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NEW INTEGRAL INEQUALITIES OF HERMITE-HADAMARD TYPE FOR n-TIMES DIFFERENTIABLE s-LOGARTHIMCALLY CONVEX WITH APPLICATIONS

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Abstract. In this paper, some new integral inequalities of Hermite-Hadamard type are presetned for functions whose nth derivatives in absolute value are s-logarthimically convex. From our results, several inequalities of Hermite-Hadamard type can be derived in terms of functions whose first and second derivatives in absolute value are s-logarthimically convex functions as special cases. Our results may provide refinements of some results for s-logarthimcally convex functions already exist in literature. Finally, applications to special means of the established results are given.

1. Introduction

A function $f : I \to \mathbb{R}$, $\emptyset \neq I \subseteq \mathbb{R}$, is said to be convex on I if the inequality

$$f(tx + (1-t)y) \leq tf(x) + (1-t)f(y),$$

holds for all $x, y \in I$ and $t \in [0,1]$.

Let $f : I \subseteq \mathbb{R} \to \mathbb{R}$ be a convex mapping and $a, b \in I$ with $a < b$. Then

$$f\left(\frac{a+b}{2}\right) \leq \frac{1}{b-a}\int_a^b f(x)dx \leq \frac{f(a)+f(b)}{2}. \tag{1.1}$$

The double inequality (1.1) is known as the Hermite-Hadamard inequality (see [8]). The inequalities (1.1) hold in reversed direction if f is concave.

For recent results on Hermite-Hadamard type integral inequalities for convex functions see [6, 12, 13, 14, 15, 17, 19, 23, 28] and closely related references therein.

The classical convexity has been generalized in diverse ways such as s-convexity, m-convexity, (α, m)-convexity, h-convexity, logarithmically-convexity, s- logarithmically convexity, (α, m)- logarithmically convexity and h-log-convexity. Many papers have been written by a number of mathematicians concerning Hermite-Hadamard type inequalities for these classes of convex functions see for instance the recent papers [2, 3, 4, 7, 8, 9, 16, 18, 24, 25, 27, 29, 31, 32, 33, 35] and the references therein.

The notion of logarithmically convex functions is defined as follows.

Definition 1. [2, 33, 34] *If a function* $f : I \subseteq \mathbb{R} \to (0, \infty)$ *satisfies*

$$f(\lambda x + (1-\lambda)y) \leq [f(x)]^{\lambda}[f(y)]^{1-\lambda}, \tag{1.2}$$

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for all x, $y \in I$, $\lambda \in [0,1]$, the function f is called logarithmically convex on I. If the inequality (1.2) reverses, the function f is called logarithmically concave on I.

The concept of logarithmically convex functions was further generalized as in the definition below.

Definition 2. [2, 33, 34] *For some $s \in (0,1]$, a positive function $f : I \subseteq \mathbb{R} \to (0,\infty)$ is said to be s-logarthmically convex on I if and only if*

$$f(\lambda x + (1-\lambda) y) \leq [f(x)]^{s\lambda} [f(y)]^{s(1-\lambda)}$$

holds for all x, $y \in I$ and $\lambda \in [0,1]$.

It is obvious that when $s = 1$ in Definition 2, the s-logarthmically convexity becomes the usual logarithmically convexity.

Xi et al. [33], obtained the following Hermite-Hadamard type inequalities for s-logarthmically convex functions.

Theorem 1. [33] *Let $f : I \subseteq [0,\infty) \to (0,\infty)$ be a differentiable function on I°, a, $b \in I^{\circ}$ with $a < b$ and $f' \in L([a,b])$. If $|f(x)|^q$ for $q \geq 1$ is s-logarithmically convex on $[a,b]$ for some given $s \in (0,1]$, then*

$$\left| f(a) - \frac{1}{b-a}\int_a^b f(x)\,dx \right| \leq \frac{(b-a)}{4}\left(\frac{1}{2}\right)^{1-1/q}\left\{3^{(q-1)/q}[L_1(\mu,q)]^{1/q} + [L_2(\mu,q,b)]^{1/q}\right\}, \tag{1.3}$$

where

$$L_1(\mu,q) \leq \begin{cases} \left|f'(a)f'(b)\right|^{sq/2} F_1(\mu_1), & 0 < \left|f^{(n)}(a)\right|, \left|f^{(n)}(b)\right| \leq 1, \\ \left|f'(a)f'(b)\right|^{q/(2s)} F_1(\mu_2), & 1 \leq \left|f^{(n)}(a)\right|, \left|f^{(n)}(b)\right|, \\ \left|f'(a)f'(b)\right|^{sq/2} F_1(\mu_3), & 0 < \left|f^{(n)}(a)\right| \leq 1 < \left|f^{(n)}(b)\right|, \\ \left|f'(a)f'(b)\right|^{q/(2s)} F_1(\mu_4), & 0 < \left|f^{(n)}(b)\right| \leq 1 < \left|f^{(n)}(a)\right|, \end{cases}$$

$$L_2(\mu,q,u) \leq \begin{cases} \left|f'(u)\right|^{sq/2} F_1(\mu_1), & 0 < \left|f^{(n)}(a)\right|, \left|f^{(n)}(b)\right| \leq 1, \\ \left|f'(u)\right|^{q/(2s)} F_1(\mu_2), & 1 \leq \left|f^{(n)}(a)\right|, \left|f^{(n)}(b)\right|, \\ \left|f'(u)\right|^{sq/2} F_1(\mu_3), & 0 < \left|f^{(n)}(a)\right| \leq 1 < \left|f^{(n)}(b)\right|, \\ \left|f'(u)\right|^{q/(2s)} F_1(\mu_4), & 0 < \left|f^{(n)}(b)\right| \leq 1 < \left|f^{(n)}(a)\right|, \end{cases}$$

$$F_1(\nu) = \begin{cases} \frac{1}{\ln\nu}\left(2\nu - 1 - \frac{\nu-1}{\ln\nu}\right) & \nu \neq 1, \\ \frac{3}{2} & \nu = 1, \end{cases}$$

$$F_2(\nu)=\begin{cases}\frac{1}{\ln\nu}\left(\nu-\frac{\nu-1}{\ln\nu}\right) & \nu\neq 1,\\ \frac{1}{2} & \nu=1,\end{cases}$$

and

$$\mu_1=\left|\frac{f^{(n)}(a)}{f^{(n)}(b)}\right|^{sq/2},\mu_2=\left|\frac{f^{(n)}(a)}{f^{(n)}(b)}\right|^{q/(2s)},\mu_3=\frac{\left|f^{(n)}(a)\right|^{sq/2}}{\left|f^{(n)}(b)\right|^{q/(2s)}},\mu_4=\frac{\left|f^{(n)}(a)\right|^{q/(2s)}}{\left|f^{(n)}(b)\right|^{qs/2}}.$$

Theorem 2. [33] *Under the conditions of Theorem 1, we have*

$$\left|f(b)-\frac{1}{b-a}\int_a^b f(x)\,dx\right| \leq \frac{(b-a)}{4}\left(\frac{1}{2}\right)^{1-1/q}\left\{\left[L_2(\mu,q,a)\right]^{1/q}+3^{(q-1)/q}\left[L_1\left(\mu^{-1},q\right)\right]^{1/q}\right\}, \tag{1.4}$$

where $L_1(\mu,q)$*,* $L_2(\mu,q,u)$*,* $F_1(\nu)$*,* $F_2(\nu)$ *and* μ_i *for* $i=1,\ 2,\ 3,\ 4$ *are defined as in Theorem 1.*

Theorem 3. [33] *Under the conditions of Theorem 1, we have*

$$\left|\frac{f(a)+f(b)}{2}-\frac{1}{b-a}\int_a^b f(x)\,dx\right| \leq \frac{(b-a)}{4}\left(\frac{1}{2}\right)^{1-1/q}\left\{\left[L_2(\mu,q,b)\right]^{1/q}+\left[L_1\left(\mu^{-1},q,a\right)\right]^{1/q}\right\}, \tag{1.5}$$

where $L_1(\mu,q)$*,* $L_2(\mu,q,u)$*,* $F_1(\nu)$*,* $F_2(\nu)$ *and* μ_i *for* $i=1,\ 2,\ 3,\ 4$ *are defined as in Theorem 1.*

Applications to special means of positive numbers of the above results can also be seen in [33].

For further results on Hermite-Hadamard type inequalities for s-logarthmically convex we refer the reader to [2, 16, 34, 35]. The main purpose of the present paper is to establish a new Hermite-Hadamard type integral inequalities in Section 2 by using the notion of s-logarthmically convexity and new identity for n-times differentiable functions from [19]. The applications of our results to special means of positive real numbers are also given in Section 3

2. Main Results

The following Lemmas are essential in establishing our main results in this section.

Lemma 1. [19] *Let $f : I \subset \mathbb{R} \to \mathbb{R}$ be a function such that $f^{(n)}$ exists on I° and $f^{(n)} \in L([a,b])$ for $n \in \mathbb{N}$, where a, $b \in I^{\circ}$ with $a < b$, we have the identity*

$$\begin{aligned} (2.1) \quad & \frac{f(a)+f(b)}{2} - \frac{1}{b-a}\int_a^b f(x)\,dx - \sum_{k=1}^{n-1} \frac{k\left[1+(-1)^k\right](b-a)^k}{2^{k+1}(k+1)!} f^{(k)}\left(\frac{a+b}{2}\right) \\ &= \frac{(b-a)^n}{2^{n+1}n!}\int_0^1 (1-t)^{n-1}(n-1+t) f^{(n)}\left(\frac{1-t}{2}a + \frac{1+t}{2}b\right) dt \\ &+ \frac{(-1)^n (b-a)^n}{2^{n+1}n!}\int_0^1 (1-t)^{n-1}(n-1+t) f^{(n)}\left(\frac{1-t}{2}b + \frac{1+t}{2}a\right) dt, \end{aligned}$$

where an empty sum is understood to be nil.

Lemma 2. [20] *If $\mu > 0$ and $\mu \neq 1$, then*

$$(2.2) \quad \int_0^1 t^n \mu^t dt = \frac{(-1)^{n+1} n!}{(\ln \mu)^{n+1}} + n!\mu \sum_{k=0}^{n} \frac{(-1)^k}{(n-k)!(\ln \mu)^{k+1}}.$$

Lemma 3. *If $\mu > 0$ and $\mu \neq 1$, then*

$$(2.3) \quad \int_0^1 (1-t)^n \mu^t dt = \frac{n!\mu}{(\ln \mu)^{n+1}} - n! \sum_{k=0}^{n} \frac{1}{(n-k)!(\ln \mu)^{k+1}}.$$

Proof. By making the substitution $t = 1-u$ in Lemma 2, we get (2.3). □

Lemma 4. [7] *For $\alpha > 0$ and $\mu > 0$, we have*

$$(2.4) \quad J(\alpha,\mu) := \int_0^1 (1-t)^{\alpha-1} \mu^t dt = \sum_{k=1}^{\infty} \frac{(\ln \mu)^{k-1}}{(\alpha)_k} < \infty,$$

where

$$(\alpha)_k = \alpha(\alpha+1)(\alpha+2)\ldots(\alpha+k-1).$$

Theorem 4. *Let $f : I \subset [0,\infty) \to (0,\infty)$ be a function such that $f^{(n)}$ exists on I° and $f^{(n)} \in L([a,b])$ for $n \in \mathbb{N}$, where a, $b \in I^{\circ}$ with $a < b$. If $\left|f^{(n)}\right|^q$ is s-logarithmically convex on $[a,b]$ for some $s \in (0,1]$ and $q \in [1,\infty)$, we have the inequality*

$$\begin{aligned} (2.5) \quad & \left| \frac{f(a)+f(b)}{2} - \frac{1}{b-a}\int_a^b f(x)\,dx \right. \\ & \left. - \sum_{k=1}^{n-1} \frac{k\left[1+(-1)^k\right](b-a)^k}{2^{k+1}(k+1)!} f^{(k)}\left(\frac{a+b}{2}\right) \right| \\ & \leq \frac{(b-a)^n}{2^{n+1}n!}\left(\frac{n}{n+1}\right)^{1-\frac{1}{q}} \left|f^{(n)}(a)\right|^{\delta} \left|f^{(n)}(b)\right|^{\theta} \left\{ \left[F_1(\mu,n)\right]^{\frac{1}{q}} + \left[F_1\left(\mu^{-1},n\right)\right]^{\frac{1}{q}} \right\}, \end{aligned}$$

where $\mu = \left|\frac{f^{(n)}(b)}{f^{(n)}(a)}\right|^{sq/2}$,

$$(\delta,\theta) = \begin{cases} (s/2, s/2), & \text{if } 0 < \left|f^{(n)}(a)\right|, \left|f^{(n)}(b)\right| \leq 1, \\ (1-s/2, 1-s/2), & \text{if } 1 \leq \left|f^{(n)}(a)\right|, \left|f^{(n)}(b)\right|, \\ (s/2, 1-s/2), & \text{if } 0 < \left|f^{(n)}(a)\right| \leq 1 \leq \left|f^{(n)}(b)\right|, \\ (1-s/2, s/2) & \text{if } 0 < \left|f^{(n)}(b)\right| \leq 1 \leq \left|f^{(n)}(a)\right|, \end{cases}$$

and

$$F_1(\nu,n)=\begin{cases}\frac{n!\nu(\ln\nu-1)}{(\ln\nu)^{n+1}}+\frac{1}{\ln\upsilon}-n!\sum_{k=1}^{n}\frac{\ln\nu-1}{(n-k)!(\ln\nu)^{k+1}}, & \nu\neq1,\\ \frac{n}{n+1}, & \nu=1.\end{cases}$$

Proof. From Lemma 1, the Hölder inequality and using the fact that $\left|f^{(n)}\right|^q$ is s-logarithmically convex on $[a,b]$, we have

$$\begin{aligned}(2.6)\quad &\left|\frac{f(a)+f(b)}{2}-\sum_{k=1}^{n-1}\frac{k\left[1+(-1)^k\right](b-a)^k}{2^{k+1}(k+1)!}f^{(k)}\left(\frac{a+b}{2}\right)\right.\\ &\left.-\frac{1}{b-a}\int_a^b f(x)\,dx\right|\leq\frac{(b-a)^n}{2^{n+1}n!}\left(\int_0^1(1-t)^{n-1}(n-1+t)\,dt\right)^{1-\frac{1}{q}}\\ &\times\left\{\left(\int_0^1(1-t)^{n-1}(n-1+t)\left|f^{(n)}(a)\right|^{q\left(\frac{1-t}{2}\right)^s}\left|f^{(n)}(b)\right|^{q\left(\frac{1+t}{2}\right)^s}dt\right)^{1/q}\right.\\ &\left.+\left(\int_0^1(1-t)^{n-1}(n-1+t)\left|f^{(n)}(b)\right|^{q\left(\frac{1-t}{2}\right)^s}\left|f^{(n)}(a)\right|^{q\left(\frac{1+t}{2}\right)^s}dt\right)^{1/q}\right\}.\end{aligned}$$

Since for $0<\xi\leq1\leq\eta$, $0\leq\lambda\leq1$ and $0<s\leq1$. Then

$$(2.7)\qquad \xi^{\lambda^s}\leq\xi^{s\lambda}\quad\text{and}\quad\eta^{\lambda^s}\leq\eta^{\lambda s+1-s}.$$

When $0<\left|f^{(n)}(a)\right|,\left|f^{(n)}(b)\right|\leq1$, by using Lemma 3 and (2.7), we have

$$\begin{aligned}(2.8)\quad &\int_0^1(1-t)^{n-1}(n-1+t)\left|f^{(n)}(a)\right|^{q\left(\frac{1-t}{2}\right)^s}\left|f^{(n)}(b)\right|^{q\left(\frac{1+t}{2}\right)^s}dt\\ &\leq\int_0^1(1-t)^{n-1}(n-1+t)\left|f^{(n)}(a)\right|^{sq\left(\frac{1-t}{2}\right)}\left|f^{(n)}(b)\right|^{sq\left(\frac{1+t}{2}\right)}dt\\ &=\left|f^{(n)}(a)f^{(n)}(b)\right|^{sq/2}\int_0^1(1-t)^{n-1}(n-1+t)\mu^t dt\\ &=\left|f^{(n)}(a)f^{(n)}(b)\right|^{sq/2}F_1(n,\mu).\end{aligned}$$

and

$$\begin{aligned}(2.9)\quad &\int_0^1(1-t)^{n-1}(n-1+t)\left|f^{(n)}(b)\right|^{q\left(\frac{1-t}{2}\right)^s}\left|f^{(n)}(a)\right|^{q\left(\frac{1+t}{2}\right)^s}dt\\ &\leq\int_0^1(1-t)^{n-1}(n-1+t)\left|f^{(n)}(b)\right|^{sq\left(\frac{1-t}{2}\right)}\left|f^{(n)}(a)\right|^{sq\left(\frac{1+t}{2}\right)}dt\\ &=\left|f^{(n)}(a)f^{(n)}(b)\right|^{sq/2}\int_0^1(1-t)^{n-1}(n-1+t)\mu^{-t}dt\\ &=\left|f^{(n)}(a)f^{(n)}(b)\right|^{sq/2}F_1\left(n,\mu^{-1}\right).\end{aligned}$$

When $\left|f^{(n)}(a)\right|,\left|f^{(n)}(b)\right|\geq 1$, by using Lemma 3 and (2.7), we have

$$\begin{aligned}(2.10)\quad &\int_0^1 (1-t)^{n-1}(n-1+t)\left|f^{(n)}(a)\right|^{q\left(\frac{1-t}{2}\right)^s}\left|f^{(n)}(b)\right|^{q\left(\frac{1+t}{2}\right)^s}dt\\ &\leq \left|f^{(n)}(a)f^{(n)}(b)\right|^{q(1-s/2)}\int_0^1(1-t)^{n-1}(n-1+t)\mu^t dt\\ &= \left|f^{(n)}(a)f^{(n)}(b)\right|^{q(1-s/2)}F_1(n,\mu).\end{aligned}$$

and

$$\begin{aligned}(2.11)\quad &\int_0^1 (1-t)^{n-1}(n-1+t)\left|f^{(n)}(b)\right|^{q\left(\frac{1-t}{2}\right)^s}\left|f^{(n)}(a)\right|^{q\left(\frac{1+t}{2}\right)^s}dt\\ &\leq \left|f^{(n)}(a)f^{(n)}(b)\right|^{q(1-s/2)}\int_0^1(1-t)^{n-1}(n-1+t)\mu^{-t} dt\\ &= \left|f^{(n)}(a)f^{(n)}(b)\right|^{q(1-s/2)}F_1\left(n,\mu^{-1}\right).\end{aligned}$$

When $0<\left|f^{(n)}(a)\right|\leq 1\leq\left|f^{(n)}(b)\right|$, by using Lemma 3 and (2.7), we have

$$\begin{aligned}(2.12)\quad &\int_0^1 (1-t)^{n-1}(n-1+t)\left|f^{(n)}(a)\right|^{q\left(\frac{1-t}{2}\right)^s}\left|f^{(n)}(b)\right|^{q\left(\frac{1+t}{2}\right)^s}dt\\ &\leq \left|f^{(n)}(a)\right|^{sq/2}\left|f^{(n)}(b)\right|^{q/2s}\int_0^1(1-t)^{n-1}(n-1+t)\mu^t dt\\ &= \left|f^{(n)}(a)\right|^{sq/2}\left|f^{(n)}(b)\right|^{q(1-s/2)}F_1(n,\mu).\end{aligned}$$

and

$$\begin{aligned}(2.13)\quad &\int_0^1 (1-t)^{n-1}(n-1+t)\left|f^{(n)}(b)\right|^{q\left(\frac{1-t}{2}\right)^s}\left|f^{(n)}(a)\right|^{q\left(\frac{1+t}{2}\right)^s}dt\\ &\leq \left|f^{(n)}(a)\right|^{sq/2}\left|f^{(n)}(b)\right|^{q/2s}\int_0^1(1-t)^{n-1}(n-1+t)\mu^{-t} dt\\ &= \left|f^{(n)}(a)\right|^{q/2s}\left|f^{(n)}(b)\right|^{q(1-s/2)}F_1\left(n,\mu^{-1}\right).\end{aligned}$$

When $0<\left|f^{(n)}(b)\right|\leq 1\leq\left|f^{(n)}(a)\right|$, by using Lemma 3 and (2.7), we have

$$\begin{aligned}(2.14)\quad &\int_0^1 (1-t)^{n-1}(n-1+t)\left|f^{(n)}(a)\right|^{q\left(\frac{1-t}{2}\right)^s}\left|f^{(n)}(b)\right|^{q\left(\frac{1+t}{2}\right)^s}dt\\ &\leq \left|f^{(n)}(a)\right|^{q(1-s/2)}\left|f^{(n)}(b)\right|^{sq/2}\int_0^1(1-t)^{n-1}(n-1+t)\mu^t dt\\ &= \left|f^{(n)}(a)\right|^{q(1-s/2)}\left|f^{(n)}(b)\right|^{sq/2}F_1(n,\mu).\end{aligned}$$

and

$$\int_0^1 (1-t)^{n-1}(n-1+t)\left|f^{(n)}(b)\right|^{q\left(\frac{1-t}{2}\right)^s}\left|f^{(n)}(a)\right|^{q\left(\frac{1+t}{2}\right)^s}dt \tag{2.15}$$
$$\leq \left|f^{(n)}(a)\right|^{q(1-s/2)}\left|f^{(n)}(b)\right|^{sq/2}\int_0^1 (1-t)^{n-1}(n-1+t)\mu^{-t}dt$$
$$= \left|f^{(n)}(a)\right|^{q(1-s/2)}\left|f^{(n)}(b)\right|^{sq/2}F_1\left(n,\mu^{-1}\right),$$

where $\mu=\left|\frac{f^{(n)}(b)}{f^{(n)}(a)}\right|^{sq/2}$. A combination of (2.8)-(2.15) into (2.6) gives the desired result. This completes the proof of the Theorem. □

Corollary 1. *Under the assumptions of Theorem 4, if $q=1$, we have the inequality*

$$\left|\frac{f(a)+f(b)}{2}-\frac{1}{b-a}\int_a^b f(x)\,dx \right. \tag{2.16}$$
$$\left. -\sum_{k=1}^{n-1}\frac{k\left[1+(-1)^k\right](b-a)^k}{2^{k+1}(k+1)!}f^{(k)}\left(\frac{a+b}{2}\right)\right|$$
$$\leq \frac{(b-a)^n}{2^{n+1}n!}\left|f^{(n)}(a)\right|^{\delta}\left|f^{(n)}(b)\right|^{\theta}\left\{F_1(\mu,n)+F_1\left(\mu^{-1},n\right)\right\},$$

where $F_1(\nu,n)$, μ and (δ,θ) are defined as in Theorem 4.

Corollary 2. *Under the assumptions of Theorem 4, if $n=1$, we have the inequality*

$$\left|\frac{f(a)+f(b)}{2}-\frac{1}{b-a}\int_a^b f(x)\,dx\right| \tag{2.17}$$
$$\leq (b-a)\left(\frac{1}{2}\right)^{3-\frac{1}{q}}\left|f^{(n)}(a)\right|^{\delta}\left|f^{(n)}(b)\right|^{\theta}\left\{\left[F_1(\mu,1)\right]^{\frac{1}{q}}+\left[F_1\left(\mu^{-1},1\right)\right]^{\frac{1}{q}}\right\},$$

where

$$F_1(\nu,1)=\begin{cases}\frac{\nu(\ln\nu-1)+1}{(\ln\nu)^2}, & \nu\neq 1\\ \frac{1}{2}, & \nu=1\end{cases}\quad,\ \mu=\left|\frac{f'(b)}{f'(a)}\right|^{sq/2}$$

and

$$(\delta,\theta)=\begin{cases}(s/2,s/2), & \text{if } 0<\left|f'(a)\right|,\left|f'(b)\right|\leq 1,\\ (1-s/2,1-s/2), & \text{if } 1\leq\left|f'(a)\right|,\left|f'(b)\right|,\\ (s/2,1-s/2), & \text{if } 0<\left|f'(a)\right|\leq 1\leq\left|f'(b)\right|,\\ (1-s/2,s/2) & \text{if } 0<\left|f'(b)\right|\leq 1\leq\left|f'(a)\right|.\end{cases}$$

Corollary 3. *If we take $q=1$ in Corollary 2, we have*

$$\left|\frac{f(a)+f(b)}{2}-\frac{1}{b-a}\int_a^b f(x)\,dx\right| \tag{2.18}$$
$$\leq\left(\frac{b-a}{4}\right)\left|f^{(n)}(a)\right|^{\delta}\left|f^{(n)}(b)\right|^{\theta}\left\{\left[F_1(\mu,1)\right]+\left[F_1\left(\mu^{-1},1\right)\right]\right\},$$

where $F_1(\nu,1)$, μ and (δ,θ) are defined as in Corollary 2.

Corollary 4. *Suppose the assumptions of Theorem 4 are fulfilled and if $n=2$, we have*

$$\left|\frac{f(a)+f(b)}{2}-\frac{1}{b-a}\int_a^b f(x)\,dx\right| \le \frac{(b-a)^2}{16}\left(\frac{2}{3}\right)^{1-\frac{1}{q}}\left|f^{(n)}(a)\right|^{\delta}\left|f^{(n)}(b)\right|^{\theta}\left\{\left[F_1(\mu,2)\right]^{\frac{1}{q}}+\left[F_1\left(\mu^{-1},2\right)\right]^{\frac{1}{q}}\right\}, \tag{2.19}$$

where

$$F_1(\nu,2)=\begin{cases}\frac{2\nu(\ln\nu-1)-(\ln\nu)^2+2}{(\ln\nu)^3}, & \nu\neq 1,\\ \frac{2}{3}, & \nu=1,\end{cases}\ ;\ \mu=\left|\frac{f''(b)}{f''(a)}\right|^{sq/2}$$

and

$$(\delta,\theta)=\begin{cases}(s/2,s/2), & \text{if } 0<\left|f''(a)\right|,\left|f''(b)\right|\le 1,\\ (1-s/2,1-s/2), & \text{if } 1\le\left|f''(a)\right|,\left|f''(b)\right|,\\ (s/2,1-s/2), & \text{if } 0<\left|f''(a)\right|\le 1\le\left|f''(b)\right|,\\ (1-s/2,s/2) & \text{if } 0<\left|f''(b)\right|\le 1\le\left|f''(a)\right|.\end{cases}$$

Corollary 5. *If $q=1$ in Corollary 4, we have*

$$\left|\frac{f(a)+f(b)}{2}-\frac{1}{b-a}\int_a^b f(x)\,dx\right| \le \frac{(b-a)^2}{16}\left(\frac{2}{3}\right)^{1-\frac{1}{q}}\left|f^{(n)}(a)\right|^{\delta}\left|f^{(n)}(b)\right|^{\theta}\left\{\left[F_1(\mu,2)\right]+\left[F_1\left(\mu^{-1},2\right)\right]\right\}, \tag{2.20}$$

where $F_1(\nu,2)$, μ and (δ,θ) are defined as in Corollary 4.

Theorem 5. *Let $f: I\subset[0,\infty)\to(0,\infty)$ be a function such that $f^{(n)}$ exists on I° and $f^{(n)}\in L([a,b])$ for $n\in\mathbb{N}$, where a, $b\in I^{\circ}$ with $a<b$. If $\left|f^{(n)}\right|^q$ is s-logarithmically convex on $[a,b]$ for some $s\in(0,1]$ and $q\in(1,\infty)$, we have the inequality*

$$\begin{aligned}&\left|\frac{f(a)+f(b)}{2}-\frac{1}{b-a}\int_a^b f(x)\,dx - \sum_{k=1}^{n-1}\frac{k\left[1+(-1)^k\right](b-a)^k}{2^{k+1}(k+1)!}f^{(k)}\left(\frac{a+b}{2}\right)\right|\\ &\le \frac{(b-a)^n\left[n^{(2q-1)/(q-1)}-(n-1)^{(2q-1)/(q-1)}\right]^{1-1/q}}{2^{n+1}n!}\left(\frac{q-1}{2q-1}\right)^{1-1/q}\\ &\quad\times\left|f^{(n)}(a)\right|^{\delta}\left|f^{(n)}(b)\right|^{\theta}\left\{\left[F_2(\mu,n)\right]^{\frac{1}{q}}+\left[F_2\left(\mu^{-1},n\right)\right]^{\frac{1}{q}}\right\},\end{aligned} \tag{2.21}$$

where

$$F_2(\nu,n)=\begin{cases}\sum_{k=1}^{\infty}\frac{(\ln\nu)^{k-1}}{(nq-q+1)_k}<\infty, & \nu\neq 1,\\ \frac{1}{nq-q+1}, & \nu=1,\end{cases}$$

$$(nq-q+1)_k=(nq-q+1)(nq-q+2)\ldots(nq-q+k),$$

μ and (δ,θ) are defined as in Theorem 4.

Proof. Using Lemma 1, the Hölder inequality and the s-log-convexity of $\left|f^{(n)}\right|^q$ on $[a,b]$, we have

$$\left|\frac{f(a)+f(b)}{2}-\frac{1}{b-a}\int_a^b f(x)\,dx \right. \tag{2.22}$$
$$\left. -\sum_{k=1}^{n-1}\frac{k\left[1+(-1)^k\right](b-a)^k}{2^{k+1}(k+1)!}f^{(k)}\left(\frac{a+b}{2}\right)\right| \le \frac{(b-a)^n}{2^{n+1}n!}\left(\int_0^1 (n-1+t)^{\frac{q}{q-1}}\,dt\right)^{1-\frac{1}{q}}$$
$$\times\left\{\left(\int_0^1 (1-t)^{q(n-1)}\left[\left|f^{(n)}(a)\right|^{q\left(\frac{1-t}{2}\right)^s}\left|f^{(n)}(b)\right|^{q\left(\frac{1+t}{2}\right)^s}\right]dt\right)^{1/q}\right.$$
$$\left.+\left(\int_0^1 (1-t)^{q(n-1)}\left[\left|f^{(n)}(b)\right|^{q\left(\frac{1-t}{2}\right)^s}\left|f^{(n)}(a)\right|^{q\left(\frac{1+t}{2}\right)^s}\right]dt\right)^{1/q}\right\}.$$

The proof follows by using similar arguments as in proving Theorem 4 and using Lemma 4. □

Corollary 6. *Under the assumptions of Theorem 5, if $n=1$, we have the inequality*

$$\left|\frac{f(a)+f(b)}{2}-\frac{1}{b-a}\int_a^b f(x)\,dx\right| \tag{2.23}$$
$$\le \frac{(b-a)}{4}\left(\frac{q-1}{2q-1}\right)^{1-1/q}\left|f'(a)\right|^{\delta}\left|f'(b)\right|^{\theta}\left\{\left[F_2(\mu,1)\right]^{\frac{1}{q}}+\left[F_2\left(\mu^{-1},1\right)\right]^{\frac{1}{q}}\right\},$$

where

$$F_2(\nu,1)=\begin{cases}\sum_{k=1}^{\infty}\frac{(\ln\nu)^{k-1}}{k!}<\infty, & \nu\neq 1,\\ 1, & \nu=1,\end{cases}\ ;\ \mu=\left|\frac{f'(b)}{f'(a)}\right|^{sq/2}$$

and (δ,θ) is defined as in Corollary 2.

Corollary 7. *Under the assumptions of Theorem 5, if $n=2$, we have the inequality*

$$\left|\frac{f(a)+f(b)}{2}-\frac{1}{b-a}\int_a^b f(x)\,dx\right| \tag{2.24}$$
$$\le \frac{(b-a)^2\left[2^{(2q-1)/(q-1)}-1\right]^{1-1/q}}{16}\left(\frac{q-1}{2q-1}\right)^{1-1/q}$$
$$\times\left|f''(a)\right|^{\delta}\left|f''(b)\right|^{\theta}\left\{\left[F_2(\mu,2)\right]^{\frac{1}{q}}+\left[F_2\left(\mu^{-1},2\right)\right]^{\frac{1}{q}}\right\},$$

where

$$F_2(\nu,2)=\begin{cases}\sum_{k=1}^{\infty}\frac{(\ln\nu)^{k-1}}{(q+1)_k}<\infty, & \nu\neq 1,\\ \frac{1}{q+1}, & \nu=1,\end{cases}$$

$$(q+1)_k=(q+1)(q+2)\ldots(q+k),\ \mu=\left|\frac{f''(b)}{f''(a)}\right|^{sq/2}$$

and (δ,θ) are defined as in Corollary 4.

Theorem 6. *Let $f : I \subset \mathbb{R} \to \mathbb{R}$ be a function such that $f^{(n)}$ exists on I° and $f^{(n)} \in L([a,b])$ for $n \in \mathbb{N}$, where $a, b \in I^{\circ}$ with $a < b$. If $\left|f^{(n)}\right|^q$ is convex on $[a,b]$ for $q \in (1,\infty)$, we have the inequality*

$$\left|\frac{f(a)+f(b)}{2} - \sum_{k=1}^{n-1} \frac{k\left[1+(-1)^k\right](b-a)^k}{2^{k+1}(k+1)!} f^{(k)}\left(\frac{a+b}{2}\right) - \frac{1}{b-a}\int_a^b f(x)\,dx\right| \le \frac{n^{n+1-\frac{1}{q}}(b-a)^n}{2^{n+1}n!}\left[B\left(\frac{1}{n};\frac{nq-1}{q-1},\frac{2q-1}{q-1}\right)\right]^{1-\frac{1}{q}} \times \left|f^{(n)}(a)\right|^{\delta}\left|f^{(n)}(b)\right|^{\theta}\left\{\left[F_3(\mu)\right]^{\frac{1}{q}} + \left[F_3\left(\mu^{-1}\right)\right]^{\frac{1}{q}}\right\}, \tag{2.25}$$

where

$$F_3(\nu) = \begin{cases} \frac{\nu-1}{\ln\nu}, & \nu \ne 1, \\ 1, & \nu = 1, \end{cases}, \mu = \left|\frac{f^{(n)}(b)}{f^{(n)}(a)}\right|^{sq/2},$$

$$B(z;\alpha,\beta) = \int_0^z z^{\alpha-1}(1-z)^{1-\beta}\,dt, 0 \le z \le 1, \alpha > 0, \beta > 0$$

is the incomplete Beta function and (δ,θ) are defined as in Theorem 4.

Proof. Using Lemma 1, the Hölder inequality and the s-log convexity of $\left|f^{(n)}\right|^q$ on $[a,b]$, we have

$$\begin{aligned} &\left|\frac{f(a)+f(b)}{2} - \frac{1}{b-a}\int_a^b f(x)\,dx - \sum_{k=1}^{n-1} \frac{k\left[1+(-1)^k\right](b-a)^k}{2^{k+1}(k+1)!} f^{(k)}\left(\frac{a+b}{2}\right)\right| \\ &\le \frac{(b-a)^n}{2^{n+1}n!}\left(\int_0^1 (1-t)^{q(n-1)/(q-1)}(n-1+t)^{q/(q-1)}\,dt\right)^{1-1/q} \\ &\times\left\{\left(\int_0^1 \left|f^{(n)}(a)\right|^{q\left(\frac{1-t}{2}\right)^s}\left|f^{(n)}(b)\right|^{q\left(\frac{1+t}{2}\right)^s}dt\right)^{1/q} + \left(\int_0^1 \left|f^{(n)}(b)\right|^{q\left(\frac{1-t}{2}\right)^s}\left|f^{(n)}(a)\right|^{q\left(\frac{1+t}{2}\right)^s}dt\right)^{1/q}\right\}. \end{aligned} \tag{2.26}$$

By using (2.7) and the fact that

$$\int_0^1 (1-t)^{q(n-1)/(q-1)}(n-1+t)^{q/(q-1)}\,dt = n^{\frac{nq+q-1}{q-1}}\int_0^{\frac{1}{n}} t^{\frac{(n-1)q}{q-1}}(1-t)^{\frac{q}{q-1}}\,dt = n^{\frac{nq+q-1}{q-1}} B\left(\frac{1}{n};\frac{nq-1}{q-1},\frac{2q-1}{q-1}\right),$$

we get the required inequality (2.25) from (2.26). □

Corollary 8. *Suppose the assumptions of Theorem 6 are satisfied and if* $n=1$*, we have the inequality*

$$\left|\frac{f(a)+f(b)}{2}-\frac{1}{b-a}\int_a^b f(x)\,dx\right| \leq \frac{(b-a)}{4}\left(\frac{q-1}{2q-1}\right)^{1-1/q}\left|f^{'}(a)\right|^{\delta}\left|f^{'}(b)\right|^{\theta}\left\{[F_3(\mu)]^{\frac{1}{q}}+\left[F_3\left(\mu^{-1}\right)\right]^{\frac{1}{q}}\right\}, \tag{2.27}$$

where

$$F_3(\nu)=\begin{cases}\frac{\nu-1}{\ln\nu}, & \nu\neq 1,\\ 1, & \nu=1,\end{cases}\ ;\mu=\left|\frac{f^{'}(b)}{f^{'}(a)}\right|^{sq/2}$$

and (δ,θ) *are defined as in Corollary 2.*

Corollary 9. *Suppose the assumptions of Theorem 6 are satisfied and if* $n=2$*, we have the inequality*

$$\left|\frac{f(a)+f(b)}{2}-\frac{1}{b-a}\int_a^b f(x)\,dx\right| \leq \frac{(b-a)^2}{2^{1+1/q}}\left[B\left(\frac{1}{2};\frac{2q-1}{q-1},\frac{2q-1}{q-1}\right)\right]^{1-\frac{1}{q}} \times\left|f^{''}(a)\right|^{\delta}\left|f^{''}(b)\right|^{\theta}\left\{[F_3(\mu)]^{\frac{1}{q}}+\left[F_3\left(\mu^{-1}\right)\right]^{\frac{1}{q}}\right\}, \tag{2.28}$$

where

$$F_3(\nu)=\begin{cases}\frac{\nu-1}{\ln\nu}, & \nu\neq 1,\\ 1, & \nu=1,\end{cases}\ ;\mu=\left|\frac{f^{''}(b)}{f^{''}(a)}\right|^{sq/2},$$

$B(z;a,b)$ *is the incomplete Beta function as defined in Theorem 6 and* (δ,θ) *are defined as in Corollary 4.*

Remark 1. *We can get several interesting inequalities for log-convex functions by setting* $s=1$ *in the above results. However, the details are left to the interested reader.*

3. Applications to Special Means

For positive numbers $a>0$, $b>0$, define

$$A(a,b)=\frac{a+b}{2},\ G(a,b)=\sqrt{ab},\ H(a,b)=\frac{2ab}{a+b},$$

$$I(a,b)=\begin{cases}\frac{1}{e}\left(\frac{b^b}{a^a}\right)^{1/(b-a)}, & a\neq b,\\ a, & a=b\end{cases}$$

and

$$L_p(a,b)=\begin{cases}\left[\frac{b^{p+1}-a^{p+1}}{(p+1)(b-a)}\right]^{1/p}, & p\neq 0,-1 \text{ and } a\neq b,\\ \frac{b-a}{\ln b-\ln a}, & p=-1 \text{ and } a\neq b,\\ I(a,b), & p=0 \text{ and } a\neq b,\\ a, & a=b.\end{cases}$$

It is well known that A, G, H, $L = L_{-1}$, $I = L_0$ and L_p are called the arithmetic, geometric, harmonic, logarithmic, exponential and generalized logarithmic means of positive numbers a and b.

In what follows we will use the above means and the established results of the previous section to obtain some interesting inequalities involving means.

Theorem 7. *Let $0 < a < b \leq 1$, $r < 0$, $r \neq -1$, $s \in (0,1]$ and $q \geq 1$.*

(1) *If $r \neq -2$, then*

$$\begin{aligned}
&\left|A\left(a^{r+1}, b^{r+1}\right) - \left[L_{r+1}(a,b)\right]^{r+1}\right| \\
&\leq (b-a)\left(\frac{1}{2}\right)^{3-\frac{1}{q}} |r+1| \left[G\left(a^r, b^r\right)\right]^2 \\
&\times \left\{a^{-rs}\left[\frac{2\left(1 - b^{-rqs/2} L\left(a^{rqs/2}, b^{rqs/2}\right)\right)}{rqs(\ln b - \ln a)}\right]^{1/q}\right. \\
&\left. + b^{-rs}\left[\frac{2\left(a^{-rqs/2} L\left(a^{rqs/2}, b^{rqs/2}\right) - 1\right)}{rqs(\ln b - \ln a)}\right]^{1/q}\right\}.
\end{aligned}$$

(2) *If $r = -2$, then*

$$\begin{aligned}
&\left|\frac{1}{H(a,b)} - \frac{1}{L(a,b)}\right| \\
&\leq (b-a)\left(\frac{1}{2}\right)^{3-\frac{1}{q}} \left[G\left(a^{-2}, b^{-2}\right)\right]^2 \\
&\times \left\{a^{2s}\left[\frac{1 - b^{qs} L\left(a^{-qs}, b^{-qs}\right)}{qs(\ln a - \ln b)}\right]^{1/q} + b^{2s}\left[\frac{a^{qs} L\left(a^{-qs}, b^{-qs}\right) - 1}{qs(\ln a - \ln b)}\right]^{1/q}\right\}.
\end{aligned}$$

Proof. Let $f(x) = \frac{x^{r+1}}{r+1}$ for $0 < x \leq 1$. Then $|f'(x)| = x^r$ and

$$\ln\left|f'(\lambda x + (1-\lambda) y)\right|^q \leq \lambda^s \ln\left|f'(x)\right|^q + (1-\lambda)^s \left|f'(y)\right|^q$$

for x, $y \in (0,1]$, $\lambda \in [0,1]$, $s \in (0,1]$ and $q \geq 1$. This shows that $|f'(x)|^q = x^{rq}$ is s-logarthmically convex function on $(0,1]$. Since$|f'(a)| > |f'(b)| = b^r \geq 1$, hence

$$\mu = \left|\frac{f'(a)}{f'(b)}\right|^{sq/2} = \left(\frac{b}{a}\right)^{rqs/2}, \ \mu^{-1} = \left(\frac{a}{b}\right)^{rqs/2}$$

and

$$\begin{aligned}
&\left|f'(a) f'(b)\right|^{(1-s/2)} \left\{\left[F_1(\mu, 1)\right]^{\frac{1}{q}} + \left[F_1\left(\mu^{-1}, 1\right)\right]^{\frac{1}{q}}\right\} \\
&= \left[G\left(a^r, b^r\right)\right]^2 \left\{a^{-rs}\left[\frac{2\left(1 - b^{-rqs/2} L\left(a^{rqs/2}, b^{rqs/2}\right)\right)}{rqs(\ln b - \ln a)}\right]^{1/q}\right. \\
&\left. + b^{-rs}\left[\frac{2\left(a^{-rqs/2} L\left(a^{rqs/2}, b^{rqs/2}\right) - 1\right)}{rqs(\ln b - \ln a)}\right]^{1/q}\right\}.
\end{aligned}$$

Substituting the above quantities in Corollary 2, we get the required inequality. □

Remark 2. *Many interesting inequalities of means of positive real numbers can be obtained by using the other results, however, the details are left to the interested reader.*

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