac{(b-x)f(b)+(x-a)f(a)}{b-a}-\frac{2}{b-a}\int_a^b f(u)\,du\right|$$
$$\leq\frac{(x-a)^2}{b-a}\left(\int_0^1\left(\frac{t}{2}\right)^p dt\right)^{\frac{1}{p}}\left(\int_0^1\left|f'\left(\frac{1+t}{2}x+\frac{1-t}{2}a\right)\right|^q dt\right)^{\frac{1}{q}}$$
$$+\frac{(x-a)^2}{b-a}\left(\int_0^1\left(\frac{t}{2}\right)^p dt\right)^{\frac{1}{p}}\left(\int_0^1\left|f'\left(\frac{1-t}{2}x+\frac{1+t}{2}a\right)\right|^q dt\right)^{\frac{1}{q}}$$
$$+\frac{(b-x)^2}{b-a}\left(\int_0^1\left(\frac{t}{2}\right)^p dt\right)^{\frac{1}{p}}\left(\int_0^1\left|f'\left(\frac{1+t}{2}x+\frac{1-t}{2}b\right)\right|^q dt\right)^{\frac{1}{q}}$$
$$+\frac{(b-x)^2}{b-a}\left(\int_0^1\left(\frac{t}{2}\right)^p dt\right)^{\frac{1}{p}}\left(\int_0^1\left|f'\left(\frac{1-t}{2}x+\frac{1+t}{2}b\right)\right|^q dt\right)^{\frac{1}{q}},\quad(2.16)$$

for all $x\in[a,b]$.
Since $|f'|^q$ is s-convex on $[a,b]$ so by using the inequality (1.2), we have

$$\int_0^1\left|f'\left(\frac{1+t}{2}x+\frac{1-t}{2}a\right)\right|^q dt\leq\frac{|f'(x)|^q+\left|f'\left(\frac{x+a}{2}\right)\right|^q}{s+1}.\quad(2.17)$$

Similarly,

$$\int_0^1\left|f'\left(\frac{1-t}{2}x+\frac{1+t}{2}a\right)\right|^q dt\leq\frac{|f'(a)|^q+\left|f'\left(\frac{x+a}{2}\right)\right|^q}{s+1},\quad(2.18)$$

$$\int_0^1\left|f'\left(\frac{1+t}{2}x+\frac{1-t}{2}b\right)\right|^q dt\leq\frac{|f'(x)|^q+\left|f'\left(\frac{x+b}{2}\right)\right|^q}{s+1}\quad(2.19)$$

and

$$\int_0^1\left|f'\left(\frac{1-t}{2}x+\frac{1+t}{2}b\right)\right|^q dt\leq\frac{|f'(b)|^q+\left|f'\left(\frac{x+b}{2}\right)\right|^q}{s+1}.\quad(2.20)$$

Using the inequalities (2.17)-(2.20) in (2.16) and the fact

$$\int_0^1\left(\frac{t}{2}\right)^p dt=\frac{1}{2^p}\frac{1}{p+1},$$

we get inequality (2.15).
This completes the proof of the theorem. □

Corollary 6. *Suppose all the conditions of Theorem 7 are satisfied and if* $x = \frac{a+b}{2}$, *then we have the inequality:*

$$\begin{aligned}
&\left| f\left(\frac{a+b}{2}\right) + \frac{f(a)+f(b)}{2} - \frac{2}{b-a}\int_a^b f(u)\,du \right| \\
&\leq \left(\frac{b-a}{8}\right)\left\{ \left[\left|f'\left(\frac{a+b}{2}\right)\right|^q + \left|f'\left(\frac{3a+b}{4}\right)\right|^q\right]^{\frac{1}{q}} + \left[|f'(a)|^q + \left|f'\left(\frac{3a+b}{4}\right)\right|^q\right]^{\frac{1}{q}} \right. \\
&\quad \left. + \left[\left|f'\left(\frac{a+b}{2}\right)\right|^q + \left|f'\left(\frac{a+3b}{4}\right)\right|^q\right]^{\frac{1}{q}} + \left[|f'(b)|^q + \left|f'\left(\frac{a+3b}{4}\right)\right|^q\right]^{\frac{1}{q}} \right\} \\
&\leq \left(\frac{b-a}{8}\right)\left\{ \left[\left(\frac{1}{2}\right)^s + \left(\frac{3}{4}\right)^s\right]^{\frac{1}{q}} + \left[1 + \left(\frac{3}{4}\right)^s\right]^{\frac{1}{q}} \right. \\
&\quad \left. + \left[\left(\frac{1}{2}\right)^s + \left(\frac{1}{4}\right)^s\right]^{\frac{1}{q}} + \left(\frac{1}{4}\right)^{\frac{s}{q}} \right\} \left[|f'(a)| + |f'(b)|\right] \qquad (2.21)
\end{aligned}$$

Proof. It follows directly from Theorem 7 and using similar arguments as that of the Corollary 5 and Remark 4. □

Theorem 8. *Let* $f : I \subseteq \mathbb{R} \to \mathbb{R}$ *be a differentiable function on* I° *such that* $f' \in L[a,b]$, *where* $a, b \in I$ *with* $a < b$. *If* $|f'|^q$ *is s-concave on* $[a,b]$ *for some fixed* $q > 1$, $s \in (0,1]$, *then the following inequality holds:*

$$\begin{aligned}
&\left| f(x) + \frac{(b-x)f(b) + (x-a)f(a)}{b-a} - \frac{2}{b-a}\int_a^b f(u)\,du \right| \leq \left(\frac{1}{p+1}\right)^{\frac{1}{p}} 2^{\frac{s-1}{q}-1} \\
&\left[\left|f'\left(\frac{3x+a}{4}\right)\right| + \left|f'\left(\frac{x+3a}{4}\right)\right| + \left|f'\left(\frac{3x+b}{4}\right)\right| + \left|f'\left(\frac{x+3b}{4}\right)\right| \right], \qquad (2.22)
\end{aligned}$$

for all $x \in [a,b]$ *and* $\frac{1}{p} + \frac{1}{q} = 1$.

Proof. From Lemma 1 and using the well-known Hölder integral inequality, we have

$$\begin{aligned}
&\left| f(x) + \frac{(b-x)f(b) + (x-a)f(a)}{b-a} - \frac{2}{b-a}\int_a^b f(u)\,du \right| \\
&\leq \frac{(x-a)^2}{b-a}\left(\int_0^1 \left(\frac{t}{2}\right)^p dt\right)^{\frac{1}{p}} \left(\int_0^1 \left|f'\left(\frac{1+t}{2}x + \frac{1-t}{2}a\right)\right|^q dt\right)^{\frac{1}{q}} \\
&+ \frac{(x-a)^2}{b-a}\left(\int_0^1 \left(\frac{t}{2}\right)^p dt\right)^{\frac{1}{p}} \left(\int_0^1 \left|f'\left(\frac{1-t}{2}x + \frac{1+t}{2}a\right)\right|^q dt\right)^{\frac{1}{q}} \\
&+ \frac{(b-x)^2}{b-a}\left(\int_0^1 \left(\frac{t}{2}\right)^p dt\right)^{\frac{1}{p}} \left(\int_0^1 \left|f'\left(\frac{1+t}{2}x + \frac{1-t}{2}b\right)\right|^q dt\right)^{\frac{1}{q}} \\
&+ \frac{(b-x)^2}{b-a}\left(\int_0^1 \left(\frac{t}{2}\right)^p dt\right)^{\frac{1}{p}} \left(\int_0^1 \left|f'\left(\frac{1-t}{2}x + \frac{1+t}{2}b\right)\right|^q dt\right)^{\frac{1}{q}}, \qquad (2.23)
\end{aligned}$$

for all $x \in [a,b]$.
Since $|f'|^q$ is s-concave on $[a,b]$, by using the Hermite-Hadamard type inequality (1.2), we have

$$\int_0^1 \left| f'\left(\frac{1+t}{2}x+\frac{1-t}{2}a\right)\right|^q dt \le 2^{s-1}\left|f'\left(\frac{3x+a}{4}\right)\right|^q. \tag{2.24}$$

Similarly,

$$\int_0^1 \left| f'\left(\frac{1-t}{2}x+\frac{1+t}{2}a\right)\right|^q dt \le 2^{s-1}\left|f'\left(\frac{x+3a}{4}\right)\right|^q, \tag{2.25}$$

$$\int_0^1 \left| f'\left(\frac{1+t}{2}x+\frac{1-t}{2}b\right)\right|^q dt \le 2^{s-1}\left|f'\left(\frac{3x+b}{4}\right)\right|^q \tag{2.26}$$

and

$$\int_0^1 \left| f'\left(\frac{1-t}{2}x+\frac{1+t}{2}b\right)\right|^q dt \le 2^{s-1}\left|f'\left(\frac{x+3b}{4}\right)\right|^q. \tag{2.27}$$

By making use of the inequalities (2.24)-(2.27) in (2.23), we get (2.22).
Hence the proof of the theorem is complete. □

Corollary 7. *Suppose all the conditions of Theorem 8 are satisfied. If we choose $x=\frac{a+b}{2}$ and assume that $|f'|$ is a linear map, then we have the inequality:*

$$\left|f\left(\frac{a+b}{2}\right)+\frac{f(a)+f(b)}{2}-\frac{2}{b-a}\int_a^b f(u)\,du\right| \le \left(\frac{1}{p+1}\right)^{\frac{1}{p}} 2^{\frac{s-1}{q}}\left[\left|f'(a+b)\right|\right]. \tag{2.28}$$

Proof. It is a direct consequence of Theorem 8. □

Theorem 9. *Let $f: I \subseteq \mathbb{R} \to \mathbb{R}$ be a differentiable function on I° such that $f' \in L[a,b]$, where $a,b \in I$ with $a<b$. If $|f'|^q$ is s-convex on $[a,b]$ for some fixed $q \ge 11$, $s \in (0,1]$, then the following inequality holds:*

$$\begin{aligned}\left|f(x)+\frac{(b-x)f(b)+(x-a)f(a)}{b-a}-\frac{2}{b-a}\int_a^b f(u)\,du\right| &\le \left(\frac{1}{4}\right)^{1-\frac{1}{q}} \\ \times\left(\frac{1}{(s+1)(s+2)}\right)^{\frac{1}{q}}\Bigg\{\frac{(x-a)^2}{b-a}\Big[&\left(\left(s+2^{-s-1}\right)|f'(x)|^q+2^{-s-1}|f'(a)|^q\right)^{\frac{1}{q}} \\ &+\left(2^{-s-1}|f'(x)|^q+\left(s+2^{-s-1}\right)|f'(a)|^q\right)^{\frac{1}{q}}\Big] \\ +\frac{(b-x)^2}{b-a}\Big[&\left(\left(s+2^{-s-1}\right)|f'(x)|^q+2^{-s-1}|f'(b)|^q\right)^{\frac{1}{q}} \\ &+\left(2^{-s-1}|f'(x)|^q+\left(s+2^{-s-1}\right)|f'(b)|^q\right)^{\frac{1}{q}}\Big]\Bigg\},\end{aligned} \tag{2.29}$$

for all $x \in [a,b]$.

Proof. From Lemma 1 and using the well-known power-mean integral inequality, we have

$$\begin{aligned}
&\left| f(x)+\frac{(b-x)f(b)+(x-a)f(a)}{b-a}-\frac{2}{b-a}\int_a^b f(u)\,du\right| \\
&\le \frac{(x-a)^2}{b-a}\left(\int_0^1 \frac{t}{2}dt\right)^{\frac{1}{q}-1}\left(\int_0^1 \frac{t}{2}\left|f'\left(\frac{1+t}{2}x+\frac{1-t}{2}a\right)\right|^q dt\right)^{\frac{1}{q}} \\
&+\frac{(x-a)^2}{b-a}\left(\int_0^1 \frac{t}{2}dt\right)^{\frac{1}{q}-1}\left(\int_0^1 \frac{t}{2}\left|f'\left(\frac{1-t}{2}x+\frac{1+t}{2}a\right)\right|^q dt\right)^{\frac{1}{q}} \\
&+\frac{(b-x)^2}{b-a}\left(\int_0^1 \frac{t}{2}dt\right)^{\frac{1}{q}-1}\left(\int_0^1 \frac{t}{2}\left|f'\left(\frac{1+t}{2}x+\frac{1-t}{2}b\right)\right|^q dt\right)^{\frac{1}{q}} \\
&+\frac{(b-x)^2}{b-a}\left(\int_0^1 \frac{t}{2}dt\right)^{\frac{1}{q}-1}\left(\int_0^1 \frac{t}{2}\left|f'\left(\frac{1-t}{2}x+\frac{1+t}{2}b\right)\right|^q dt\right)^{\frac{1}{q}}, \qquad (2.30)
\end{aligned}$$

for all $x\in[a,b]$.
Since $|f'|^q$ is s-convex on $[a,b]$, we have

$$\begin{aligned}
&\int_0^1 \frac{t}{2}\left|f'\left(\frac{1+t}{2}x+\frac{1-t}{2}a\right)\right|^q dt \\
&\le |f'(x)|^q\int_0^1 \frac{t}{2}\left(\frac{1+t}{2}\right)^s dt+|f'(a)|^q\int_0^1 \frac{t}{2}\left(\frac{1-t}{2}\right)^s dt \\
&=\frac{s+2^{-s-1}}{(s+1)(s+2)}|f'(x)|^q+\frac{2^{-s-1}}{(s+1)(s+2)}|f'(a)|^q. \qquad (2.31)
\end{aligned}$$

Similarly,

$$\int_0^1 \left|f'\left(\frac{1-t}{2}x+\frac{1+t}{2}a\right)\right|^q dt\le \frac{2^{-s-1}}{(s+1)(s+2)}|f'(x)|^q+\frac{s+2^{-s-1}}{(s+1)(s+2)}|f'(a)|^q, \qquad (2.32)$$

$$\int_0^1 \left|f'\left(\frac{1+t}{2}x+\frac{1-t}{2}b\right)\right|^q dt\le \frac{s+2^{-s-1}}{(s+1)(s+2)}|f'(x)|^q+\frac{2^{-s-1}}{(s+1)(s+2)}|f'(b)|^q \qquad (2.33)$$

and

$$\int_0^1 \left|f'\left(\frac{1-t}{2}x+\frac{1+t}{2}b\right)\right|^q dt\le \frac{2^{-s-1}}{(s+1)(s+2)}|f'(x)|^q+\frac{s+2^{-s-1}}{(s+1)(s+2)}|f'(a)|^q. \qquad (2.34)$$

Using the inequalities (2.31)-(2.34) in (2.30) and the fact

$$\int_0^1 \frac{t}{2}dt=\frac{1}{4},$$

we get inequality (2.29).
This completes the proof of the theorem. □

Corollary 8. *Suppose all the conditions of Theorem 7 are satisfied. If we choose* $x=\frac{a+b}{2}$ *and using similar arguments as in Corollary 5, we have the inequalities:*

$$\left|f(x)+\frac{(b-x)f(b)+(x-a)f(a)}{b-a}-\frac{2}{b-a}\int_a^b f(u)\,du\right|\leq\left(\frac{1}{4}\right)^{1-\frac{1}{q}}$$
$$\times\left(\frac{1}{(s+1)(s+2)}\right)^{\frac{1}{q}}\left(\frac{b-a}{4}\right)\left\{\left[\left(s+2^{-s-1}\right)\left|f'\left(\frac{a+b}{2}\right)\right|^q+2^{-s-1}\left|f'(a)\right|^q\right]^{\frac{1}{q}}\right.$$
$$+\left[2^{-s-1}\left|f'\left(\frac{a+b}{2}\right)\right|^q+\left(s+2^{-s-1}\right)\left|f'(a)\right|^q\right]^{\frac{1}{q}}$$
$$+\left[\left[\left(s+2^{-s-1}\right)\left|f'\left(\frac{a+b}{2}\right)\right|^q+2^{-s-1}\left|f'(b)\right|^q\right]^{\frac{1}{q}}\right.$$
$$\left.+\left[2^{-s-1}\left|f'\left(\frac{a+b}{2}\right)\right|^q+\left(s+2^{-s-1}\right)\left|f'(b)\right|^q\right]^{\frac{1}{q}}\right\}$$
$$\leq\left(\frac{1}{4}\right)^{1-\frac{1}{q}}\left(\frac{1}{(s+1)(s+2)}\right)^{\frac{1}{q}}\left(\frac{b-a}{4}\right)\left[\left(2^{-s}s+2^{-2s-1}+2^{-s-1}\right)^{\frac{1}{q}}\right.$$
$$\left.\left(2^{-2s-1}+s+2^{-s-1}\right)^{\frac{1}{q}}+\left(2^{-s}s+2^{-2s-1}\right)^{\frac{1}{q}}+2^{\frac{-2s-1}{q}}\right]\left[\left|f'(a)\right|+\left|f'(b)\right|\right],\quad(2.35)$$

Remark 5. *If we take* $s=1$ *in Theorem 9 and Corollary 8, we get the inequalities (1.7) and (1.8) respectively.*

3. Applications to Special Means

In [8], the following example is given:

Let $s\in(0,1)$ and $a,b,c\in\mathbb{R}$. We define function $f:[0,\infty)\to\mathbb{R}$ as

$$f(t)=\begin{cases}a, & t=0,\\ bt^s+c, & t>0.\end{cases}$$

If $b\geq 0$ and $0\leq c\leq a$, then $f\in K_s^2$. Hence for $b=1$ and $a=c=0$, we have $f:[0,1]\to[0,1]$, $f(t)=t^s$, $f\in K_s^2$.

Now, using the results of Section 2, we give some applications to special means of real numbers.

We shall consider the means for arbitrary real numbers a,b $(a\neq b)$. We take

(1) The arithmetic mean:

$$A(a,b)=\frac{a+b}{2};\ a,\,b\in\mathbb{R}.$$

(2) Generalized log-mean:

$$L_n(a,b)=\left[\frac{b^{n+1}-a^{n+1}}{(n+1)(b-a)}\right]^{\frac{1}{n}};\ a,\,b\in\mathbb{R},\ n\in\mathbb{Z}\backslash\{-1,0\},\ a\neq b.$$

Therefore, by considering the s-convex mapping $f:[0,1]\to[0,1], f(x)=x^s$, $s\in(0,1)$, the following results hold:

Proposition 1. *Let* $a,\,b\in(0,1)$ *with* $a<b$ *and* $0<s<1$. *Then, we have*

$$\left|A^s(a,b)+A(a^s,b^s)-2L_s^s(a,b)\right|\leq\frac{s\left(s+2^{-s}\right)(b-a)}{(s+1)(s+2)}A\left(|a|^{s-1},|b|^{s-1}\right).\quad(3.1)$$

Proof. It follows from Corollary 4 when applied to the s-convex function $f:[0,1]\to[0,1]$, $f(x)=x^s$, $s\in(0,1)$. □

Proposition 2. *Let $a,\ b\in(0,1)$ with $a<b$ and $0<s<1$. Then for $q>1$, we have*

$$\begin{aligned}
&\left|A^s(a,b)+A(a^s,b^s)-2L_s^s(a,b)\right| \\
&\leq s\left(\frac{b-a}{8}\right)\Bigg\{\left(\left(2-2^{-s}\right)\left|\frac{a+b}{2}\right|^{q(s-1)}+2^{-s}|a|^{q(s-1)}\right)^{\frac{1}{q}} \\
&\quad+\left(2^{-s}\left|\frac{a+b}{2}\right|^{q(s-1)}+\left(2-2^{-s}\right)|a|^{q(s-1)}\right)^{\frac{1}{q}} \\
&\quad+\left(\left(2-2^{-s}\right)\left|\frac{a+b}{2}\right|^{q(s-1)}+2^{-s}|b|^{q(s-1)}\right)^{\frac{1}{q}} \\
&\quad+\left(2^{-s}\left|\frac{a+b}{2}\right|^{q(s-1)}+\left(2-2^{-s}\right)|b|^{q(s-1)}\right)^{\frac{1}{q}}\Bigg\} \\
&\leq s\left(\frac{b-a}{4}\right)\Big[2^{-\frac{2s}{q}}+\left(2^{1-s}-2^{-2s}\right)^{\frac{1}{q}}+\left(2^{-2s}-2^{-s}+2\right)^{\frac{1}{q}} \\
&\quad+\left(2^{1-s}-2^{-2s}+2^{-s}\right)^{\frac{1}{q}}\Big]A\left(|a|^{s-1},|b|^{s-1}\right). \qquad (3.2)
\end{aligned}$$

Proof. The assertion follows from Remark 4 when applied to the s-convex function $f:[0,1]\to[0,1]$, $f(x)=x^s$, $s\in(0,1)$. □

Proposition 3. *Let $a,\ b\in(0,1)$ with $a<b$ and $0<s<1$. Then for $q>1$, we have*

$$\begin{aligned}
&\left|A^s(a,b)+A(a^s,b^s)-2L_s^s(a,b)\right| \\
&\leq s\left(\frac{b-a}{8}\right)\Bigg\{\left[\left|\frac{a+b}{2}\right|^{q(s-1)}+\left|\frac{3a+b}{4}\right|^{q(s-1)}\right]^{\frac{1}{q}}+\left[|a|^{q(s-1)}+\left|\frac{3a+b}{4}\right|^{q(s-1)}\right]^{\frac{1}{q}} \\
&\quad+\left[\left|\frac{a+b}{2}\right|^{q(s-1)}+\left|\frac{a+3b}{4}\right|^{q(s-1)}\right]^{\frac{1}{q}}+\left[|b|^{q(s-1)}+\left|\frac{a+3b}{4}\right|^{q(s-1)}\right]^{\frac{1}{q}}\Bigg\} \\
&\leq s\left(\frac{b-a}{4}\right)\Bigg\{\left[\left(\frac{1}{2}\right)^s+\left(\frac{3}{4}\right)^s\right]^{\frac{1}{q}}+\left[1+\left(\frac{3}{4}\right)^s\right]^{\frac{1}{q}} \\
&\quad+\left[\left(\frac{1}{2}\right)^s+\left(\frac{1}{4}\right)^s\right]^{\frac{1}{q}}+\left(\frac{1}{4}\right)^{\frac{s}{q}}\Bigg\}A\left(|a|^{s-1},|b|^{s-1}\right) \qquad (3.3)
\end{aligned}$$

Proof. The assertion follows from Corollary 6 when applied to the s-convex function $f:[0,1]\to[0,1]$, $f(x)=x^s$, $s\in(0,1)$. □

Proposition 4. *Let $a,\ b \in (0,1)$ with $a < b$ and $0 < s < 1$. Then for $q \geq 1$, we have*

$$\left|A^{s}\left(a,b\right)+A\left(a^{s},b^{s}\right)-2L_{s}^{s}\left(a,b\right)\right| \leq s\left(\frac{1}{4}\right)^{1-\frac{1}{q}}$$
$$\times\left(\frac{1}{\left(s+1\right)\left(s+2\right)}\right)^{\frac{1}{q}}\left(\frac{b-a}{4}\right)\left\{\left[\left(s+2^{-s-1}\right)\left|\frac{a+b}{2}\right|^{q(s-1)}+2^{-s-1}\left|a\right|^{q(s-1)}\right]^{\frac{1}{q}}\right.$$
$$+\left[2^{-s-1}\left|\frac{a+b}{2}\right|^{q(s-1)}+\left(s+2^{-s-1}\right)\left|a\right|^{q(s-1)}\right]^{\frac{1}{q}}$$
$$+\left[\left[\left(s+2^{-s-1}\right)\left|\frac{a+b}{2}\right|^{q(s-1)}+2^{-s-1}\left|b\right|^{q(s-1)}\right]^{\frac{1}{q}}\right.$$
$$\left.+\left[2^{-s-1}\left|\frac{a+b}{2}\right|^{q(s-1)}+\left(s+2^{-s-1}\right)\left|b\right|^{q(s-1)}\right]^{\frac{1}{q}}\right\}$$
$$\leq s\left(\frac{1}{4}\right)^{1-\frac{1}{q}}\left(\frac{1}{\left(s+1\right)\left(s+2\right)}\right)^{\frac{1}{q}}\left(\frac{b-a}{2}\right)\left[\left(2^{-s}s+2^{-2s-1}+2^{-s-1}\right)^{\frac{1}{q}}\right.$$
$$\left.\left(2^{-2s-1}+s+2^{-s-1}\right)^{\frac{1}{q}}+\left(2^{-s}s+2^{-2s-1}\right)^{\frac{1}{q}}+2^{\frac{-2s-1}{q}}\right]A\left(\left|a\right|^{s-1},\left|b\right|^{s-1}\right), \quad (3.4)$$

Proof. The assertion follows from Corollary 8 when applied to the s-convex function $f:[0,1]\to[0,1]$, $f(x)=x^{s}$, $s\in(0,1)$. □

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