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NEW HADAMARD-TYPE INEQUALITIES FOR $m-$CONVEX AND $(\alpha, m)-$CONVEX FUNCTIONS

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Abstract. In this paper some new inequalities are presented related to right hand side of Hermite-Hadamard inequality for the classes of functions whose derivatives of absolute values are $m-$convex and $(\alpha, m)-$convex.

1. INTRODUCTION

The function $f : [a, b] \to \mathbb{R}$, is said to be convex, if we have

$$f(tx + (1-t)y) \le tf(x) + (1-t)f(y)$$

for all $x, y \in [a, b]$ and $t \in [0, 1]$. We can define starshaped functions on $[0, b]$ which satisfy the condition

$$f(tx) \le tf(x)$$

for $t \in [0, 1]$. The classical Hermite-Hadamard inequality gives us an estimate of the mean value of a convex function $f : I \subseteq \mathbb{R} \to \mathbb{R}$ which is well-known in the literature as following;

$$f\left(\frac{a+b}{2}\right) \le \frac{1}{b-a}\int_a^b f(x)dx \le \frac{f(a)+f(b)}{2}.$$

Several results related to right hand side and left hand side of the above inequality have been proved in recent papers. In [17], Dragomir and Agarwal proved following inequality for convex functions;

Theorem 1. *Let $f : I \subseteq \mathbb{R} \to \mathbb{R}$, be a differentiable mapping on I^0 and $a, b \in I$, where $a < b$. If $|f'|^q$ is convex on $[a, b]$, then the following inequality holds;*

$$\left|\frac{f(a)+f(b)}{2} - \frac{1}{b-a}\int_a^b f(x)dx\right| \le \frac{(b-a)(|f'(a)| + |f'(b)|)}{8}. \tag{1.1}$$

In [2], Pearce and Pečarić proved following inequalities for convex functions;

Theorem 2. *Let $f : I \subseteq \mathbb{R} \to \mathbb{R}$, be a differentiable mapping on I^0 and $a, b \in I$, where $a < b$. If $|f'|^q$ is convex on $[a, b]$ for some $q \ge 1$, then*

$$\left|\frac{f(a)+f(b)}{2} - \frac{1}{b-a}\int_a^b f(x)dx\right| \le \frac{b-a}{4}\left(\frac{|f'(a)|^q + |f'(b)|^q}{2}\right)^{\frac{1}{q}} \tag{1.2}$$

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and

$$\left| f\left(\frac{a+b}{2}\right) - \frac{1}{b-a}\int_a^b f(x)dx \right| \leq \frac{b-a}{4}\left(\frac{|f'(a)|^q + |f'(b)|^q}{2}\right)^{\frac{1}{q}}. \tag{1.3}$$

The concept of $m-$convexity has been introduced by Toader in [7], an intermediate between the ordinary convexity and starshaped property, as following:

Definition 1. *The function $f : [0, b] \to \mathbb{R}$, $b > 0$, is said to be $m-$convex, where $m \in [0, 1]$, if we have*

$$f(tx + m(1-t)y) \leq tf(x) + m(1-t)f(y)$$

for all $x, y \in [0, b]$ and $t \in [0, 1]$. We say that f is $m-$concave if $-f$ is $m-$convex.

Several papers have been written on $m-$convex functions and we refer the papers [3], [4], [5], [7], [9], [10], [11], [13], [14] and [15]. In [5], Dragomir and Toader proved following inequality for $m-$convex functions.

Theorem 3. *Let $f : [0, \infty) \to \mathbb{R}$ be a $m-$convex function with $m \in (0, 1]$. If $0 \leq a < b < \infty$ and $f \in L_1[a, b]$, then one has the inequality:*

$$\frac{1}{b-a}\int_a^b f(x)\,dx \leq \min\left\{\frac{f(a) + mf\left(\frac{b}{m}\right)}{2}, \frac{f(b) + mf\left(\frac{a}{m}\right)}{2}\right\}. \tag{1.4}$$

In [11], Dragomir established following inequalities of Hadamard-type similar to above.

Theorem 4. *Let $f : [0, \infty) \to \mathbb{R}$ be a $m-$convex function with $m \in (0, 1]$. If $0 \leq a < b < \infty$ and $f \in L_1[a, b]$, then one has the inequality:*

$$\begin{aligned} f\left(\frac{a+b}{2}\right) &\leq \frac{1}{b-a}\int_a^b \frac{f(x) + mf\left(\frac{x}{m}\right)}{2}dx \\ &\leq \frac{m+1}{4}\left[\frac{f(a) + f(b)}{2} + m\frac{f\left(\frac{a}{m}\right) + f\left(\frac{b}{m}\right)}{2}\right]. \end{aligned} \tag{1.5}$$

Theorem 5. *Let $f : [0, \infty) \to \mathbb{R}$ be a $m-$convex function with $m \in (0, 1]$. If $f \in L_1[am, b]$ where $0 \leq a < b < \infty$, then one has the inequality:*

$$\frac{1}{m+1}\left[\int_a^{mb} f(x)dx + \frac{mb-a}{b-ma}\int_{ma}^b f(x)dx\right] \leq (mb-a)\frac{f(a) + f(b)}{2}. \tag{1.6}$$

In [6], Miheşan gave definition of $(\alpha, m)-$convexity as following;

Definition 2. *The function $f : [0, b] \to \mathbb{R}$, $b > 0$ is said to be $(\alpha, m)-$convex, where $(\alpha, m) \in [0, 1]^2$, if we have*

$$f(tx + m(1-t)y) \leq t^\alpha f(x) + m(1-t^\alpha)f(y)$$

for all $x, y \in [0, b]$ and $t \in [0, 1]$.

Denote by $K_m^\alpha(b)$ the class of all $(\alpha, m)-$convex functions on $[0, b]$ for which $f(0) \leq 0$. If we choose $(\alpha, m) = (1, m)$, it can be easily seen that $(\alpha, m)-$convexity reduces to $m-$convexity and for $(\alpha, m) = (1, 1)$, we have ordinary convex functions on $[0, b]$. For the recent results based on the above definition see the papers [3], [4], [8], [12], [14] and [16].

In this paper, we prove some new Hadamard-type inequalities for functions whose derivatives of absolute values are $m-$convex and $(\alpha, m)-$convex functions.

2. THE NEW RESULTS FOR $m-$CONVEX FUNCTIONS

To prove our main results, we use following Lemma which was used by Alomari *et al.* (see [1]) to prove Hadamard-type inequalities for quasi-convex functions.

Lemma 1. *Let $f : I \subseteq \mathbb{R} \to \mathbb{R}$, be a differentiable mapping on I where $a, b \in I$, with $a < b$. Let $f' \in L[a,b]$, then the following equality holds;*

$$\begin{aligned} & \frac{f(a)+f(b)}{2} - \frac{1}{b-a}\int_a^b f(x)dx \\ = & \frac{b-a}{4}\left[\int_0^1 (-t)\, f'\left(\frac{1+t}{2}a + \frac{1-t}{2}b\right)dt + \int_0^1 t f'\left(\frac{1+t}{2}b + \frac{1-t}{2}a\right)dt\right]. \end{aligned}$$

Theorem 6. *Let $f : I \subseteq [0,\infty) \to \mathbb{R}$, be a differentiable mapping on I^0 such that $f' \in L[a,b]$. If $|f'|$ is $m-$convex on $[a,b]$ for some fixed $m \in (0,1]$, then the following inequality holds;*

$$\begin{aligned} & \left|\frac{f(a)+f(b)}{2} - \frac{1}{b-a}\int_a^b f(x)dx\right| \\ \leq & \frac{b-a}{4}\left[\frac{5\left(|f'(a)| + |f'(b)|\right) + m\left(\left|f'\left(\frac{a}{m}\right)\right| + \left|f'\left(\frac{b}{m}\right)\right|\right)}{12}\right]. \end{aligned} \tag{2.1}$$

Proof. From Lemma 1 and by using the properties of modulus, we have

$$\begin{aligned} & \left|\frac{f(a)+f(b)}{2} - \frac{1}{b-a}\int_a^b f(x)dx\right| \\ \leq & \frac{b-a}{4}\left[\int_0^1 |-t|\left|f'\left(\frac{1+t}{2}a + \frac{1-t}{2}b\right)\right|dt + \int_0^1 |t|\left|f'\left(\frac{1+t}{2}b + \frac{1-t}{2}a\right)\right|dt\right]. \end{aligned} \tag{2.2}$$

Since $|f'|$ is $m-$convex on $[a,b]$, we can write

$$\left|f'\left(\frac{1+t}{2}a + \frac{1-t}{2}b\right)\right| \leq \frac{1+t}{2}|f'(a)| + m\frac{1-t}{2}\left|f'\left(\frac{b}{m}\right)\right| \tag{2.3}$$

and

$$\left|f'\left(\frac{1+t}{2}b + \frac{1-t}{2}a\right)\right| \leq \frac{1+t}{2}|f'(b)| + m\frac{1-t}{2}\left|f'\left(\frac{a}{m}\right)\right| \tag{2.4}$$

for any $t\in[0,1]$. Therefore, by using the inequalities (2.3) and (2.4) in (2.2), we obtain

$$\left|\frac{f(a)+f(b)}{2}-\frac{1}{b-a}\int_a^b f(x)dx\right|$$
$$\leq \frac{b-a}{4}\left[\int_0^1 |-t|\left(\frac{1+t}{2}\left|f'(a)\right|+m\frac{1-t}{2}\left|f'\left(\frac{b}{m}\right)\right|\right)dt\right.$$
$$\left.+\int_0^1 |t|\left(\frac{1+t}{2}\left|f'(b)\right|+m\frac{1-t}{2}\left|f'\left(\frac{a}{m}\right)\right|\right)dt\right].$$

By calculating the above integrals, we get the desired result. □

Remark 1. *If we choose $m=1$ in (2.1), we obtain the inequality (1.1).*

Corollary 1. *If we choose $m=1$ and $|f'|\leq M$ in (2.1), we obtain a version of the inequality (1.2) as following;*

$$\left|\frac{f(a)+f(b)}{2}-\frac{1}{b-a}\int_a^b f(x)dx\right|\leq M\frac{b-a}{4}.$$

Theorem 7. *Let $f:I\subseteq[0,\infty)\to\mathbb{R}$, be a differentiable mapping on I^0 such that $f'\in L[a,b]$. If $|f'|^q$ is $m-$convex on $[a,b]$ for some fixed $m\in(0,1]$ and $q>1$, $0\leq p\leq q$, then the following inequality holds;*

(2.5)
$$\left|\frac{f(a)+f(b)}{2}-\frac{1}{b-a}\int_a^b f(x)dx\right|$$
$$\leq \left(\frac{b-a}{4}\right)\left(\frac{q-1}{2q-p-1}\right)^{\frac{q-1}{q}}\left[\left(\frac{2p+3}{2(p+1)(p+2)}\left|f'(a)\right|^q+\frac{m}{2(p+1)(p+2)}\left|f'\left(\frac{b}{m}\right)\right|^q\right)^{\frac{1}{q}}\right.$$
$$\left.+\left(\frac{2p+3}{2(p+1)(p+2)}\left|f'(b)\right|^q+\frac{m}{2(p+1)(p+2)}\left|f'\left(\frac{a}{m}\right)\right|^q\right)^{\frac{1}{q}}\right].$$

Proof. From Lemma 1 and by using the properties of modulus, we have

$$\left|\frac{f(a)+f(b)}{2}-\frac{1}{b-a}\int_a^b f(x)dx\right|$$
$$\leq \frac{b-a}{4}\left[\int_0^1 |-t|\left|f'\left(\frac{1+t}{2}a+\frac{1-t}{2}b\right)\right|dt+\int_0^1 |t|\left|f'\left(\frac{1+t}{2}b+\frac{1-t}{2}a\right)\right|dt\right].$$

By applying the Hölder inequality for $q > 1$, $0 \leq p \leq q$, we get

$$\left| \frac{f(a)+f(b)}{2} - \frac{1}{b-a} \int_a^b f(x)dx \right|$$
$$\leq \frac{b-a}{4} \left[\left(\int_0^1 t^{\frac{q-p}{q-1}} dt \right)^{\frac{q-1}{q}} \left(\int_0^1 t^p \left| f'\left(\frac{1+t}{2} a + \frac{1-t}{2} b \right) \right|^q dt \right)^{\frac{1}{q}} \right.$$
$$\left. + \left(\int_0^1 t^{\frac{q-p}{q-1}} dt \right)^{\frac{q-1}{q}} \left(\int_0^1 t^p \left| f'\left(\frac{1+t}{2} b + \frac{1-t}{2} a \right) \right|^q dt \right)^{\frac{1}{q}} \right].$$

It is to see that

$$\int_0^1 t^{\frac{q-p}{q-1}} dt = \frac{q-1}{2q-p-1}.$$

Hence, by $m-$convexity of $|f'|^q$ on $[a,b]$, we obtain the inequality (2.5). □

Corollary 2. *Under the assumptions of Theorem 7, if we choose $p = m = 1$, we obtain the inequality;*

$$\left| \frac{f(a)+f(b)}{2} - \frac{1}{b-a} \int_a^b f(x)dx \right|$$
$$\leq \left(\frac{b-a}{4} \right) \left(\frac{1}{2} \right)^{\frac{q-1}{q}} \left[\left(\frac{5}{12} |f'(a)|^q + \frac{1}{12} |f'(b)|^q \right)^{\frac{1}{q}} \right.$$
$$\left. + \left(\frac{5}{12} |f'(b)|^q + \frac{1}{12} |f'(a)|^q \right)^{\frac{1}{q}} \right].$$

3. THE NEW RESULTS FOR $(\alpha, m)-$CONVEX FUNCTIONS

Theorem 8. *Let $f : I \subseteq [0, \infty) \to \mathbb{R}$, be a differentiable mapping on I^0 such that $f' \in L[a,b]$. If $|f'|$ is $(\alpha, m)-$convex on $[a,b]$ for some fixed $(\alpha, m) \in (0,1]^2$, then the following inequality holds;*

$$\left| \frac{f(a)+f(b)}{2} - \frac{1}{b-a} \int_a^b f(x)dx \right| \tag{3.1}$$
$$\leq \frac{b-a}{4} \left[\frac{2^{\alpha+1}\alpha + 1}{2^\alpha (\alpha+2)(\alpha+1)} \left(|f'(a)| + |f'(b)| \right) \right.$$
$$\left. + m \frac{2^{\alpha-1}(\alpha+2)(\alpha+1) - 1}{2^\alpha (\alpha+2)(\alpha+1)} \left(\left| f'\left(\frac{a}{m} \right) \right| + \left| f'\left(\frac{b}{m} \right) \right| \right) \right].$$

Proof. From Lemma 1 and by using the properties of modulus, we can write

$$\left|\frac{f(a)+f(b)}{2}-\frac{1}{b-a}\int_a^b f(x)dx\right| \tag{3.2}$$
$$\leq \frac{b-a}{4}\left[\int_0^1 |-t|\left|f'\left(\frac{1+t}{2}a+\frac{1-t}{2}b\right)\right|dt+\int_0^1 |t|\left|f'\left(\frac{1+t}{2}b+\frac{1-t}{2}a\right)\right|dt\right].$$

Since $|f'|$ is $(\alpha,m)-$convex on $[a,b]$, we have

$$\left|f'\left(\frac{1+t}{2}a+\frac{1-t}{2}b\right)\right| \leq \left(\frac{1+t}{2}\right)^{\alpha}|f'(a)|+m\left(1-\left(\frac{1-t}{2}\right)^{\alpha}\right)\left|f'\left(\frac{b}{m}\right)\right| \tag{3.3}$$

and

$$\left|f'\left(\frac{1+t}{2}b+\frac{1-t}{2}a\right)\right| \leq \left(\frac{1+t}{2}\right)^{\alpha}|f'(b)|+m\left(1-\left(\frac{1-t}{2}\right)^{\alpha}\right)\left|f'\left(\frac{a}{m}\right)\right| \tag{3.4}$$

for some fixed $t\in[0,1]$ and $(\alpha,m)\in[0,1]^2$. By using the inequalities (3.3) and (3.4) in (3.2), we get

$$\left|\frac{f(a)+f(b)}{2}-\frac{1}{b-a}\int_a^b f(x)dx\right|$$
$$\leq \frac{b-a}{4}\left[\int_0^1 t\left(\left(\frac{1+t}{2}\right)^{\alpha}|f'(a)|+m\left(1-\left(\frac{1-t}{2}\right)^{\alpha}\right)\left|f'\left(\frac{b}{m}\right)\right|\right)dt\right.$$
$$\left.+\int_0^1 t\left(\left(\frac{1+t}{2}\right)^{\alpha}|f'(b)|+m\left(1-\left(\frac{1-t}{2}\right)^{\alpha}\right)\left|f'\left(\frac{a}{m}\right)\right|\right)dt\right].$$

By a simple computation, we obtain

$$\left|\frac{f(a)+f(b)}{2}-\frac{1}{b-a}\int_a^b f(x)dx\right|$$
$$\leq \frac{b-a}{4}\left[\frac{2^{\alpha+1}\alpha+1}{2^{\alpha}(\alpha+2)(\alpha+1)}|f'(a)|+m\frac{2^{\alpha-1}(\alpha+2)(\alpha+1)-1}{2^{\alpha}(\alpha+2)(\alpha+1)}\left|f'\left(\frac{b}{m}\right)\right|\right.$$
$$\left.+\frac{2^{\alpha+1}\alpha+1}{2^{\alpha}(\alpha+2)(\alpha+1)}|f'(b)|+m\frac{2^{\alpha-1}(\alpha+2)(\alpha+1)-1}{2^{\alpha}(\alpha+2)(\alpha+1)}\left|f'\left(\frac{a}{m}\right)\right|\right]$$

which is the inequality (3.1). □

Corollary 3. *Under the assumptions of Theorem 8, if we choose $\alpha=m=1$, we obtain the inequality;*

$$\left|\frac{f(a)+f(b)}{2}-\frac{1}{b-a}\int_a^b f(x)dx\right| \leq \frac{b-a}{4}\left[\frac{5}{6}|f'(a)|+\frac{5}{6}|f'(b)|\right].$$

Theorem 9. *Let $f : I \subseteq [0, \infty) \to \mathbb{R}$, be a differentiable mapping on I^0 such that $f' \in L[a, b]$. If $|f'|^{\frac{p}{p-1}}$ is $(\alpha, m)-$convex on $[a, b]$ for some fixed $(\alpha, m) \in (0, 1]^{2}2$ and $p > 1$, then the following inequality holds;*

$$\left|\frac{f(a)+f(b)}{2}-\frac{1}{b-a}\int_a^b f(x)dx\right| \tag{3.5}$$

$$\leq \frac{b-a}{4}\left(\frac{1}{p+1}\right)^{\frac{1}{p}}\left(\frac{2^{\alpha+1}-1}{2^{\alpha}(\alpha+1)}\right)^{\frac{1}{q}}\left[\left(\left|f'(a)\right|^q+m\left|f'\left(\frac{b}{m}\right)\right|^q\right)^{\frac{1}{q}}\right.$$

$$\left.+\left(\left|f'(b)\right|^q+m\left|f'\left(\frac{a}{m}\right)\right|^q\right)^{\frac{1}{q}}\right].$$

where $\frac{1}{q}+\frac{1}{p}=1$.

Proof. From Lemma 1, we can write

$$\left|\frac{f(a)+f(b)}{2}-\frac{1}{b-a}\int_a^b f(x)dx\right|$$

$$\leq \frac{b-a}{4}\left[\int_0^1 |-t|\left|f'\left(\frac{1+t}{2}a+\frac{1-t}{2}b\right)\right|dt+\int_0^1 |t|\left|f'\left(\frac{1+t}{2}b+\frac{1-t}{2}a\right)\right|dt\right].$$

By applying the Hölder inequality for $q > 1$, we get

$$\left|\frac{f(a)+f(b)}{2}-\frac{1}{b-a}\int_a^b f(x)dx\right|$$

$$\leq \frac{b-a}{4}\left[\left(\int_0^1 t^p dt\right)^{\frac{1}{p}}\left(\int_0^1\left|f'\left(\frac{1+t}{2}a+\frac{1-t}{2}b\right)\right|^q dt\right)^{\frac{1}{q}}\right.$$

$$\left.+\left(\int_0^1 t^p dt\right)^{\frac{1}{p}}\left(\int_0^1\left|f'\left(\frac{1+t}{2}b+\frac{1-t}{2}a\right)\right|^q dt\right)^{\frac{1}{q}}\right].$$

Now by using $(\alpha, m)-$convexity of $|f'|^{\frac{p}{p-1}}$ on $[a, b]$ and by computing the integrals, we obtain the inequality (3.5). □

Corollary 4. *Under the assumptions of Theorem 9, if we choose $\alpha = m = 1$, we obtain the inequality;*

$$\left|\frac{f(a)+f(b)}{2}-\frac{1}{b-a}\int_a^b f(x)dx\right| \leq \frac{b-a}{2}\left(\frac{1}{p+1}\right)^{\frac{1}{p}}\left[\left(\frac{3}{4}\left(\left|f'(a)\right|^q+\left|f'(b)\right|^q\right)\right)^{\frac{1}{q}}\right].$$

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