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SEVERAL INTEGRAL INEQUALITIES ON TIME SCALES VIA ISOTONIC LINEAR FUNCTIONALS

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ABSTRACT. The aim of this paper is to establish some extensions of Minkowski's inequality and Qi's inequality to isotonic linear functionals taking into account that the time scale Cauchy delta, Cauchy nabla, α-diamond, multiple Riemann, and multiple Lebesque integrals all are isotonic linear functionals. Several applications of these results for other particular isotonic linear functionals will be also obtaned.

1. Introduction

In this paper we adopt the notations from the monograph [4] of Bohner and Peterson. For further information concerning time scales, see [4]. The following results will be useful below in order to establish the main results of this paper, and can be found in [4], in [15], in [3] and in [7].

Many classical inequalities are proved in the monograph [11] for the so-called isotonic linear functionals and using the fact that the time-scales integral is an isotonic linear functional, we can use these results to these integrals. Also in [8] appear other usual examples of isotonic linear functionals that are normalised. Therefore the results from this paper which take place for such functionals can be rewritten for these particular examples. Moreover, as applications, Gruss type inequality and some of its refinements which take place for normalised isotonic linear functionals will be true for the time scale Cauchy delta, Cauchy nabla, α-diamond, multiple Riemann, and multiple Lebesque integrals.

We need also to recall the following results which will be used below.

Lemma 1. *([9], Corollary 3.3) If f is Δ- integrable on $[a,b)$ then for an arbitrary positive number α the function $|f|^{\alpha}$ is Δ-integrable on $[a,b)$.*

Lemma 2. *([9], Theorem 3.6) Let f and g be Δ-integrable functions on $[a,b)$. then their product fg is Δ-integrable on $[a,b)$.*

The following definition is given in [3], [7] and it is necessary to recall it here.

Definition 1. *Let E be a nonempty set and L be a class of real-valued functions $f : E \to \mathbb{R}$ having the following properties:*

(L1) If $f, g \in L$ and $a, b \in \mathbb{R}$, then $(af + bg) \in L$.

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(L2) $\mathbf{1} \in L$ *i.e. if* $f(t) = 1$ *for all* $t \in E$, *then* $f \in L$.

An isotonic linear functional is a functional $A : L \to \mathbb{R}$ *having the following properties:*

(A1) If $f, g \in L$ *and* $a, b \in \mathbb{R}$, *then* $A(af + bg) = aA(f) + bA(g)$.

(A2) If $f \in L$ *and* $f(t) \geq 0$ *for all* $t \in E$, *then* $A(f) \geq 0$.

The mapping A is said to be normalised if

(A3) $A(\mathbf{1}) = 1$.

Now we will recall Holder's inequality for isotonic linear functionals as it appears in [11].

Theorem 1. *([3]) Let* E, L, *and A such that (L1), (L2), (A1) and (A2) are satisfied. For* $p \neq 1$, *define* $q = \frac{p}{p-1}$. *Assume* $|w||f|^p$, $|w||g|^q$, $|wfg| \in L$. *If* $p > 1$, *then*

$$A(|wfg|) \leq A^{\frac{1}{p}}(|w||f|^p)A^{\frac{1}{q}}(|w||g|^q). \tag{4}$$

Then inequality is reversed if $0 < p < 1$ *and* $A(|w||g|^q) > 0$, *and it is also reversed if* $p < 0$ *and* $A(|w||f|^p) > 0$.

We enunciate Theorem 2.2 from [1], in the case of these functionals when $p > 1$ and then the same kind of theorem, Theorem 1 from [2] in the case of these functionals when $0 < p < 1$. The proof of the result from below is given in [6].

Theorem 2. *Let* $1 < p < \infty$ *and let* $q = \frac{p}{p-1}$ *be its conjugate exponent, L satisfy conditions L1, L2 and A satisfy A1, A2 on the set E. If* $|f|^p$, $|g|^q$, $|fg|$, $|f|^{\frac{p}{2}}|g|^{\frac{q}{2}} \in L$, $A(|f|^p) > 0$, $A(|g|^q) > 0$ *and if* $1 < p \leq 2$, *then*

$$A^{\frac{1}{p}}(|f|^p)A^{\frac{1}{q}}(|g|^q)\left(1 - \frac{1}{p}A\left[\left(\frac{|f|^{\frac{p}{2}}}{A^{\frac{1}{2}}(|f|^p)} - \frac{|g|^{\frac{q}{2}}}{A^{\frac{1}{2}}(|g|^q)}\right)^2\right]\right)_+ \leq A(|fg|) \leq$$

$$\leq A^{\frac{1}{p}}(|f|^p)A^{\frac{1}{q}}(|g|^q)\left(1 - \frac{1}{q}A\left[\left(\frac{|f|^{\frac{p}{2}}}{A^{\frac{1}{2}}(|f|^p)} - \frac{|g|^{\frac{q}{2}}}{A^{\frac{1}{2}}(|g|^q)}\right)^2\right]\right) \tag{5}$$

while if $2 \leq p < \infty$, *the terms* $\frac{1}{p}$ *and* $\frac{1}{q}$ *exchange their positions in the preceeding inequalities.*

2. **Subdividing of Holder's inequality for isotonic linear functionals**

In this section, we give some refinements of Holder's inequalty and of subdividing of Holder's inequality in the case when $0 < r < 1$ and Minkowski's inequality for isotonic linear functionals and as applications the corresponding inequalities for on time scales Cauchy delta, Cauchy nabla, α-diamond integrals.

Theorem 3. *Let* $0 < r < 1$ *and let* $s = \frac{r}{r-1}$ *be its conjugate exponent, L satisfies conditions L1, L2 and A satisfies A1, A2 on the set E.*

If $hk,\ k^s,\ h^r \in L$ are nonnegative functions, $A(hk) > 0,\ A(k^s) > 0,\ A(h^r) > 0$ and if $\frac{1}{2} < r < 1$, then

$$A(hk)\left[1 - rA\left[\left(\frac{h^{\frac{1}{2}}k^{\frac{1}{2}}}{A^{\frac{1}{2}}(hk)} - \frac{k^{\frac{s}{2}}}{A^{\frac{1}{2}}(k^s)}\right)^2\right]\right]_+^{\frac{1}{r}} \le A^{\frac{1}{r}}(h^r)A^{\frac{1}{s}}(k^s) \le$$

$$\le A(hk)\left[1 - (1-r)A\left[\left(\frac{h^{\frac{1}{2}}k^{\frac{1}{2}}}{A^{\frac{1}{2}}(hk)} - \frac{k^{\frac{s}{2}}}{A^{\frac{1}{2}}(k^s)}\right)^2\right]\right]^{\frac{1}{r}} \tag{6}$$

while if $0 \le r < \frac{1}{2}$, the terms r and $1-r$ exchange their positions in the preceeding inequalities.

Proof. Suppose $\frac{1}{2} \le r < 1$ and take $p = \frac{1}{r}$. Because $1 < p \le 2$ we can apply inequality (5) to the functions $f = h^r k^r$ and $g = k^{-r}$, obtaining:

$$A^r(hk)A^{1-r}(k^s)\left(1 - rA\left[\left(\frac{(hk)^{\frac{1}{2}}}{A^{\frac{1}{2}}(hk)} - \frac{k^{\frac{s}{2}}}{A^{\frac{1}{2}}(k^s)}\right)^2\right]\right)_+ \le A(h^r) \le$$

$$\le A^r(hk)A^{1-r}(k^s)\left(1 - (1-r)A\left[\left(\frac{(hk)^{\frac{1}{2}}}{A^{\frac{1}{2}}(hk)} - \frac{k^{\frac{s}{2}}}{A^{\frac{1}{2}}(k^s)}\right)^2\right]\right).$$

Then the inequality (6) is obvious. As in [2], if $0 < r \le \frac{1}{2}$ then $2 \le p < \infty$ and it is enough to interchange $\frac{1}{p}$ and $\frac{1}{q}$ in inequality (5).

∎

Theorem 4. *Let $0 < r < 1$ and L satisfying conditions L1, L2 and A satisfying A1, A2 on the set E. Considering the nonnegative functions h, w with $h^r,\ w^r,\ (h+w)^r,\ h^{\frac{1}{2}}(h+w)^{\frac{r}{2}},\ w^{\frac{1}{2}}(h+w)^{\frac{r}{2}} \in L$, $A(h^r) > 0,\ A(w^r) > 0,\ A(w^{\frac{1}{2}}(h+w)^{\frac{r}{2}}) > 0$, $A(h^{\frac{1}{2}}(h+w)^{\frac{r}{2}}) > 0$ and*

$$k = \frac{(h+w)^{r-1}}{A^{\frac{1}{s}}((h+w)^{s(r-1)})}$$

we have

$$A^{\frac{1}{r}}((h+w)^r) \ge A^{\frac{1}{r}}(h^r)\left[1 - (1-r)A\left[\left(\frac{h^{\frac{1}{2}}k^{\frac{1}{2}}}{A^{\frac{1}{2}}(hk)} - \frac{k^{\frac{s}{2}}}{A^{\frac{1}{2}}(k^s)}\right)^2\right]\right]^{-\frac{1}{r}} +$$

$$+A^{\frac{1}{r}}(w^r)\left[1 - (1-r)A\left[\left(\frac{w^{\frac{1}{2}}k^{\frac{1}{2}}}{A^{\frac{1}{2}}(wk)} - \frac{k^{\frac{s}{2}}}{A^{\frac{1}{2}}(k^s)}\right)^2\right]\right]^{-\frac{1}{r}}, \tag{7}$$

if $\frac{1}{2} \le r < 1$ and if $0 < r \le \frac{1}{2}$ then the same inequality takes place but with $1-r$ replaced by r.

Proof. When $\frac{1}{2} \leq r < 1$, taking into account hypothesis, as in [2], we notice that

$$A^{\frac{1}{r}}((h+w)^r) = A\left(\frac{(h+w)^{r-1}}{A^{\frac{1}{s}}((h+w)^{s(r-1)})}(h+w)\right) = A(hk) + A(wk) \geq$$

$$\geq A^{\frac{1}{r}}(h^r)\left[1-(1-r)A\left[\left(\frac{h^{\frac{1}{2}}k^{\frac{1}{2}}}{A^{\frac{1}{2}}(hk)} - \frac{k^{\frac{s}{2}}}{A^{\frac{1}{2}}(k^s)}\right)^2\right]\right]^{-\frac{1}{r}} +$$

$$+A^{\frac{1}{r}}(w^r)\left[1-(1-r)A\left[\left(\frac{w^{\frac{1}{2}}k^{\frac{1}{2}}}{A^{\frac{1}{2}}(wk)} - \frac{k^{\frac{s}{2}}}{A^{\frac{1}{2}}(k^s)}\right)^2\right]\right]^{-\frac{1}{r}}$$

by inequality (6).

∎

Theorem 5. *Let $1 < p < \infty$ and L satisfying conditions L1, L2 and A satisfying A1, A2 on the set E. Considering the nonnegative functions f, h with f^p, h^p, $(f+h)^{\frac{p}{2}}f^{\frac{p}{2}}$, $(f+h)^{\frac{p}{2}}h^{\frac{p}{2}}$, $(f+h)^{p-1}f$, $(f+h)^{p-1}h \in L$ and $A(f^p) > 0$, $A(h^p) > 0$, $A((f+h)^p) > 0$ we have*

$$A^{\frac{1}{p}}((f+h)^p) \leq A^{\frac{1}{p}}(f^p)\left[1-\frac{1}{q}A\left[\left(\frac{(f+h)^{\frac{p}{2}}}{A^{\frac{1}{2}}((f+h)^p)} - \frac{f^{\frac{p}{2}}}{A^{\frac{1}{2}}(f^p)}\right)^2\right]\right] +$$

$$+A^{\frac{1}{p}}(h^p)\left[1-\frac{1}{q}A\left[\left(\frac{(f+h)^{\frac{p}{2}}}{A^{\frac{1}{2}}((f+h)^p)} - \frac{h^{\frac{p}{2}}}{A^{\frac{1}{2}}(h^p)}\right)^2\right]\right], \tag{8}$$

when $1 < p \leq 2$, while if $2 \leq p < \infty$ then the same inequality takes place but with $\frac{1}{p}$ replaced by $\frac{1}{q}$.

Proof. We suppose $1 < p \leq 2$ and by applying inequality (5) from Theorem 2 first time for $f$and $\frac{(f+h)^{p-1}}{A^{\frac{1}{q}}((f+h)^{q(p-1)})}$ and then for h and $\frac{(f+h)^{p-1}}{A^{\frac{1}{q}}((f+h)^{q(p-1)})}$ we will obtain:

$$A^{\frac{1}{p}}((f+h)^p) = A\left(\frac{(f+h)^{p-1}}{A^{\frac{1}{q}}((f+h)^{q(p-1)})}(f+h)\right) =$$

$$= A\left(\frac{(f+h)^{p-1}}{A^{\frac{1}{q}}((f+h)^{q(p-1)})}f)\right) + A\left(\frac{(f+h)^{p-1}}{A^{\frac{1}{q}}((f+h)^{q(p-1)})}h)\right) \leq$$

$$\leq A^{\frac{1}{p}}(f^p)A^{\frac{1}{q}}\left(\frac{(f+h)^{q(p-1)}}{A((f+h)^{q(p-1)})}\right)\left[1-\frac{1}{q}A\left[\left(\frac{(f+h)^{\frac{p}{2}}}{A^{\frac{1}{2}}((f+h)^p)} - \frac{f^{\frac{p}{2}}}{A^{\frac{1}{2}}(f^p)}\right)^2\right]\right] +$$

$$+A^{\frac{1}{p}}(h^p)A^{\frac{1}{q}}\left(\frac{(f+h)^{q(p-1)}}{A((f+h)^{q(p-1)})}\right)\left[1-\frac{1}{q}A\left[\left(\frac{(f+h)^{\frac{p}{2}}}{A^{\frac{1}{2}}((f+h)^p)} - \frac{h^{\frac{p}{2}}}{A^{\frac{1}{2}}(h^p)}\right)^2\right]\right] =$$

$$= A^{\frac{1}{p}}(f^p)\left[1-\frac{1}{q}A\left[\left(\frac{(f+h)^{\frac{p}{2}}}{A^{\frac{1}{2}}((f+h)^p)} - \frac{f^{\frac{p}{2}}}{A^{\frac{1}{2}}(f^p)}\right)^2\right]\right] +$$

$$+A^{\frac{1}{p}}(h^p)\left[1-\frac{1}{q}A\left[\left(\frac{(f+h)^{\frac{p}{2}}}{A^{\frac{1}{2}}((f+h)^p)} - \frac{h^{\frac{p}{2}}}{A^{\frac{1}{2}}(h^p)}\right)^2\right]\right].$$

∎

Theorem 6. *Let $s,t \in \mathbb{R}$, $p = \frac{s-t}{1-t}$, and $q = \frac{s-t}{s-1}$ such that $s > t > 1$ or $s < t < 1$; $t > s > 1$ or $t < s < 1$, and L satisfies conditions L1, L2 and A satisfies conditions A1, A2 on the set E. If $fg,\ (fg)^t,\ (fg)^s,\ (fg)^{\frac{t+1}{2}},\ fg^{q+1},\ (fg^{q+1})^{\frac{t}{2}},\ f^{sp},\ f^{tp},\ g^{sq},\ g^{tq} \in L$, $A(fg) > 0,\ A(f^{sp}) > 0,\ A(g^{sq}) > 0,\ A(f^{tp}) > 0,\ A(g^{tq}) > 0,\ A((fg)^t) > 0,\ A((fg)^s) > 0$ and f,g are positive functions then*

$$A(fg)\left[1-\frac{2}{\min\{\frac{1}{p},\frac{1}{1-p}\}}\left(1-\frac{A((fg)^{\frac{t+1}{2}})}{A^{\frac{1}{2}}(fg)A^{\frac{1}{2}}((fg)^t)}\right)\right]_+^{\frac{1}{p}}\cdot$$
$$\cdot\left[1-\frac{2}{\min\{\frac{1}{p},\frac{1}{1-p}\}}\left(1-\frac{A(f^{\frac{s}{2}}g^{s\frac{q+1}{2}})}{A^{\frac{1}{2}}((fg)^s)A^{\frac{1}{2}}(g^{sq})}\right)\right]_+^{\frac{1}{p^2}}\cdot$$
$$\cdot\left[1-\frac{2}{\min\{\frac{1}{p},\frac{1}{1-p}\}}\left(1-\frac{A(f^{\frac{t}{2}}g^{t\frac{q+1}{2}})}{A^{\frac{1}{2}}((fg)^t)A^{\frac{1}{2}}(g^{tq})}\right)\right]_+^{\frac{1}{pq}}\le$$
$$\le A^{\frac{1}{p^2}}(f^{sp})A^{\frac{1}{pq}}(g^{sq})A^{\frac{1}{pq}}(f^{tp})A^{\frac{1}{q^2}}(g^{tq}),$$

and

$$A^{\frac{1}{p^2}}(f^{sp})A^{\frac{1}{pq}}(g^{sq})A^{\frac{1}{pq}}(f^{tp})A^{\frac{1}{q^2}}(g^{tq})\le$$
$$\le A(fg)\left[1-\frac{2}{\max\{\frac{1}{p},\frac{1}{1-p}\}}\left(1-\frac{A((fg)^{\frac{t+1}{2}})}{A^{\frac{1}{2}}(fg)A^{\frac{1}{2}}((fg)^t)}\right)\right]^{\frac{1}{p}}$$
$$\cdot\left[1-\frac{2}{\max\{\frac{1}{p},\frac{1}{1-p}\}}\left(1-\frac{A(f^{\frac{s}{2}}g^{s\frac{q+1}{2}})}{A^{\frac{1}{2}}((fg)^s)A^{\frac{1}{2}}(g^{sq})}\right)\right]^{\frac{1}{p^2}}\cdot$$
$$\cdot\left[1-\frac{2}{\max\{\frac{1}{p},\frac{1}{1-p}\}}\left(1-\frac{A(f^{\frac{t}{2}}g^{t\frac{q+1}{2}})}{A^{\frac{1}{2}}((fg)^t)A^{\frac{1}{2}}(g^{tq})}\right)\right]^{\frac{1}{pq}}.$$

Proof. By inequality (3) given in Theorem 3, applied for $p = \frac{s-t}{1-t}$, $q = \frac{s-t}{s-1}$ we have

$$A(fg) = A\left([(fg)^s]^{\frac{1-t}{s-t}}[(fg)^t]^{\frac{s-1}{s-t}}\right)$$

and

$$A^{\frac{1}{p}}((fg)^s)\cdot A^{\frac{1}{q}}((fg)^t) = A^{\frac{1-t}{s-t}}((fg)^s)\cdot A^{\frac{s-1}{s-t}}((fg)^t)\le$$
$$\le A\left([(fg)^s]^{\frac{1-t}{s-t}}[(fg)^t]^{\frac{s-1}{s-t}}\right)\left[1-\frac{2}{\max\{\frac{1}{p},\frac{1}{1-p}\}}\left(1-\frac{A((fg)^{\frac{s}{2p}}(fg)^{\frac{t(q+1)}{2q}})}{A^{\frac{1}{2}}(fg)A^{\frac{1}{2}}((fg)^t)}\right)\right]^{\frac{1}{p}}=$$
$$=A(fg)\left[1-\frac{2}{\max\{\frac{1}{p},\frac{1}{1-p}\}}\left(1-\frac{A((fg)^{\frac{t+1}{2}})}{A^{\frac{1}{2}}(fg)A^{\frac{1}{2}}((fg)^t)}\right)\right]^{\frac{1}{p}}.$$

Applying again Theorem 3 for $0 < \frac{s-t}{1-t} < 1$ or $\frac{s-t}{1-t} < 0$ we get

$$A^{\frac{1-t}{s-t}}(f^{s\frac{s-t}{1-t}})A^{\frac{s-1}{s-t}}(g^{s\frac{s-t}{s-1}})\le A((fg)^s)\cdot\left[1-\frac{2}{\max\{\frac{1}{p},\frac{1}{1-p}\}}\left(1-\frac{A(f^{\frac{s}{2}}g^{s\frac{q+1}{2}})}{A^{\frac{1}{2}}((fg)^s)A^{\frac{1}{2}}(g^{sq})}\right)\right]^{\frac{1}{p}}$$

and

$$A^{\frac{1-t}{s-t}}(f^{t\frac{s-t}{1-t}})A^{\frac{s-1}{s-t}}(g^{t\frac{s-t}{s-1}})\le A((fg)^t)\cdot\left[1-\frac{2}{\max\{\frac{1}{p},\frac{1}{1-p}\}}\left(1-\frac{A(f^{\frac{t}{2}}g^{t\frac{q+1}{2}})}{A^{\frac{1}{2}}((fg)^t)A^{\frac{1}{2}}(g^{tq})}\right)\right]^{\frac{1}{q}}.$$

Taking into account these three inequalities we obtain the desired inequality.

For second inequality we taking into account the first inequality from Theorem 3 and use the same reason like before. ∎

We can also state the following improvement of Minkowski's inequality for isotonic linear functionals giving in this way a refinement of Minkowski's inequality for the delta integral, the nabla integral and the diamond-α integral from [5].

Theorem 7. *Let $p>0$, $s,t\in\mathbb{R}-\{0\}$, and $s\neq t$. We consider $p,s,t\in\mathbb{R}$ different numbers, such that $s,t>1$, $\frac{s-t}{p-t}>1$, L satisfies conditions L1, L2 and A satisfies A1, A2 on the set E. If $f,g\geq 0$ on E with $(f+g)^{\frac{s+t}{2}}$, $(f+g)^s$, $(f+g)^t$, f^s, g^s, f^t, g^t, $f^{\frac{s}{2}}(f+g)^{\frac{s}{2}}$, $f^{\frac{t}{2}}(f+g)^{\frac{t}{2}}$, $g^{\frac{t}{2}}(f+g)^{\frac{t}{2}}$, $g^{\frac{s}{2}}(f+g)^{\frac{s}{2}}\in L$ and $A(f^s)>0$, $A(f^t)>0$, $A(g^s)>0$, $A(g^t)>0$, $A((f+g)^s)>0$, $A((f+g)^t)>0$ then*

$$A((f+g)^p)\leq\left[1-2\frac{s-p}{s-t}\left(1-\frac{A((f+g)^{\frac{s+t}{2}})}{A^{\frac{1}{2}}((f+g)^s)A^{\frac{1}{2}}((f+g)^t)}\right)\right]$$

$$\cdot\{A^{\frac{1}{s}}(f^s)\left[1-\frac{s-1}{s}A\left[\left(\frac{(f+g)^{\frac{s}{2}}}{A^{\frac{1}{2}}((f+g)^s)}-\frac{f^{\frac{s}{2}}}{A^{\frac{1}{2}}(f^s)}\right)^2\right]\right]+$$

$$+A^{\frac{1}{s}}(g^s)\left[1-\frac{s-1}{s}A\left[\left(\frac{(f+g)^{\frac{s}{2}}}{A^{\frac{1}{2}}((f+g)^s)}-\frac{g^{\frac{s}{2}}}{A^{\frac{1}{2}}(g^s)}\right)^2\right]\right]\}^{\frac{s(p-t)}{s-t}}\cdot$$

$$\cdot\{A^{\frac{1}{t}}(f^t)\left[1-\frac{t-1}{t}A\left[\left(\frac{(f+g)^{\frac{t}{2}}}{A^{\frac{1}{2}}((f+g)^t)}-\frac{f^{\frac{t}{2}}}{A^{\frac{1}{2}}(f^t)}\right)^2\right]\right]+$$

$$+A^{\frac{1}{t}}(g^t)\left[1-\frac{t-1}{t}A\left[\left(\frac{(f+g)^{\frac{t}{2}}}{A^{\frac{1}{2}}((f+g)^t)}-\frac{g^{\frac{t}{2}}}{A^{\frac{1}{2}}(g^t)}\right)^2\right]\right]\}^{\frac{t(s-p)}{s-t}},$$

when $1<s\leq 2$ and $1<t\leq 2$, while if $2\leq s<\infty$ and $2\leq t<\infty$ then the same inequality holds, but with $\frac{s-1}{s}$ replacing $\frac{1}{s}$ throughout and $\frac{t-1}{t}$ replacing $\frac{1}{t}$ throughout.

Proof. We will use first inequality (5) from Theorem 2 and then the inequality (8) from Theorem 5. For inequality (5) we use indices $\frac{s-t}{p-t}>1$ and $\frac{s-t}{s-p}$ obtaining:

$$A((f+g)^p)=A((f+g)^{s\frac{p-t}{s-t}}(f+g)^{t\frac{s-p}{s-t}})\leq$$

$$\leq A^{\frac{p-t}{s-t}}((f+g)^s)A^{\frac{s-p}{s-t}}((f+g)^t)\left[1-2\frac{s-p}{s-t}\left(1-\frac{A((f+g)^{\frac{s+t}{2}})}{A^{\frac{1}{2}}((f+g)^s)A^{\frac{1}{2}}((f+g)^t)}\right)\right].$$

We use first time $s>1$ for inequality (8) and the second time $t>1$, obtaining:

$$A^{\frac{1}{s}}((f+g)^s)\leq A^{\frac{1}{s}}(f^s)\left[1-\frac{s-1}{s}A\left[\left(\frac{(f+g)^{\frac{s}{2}}}{A^{\frac{1}{2}}((f+g)^s)}-\frac{f^{\frac{s}{2}}}{A^{\frac{1}{2}}(f^s)}\right)^2\right]\right]+$$

$$+A^{\frac{1}{s}}(g^s)\left[1-\frac{s-1}{s}A\left[\left(\frac{(f+g)^{\frac{s}{2}}}{A^{\frac{1}{2}}((f+g)^s)}-\frac{g^{\frac{s}{2}}}{A^{\frac{1}{2}}(g^s)}\right)^2\right]\right]$$

and

$$A^{\frac{1}{t}}((f+g)^t) \leq A^{\frac{1}{t}}(f^t)\left[1-\frac{t-1}{t}A\left[\left(\frac{(f+g)^{\frac{t}{2}}}{A^{\frac{1}{2}}((f+g)^t)}-\frac{f^{\frac{t}{2}}}{A^{\frac{1}{2}}(f^t)}\right)^2\right]\right]+$$

$$+A^{\frac{1}{t}}(g^t)\left[1-\frac{t-1}{t}A\left[\left(\frac{(f+g)^{\frac{t}{2}}}{A^{\frac{1}{2}}((f+g)^t)}-\frac{g^{\frac{t}{2}}}{A^{\frac{1}{2}}(g^t)}\right)^2\right]\right].$$

These three inequalities completes the proof, taking into account the hypotheses of the theorem.

∎

Consequence 1. *(i) Let $f,g \in C_{rd}([a,b],\mathbf{R})$, $p>0$, $s,t \in \mathbf{R}-\{0\}$ and $s \neq t$. We consider $p,s,t \in \mathbf{R}$ three different numbers such that $s,t<1$ and $s,t \neq 0$ and $\frac{s-t}{p-t}<1$. Then we have,*

$$\int_a^b (f(x)+g(x))^p \Delta x \leq \left[1-2\frac{s-p}{s-t}\left(1-\frac{\int_a^b (f(x)+g(x))^{\frac{s+t}{2}}\Delta x}{(\int_a^b (f(x)+g(x))^s \Delta x \int_a^b (f(x)+g(x))^t \Delta x)^{\frac{1}{2}}}\right)\right]$$

$$\cdot\{(\int_a^b f^s(x)\Delta x)^{\frac{1}{s}}\left[1-2\frac{s-1}{s}\left(1-\frac{\int_a^b (f(x)+g(x))^{\frac{s}{2}} f^{\frac{s}{2}}(x)\Delta x}{(\int_a^b (f(x)+g(x))^s \Delta x \int_a^b f^s(x)\Delta x)^{\frac{1}{2}}}\right)\right]+$$

$$+(\int_a^b g^s(x)\Delta x)^{\frac{1}{s}}\left[1-2\frac{s-1}{s}\left(1-\frac{\int_a^b (f(x)+g(x))^{\frac{s}{2}} g^{\frac{s}{2}}(x)\Delta x}{(\int_a^b (f(x)+g(x))^s \Delta x \int_a^b g^s(x)\Delta x)^{\frac{1}{2}}}\right)\right]\}^{\frac{s(p-t)}{s-t}}\cdot$$

$$\cdot\{(\int_a^b f^t(x)\Delta x)^{\frac{1}{t}}\left[1-2\frac{t-1}{t}\left(1-\frac{\int_a^b (f(x)+g(x))^{\frac{t}{2}} f^{\frac{t}{2}}(x)\Delta x}{(\int_a^b (f(x)+g(x))^t \Delta x \int_a^b f^t(x)\Delta x)^{\frac{1}{2}}}\right)\right]+$$

$$+(\int_a^b g^t(x)\Delta x)^{\frac{1}{t}}\left[1-2\frac{t-1}{t}\left(1-\frac{\int_a^b (f(x)+g(x))^{\frac{t}{2}} g^{\frac{t}{2}}(x)\Delta x}{(\int_a^b (f(x)+g(x))^t \Delta x \int_a^b g^t(x)\Delta x)^{\frac{1}{2}}}\right)\right]\}^{\frac{t(s-p)}{s-t}},$$

when $1<s\leq 2$ and $1<t\leq 2$, while if $2\leq s<\infty$ and $2\leq t<\infty$ then the same inequality holds, but with $\frac{s-1}{s}$ replacing $\frac{1}{s}$ throughout and $\frac{t-1}{t}$ replacing $\frac{1}{t}$ throughout.

(ii) Let $f,g \in C_{ld}([a,b],\mathbf{R})$, $p>0$, $s,t \in \mathbf{R}-\{0\}$ and $s \neq t$. We consider $p,s,t \in \mathbf{R}$ three different numbers such that $s,t<1$ and $s,t \neq 0$ and $\frac{s-t}{p-t}<1$. Then we have,

$$\int_a^b (f(x)+g(x))^p \nabla x \leq \left[1-2\frac{s-p}{s-t}\left(1-\frac{\int_a^b (f(x)+g(x))^{\frac{s+t}{2}}\nabla x}{(\int_a^b (f(x)+g(x))^s \nabla x \int_a^b (f(x)+g(x))^t \nabla x)^{\frac{1}{2}}}\right)\right]$$

$$\cdot\{(\int_a^b f^s(x)\nabla x)^{\frac{1}{s}}\left[1-2\frac{s-1}{s}\left(1-\frac{\int_a^b (f(x)+g(x))^{\frac{s}{2}} f^{\frac{s}{2}}(x)\nabla x}{(\int_a^b (f(x)+g(x))^s \nabla x \int_a^b f^s(x)\nabla x)^{\frac{1}{2}}}\right)\right]+$$

$$+(\int_a^b g^s(x)\nabla x)^{\frac{1}{s}}\left[1-2\frac{s-1}{s}\left(1-\frac{\int_a^b (f(x)+g(x))^{\frac{s}{2}} g^{\frac{s}{2}}(x)\nabla x}{(\int_a^b (f(x)+g(x))^s \nabla x \int_a^b g^s(x)\nabla x)^{\frac{1}{2}}}\right)\right]\}^{\frac{s(p-t)}{s-t}}\cdot$$

$$\cdot\{(\int_a^b f^t(x)\nabla x)^{\frac{1}{t}}\left[1-2\frac{t-1}{t}\left(1-\frac{\int_a^b (f(x)+g(x))^{\frac{t}{2}} f^{\frac{t}{2}}(x)\nabla x}{(\int_a^b (f(x)+g(x))^t \nabla x \int_a^b f^t(x)\nabla x)^{\frac{1}{2}}}\right)\right]+$$

$$+(\int_a^b g^t(x)\nabla x)^{\frac{1}{t}}\left[1-2\frac{t-1}{t}\left(1-\frac{\int_a^b (f(x)+g(x))^{\frac{t}{2}} g^{\frac{t}{2}}(x)\nabla x}{(\int_a^b (f(x)+g(x))^t\nabla x \int_a^b g^t(x)\nabla x)^{\frac{1}{2}}}\right)\right]\}^{\frac{t(s-p)}{s-t}},$$

when $1 < s \leq 2$ and $1 < t \leq 2$, while if $2 \leq s < \infty$ and $2 \leq t < \infty$ then the same inequality holds, but with $\frac{s-1}{s}$ replacing $\frac{1}{s}$ throughout and $\frac{t-1}{t}$ replacing $\frac{1}{t}$ throughout.

(iii) Let $f, g : [a, b] \to \mathbf{R}$ be $\diamond_\alpha$-integrable functions, $p > 0$, $s, t \in \mathbf{R} - \{0\}$ and $s \neq t$. We consider $p, s, t \in \mathbf{R}$ three different numbers such that $s, t < 1$ and $s, t \neq 0$ and $\frac{s-t}{p-t} < 1$. Then we have,

$$\int_a^b (f(x)+g(x))^p \diamond_\alpha x \leq \left[1-2\frac{s-p}{s-t}\left(1-\frac{\int_a^b (f(x)+g(x))^{\frac{s+t}{2}} \diamond_\alpha x}{(\int_a^b (f(x)+g(x))^s \diamond_\alpha x \int_a^b (f(x)+g(x))^t \diamond_\alpha x)^{\frac{1}{2}}}\right)\right]$$

$$\cdot\{(\int_a^b f^s(x)\diamond_\alpha x)^{\frac{1}{s}}\left[1-2\frac{s-1}{s}\left(1-\frac{\int_a^b (f(x)+g(x))^{\frac{s}{2}} f^{\frac{s}{2}}(x)\diamond_\alpha x}{(\int_a^b (f(x)+g(x))^s \diamond_\alpha x \int_a^b f^s(x)\diamond_\alpha x)^{\frac{1}{2}}}\right)\right]+$$

$$+(\int_a^b g^s(x)\diamond_\alpha x)^{\frac{1}{s}}\left[1-2\frac{s-1}{s}\left(1-\frac{\int_a^b (f(x)+g(x))^{\frac{s}{2}} g^{\frac{s}{2}}(x)\diamond_\alpha x}{(\int_a^b (f(x)+g(x))^s \diamond_\alpha x \int_a^b g^s(x)\diamond_\alpha x)^{\frac{1}{2}}}\right)\right]\}^{\frac{s(p-t)}{s-t}}.$$

$$\cdot\{(\int_a^b f^t(x)\diamond_\alpha x)^{\frac{1}{t}}\left[1-2\frac{t-1}{t}\left(1-\frac{\int_a^b (f(x)+g(x))^{\frac{t}{2}} f^{\frac{t}{2}}(x)\diamond_\alpha x}{(\int_a^b (f(x)+g(x))^t \diamond_\alpha x \int_a^b f^t(x)\diamond_\alpha x)^{\frac{1}{2}}}\right)\right]+$$

$$+(\int_a^b g^t(x)\diamond_\alpha x)^{\frac{1}{t}}\left[1-2\frac{t-1}{t}\left(1-\frac{\int_a^b (f(x)+g(x))^{\frac{t}{2}} g^{\frac{t}{2}}(x)\diamond_\alpha x}{(\int_a^b (f(x)+g(x))^t \diamond_\alpha x \int_a^b g^t(x)\diamond_\alpha x)^{\frac{1}{2}}}\right)\right]\}^{\frac{t(s-p)}{s-t}},$$

when $1 < s \leq 2$ and $1 < t \leq 2$, while if $2 \leq s < \infty$ and $2 \leq t < \infty$ then the same inequality holds, but with $\frac{s-1}{s}$ replacing $\frac{1}{s}$ throughout and $\frac{t-1}{t}$ replacing $\frac{1}{t}$ throughout.

(iv) Inequalities from Theorem 6 and Theorem 7 remain true for examples of normalised isotonic linear functionals given in [8] *if we write again hypothesis of these theorems in these cases.*

(v) Inequalities from Theorem 4 and Theorem 5 remain true for examples of normalised isotonic linear functionals given in [8] *if we write again hypothesis of these theorems in these cases.*

3. Some variants of Qi's inequality for isotonic linear functionals

In this section we give several variants of some inequalities from [12] in the case of isotonic linear functionals for $p > 1$ and $0 < r < 1$ using the corresponding Holder's inequalities.

Lemma 3. *Let E, L and A be such that L1, L2, A1, A2 are satisfied. If f, g, $\frac{f^p}{g^{\frac{p}{q}}}$, $\frac{f^{\frac{p}{2}}}{g^{\frac{p}{2q}}} g^{\frac{1}{2}} \in L$ are positive functions then*

$$\left[1-\min\{\frac{1}{p},\frac{1}{q}\}A\left[\left(\frac{\left(\frac{f^p}{g^{\frac{p}{q}}}\right)^{\frac{1}{2}}}{A^{\frac{1}{2}}\left(\frac{f^p}{g^{\frac{p}{q}}}\right)}-\frac{g^{\frac{1}{2}}}{A^{\frac{1}{2}}(g)}\right)^2\right]\right]^p A\left(\frac{f^p}{g^{\frac{p}{q}}}\right) \geq \frac{A^p(f)}{A^{\frac{p}{q}}(g)},$$

where $p > 1$ or $p < 0$ while $\frac{1}{p}+\frac{1}{q} = 1$.

Theorem 8. *Let E, L and A be such that L1, L2, A1, A2 are satisfied. If $f,\ f^{p+2},\ f^{\frac{p+2}{2}} \in L$, f is positive and $A(f) \geq A^2(\mathbf{1})$ then*

$$A^{p-1}(\mathbf{1})A(f^{p+2})\left[1-\frac{1}{p+2}A\left[\left(\frac{f^{\frac{p+2}{2}}}{A^{\frac{1}{2}}(f^{p+2})}-\frac{\mathbf{1}}{A^{\frac{1}{2}}(\mathbf{1})}\right)^2\right]\right]^{p+2} \geq A^{p+1}(f),$$

takes place for $p > 1$.

Proof. By Lemma 3 and hypothesis we have,

$$A(f^{p+2})\left[1-\frac{1}{p+2}A\left[\left(\frac{f^{\frac{p+2}{2}}}{A^{\frac{1}{2}}(f^{p+2})}-\frac{\mathbf{1}}{A^{\frac{1}{2}}(\mathbf{1})}\right)^2\right]\right]^{p+2} =$$

$$= A\left(\frac{f^{p+2}}{1^{p+1}}\right)\left[1-\frac{1}{p+2}A\left[\left(\frac{f^{\frac{p+2}{2}}}{A^{\frac{1}{2}}(f^{p+2})}-\frac{\mathbf{1}}{A^{\frac{1}{2}}(\mathbf{1})}\right)^2\right]\right]^{p+2} \geq$$

$$\geq \frac{A^{p+2}(f)}{A^{p+1}(\mathbf{1})} = \frac{A^{p+1}(f)A(f)}{A^{p-1}(\mathbf{1})A^2(\mathbf{1})} \geq \frac{A^{p+1}(f).}{A^{p-1}(\mathbf{1})}.$$

∎

As applications, next results present some refinements of some inequalities given by Qi and Yin, [12], in the cases of delta time-scale integral, the Cauchy nabla time-scales integrals and the Cauchy α-diamond time scale integrals.

Remark 1. *(i) Let $a, b \in \mathbb{T}$. If $f \in C_{rd}(\mathbb{T}, \mathbb{R})$ is positive, Δ-differentiable on (a, b) and*

$$\int_a^b f(x)\Delta x \geq (b-a)^2$$

then

$$\int_a^b f^{p+2}(x)\Delta x\left[1-\frac{2}{p+2}\left(1-\frac{\int_a^b f^{\frac{p+2}{2}}(x)\Delta x}{(b-a)^{\frac{1}{2}}(\int_a^b f^{p+2}(x)\Delta x)^{\frac{1}{2}}}\right)\right]^{p+2} \geq$$

$$\geq \frac{1}{(b-a)^{p-1}}\left[\int_a^b f(x)\Delta x\right]^{p+1},$$

where $p > 1$.

(ii) In the case of the Cauchy nabla time-scales integrals and the Cauchy α-diamond time scale integrals similary inequalities can be stated as above.

Lemma 4. *Let E, L and A be such that L1, L2, A1, A2 are satisfied. If $0 < r < 1$, $s = \frac{r}{r-1}$ and f, g, $\frac{f^r}{g^{\frac{r}{s}}}$, $f^{\frac{1}{2}}g^{\frac{1}{2}} \in L$ are positive functions and $A(f) > 0$, $A(g) > 0$ then*

$$A\left(\frac{f^r}{g^{\frac{r}{s}}}\right) \leq \frac{A^r(f)}{A^{\frac{r}{s}}(g)}\left[1-2(1-r)\left(1-\frac{f^{\frac{1}{2}}g^{\frac{1}{2}}}{A^{\frac{1}{2}}(f)A^{\frac{1}{2}}(g)}\right)\right],$$

when $\frac{1}{2} < r < 1$ while if $0 \leq r < \frac{1}{2}$ the terms $1-r$ and r exchange their positions in the preceeding inequalities.

Proof. We apply Holder' s inequality from Theorem 3 when $\frac{1}{2} < r < 1$ and $f,\ g,\ \frac{f^r}{g^{\frac{r}{s}}},\ f^{\frac{1}{2}}g^{\frac{1}{2}} \in L$ are positive functions, obtaining:

$$A(f) = A\left(\frac{f}{g^{\frac{1}{s}}}g^{\frac{1}{s}}\right)$$

and

$$A(f)\left[1-2(1-r)\left(1-\frac{\left(\frac{f}{g^{\frac{1}{s}}}\right)^{\frac{1}{2}}(g^{\frac{1}{s}})^{\frac{s+1}{2}}}{A^{\frac{1}{2}}(f)A^{\frac{1}{2}}(g)}\right)\right]^{\frac{1}{r}} \geq$$

$$\geq A^{\frac{1}{r}}\left(\frac{f^r}{g^{\frac{r}{s}}}\right)A^{\frac{1}{s}}(g)$$

or

$$A^{\frac{1}{r}}\left(\frac{f^r}{g^{\frac{r}{s}}}\right) \leq \frac{A(f)}{A^{\frac{1}{s}}(g)}\left[1-2(1-r)\left(1-\frac{f^{\frac{1}{2}}g^{\frac{1}{2}}}{A^{\frac{1}{2}}(f)A^{\frac{1}{2}}(g)}\right)\right]^{\frac{1}{r}}.$$

Then we take the r-th power on both sides of the inequalities and have:

$$A\left(\frac{f^r}{g^{\frac{r}{s}}}\right) \leq \frac{A^r(f)}{A^{\frac{r}{s}}(g)}\left[1-2(1-r)\left(1-\frac{f^{\frac{1}{2}}g^{\frac{1}{2}}}{A^{\frac{1}{2}}(f)A^{\frac{1}{2}}(g)}\right)\right].$$

∎

Consequence 2. *Let* $a, b \in \mathbb{T}$ *and* $0 < r < 1,\ s = \frac{r}{r-1}$. *If* $f \in C_{rd}(\mathbb{T}, \mathbb{R})$ *is positive then*

$$\int_a^b \frac{f^r(x)}{g^{\frac{r}{s}}(x)}\Delta x \leq \frac{(\int_a^b f(x)\Delta x)^r}{(\int_a^b f(x)\Delta x)^{\frac{r}{s}}}\left[1-2(1-r)\left(1-\frac{f^{\frac{1}{2}}(x)g^{\frac{1}{2}}(x)}{(\int_a^b f(x)\Delta x\int_a^b g(x)\Delta x)^{\frac{1}{2}}}\right)\right]$$

when $\frac{1}{2} < r < 1$ *while if* $0 \leq r < \frac{1}{2}$ *the terms* $1-r$ *and* r *exchange their positions in the preceeding inequalities.*

A variant of Theorem 3.1 from [12] for isotonic linear functional can be the following:

Theorem 9. *Let* E, L *and* A *be such that L1, L2, A1, A2 are satisfied. If* $0 < r < 1,\ s = \frac{r}{r-1},\ f,\ f^r,\ f^{\frac{1}{2}} \in L$, f *is positive,* $A(f) > 0$ *and* $A(f) \leq A^{r-1}(\mathbf{1})$ *then*

$$A(f^r) \leq A^{r-1}(f)\left[1-2(1-r)\left(1-\frac{f^{\frac{1}{2}}}{A^{\frac{1}{2}}(f)A^{\frac{1}{2}}(\mathbf{1})}\right)\right]$$

when $\frac{1}{2} < r < 1$ *while if* $0 \leq r < \frac{1}{2}$ *the terms* $1-r$ *and* r *exchange their positions in the preceeding inequalities.*

Proof. By Lemma 4 and hypothesis we have,

$$A(f^r) = A\left(\frac{f^r}{\mathbf{1}^{r-1}}\right) = A\left(\frac{f^r}{1^{\frac{r}{s}}}\right)$$

$$A\left(\frac{f^r}{\mathbf{1}^{\frac{r}{s}}}\right) \leq \frac{A^r(f)}{A^{\frac{r}{s}}(\mathbf{1})}\left[1-2(1-r)\left(1-\frac{f^{\frac{1}{2}}}{A^{\frac{1}{2}}(f)A^{\frac{1}{2}}(\mathbf{1})}\right)\right] \leq$$

$$\leq A^{r-1}(\mathbf{1})\frac{A^{r-1}(f)}{A^{\frac{r}{s}}(\mathbf{1})}\left[1-2(1-r)\left(1-\frac{f^{\frac{1}{2}}}{A^{\frac{1}{2}}(f)A^{\frac{1}{2}}(\mathbf{1})}\right)\right]=$$

$$=A^{r-1}(f)\left[1-2(1-r)\left(1-\frac{f^{\frac{1}{2}}}{A^{\frac{1}{2}}(f)A^{\frac{1}{2}}(\mathbf{1})}\right)\right],$$

when $\frac{1}{2}<r<1$.
∎

As an application for time scales integrals of previous result we obtain:

Consequence 3. *(i) Let $a,b\in\mathbb{T}$ and $0<r<1$. If $f\in C_{rd}(\mathbb{T},\mathbb{R})$ is positive, and*

$$\int_a^b f(x)\Delta x\leq(b-a)^{r-1}$$

then

$$\int_a^b f^r(x)\Delta x\leq\left(\int_a^b f(x)\Delta x\right)^{r-1}\left[1-2(1-r)\left(\frac{f^{\frac{1}{2}}(x)}{(b-a)^{\frac{1}{2}}(\int_a^b f(x)\Delta x)^{\frac{1}{2}}}\right)\right]$$

when $\frac{1}{2}<r<1$ while if $0\leq r<\frac{1}{2}$ the terms $1-r$ and r exchange their positions in the preceeding inequalities.

(ii) In the case of the Cauchy nabla time-scales integrals and the Cauchy α-diamond time scale integrals similary inequalities can be stated as above.

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