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Intervally Distributed, Palindromic, Selfie Magic Squares, and Double Colored Patterns

Inder J. Taneja[1]

Abstract

*In this paper, we shall work with examples of magic squares with palindromic numbers. Representations in colored and/or double (superimposed) colored pattern are studied. The idea of intervally distributed magic squares is also defined. Whenever possible, decomposition in **"pairwise mutually orthogonal diagonalized Latin squares"** are presented. Self-orthogonal diagonal Latin squares resulting to intervally distributed magic square are given. Upside-down and mirror looking magic squqres defined as **Selfie magic squares** are studied. The work is limited only to magic squares of orders 3 to 10. Further extensions shall be given in another work [16].*

1. Introduction

We shall work with different kinds of magic squares, such as palindromic, Selfie, uposide down, and double colored. These ideas are divided parts:

(i) Intervally distributed magic squares;
(ii) Palindromic representations;
(iii) Selfie magic squares;
(iv) Orthogonal Latin square decompositions;
(v) Colored pattern representation;

Before starting, here below are some explainations.

1.1. Intervally Distributed Magic Squares

Magic Square. *A magic square of order $n > 1$ is an $n \times n$ matrix $A = (a_{ij})$ in which n^2 distinct numbers from a set S are arranged, such that the sum of the numbers in each row, each column and in principal diagonals is the same. This sum is called the magic sum, which will be denoted by S. When the sum of square of each element is same, in each row, each column and in principal diagonal, then it is called bimagic square. The bimagic sum is denoted by Sb.*

Intervally Distributed Magic Squares. *Let us consider in a sequence of n^2 numbers. Put these nembers in increasing order. Divide de n^2 elements in n interval. If elements of each inverval appears only once in row, column and in principal diagonals, then the we call it intervally distribuited. If the distribution is a magic square, we call it a **intervally distributed magic squares.***

[1]Formerly, Professor of Mathematics, Federal University of Santa Catarina, 88040-400 Florianópolis, SC, Brazil. Email: ijtaneja@gmail.com

For simplicity let us consider 100 (10^2) numbers in a sequence, i.e., from 1 to 100. Let us divide these numbers in the 10 intervals:

1st interval	01-10	1	2	3	4	5	6	7	8	9	10
2nd interval	11-20	11	12	13	14	15	16	17	18	19	20
3rd interval	21-30	21	22	23	24	25	26	27	28	29	30
4th interval	31-40	31	32	33	34	35	36	37	38	39	40
5th interval	41-50	41	42	43	44	45	46	47	48	49	50
6th interval	51-60	51	52	53	54	55	56	57	58	59	60
7th interval	61-70	61	62	63	64	65	66	67	68	69	70
8th interval	71-80	71	72	73	74	75	76	77	78	79	80
9th interval	81-90	81	82	83	84	85	86	87	88	89	90
10th interval	91-100	91	92	93	94	95	96	97	98	99	100

Intervally distributed elements are understood as members of each interval should appear only once in *each row* and in *each column* and in *principal diagonals*, for example,

1							61		
		65		4					
	6				66				
			62			8			
	68								10
			5			63			
		7						64	
					9				69
70							2		
				67				3	

It is not necessary that all *intervally distribuited numbers* always forms a magic square. For example,

				30
1	12	7	13	33
5	14	2	11	32
15	6	10	4	35
9	3	16	8	36
30	35	35	36	33

The above grid is *intervally distributed* but not a magic square. Later, it is explained, when we can have *intervally distributed magic squares*.

1.2. Latin Squares

Here below are some basic definitions of Latin squares [1], [6].

Latin Square. *A latin square of order n is an* $n \times n$ *matrix in which n distinct symbols from a symbol set* **S** *are arranged, such that each symbol occurs exactly once in each row and in each column, i.e., no symbol is repeated in a row or a column.*

[1]Formerly, Professor of Mathematics, Federal University of Santa Catarina, 88040-400 Florianópolis, SC, Brazil. Email: ijtaneja@gmail.com

Orthogonal Latin Squares. *Two Latin squares of order n are orthogonal if each symbol in the first square meets each symbol in the second square exactly once when they are superposed. When two latin arrays are orthogonal to each other, we call them orthogonal mates.*

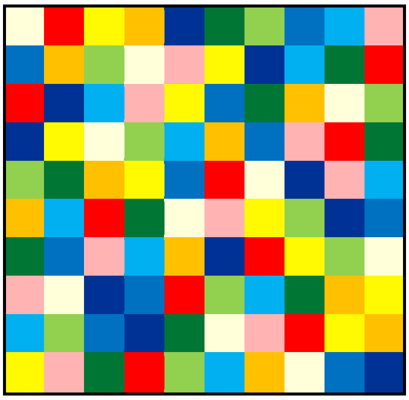
Mutually Orthogonal Diagonalized Latin Squares. *A set of mutually orthogonal latin arrays is a collection of latin arrays that are pairwise orthogonal to each other. A diagonal Latin square of order n is a Latin square such that no symbol appears more than once in any of its two main diagonals.*

Whe we are working with *"pairwise mutually orthogonal Latin squares"*, it always give *intervally distributed magic square.* When the Latin squares are *diagonalized*, the condition becomes more stronger. We shall see that there are examples, where the pair of Latin squares are not necessarily diagonalized, but still the we have magic squares intervally distributed, specially in case of magic square of order 6. Also in case of *self-orthogonal diagonalized Latin squares* we are lead to *intervally distributed magic squares*.

We shall apply *intervally distributed magic square* to bring *palindromic* and *upside down magic squares*. Based on *pairwise mutually diagonalized orthogonal Latin squares*.

1.3. Colored Patterns

Here below is an example of colored pattern, where same color don't appears again in the same row or same column or in principal diagonal.



Every *intervally distributed magic square* always give colored pattern, but there are colored patterns not necessary magic squares. Every *diagonized Latin square* always bears colored pattern. See examples below of order 4:

				30
1	12	7	13	33
5	14	2	11	32
15	6	10	4	35
9	3	16	8	36
30	35	35	36	33

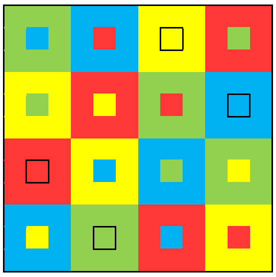
				10
2	3	1	4	10
1	4	2	3	10
4	1	3	2	10
3	2	4	1	10
10	10	10	10	10

First example shows that colored pattern is not necessarily a magic square, and second example shows that every *"diagonalized Latin square"* lead us to a colored pattern.

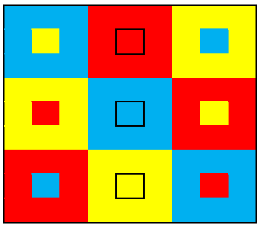
[1]Formerly, Professor of Mathematics, Federal University of Santa Catarina, 88040-400 Florianópolis, SC, Brazil. Email: ijtaneja@gmail.com

1.4. Double Colored Patterns

Double colored patterns are combination *superimposed double colors* with the property that there is no same color in the same row, or column or in principal diagonal. This is only possibly, when we are woking with "*mutually orthogonal diagonalized Latin squares*". See example below:



We call *semi-double colored pattern* if *superimposed property* is not applicable for diagonals. For example,



The above grid is *semi-double colored pattern pattern*.

1.5. Selfie Magic Squares

Selfie Magic Square. *It is a magic square that remains the same after making the rotation of* 180^0 *and looking through the mirror. Most of the mirror looking magic squares are rervesible, but there are* rotatable *magic square not mirror looking. In this case we call as* ***rotatable or upside down magic squares****. Seflie magic squares are also known by* ***Universal magic squares****.*

More precisely, a *Selfie magic square* should have the following three properties:

(i) Upside down (180^0 rotatable);
(ii) Mirror looking;
(iii) Having the same sum (sum of rows, columns and of principal diagonals).

By *rotatable or upside down magic square*, it is understood that when we make a rotation of 180^0, it remais the same. *Mirror looking* magic square are when seen through mirror or their reflextion in water, again remains as magic square. It may happen that when rotating to 180^0 or looking through mirror, they are magic squares, but with different sums. In this case, they are not *Selfie magic square*. *Selfie magic square* with letters are famous as IXOHOXI magic squares.

Working with numbers, there are 7 *upside down digits*, i.e., **0**, **1**, **2**, **5**, **6**, **8** and **9**. Digits **0**, **1** and **8** are always *upside down* and *mirror looking*. After making rotation of 180^0, **6** becomes as **9** and **9** as **6**, but these two are not mirror looking. Also, 2 and 5 are not rotatable.

[1]Formerly, Professor of Mathematics, Federal University of Santa Catarina, 88040-400 Florianópolis, SC, Brazil. Email: ijtaneja@gmail.com

When we write them in digital form, like 2 and 5, they becomes rotatable and mirror looking. When we see them through mirror, 2 becomes as 5, and 5 as 2. Here below are examples of magic square of order 4 in both the situations:

- **Upside down**

68	89	11	96
16	91	69	88
99	18	86	61
81	66	98	19

61	86	99	18
19	98	81	66
88	69	16	91
96	11	68	89

The above two magic squares are upside down, i.e., one is 180^0 rotation of another.

- **Mirror looking**

25	58	11	82
12	81	28	55
88	15	52	21
51	22	85	18

58	11	82	25
22	85	18	51
15	52	21	88
81	28	55	12

Above two magic squares are mirror looking, one is reflexion of another. In both the situations they have the same sum. Moreover, they are also rotatable with same sum. Thus according the above definition, it is a *Selfie magic square*.

We observe that, there are only five digits 0, 1, 2, 5 and 8 written in digital form, give us conditions to bring *Selfie magic square*. In subsequent sections, we have more study of *Selfie magic squares*.

1.6. Palindromic numbers

Palindormic Numbers. *Numbers that read the same backwords and forwards, i.e, it remains the same when its digits are reversed, for example, 121, 3883, 19991, etc.*

Table below give the quantity of palindromes for each number of digit:

Digits	Palindormes	Quantity
1-digit	*a*	9
2-digits	*aa*	9
3-digits	*aba*	90
4-digits	*abba*	90
5-digits	*abcba*	900
6-digits	*abccba*	900
7-digits	*abcdcba*	9000
8-digits	*abcddcba*	9000
…..	…..	…..

[1]Formerly, Professor of Mathematics, Federal University of Santa Catarina, 88040-400 Florianópolis, SC, Brazil. Email: ijtaneja@gmail.com

If we want to write 3-digits palindormes using only 1 and 2 we have exactly four (2^2) palindormes, i.e., 111, 121, 212 and 222. The same is true for 4-digits. The table below give exact quantity in each case:

3 or 4-digits using numbers	Total Palindromes
1, 2	$4 = 2^2$
1, 2, 3	$9 = 3^2$
1, 2, 3, 4	$16 = 4^2$
1, 2, 3, 4, 5	$25 = 5^2$
1, 2, 3, 4, 5, 6	$36 = 6^2$
1, 2, 3, 4, 5, 6, 7	$49 = 7^2$
1, 2, 3, 4, 5, 6, 7,8	$64 = 8^2$
1, 2, 3, 4, 5, 6, 7, 8, 9	$81 = 9^2$
0, 1, 2, 3, 4, 5, 6, 7, 8, 9	90

This idea, we shall use in constructing *palindromic magic squares* in each case.

1.6.1. Reversible Number Magic Squares

Reversible number magic squares are magic square obtained by inverting the order of elements in a magic squares. Every 4-digits palindromic magic square can always be written as two *reversible number magic squares*, i.e.,

$$M_S = 1000 \times A + B .$$

where M_S is a 4-digits *palindromic magic square*, A and B are *reversible number magic squares*. The same is always true for magic squares formed by even order palindromes. The *reversible number magic squares* are different from *upside down magic squares*. In the first case we just reverse the order of numbers, while in the second case, we give rotation of 180^0 and again get a magic square.

2. Magic Squares of Different Orders

In this section, we shall present magic squares of orders 3 to 10. From order 4 onwards, we can always bring *intervally distributed magic squares*. The idea of orthoganl Latins squares and *double colored patterns* are also given, except for 6x6. Whenver possible, *Selfie and upside down magic squares* are also presented.

2.1. Magic Squares of Order 3

Let us consider a classical Lo-Shu magic square of order 3

			15
4	9	2	15
3	5	7	15
8	1	6	15
15	15	15	15

... (1)

[1]Formerly, Professor of Mathematics, Federal University of Santa Catarina, 88040-400 Florianópolis, SC, Brazil. Email: ijtaneja@gmail.com

2.1.1. Palindromic Representations

Let us consider three letters a, b and c with $a, b, c \in \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$.We can make exactly 9 palindrome of 3-digits with these three letters:

1	2	3	4	5	6	7	8	9
aaa	*aba*	*aca*	*bab*	*bbb*	*bcb*	*cac*	*cbc*	*ccc*

… (2)

Let us replace 1 to 9 with respective palindromes given in (2) in (1), we get the following grid of order 3:

bab	*ccc*	*aba*
aca	*bbb*	*cac*
cbc	*aaa*	*bcb*

… (3)

Above grid always lead us to a *semi-palindromic magic square* for all $a, b, c \in \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$. *Semi-magic sum* (sum of rows and columns) is given by

$$S_{3\times3}(a,b,c) = (a+b+c)\times 111 \text{ or } \frac{S_{3\times3}(a,b,c)}{a+b+c} = 111.$$

For $b = (a+c)/2$, it is always a *palindromic magic square*. Below are some interesting examples:

Example 1. For $a = 1, b = 2$ and $c = 3$ in (1), we have 3-digits *palindromic magic square* of order 3 with sum $S_{3\times3}(1,2,3) = (1+2+3)\times 111 = 666$:

212	333	121
131	222	313
323	111	232

Repeating the middle value we get a 4-digits *palindromic magic* square of order 3 with sum $S_{3\times3}(1,2,3) = 6666$:

2112	3333	1221
1331	2222	3113
3223	1111	2332

Example 2. For $a = 2, b = 5$ and $c = 8$ in (1), we have regular 3-digits *palindromic Selfie magic square* of order 3 with sum $S_{3\times3}(2,5,8) = (2+5+8)\times 111 = 1665$:

525	888	252
282	555	828
858	222	585

[1]Formerly, Professor of Mathematics, Federal University of Santa Catarina, 88040-400 Florianópolis, SC, Brazil. Email: ijtaneja@gmail.com

Example 3. For $a=1, b=6$ and $c=9$ in (1), we have 3-digits *palindromic upside down semi-magic square* of order 3:

616	999	161
191	666	919
969	111	696

The semi-magic sum is $S_{3\times3}(1,6,9)=(1+6+9)\times111=1776$ (sums of rows and columns). The diagonals sums are 1796 and 1978 respectively.

A general way to get 4 and 5-digits *palindromic magic square* of order 3 is just put "*a*" in front and in middle of grid (1):

baab	*cccc*	*abba*
acca	*bbbb*	*caac*
cbbc	*aaaa*	*bccb*

ababa	*accca*	*aabaa*
aacaa	*abbba*	*acaca*
acbca	*aaaaa*	*abcba*

baaab	*ccacc*	*ababa*
acaca	*bbabb*	*caaac*
cbabc	*aaaaa*	*bcacb*

For all $a \in \{0,1,2,3,4,5,6,7,8,9\}$. For $a=0$ it becomes little artificial.

2.1.2. Composite Magic Square of Order 3

Eliminating the third value in (3), and then splitting in two Latin squares, we get

b	*c*	*a*
a	*b*	*c*
c	*a*	*b*

A

a	*c*	*b*
c	*b*	*a*
b	*a*	*c*

B

ba	*cc*	*ab*
ac	*bb*	*ca*
cb	*aa*	*bc*

AB

In particular for $a=1, b=2$ and $c=3$, we get

2	3	1
1	2	3
3	1	2

A

1	3	2
3	2	1
2	1	3

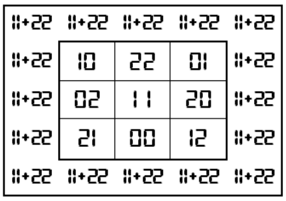
B

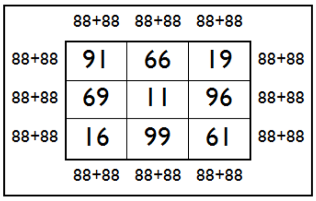
21	33	12
13	22	31
32	11	23

AB

…. (4)

Example 4. As an example of composite magic square (4) we have following *upside down magic squares*:





First one with 0, 1 and 2 is *upside down magic square* while second with 1, 6 and 9 is *upside down semi-magic square* of order 3, but both are rotatable, i.e., remais the same after making

[1]Formerly, Professor of Mathematics, Federal University of Santa Catarina, 88040-400 Florianópolis, SC, Brazil. Email: ijtaneja@gmail.com

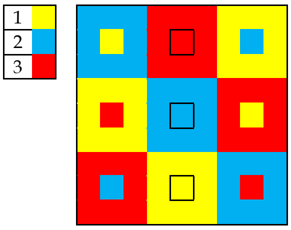
the rotation of 180⁰. First one is mirror looking, but the magic square obtained is not same, i.e., instead of **2** we have **5**. See below:

11+22	11+22	11+22	11+22	11+22
11+22	10	22	01	11+22
11+22	02	11	20	11+22
11+22	21	00	12	11+22
11+22	11+22	11+22	11+22	11+22

55+11	55+11	55+11	55+11	55+11
55+11	10	55	01	55+11
55+11	05	11	50	55+11
55+11	51	00	15	55+11
55+11	55+11	55+11	55+11	55+11

The above magic square is *upside down* and *mirror looking* with different sums. We can't say it *Selfie*, as both are with different magic sums.

Applying the procedure $3\times(A-1)+B$ over (4), we get the magic square (1). The grid AB, we call *"composite magic square"*. Based on it, here below is *"semi-double colored pattern"*.



…. (5)

Observing (4), the *superimposed colored pattern* (5) becomes *semi* as it don't have diagonalized property.

2.2. Magic squares of order 4

Let us consider classical *uniformy distributed pan diagonal* (*Khujurao magic square)* of order 4:

		34	34	34	34
	7	12	1	14	34
34	2	13	8	11	34
34	16	3	10	5	34
34	9	6	15	4	34
	34	34	34	34	34

…. (6)

This is one of the most *perfect magic square* of order 4 studied around 10th century. It is *pan diagonal*

7	12	1	14
2	13	8	11
16	3	10	5
9	6	15	4

7	12	1	14
2	13	8	11
16	3	10	5
9	6	15	4

More precisely, consider the following colored pattern, where each block of order 2 has the sum as of magic square:

[1]Formerly, Professor of Mathematics, Federal University of Santa Catarina, 88040-400 Florianópolis, SC, Brazil. Email: ijtaneja@gmail.com

7	12	1	14
2	13	8	11
16	3	10	5
9	6	15	4

7	12	1	14
2	13	8	11
16	3	10	5
9	6	15	4

7	12	1	14
2	13	8	11
16	3	10	5
9	6	15	4

7	12	1	14
2	13	8	11
16	3	10	5
9	6	15	4

7	12	1	14
2	13	8	11
16	3	10	5
9	6	15	4

7	12	1	14
2	13	8	11
16	3	10	5
9	6	15	4

…. (7)

We have 22 blocks of order 2 with same sum as of magic square. In view of above properties, the magic square (6) is famous as ***"compact magic square".*** It is also called ***perfect magic square***. For the case of *compactness* first four colored patterns are sufficient, i.e.,

7	12	1	14
2	13	8	11
16	3	10	5
9	6	15	4

7	12	1	14
2	13	8	11
16	3	10	5
9	6	15	4

7	12	1	14
2	13	8	11
16	3	10	5
9	6	15	4

7	12	1	14
2	13	8	11
16	3	10	5
9	6	15	4

… (8)

These 16 subgroups of order 2 have the same sum as of magic square sum i.e., 34, and generally represented as

34	34	34	34
34	34	34	34
34	34	34	34
34	34	34	34

The 1st magic square of order 4 is found in India and is created by the mathematician Nagarajuna in the 10th century. It can be found in Khajuraho (India) in the Parshvanath Jain temple (10th century). This magic square is not only a normal magic square, but also a pandiagonal. It has lot of other properties specified above. It is considered as one of the most perfect magic square of order 4. More studies and historical notes on this magic square can be seen in Taneja [10].

2.2.1. Palindromic Magic Squares of Order 4

Let us consider four letters a, b, c and d with $a, b, c, d \in \{0,1,2,3,4,5,6,7,8,9\}$. We can make exactly 16 palindromes of 3-digits with these four letters:

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
aaa	*aba*	*aca*	*ada*	*bab*	*bbb*	*bcb*	*bdb*	*cac*	*cbc*	*ccc*	*cdc*	*dad*	*dbd*	*dcd*	*ddd*

Replacing the above values with their respective palindromes in (7), we get the following grid of order 4:

[1]Formerly, Professor of Mathematics, Federal University of Santa Catarina, 88040-400 Florianópolis, SC, Brazil. Email: ijtaneja@gmail.com

bcb	*cdc*	*aaa*	*dbd*
aba	*dad*	*bdb*	*ccc*
ddd	*aca*	*cbc*	*bab*
cac	*bbb*	*dcd*	*ada*

… (9)

For all $a, b, c, d \in \{0,1,2,3,4,5,6,7,8,9\}$, the grid (9) represent a *palindromic magic square* of order 4. Its sum is

$$S_{4\times4}(a,b,c,d) = (a+b+c+d)\times 111 \text{ or } \frac{S_{4\times4}(a,b,c,d)}{a+b+c+d} = 111.$$

As a consequence of (9), we have following 4 and 5-digits *palindromic grids* with a, b, c and d:

bccb	*cddc*	*aaaa*	*dbbd*
abba	*daad*	*bddb*	*cccc*
dddd	*acca*	*cbbc*	*baab*
caac	*bbbb*	*dccd*	*adda*

abcba	*acdca*	*aaaaa*	*adbda*
aabaa	*adada*	*abdba*	*accca*
addda	*aacaa*	*acbca*	*ababa*
acaca	*abbba*	*adcda*	*aadaa*

bcacb	*cdadc*	*aaaaa*	*dbabd*
ababa	*daaad*	*bdabb*	*ccacc*
ddadd	*acaca*	*cbabc*	*baaab*
caaac	*bbabb*	*dcacd*	*adada*

Making proper choices of $a, b, c, d \in \{0,1,2,3,4,5,6,7,8,9\}$, the above three grids lead us to *palindromic magic squares* of order 4. For the choice zero, some numbers may be considred as 010, 060, etc. Here below are some examples:

Example 5. For $a=1, b=2, c=3$ and $d=4$ in (2), we have 3-digits *palindromic pan diagonal magic square* of order 4 with sum $S_{4\times4}(1,2,3,4) = (1+2+3+4)\times 111 = 1110$:

		1110	1110	1110	1110
	232	343	111	424	1110
1110	121	414	242	333	1110
1110	444	131	323	212	1110
1110	313	222	434	141	1110
	1110	1110	1110	1110	1110

…. (10)

We observe that not all the particular cases of (9) are pan diagonal. It depends on the choice of numbers. See the examples below. When we choose numbers with same difference, the result is always pan diagonal, for example, (1,2,3,4); (2,4,6,8), (1,3,5,7), etc.

Example 6. For $a=1, b=2, c=5$ and $d=8$ in (2), we have 4-digits *palindromic Selfie magic square* of order 4 with sum $S_{4\times4}(1,2,5,8) = (1+2+5+8)\times 111 = 1776 = 888+888$:

252	585	111	828
121	818	282	555
888	151	525	212
515	222	858	181

[1]Formerly, Professor of Mathematics, Federal University of Santa Catarina, 88040-400 Florianópolis, SC, Brazil. Email: ijtaneja@gmail.com

Example 7. For $a = 1, b = 6, c = 8$ and $d = 9$ in (2), we have 3-digits *palindromic upside down magic square* of order 4 with sum $S_{4\times4}(1,6,8,9) = (1+6+8+9)\times111 = 2664 = 3\times888$:

686	898	111	969
161	919	696	888
999	181	868	616
818	666	989	191

Examples 6 and 7 are not *pan diagonal*.

2.2.2. Reversible Number Magic Squares of Order 4

Decomposing 4-digits *palindromic grid* given in (9), we get two grids with *reversible numbers*:

bc	*cd*	*aa*	*db*
ab	*da*	*bd*	*cc*
dd	*ac*	*cb*	*ba*
ca	*bb*	*dc*	*ad*

cb	*dc*	*aa*	*bd*
ba	*ad*	*db*	*cc*
dd	*ca*	*bc*	*ab*
ac	*bb*	*cd*	*da*

…. (11)

Example 8. Considering $a = 1, b = 2, c = 3$ and $d = 4$ in (11), we have

23	34	11	42
12	41	24	33
44	13	32	21
31	22	43	14

32	43	11	24
21	14	42	33
44	31	23	12
13	22	34	41

2.2.3. Double Colored Pattern

Excluding last digits in each cell in (9), we get *mutually orthogonal diagonized Latin squares* decomposition:

b	*c*	*a*	*d*
a	*d*	*b*	*c*
d	*a*	*c*	*b*
c	*b*	*d*	*a*

A

c	*d*	*a*	*b*
b	*a*	*d*	*c*
d	*c*	*b*	*a*
a	*b*	*c*	*d*

B

bc	*cd*	*aa*	*db*
ab	*da*	*bd*	*cc*
dd	*ac*	*cb*	*ba*
ca	*bb*	*dc*	*ad*

AB

In particular $a = 1, b = 2, c = 3$ and $d = 4$, we have

2	3	1	4
1	4	2	3
4	1	3	2
3	2	4	1

A

3	4	1	2
2	1	4	3
4	3	2	1
1	2	3	4

B

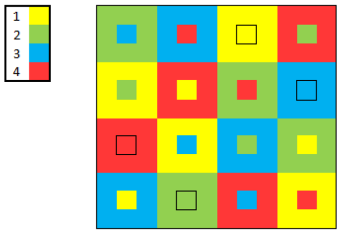
23	34	11	42
12	41	24	33
44	13	32	21
31	22	43	14

AB

…. (12)

[1]Formerly, Professor of Mathematics, Federal University of Santa Catarina, 88040-400 Florianópolis, SC, Brazil. Email: ijtaneja@gmail.com

Applying the procedure $4\times(A-1)+B$, we get magic square of order 4 given in (1). Moreover, A and B are *"pairwise mutually diagonalized orthogonal Latin squares"*. The magic square AB given in (12) is a *composite magic square* of order 4. Based on it, here below is *"double colored pattern"*.



2.2.4. Self-Orthogonal Diagonalized Latin Square

We observe that the the Latin squares A and B are not self-orthogonal. The grid A can be considered as self-orthogonal. In this case, the grid B is obtained by transposing the members of A changing rows by colunms. It lead us to following magic square of order 4.

Example 9. Magic square arising due to self-orthogonal diagonalized Latin square:

2	3	1	4
1	4	2	3
4	1	3	2
3	2	4	1

A

2	1	4	3
3	4	1	2
1	2	3	4
4	3	2	1

B

		34	34	34	34
	6	9	4	15	34
34	3	16	5	10	34
34	13	2	11	8	34
34	12	7	14	1	34
	34	34	34	34	34

2.2.5. Selfie Magic Squares of Order 4

Example 6 is *Seflie palindromic magic squares*, while the example 7 is *upside down palindromic magic square*. Here below are examples of *Selfie* and *upside down non palindromic* magic squares.

Example 10. The following two examples are *upside down* and *Seflie magic squares* just with four numbers with sums $S_{4\times4}(1,6,8,9)=264$ and $S_{4\times4}(1,2,5,8)=176$ respectively:

61	86	99	18
19	98	81	66
88	69	16	91
96	11	68	89

58	11	82	25
22	85	18	51
15	52	21	88
81	28	55	12

Example 11. The following two examples are *upside down* and *Seflie magic squares* just with two numbers with sums $S_{4\times4}(6,9)=33330$ and $S_{4\times4}(2,5)=15554$ respectively:

[1]Formerly, Professor of Mathematics, Federal University of Santa Catarina, 88040-400 Florianópolis, SC, Brazil. Email: ijtaneja@gmail.com

6996	9699	6666	9969
6669	9966	6999	9696
9999	6696	9669	6966
9666	6969	9996	6699

2552	5255	2222	5525
2225	5522	2555	5252
5555	2252	5225	2522
5222	2525	5552	2255

…. (13)

In both the examples 10 and 11, the first one is just *upside down* and is *Selfie magic square*.

2.2.6. 7-Digits Palindromic Pan Diagonal Magic Squares of Order 4

Palindormic grids given in (8) and (9) are with four letters. Let us consider the following *palindromic grid* of order 4 with only two letters *a* and *b*:

abbabba	*babbbab*	*aaaaaaa*	*bbababb*
aaabaaa	*bbaaabb*	*abbbbba*	*bababab*
bbbbbbb	*aababaa*	*baabaab*	*abaaaba*
baaaaab	*abababa*	*bbbabbb*	*aabbbaa*

… (14)

For all $a, b \in \{1,2,3,4,5,6,7,8,9\}$, above grid represent a *palindromic pan diagonal magic square of order* 4 with sum

$$S_{4\times 4}(a,b) = 2\times(aaaaaaa + bbbbbbb) = (a+b)\times 2222222 \text{ or } \frac{S_{4\times 4}(a,b)}{a+b} = 2222222.$$

Here below are some examples:

Example 12. For $a=2$ and $b=5$ in (3), we have 7-digits *palindromic pan diagonal Selfie magic square* of order 4 with sum $S_{4\times 4}(2,5) = (2+5)\times 2222222 = 15555554$:

2552552	5255525	2222222	5525255
2225222	5522255	2555552	5252525
5555555	2252522	5225225	2522252
5222225	2525252	5552555	2255522

Example 13. For $a=1$ and $b=8$ in (3), we have 7-digits *palindromic pan diagonal Selfie magic square* of order 4 with sum $S_{4\times 4}(1,8) = (1+8)\times 2222222 = 19999998$:

1881881	8188818	1111111	8818188
1118111	8811188	1888881	8181818
8888888	1181811	8118118	1811181
8111118	1818181	8881888	1188811

Example 14. For $a=6$ and $b=9$, we have 7-digits *palindromic pan diagonal upside down magic square* of order 4 with sum $S_{4\times 4}(6,9) = (6+9)\times 2222222 = 33333330$.

[1]Formerly, Professor of Mathematics, Federal University of Santa Catarina, 88040-400 Florianópolis, SC, Brazil. Email: ijtaneja@gmail.com

6996996	9699969	6666666	9969699
6669666	9966699	6999996	9696969
9999999	6696966	9669669	6966696
9666669	6969696	9996999	6699966

Above three examples give us following symmetry:

$$\frac{S_{4\times4}(2,5)}{2+5}=\frac{S_{4\times4}(1,8)}{1+8}=\frac{S_{4\times4}(6,9)}{6+9}=2222222.$$

2.3. Magic Squares of Order 5

Let consider following *intervally distributed pan diagonal magic square* of order 5.

		65	65	65	65	65
	1	7	13	19	25	65
65	18	24	5	6	12	65
65	10	11	17	23	4	65
65	22	3	9	15	16	65
65	14	20	21	2	8	65
	65	65	65	65	65	65

…. (15)

2.3.1. Palindromic Magic Squares of Order 5

Let us consider five letters a, b, c, d and e such that $a,b,c,d,e \in \{1,2,3,4,5,6,7,8,9\}$. We can make exactly 25 palindromes of 3-digits with these four letters:

1	2	3	4	5	6	7	8	9	10	11	12	13
aaa	*aba*	*aca*	*ada*	*aea*	*bab*	*bbb*	*bcb*	*bdb*	*beb*	*cac*	*cbc*	*ccc*
14	15	16	17	18	19	20	21	22	23	24	25	
cdc	*cec*	*dad*	*dbd*	*dcd*	*ddd*	*ded*	*eae*	*ebe*	*ece*	*ede*	*eee*	

Replacing the above values with their respective palindromes in (12), we get the following grid of order 5:

aaa	*bbb*	*ccc*	*ddd*	*eee*
dcd	*ede*	*aea*	*bab*	*cbc*
beb	*cac*	*dbd*	*ece*	*ada*
ebe	*aca*	*bdb*	*cec*	*dad*
cdc	*ded*	*eae*	*aba*	*bcb*

… (16)

For all $a,b,c,d,e \in \{1,2,3,4,5,6,7,8,9\}$, above grid represent a *palindromic magic square* of order 5. Its sum is

[1]Formerly, Professor of Mathematics, Federal University of Santa Catarina, 88040-400 Florianópolis, SC, Brazil. Email: ijtaneja@gmail.com

$$S_{5\times 5}(a,b,c,d,e) = (a+b+c+d+e)\times 111 \text{ or } \frac{S_{4\times 4}(a,b,c,d,e)}{a+b+c+d+e} = 111.$$

As a consequence of (16), we have following 4 and 5-digits *palindromic grids* with a, b, c, d and e:

aaaa	*bbbb*	*cccc*	*dddd*	*eeee*
dccd	*edde*	*aeea*	*baab*	*cbbc*
beeb	*caac*	*dbbd*	*ecce*	*adda*
ebbe	*acca*	*bddb*	*ceec*	*dadd*
cddc	*deed*	*eaae*	*abba*	*bccb*

aaaaa	*abbba*	*accca*	*addda*	*aeeea*
adcda	*aedea*	*aaeaa*	*ababa*	*acbca*
abeba	*acaca*	*adbda*	*aecea*	*aadaa*
aebea	*aacaa*	*abdba*	*aceca*	*adada*
acdca	*adeda*	*aeaea*	*aabaa*	*abcba*

aaaaa	*bbabb*	*ccacc*	*ddadd*	*eeaee*
dcacd	*edade*	*aeaea*	*baaab*	*cbabc*
beaeb	*caaac*	*dbabd*	*ecace*	*adada*
ebabe	*acaca*	*bdadb*	*ceaec*	*daaad*
cdadc	*deaed*	*eaaae*	*ababa*	*bcacb*

… (17)

Making proper choices of $a,b,c,d \in \{0,1,2,3,4,5,6,7,8,9\}$, the above three situations lead us to a *palindromic magic squares* of order 5. For the case zero, some numbers may be considered as 010, 060, etc. Here below are some examples:

Example 15. For $a=1, b=2, c=3, d=4$ and $e=5$ in (17), we have 3-digits *palindromic pan diagonal magic square* of order 5 with sum $S_{5\times 5}(1,2,3,4,5) = (1+2+3+4+5)\times 111 = 1665$:

111	222	333	444	555
434	545	151	212	323
252	313	424	535	141
525	131	242	353	414
343	454	515	121	232

Example 16. For $a=0, b=1, c=2, d=5$ and $e=8$ in (17), we have 3-digits *palindromic pan diagonal Selfie magic square* of order 5 with sum $S_{5\times 5}(0,1,2,5,8) = (0+1+2+5+8)\times 111 = 1776 = 888+888$:

000	111	222	555	888
525	858	080	101	212
181	202	515	828	050
818	020	151	282	505
252	585	808	010	121

Example 17. For $a=0, b=1, c=6, d=8$ and $e=9$ in (17), we have 3-digits *palindromic pan diagonal upside down magic square* of order 5 with sum $S_{5\times 5}(2,5,6,8,9) = (2+5+6+8+9)\times 111 = 3330$:

222	555	666	888	999
868	989	292	525	656
595	626	858	969	282
959	262	585	696	828
686	898	929	252	565

[1]Formerly, Professor of Mathematics, Federal University of Santa Catarina, 88040-400 Florianópolis, SC, Brazil. Email: ijtaneja@gmail.com

2.3.2. Double Colored Pattern of Order 5

The grid (16) lead us to following mutually orthogonal diagonized Latin squares:

a	*b*	*c*	*d*	*e*
d	*e*	*a*	*b*	*c*
b	*c*	*d*	*e*	*a*
e	*a*	*b*	*c*	*d*
c	*d*	*e*	*a*	*b*

A

a	*b*	*c*	*d*	*e*
c	*d*	*e*	*a*	*b*
e	*a*	*b*	*c*	*d*
b	*c*	*d*	*e*	*a*
d	*e*	*a*	*b*	*c*

B

aa	*bb*	*cc*	*dd*	*ee*
dc	*ed*	*ae*	*ba*	*cb*
be	*ca*	*db*	*ec*	*ad*
eb	*ac*	*bd*	*ce*	*da*
cd	*de*	*ea*	*ab*	*bc*

AB

In particular $a=1, b=2, c=3, d=4$ and $e=5$, we have

1	2	3	4	5
4	5	1	2	3
2	3	4	5	1
5	1	2	3	4
3	4	5	1	2

A

1	2	3	4	5
3	4	5	1	2
5	1	2	3	4
2	3	3	5	1
4	5	1	2	3

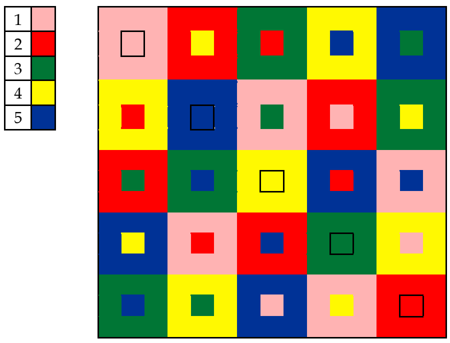
B

11	22	33	44	55
43	54	15	21	32
25	31	42	53	14
52	13	23	35	41
34	45	51	12	23

AB

…. (18)

Appying the procedure $5\times(A-1)+B$ in (18), we get magic square of order 5 given in (15). A and B are *"mutually diagonalized orthogonal Latin squares"*. The magic square AB given in (18) is a *composite magic square* of order 5. Based on it, here below is a *"double colored pattern"*.



2.3.3. Self-orthogonal Diagonalized Latin Square of Order 5

We observe that the the Latin squares A and B are not self-orthogonal with each other. The grid A can be considered as self-orthogonal. In this case, the grid B is obtained by transposing the members of A changing rows by colunms. It lead us to following magic square of order 5.

Example 18. The magic square is obtained by using $5\times(A-1)+B$:

[1]Formerly, Professor of Mathematics, Federal University of Santa Catarina, 88040-400 Florianópolis, SC, Brazil. Email: ijtaneja@gmail.com

1	2	3	4	5
4	5	1	2	3
2	3	4	5	1
5	1	2	3	4
3	4	5	1	2

A

1	4	2	5	3
2	5	3	1	4
3	1	4	2	5
4	2	5	3	1
5	3	1	4	2

B

		65	65	65	65	65
	1	9	12	20	23	65
65	17	25	3	6	14	65
65	8	11	19	22	5	65
65	24	2	10	13	16	65
65	15	18	21	4	7	65
	65	65	65	65	65	65

…. (19)

Here we have two different kind of *mutually diagonalized orthogonal Latin squares* given in (18) and (19). Later we shall use them to produce bimagic square of order 25.

2.3.4. Non Palindromic Magic Squares of Order 5

Examples 16 and 17 are the examples of *Selfie* and *upside down palindromic magic squares* respectively. Still, we can have these kind of magic squares not necessarily palindromic. See the examples below.

Example 19. Here below are two examples of *non palindromic Selfie* and *upside down magic squares* of order 5 with sums $S_{5\times5}(2,5,8)=24442$ and $S_{5\times5}(1,6,9)=34441$ respectively.

2222	2555	5225	5588	8852
5525	8888	2252	2522	5255
2552	5222	5555	8825	2288
8855	2225	2588	5252	5522
5288	5552	8822	2255	2525

1111	6696	6966	9699	9969
9666	9999	1169	6611	6996
6669	6911	9696	9966	1199
9996	1166	6699	6969	9611
6999	9669	9911	1196	6666

The first is *Selfie magic square*, while second is only *upside down*.

2.4. Magic Squares of Order 6

Let consider the *intervally distributed magic square of order* 6.

						111
1	23	28	34	17	8	111
29	7	35	14	21	5	111
12	6	13	27	31	22	111
32	16	4	24	10	25	111
19	33	11	3	30	15	111
18	26	20	9	2	36	111
111	111	111	111	111	111	111

…. (20)

[1]Formerly, Professor of Mathematics, Federal University of Santa Catarina, 88040-400 Florianópolis, SC, Brazil. Email: ijtaneja@gmail.com

2.4.1. Plaindromic Magic Squares of Order 6

Let us consider six letters a, b, c, d, e and f with $a, b, c, d, e, f \in \{0,1,2,3,4,5,6,7,8,9\}$. We can make exactly 36 palindromes of 3-digits with these six letters:

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18
aaa	*aba*	*aca*	*ada*	*aea*	*afa*	*bab*	*bbb*	*bcb*	*bdb*	*beb*	*bfb*	*cac*	*cbc*	*ccc*	*cdc*	*cec*	*cfc*
19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36
dad	*dbd*	*dcd*	*ddd*	*ded*	*dfd*	*eae*	*ebe*	*ece*	*ede*	*eee*	*efe*	*faf*	*fbf*	*fcf*	*fdf*	*fef*	*fff*

Replacing above values with their respective palindromes in (20), we get the following grid of order 6:

beb	*aba*	*dcd*	*ede*	*fbf*	*cec*
aaa	*ccc*	*fef*	*ded*	*eae*	*bfb*
cac	*fcf*	*ece*	*bbb*	*dfd*	*afa*
dbd	*bdb*	*aca*	*fdf*	*cfc*	*ebe*
efe	*ddd*	*bcb*	*cbc*	*aea*	*faf*
fff	*eee*	*cdc*	*ada*	*bab*	*dad*

… (21)

In this case we don't have many options to be a magic square. See the examples below.

Example 20. For $a=1, b=2, c=3, d=4, e=5$ and $f=6$ in (21), we have 3-digits *palindromic magic square* of order 6 with sum $S_{6\times6}(1,2,3,4,5,6)=(1+2+3+4+5+6)\times111=2331$:

252	121	434	545	626	353
111	333	656	454	515	262
313	636	535	222	464	161
424	242	131	646	363	525
565	444	232	323	151	616
666	555	343	141	212	414

…. (22)

In case of non-squential number it is not necessary that we always have a magic square. See the example below:

Example 21. For $a=1, b=2, c=3, d=5, e=7$ and $f=9$ in (21), we have following grid of order 6.

						2997
272	121	535	757	929	373	2987
111	333	979	575	717	292	3007
313	939	737	222	595	191	2997
525	252	131	959	393	727	2987
797	555	232	323	171	919	2997
999	777	353	151	212	515	3007
3017	2977	2967	2987	3017	3017	2987

[1]Formerly, Professor of Mathematics, Federal University of Santa Catarina, 88040-400 Florianópolis, SC, Brazil. Email: ijtaneja@gmail.com

Here we observe that the example 20 is just a special case. The grid (21) don't give always a magic square. In case of sequential numbers we always have a *palindromic magic square*. See another example below:

Example 22. For $a=3, b=4, c=5, d=6, e=7$ and $f=8$ in (21), we have 3-digits *palindromic magic square* of order 6 with *palindromic* sum $S_{6\times 6}(3,4,5,6,7,8)=(3+4+5+6+7+8)\times 111$ $=33\times 111=3663.$

						3663
474	343	656	767	848	575	3663
333	555	878	676	737	484	3663
535	858	757	444	686	383	3663
646	464	353	868	585	747	3663
787	666	454	545	373	838	3663
888	777	565	363	434	636	3663
3663	3663	3663	3663	3663	3663	3663

2.4.2. Composite Magic Square of Order 6

Removing third value in each cell of magic square (22), then spitting the result, we get

2	1	4	5	6	3
1	3	6	4	5	2
3	6	5	2	4	1
4	2	1	6	3	5
5	4	2	3	1	6
6	5	3	1	2	4

A

5	2	3	4	2	5
1	3	5	5	1	6
1	3	3	2	6	6
2	4	3	4	6	2
6	4	3	2	5	1
6	5	4	4	1	1

B

25	12	43	54	62	35
11	33	65	45	51	26
31	63	53	22	46	16
42	24	13	64	36	52
56	44	23	32	15	61
66	55	34	14	21	41

AB

…. (23)

Applying the procedure $6\times(A-1)+B$, we get magic square (20). The composite grid AB is also a *magic square* with magic sum 231.

Here we observe that even though the magic square is *intervally distributed*, but the decomposition given above doesn't lead us to *mutually orthogonal diagonalized Latin squares*. The grid A is *diagonalized Latin square*. B is just a grid of order 6, as there are numbers repeating in same row or same column.

2.5. Magic Squares of Order 7

Let consider following *intervally distributed pan diagonal* magic square of order 7:

[1]Formerly, Professor of Mathematics, Federal University of Santa Catarina, 88040-400 Florianópolis, SC, Brazil. Email: ijtaneja@gmail.com

		175	175	175	175	175	175	175
	1	9	17	25	33	41	49	175
175	40	48	7	8	16	24	32	175
175	23	31	39	47	6	14	15	175
175	13	21	22	30	38	46	5	175
175	45	4	12	20	28	29	37	175
175	35	36	44	3	11	19	27	175
175	18	26	34	42	43	2	10	175
	175	175	175	175	175	175	175	175

…. (24)

2.5.1. Palindromic Magic Squares of Order 7

Let us consider seven letters a, b, c, d, e, f and g such that $a,b,c,d,e,f,g \in \{0,1,2,3,4,5,6,7,8,9\}$. We can make exactly 49 palindromes of 3-digits with these seven letters:

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25
aaa	*aba*	*aca*	*ada*	*aea*	*afa*	*aga*	*bab*	*bbb*	*bcb*	*bdb*	*beb*	*bfb*	*bgb*	*cac*	*cbc*	*ccc*	*cdc*	*cec*	*cfc*	*cgc*	*dad*	*dbd*	*dcd*	*ddd*
26	27	28	29	30	31	32	33	34	35	36	37	38	39	40	41	42	43	24	45	46	47	48	49	
ded	*dfd*	*dgd*	*faf*	*fbf*	*fcf*	*fdf*	*fef*	*fff*	*fgf*	*faf*	*fbf*	*fcf*	*fdf*	*fef*	*fff*	*fgf*	*gag*	*gbg*	*gcg*	*gdg*	*geg*	*gfg*	*ggg*	

Replacing the above values with their respective palindromes in (24), we get the following grid of order 7:

aaa	*bbb*	*ccc*	*ddd*	*eee*	*fff*	*ggg*
fef	*gfg*	*aga*	*bab*	*cbc*	*dcd*	*ede*
dbd	*ece*	*fdf*	*geg*	*afa*	*bgb*	*cac*
bfb	*cgc*	*dad*	*ebe*	*fcf*	*gdg*	*aea*
gcg	*ada*	*beb*	*cfc*	*dgd*	*eae*	*fbf*
ege	*faf*	*gbg*	*aca*	*bdb*	*cec*	*dfd*
cdc	*ded*	*efe*	*fgf*	*gag*	*aba*	*bcb*

…. (25)

For all $a,b,c,d,e,f,g \in \{0,1,2,3,4,5,6,7,8,9\}$, above grid represent a 3-digits *palindromic pan diagonal magic square of order* 7 with sum

$$S_{7\times 7}(a,b,c,d,e,f,g) = (a+b+c+d+e+f+g) \times 111.$$

As a consequence of (25), we have following 4 and 5-digits *palindromic grids* with a, b, c, d, e, f and g:

[1]Formerly, Professor of Mathematics, Federal University of Santa Catarina, 88040-400 Florianópolis, SC, Brazil. Email: ijtaneja@gmail.com

aaaa	*bbbb*	*cccc*	*dddd*	*eeee*	*ffff*	*gggg*
feef	*gffg*	*agga*	*baab*	*cbbc*	*dccd*	*edde*
dbbd	*ecce*	*fddf*	*geeg*	*affa*	*bggb*	*caac*
bffb	*cggc*	*daad*	*ebbe*	*fccf*	*gddg*	*aeea*
gccg	*adda*	*beeb*	*cffc*	*dggd*	*eaae*	*fbbf*
egge	*faaf*	*gbbg*	*acca*	*bddb*	*ceec*	*dffd*
cddc	*deed*	*effe*	*fggf*	*gaag*	*abba*	*bccb*

aaaaa	*abbba*	*accca*	*addda*	*aeeea*	*afffa*	*aggga*
afefa	*agfga*	*aagaa*	*ababa*	*acbca*	*adcda*	*aedea*
adbda	*aecea*	*afdfa*	*agega*	*aafaa*	*abgba*	*acaca*
abfba	*acgca*	*adada*	*aebea*	*afcfa*	*agdga*	*aaeaa*
agcga	*aadaa*	*abeba*	*acfca*	*adgda*	*aeaea*	*afbfa*
aegea	*afafa*	*agbga*	*aacaa*	*abdba*	*aceca*	*adfda*
acdca	*adeda*	*aefea*	*afgfa*	*agaga*	*aabaa*	*abcba*

aaaaa	*bbabb*	*ccacc*	*ddadd*	*eeaee*	*ffaff*	*ggagg*
feaef	*gfafg*	*agaga*	*baaab*	*cbabc*	*dcacd*	*edade*
dbabd	*ecace*	*fdadf*	*geaeg*	*afafa*	*bgagb*	*caaac*
bfafb	*cgagc*	*daaad*	*ebabe*	*fcacf*	*gdadg*	*aeaea*
gcacg	*adada*	*beaeb*	*cfafc*	*dgagd*	*eaaae*	*fbabf*
egage	*faaaf*	*gbabg*	*acaca*	*bdadb*	*ceaec*	*dfafd*
cdadc	*deaed*	*efafe*	*fgagf*	*gaaag*	*ababa*	*bcacb*

…. (26)

For $a,b,c,d,e,f,g \in \{0,1,2,3,4,5,6,7,8,9\}$, the above three grids lead us to a *palindromic magic squares* of order 7. For the case zero, some numbers are considered as 010, 060, etc. Here below are some examples of (25):

Example 23. For $a=1, b=2, c=3, d=4, e=5, f=6$ and $g=7$ in (6), we have 3-digits *palindromic pan diagonal magic square* of order 7 with sum $S_{7\times 7}(1,2,3,4,5,6,7) = (1+2+3+4+5+6+7)\times 111 = 3107$:

111	222	333	444	555	666	777
656	767	171	212	323	434	545
424	535	646	757	161	272	313
262	373	414	525	636	747	151
737	141	252	363	474	515	626
575	616	727	131	242	353	464
343	454	565	676	717	121	232

Example 24. For $a=1, b=2, c=3, d=5, e=6, f=8$ and $g=9$ in (6), we have 3-digits *palindromic pan diagonal magic square* of order 7 with sum $S_{7\times 7}(1,2,3,5,6,8,9) = (1+2+3+5+6+8+9)\times 111 = 3774$:

111	222	333	555	666	888	999
868	989	191	212	323	535	656
525	636	858	969	181	292	313
282	393	515	626	838	959	161
939	151	262	383	595	616	828
696	818	929	131	252	363	585
353	565	686	898	919	121	232

[1]Formerly, Professor of Mathematics, Federal University of Santa Catarina, 88040-400 Florianópolis, SC, Brazil. Email: ijtaneja@gmail.com

In this case, still we don't have *Selfie palindromic magic square*, but here below is an example of *upside down palindromic magic square.*

Example 25. For $a=0, b=1, c=2, d=5, e=6, f=8$ and $g=9$ in (6), we have 3-digits *palindromic pan diagonal upside down magic square* of order 7 with sum $S_{7\times7}(0,1,2,5,6,8,9)=$ $(0+1+2+5+6+8+9)\times111=3441$:

000	111	222	555	666	888	999
868	989	090	101	212	525	656
515	626	858	969	080	191	202
181	292	505	616	828	959	060
929	050	161	282	595	606	818
696	808	919	020	151	262	585
252	565	686	898	909	010	121

In order to make it upside down, we considered *palindromic symmetry* in numbers, such as: 000, 010, 020, 050, 060, 080 and 090.

2.5.2. Double Colored Patterns

Removing the last term in each cell in (25), then decomposing lead us to following *pairwise mutually orthogonal diagonized Latin squares*:

a	*b*	*c*	*d*	*e*	*f*	*g*
f	*g*	*a*	*b*	*c*	*d*	*e*
d	*e*	*f*	*g*	*a*	*b*	*c*
b	*c*	*d*	*e*	*f*	*g*	*a*
g	*a*	*b*	*c*	*d*	*e*	*g*
e	*f*	*g*	*a*	*b*	*c*	*d*
c	*d*	*e*	*f*	*g*	*a*	*b*

A

a	*b*	*c*	*d*	*e*	*f*	*g*
e	*f*	*g*	*a*	*b*	*c*	*d*
b	*c*	*d*	*e*	*f*	*g*	*a*
f	*g*	*a*	*b*	*c*	*d*	*e*
c	*d*	*e*	*f*	*g*	*a*	*b*
g	*a*	*b*	*c*	*d*	*e*	*f*
d	*e*	*f*	*g*	*a*	*b*	*c*

B

aa	*bb*	*cc*	*dd*	*ee*	*ff*	*gg*
fe	*gf*	*ag*	*ba*	*cb*	*dc*	*ed*
db	*ec*	*fd*	*ge*	*af*	*bg*	*ca*
bf	*cg*	*da*	*eb*	*fc*	*gd*	*ae*
gc	*ad*	*be*	*cf*	*dg*	*ea*	*fb*
eg	*fa*	*gb*	*ac*	*bd*	*ce*	*df*
cd	*de*	*ef*	*fg*	*ga*	*ab*	*bc*

AB

In particular, for $a=1, b=2, c=3, d=4, e=5, f=6$ and $g=7$, we get

1	2	3	4	5	6	7
6	7	1	2	3	4	5
4	5	6	7	1	2	3
2	3	4	5	6	7	1
7	1	2	3	4	5	6
5	6	7	1	2	3	4
3	4	5	6	7	1	2

A

1	2	3	4	5	6	7
5	6	7	1	2	3	4
2	3	4	5	6	7	1
6	7	1	2	3	4	5
3	4	5	6	7	1	2
7	1	2	3	4	5	6
4	5	6	7	1	2	3

B

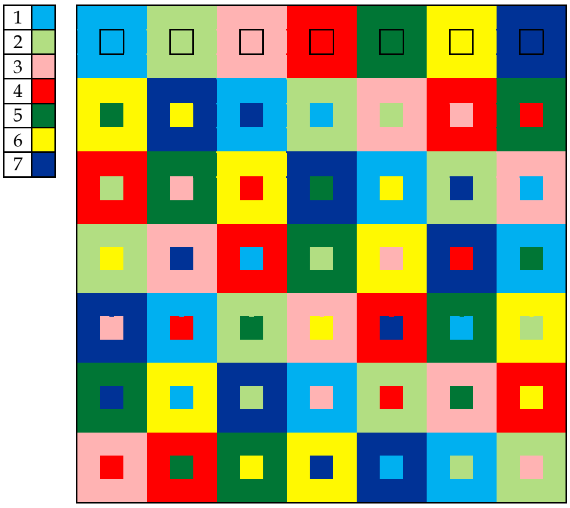
11	22	33	44	55	66	77
65	76	17	21	32	43	54
42	53	64	75	16	27	31
26	37	41	52	63	74	15
73	14	25	36	47	51	62
57	61	72	13	24	35	46
34	45	56	67	71	12	23

AB

…. (27)

[1]Formerly, Professor of Mathematics, Federal University of Santa Catarina, 88040-400 Florianópolis, SC, Brazil. Email: ijtaneja@gmail.com

If apply the procedure $7\times(A-1)+B$ in (27), we get magic square of order 7 given in (24). Moreover, A and B are *"pairwise mutually orthogonal diagonalized Latin squares"*. According to AB given in (27), below is a *double colored pattern*:



2.5.3. Self-orthogonal Diagonalized Latin Square of Order 7

The grid A appearing in (27) can be considered as *self-orthogonal*. In this case, the grid B is obtained by transposing the members of A changing rows by colunms. It lead us to following magic square of order 7.

Example 26.

1	2	3	4	5	6	7
6	7	1	2	3	4	5
4	5	6	7	1	2	3
2	3	4	5	6	7	1
7	1	2	3	4	5	6
5	6	7	1	2	3	4
3	4	5	6	7	1	2

A

1	6	4	2	7	5	3
2	7	5	3	1	6	4
3	1	6	4	2	7	5
4	2	7	5	3	1	6
5	3	1	6	4	2	7
6	4	2	7	5	3	1
7	5	3	1	6	4	2

B

		175	175	175	175	175	175	175
	1	13	18	23	35	40	45	175
175	37	49	5	10	15	27	32	175
175	24	29	41	46	2	14	19	175
175	11	16	28	33	38	43	6	175
175	47	3	8	20	25	30	42	175
175	34	39	44	7	12	17	22	175
175	21	26	31	36	48	4	9	175
	175	175	175	175	175	175	175	175

….. (28)

The grid B is transpose of A. We get the magic square of order 7 applying the procedure $7\times(A-1)+B$. Later we shall use (27) and (28) to produce bimagic square of order 49.

2.5.4. Non Palindromic Selfie Magic Squares of Order 7

The example below give two magic squares of order 7. First one *is Selfie magic square* while second one is just *upside down.*

Example 27. The following magic squares are *Selfie* and *upside down* with magic sums $S_{7\times7}(1,2,5,8):=34441$ and $S_{7\times7}(1,6,8,9):=52217$ respectively:

[1]Formerly, Professor of Mathematics, Federal University of Santa Catarina, 88040-400 Florianópolis, SC, Brazil. Email: ijtaneja@gmail.com

1818	2222	2525	5252	5555	8181	8888
8155	8881	1888	2218	2522	5225	5552
5222	5525	8152	8855	1881	2288	2518
2281	2588	5218	5522	8125	8852	1855
8825	1852	2255	2581	5288	5518	8122
5588	8118	8822	1825	2252	2555	5281
2552	5255	5581	8188	8818	1822	2225

1818	6666	6969	8181	8888	9696	9999
9688	9996	1899	6618	6966	8169	8881
8166	8869	9681	9988	1896	6699	6918
6696	6999	8118	8866	9669	9981	1888
9969	1881	6688	6996	8199	8818	9666
8899	9618	9966	1869	6681	6988	8196
6981	8188	8896	9699	9918	1866	6669

…. (29)

First one is *Selfie*, while second is just *upside down*. Still we don't have *Selfie palindromic magic squares*. We can make (29) as *Selfie palindromic magic square* just increasing members in reverse order, such as 18188181, 81555518, etc. In this case it becomes 8-digits *palindromic magic square* with four numbers 1, 2, 5 and 8. The idea is to make with less number of palindromic digits. This shall be dealt elsewhere.

2.6. Magic Squares of Order 8

Let us consider following well know Pfeffermann's [2,3,5] *intervally distributed pan diagonal bimagic square* of order 8:

		260	260	260	260	260	260	260	260
	16	41	36	5	27	62	55	18	260
260	26	63	54	19	13	44	33	8	260
260	1	40	45	12	22	51	58	31	260
260	23	50	59	30	4	37	48	9	260
260	38	3	10	47	49	24	29	60	260
260	52	21	32	57	39	2	11	46	260
260	43	14	7	34	64	25	20	53	260
260	61	28	17	56	42	15	6	35	260
	260	260	260	260	260	260	260	260	260

								11180
256	1681	1296	25	729	3844	3025	324	11180
676	3969	2916	361	169	1936	1089	64	11180
1	1600	2025	144	484	2601	3364	961	11180
529	2500	3481	900	16	1369	2304	81	11180
1444	9	100	2209	2401	576	841	3600	11180
2704	441	1024	3249	1521	4	121	2116	11180
1849	196	49	1156	4096	625	400	2809	11180
3721	784	289	3136	1764	225	36	1225	11180
11180	11180	11180	11180	11180	11180	11180	11180	11180

…. (30)

The magic sums are $S_{8\times8} := 260$ and $Sb_{8\times8} := 11180$. By Sb we mean *bimagic sum*. Each block of order 2×4 in rows and columns has the same sum as of magic square.

The above magic square is *partially compact* as some blocks of order 2×4 are not of same sum as of magic square. It is *compact* in each block of order 4×4. Following the lines of (8) we have exactly 64 blocks of order 4 having sum 520. See below:

520	520	520	520	520	520	520	520
520	520	520	520	520	520	520	520
520	520	520	520	520	520	520	520
520	520	520	520	520	520	520	520
520	520	520	520	520	520	520	520
520	520	520	520	520	520	520	520
520	520	520	520	520	520	520	520
520	520	520	520	520	520	520	520

[1]Formerly, Professor of Mathematics, Federal University of Santa Catarina, 88040-400 Florianópolis, SC, Brazil. Email: ijtaneja@gmail.com

These 64 blocks of order 4×4 are not necessarily magic squares but the sum all 16 members each block is always 520.

2.6.1. Palindromic Magic Squares of Order 8

Let us consider eight letters a, b, c, d, e, f, g and h such that $a,b,c,d,e,f,g,h \in \{0,1,2,3,4,5,6,7,8,9\}$. We can make exactly 64 palindromes of 3-digits with these eight letters:

1 *aaa*	2 *aba*	3 *aca*	4 *ada*	5 *aea*	6 *afa*	7 *aga*	8 *aha*	9 *bab*	10 *bbb*	11 *bcb*	12 *bdb*	13 *beb*	14 *bfb*	15 *bgb*	16 *bhb*
17 *cac*	18 *cbc*	19 *ccc*	20 *cdc*	21 *cec*	22 *cfc*	23 *cgc*	24 *chc*	25 *dad*	26 *dbd*	27 *dcd*	28 *ddd*	29 *ded*	30 *dfd*	31 *dgd*	32 *dhd*
33 *eae*	34 *ebe*	35 *ece*	36 *ede*	37 *eee*	38 *efe*	39 *ege*	40 *ehe*	41 *faf*	42 *fbf*	43 *fcf*	44 *fdf*	45 *fef*	46 *fff*	47 *fgf*	48 *fhf*
49 *gag*	50 *gbg*	51 *gcg*	52 *gdg*	53 *geg*	54 *gfg*	55 *ggg*	56 *ghg*	57 *hah*	58 *hbh*	59 *hch*	60 *hdh*	61 *heh*	62 *hfh*	63 *hgh*	64 *hhh*

Replacing the above values with their respective palindromes in (30), we get the following grid of order 8:

bhb	*faf*	*ede*	*aea*	*dcd*	*hfh*	*ggg*	*cbc*
dbd	*hgh*	*gfg*	*ccc*	*beb*	*fdf*	*eae*	*aha*
aaa	*ehe*	*fef*	*bdb*	*cfc*	*gcg*	*hbh*	*dgd*
cgc	*gbg*	*hch*	*dfd*	*ada*	*eee*	*fhf*	*bab*
efe	*aca*	*bbb*	*fgf*	*gag*	*chc*	*ded*	*hdh*
gdg	*cec*	*dhd*	*hah*	*ege*	*aba*	*bcb*	*fff*
fcf	*bfb*	*aga*	*ebe*	*hhh*	*dad*	*cdc*	*geg*
heh	*ddd*	*cac*	*ghg*	*fbf*	*bgb*	*afa*	*ece*

… (31)

For all $a,b,c,d,e,f,g,h \in \{0,1,2,3,4,5,6,7,8,9\}$, above grid represent a *palindromic magic square of order* 8. Its sum is

$$S_{8\times 8}(a,b,c,d,e,f,g,h) = (a+b+c+d+e+f+g+h)\times 111 .$$

As a consequence of (31), we have following 4 and 5-digits *palindromic grids* with eight digits a, b, c, d, e, f, g and h:

bhhb	*faaf*	*edde*	*aeea*	*dccd*	*hffh*	*gggg*	*cbbc*
dbbd	*hggh*	*gffg*	*cccc*	*beeb*	*fddf*	*eaae*	*ahha*
aaaa	*ehhe*	*feef*	*bddb*	*cffc*	*gccg*	*hbbh*	*dggd*
cggc	*gbbg*	*hcch*	*dffd*	*adda*	*eeee*	*fhhf*	*baab*
effe	*acca*	*bbbb*	*fggf*	*gaag*	*chhc*	*deed*	*hddh*
gddg	*ceec*	*dhhd*	*haah*	*egge*	*abba*	*bccb*	*ffff*
fccf	*bffb*	*agga*	*ebbe*	*hhhh*	*daad*	*cddc*	*geeg*
heeh	*dddd*	*caac*	*ghhg*	*fbbf*	*bggb*	*affa*	*ecce*

[1]Formerly, Professor of Mathematics, Federal University of Santa Catarina, 88040-400 Florianópolis, SC, Brazil. Email: ijtaneja@gmail.com

abhba	*afafa*	*aedea*	*aaeaa*	*adcda*	*ahfha*	*aggga*	*acbca*
adbda	*ahgha*	*agfga*	*accca*	*abeba*	*afdfa*	*aeaea*	*aahaa*
aaaaa	*aehea*	*afefa*	*abdba*	*acfca*	*agcga*	*ahbha*	*adgda*
acgca	*agbga*	*ahcha*	*adfda*	*aadaa*	*aeeea*	*afhfa*	*ababa*
aefea	*aacaa*	*abbba*	*afgfa*	*agaga*	*achca*	*adeda*	*ahdha*
agdga	*aceca*	*adhda*	*ahaha*	*aegea*	*aabaa*	*abcba*	*afffa*
afcfa	*abfba*	*aagaa*	*aebea*	*ahhha*	*adada*	*acdca*	*agega*
aheha	*addda*	*acaca*	*aghga*	*afbfa*	*abgba*	*aafaa*	*aecea*

bhahb	*faaaf*	*edade*	*aeaea*	*dcacd*	*hfafh*	*ggagg*	*cbabc*
dbabd	*hgagh*	*gfafg*	*ccacc*	*beaeb*	*fdadf*	*eaaae*	*ahaha*
aaaaa	*ehahe*	*feaef*	*bdadb*	*cfafc*	*gcacg*	*hbabh*	*dgagd*
cgagc	*gbabg*	*hcach*	*dfafd*	*adada*	*eeaee*	*fhahf*	*baaab*
efafe	*acaca*	*bbabb*	*fgagf*	*gaaag*	*chahc*	*deaed*	*hdadh*
gdadg	*ceaec*	*dhahd*	*haaah*	*egage*	*ababa*	*bcacb*	*ffaff*
fcacf	*bfafb*	*agaga*	*ebabe*	*hhahh*	*daaad*	*cdadc*	*geaeg*
heaeh	*ddadd*	*caaac*	*ghahg*	*fbabf*	*bgagb*	*afafa*	*ecace*

… (32)

Example 28. For $a=1, b=2, c=3, d=4, e=5, f=6, g=7$ and $h=8$ in (31), we have 3-digits *palindromic pan diagonal bimagic square* of order 8:

282	616	545	151	434	868	777	323
424	878	767	333	252	646	515	181
111	585	656	242	363	737	828	474
373	727	838	464	141	555	686	212
565	131	222	676	717	383	454	848
747	353	484	818	575	121	232	666
636	262	171	525	888	414	343	757
858	444	313	787	626	272	161	535

Its sums are

$$S_{8\times 8}(1,2,3,4,5,6,7,8) = (1+2+3+4+5+6+7+8)\times 111 = 3996$$

and

$$Sb_{8\times 8}(1,2,3,4,5,6,7,8) = 2428644.$$

We have total 10 digits, i.e., $\{0,1,2,3,4,5,6,7,8,9\}$. Above example considered first eight numbers in a sequence. If we consider, number not in a sequence, still we have *pan diagonal bimagic squares*. See example below:

Example 29. For $a=1, b=2, c=3, d=4, e=6, f=7, g=8$ and $h=9$ in (31), we have 3-digits *palindromic pan diagonal bimagic square* of order 8:

292	717	646	161	434	979	888	323
424	989	878	333	262	747	616	191
111	696	767	242	373	838	929	484
383	828	939	474	141	666	797	212
676	131	222	787	818	393	464	949
848	363	494	919	686	121	232	777
737	272	181	626	999	414	343	868
969	444	313	898	727	282	171	636

Its sums are

$$S_{8\times 8}(1,2,3,4,6,7,8,9) = (1+2+3+4+6+7+8+9)\times 111 = 4440$$

and

$$Sb_{8\times 8}(1,2,3,4,6,7,8,9) = 3082260 .$$

[1]Formerly, Professor of Mathematics, Federal University of Santa Catarina, 88040-400 Florianópolis, SC, Brazil. Email: ijtaneja@gmail.com

We have only 7 upside down numbrs, i.e., 0, 1, 2, 5, 6, 8 and 9. In order to bring *Selfie magic square* of order 8, we shall try to reduce this number and increase palindromic digits. This is done in the following two subsections. Magic squares appearing in examples 31 and 32 are *compact* of order 4

2.6.2. 5-Digits Palindromic Magic Square of Order 8

Above subsection is with 3-digits palindromes having eight letters. Let us consider the following 5-digits *palindromic grid* of order 8 only with four letters *a, b, c* and *d*:

addda	*ccacc*	*cadac*	*ababa*	*bcccb*	*ddbdd*	*dbcbd*	*babab*
bcbcb	*ddcdd*	*dbbbd*	*bacab*	*adada*	*ccdcc*	*caaac*	*abdba*
aaaaa	*cbdbc*	*cdadc*	*acdca*	*bbbbb*	*dacad*	*dcbcd*	*bdcdb*
bbcbb	*dabad*	*dcccd*	*bdbdb*	*aadaa*	*cbabc*	*cdddc*	*acaca*
cbbbc	*aacaa*	*acbca*	*cdcdc*	*daaad*	*bbdbb*	*bdadb*	*dcdcd*
dadad	*bbabb*	*bdddb*	*dcacd*	*cbcbc*	*aabaa*	*accca*	*cdbdc*
ccccc	*adbda*	*abcba*	*cabac*	*ddddd*	*bcacb*	*badab*	*dbabd*
ddadd	*bcdcb*	*baaab*	*dbdbd*	*ccbcc*	*adcda*	*abbba*	*cacac*

… (33)

Above grid is not always a magic square? In some cases it is a semi-magic. The grid (31) is with eight letters (a,b,c,d,e,f,g,h), while (33) is just with four letter (a,b,c,d), but both are *palindromic*.

Example 30. For $a=1, b=2, c=3$ and $d=4$ in (33), we have 4-digits *palindromic pan diagonal magic square* of order 8 with sum $S_{8\times 8}(1,2,3,4)=(1+2+3+4)\times 22222=222220$:

14441	33133	31413	12121	23332	44244	42324	21212
23232	44344	42224	21312	14141	33433	31113	12421
11111	32423	34143	13431	22222	41314	43234	24342
22322	41214	43334	24242	11411	32123	34443	13131
32223	11311	13231	34343	41114	22422	24142	43434
41414	22122	24442	43134	32323	11211	13331	34243
33333	14241	12321	31213	44444	23132	21412	42124
44144	23432	21112	42424	33233	14341	12221	31313

Each block of order 2×4 is also have the same sum as of magic square. It is *semi-bimagic*. Bimagic sum of rows and columns is $Sb_{8\times 8}(1,2,3,4)=7183217060$, and main diagonals bimagic sum is 7183377060.

The following example is a *Selfie palindromic semi-magic square*.

Example 31. For $a=1, b=2, c=5$ and $d=8$ in (33), we have 5-digits *palindromic semi-magic square* of order 8:

[1]Formerly, Professor of Mathematics, Federal University of Santa Catarina, 88040-400 Florianópolis, SC, Brazil. Email: ijtaneja@gmail.com

18881	55155	51815	12121	25552	88288	82528	21212
25252	88588	82228	21512	18181	55855	51115	12821
11111	52825	58185	15851	22222	81518	85258	28582
22522	81218	85558	28282	11811	52125	58885	15151
52225	11511	15251	58585	81118	22822	28182	85858
81818	22122	28882	85158	52525	11211	15551	58285
55555	18281	12521	51215	88888	25152	21812	82128
88188	25852	21112	82828	55255	18581	12221	51515

Semi-magic sum is $S_{5\times5}(1,2,5,8) = (1+2+5+8)\times 22222 = 355552$ (sums of rows and columns). The main diagonals sum is 359592.

Examples 28 and 29 are *bimagic* squares. Example 30 is *semi-bimagic*, while example 31 is just *semi-magic*. Moreover, example 31 is *Selfie palindromic semi-magic square*. In order to bring *Selfie palindromic magic square* of order 8, we will increase palindromic digits. Here below are examples of 11-digits *Selfie palindromic magic square* of order 8 just with two letters.

2.6.3. 11-Digits Palindromic Magic Squares of Order 8

Above subsection is with 5-digits palindromes having four letters. Let us consider the following 11-digits *palindromic grid* of order 8 only with two letters *a* and *b*:

aabbbbbbbaa	*babaaaaabab*	*baaabbbaaab*	*aaabaaabaaa*	*abbababbba*	*bbbbababbbb*	*bbabbabbabb*	*abaaabaaaba*
abbaabaabba	*bbbbbabbbbb*	*bbababababb*	*abaababaaba*	*aabbaaabbaa*	*bababbbabab*	*baaaaaaaaab*	*aaabbbbbaaa*
aaaaaaaaaaa	*baabbbbbaab*	*babbaaabbab*	*aababbbabaa*	*ababababa*	*bbaababaabb*	*bbbaabaabbb*	*abbbbabbbba*
ababbabbaba	*bbaaabaaabb*	*bbbababbbb*	*abbbababbba*	*aaaabbbaaaa*	*baabaaabaab*	*babbbbbbbab*	*aabaaaaabaa*
baabababaab	*aaaababaaaa*	*aabaabaabaa*	*babbbabbbab*	*bbaaaaaaabb*	*ababbbbbaba*	*abbbaaabbba*	*bbbabbbabbb*
bbaabbbaabb	*ababaaababa*	*abbbbbbbbba*	*bbbaaaaabbb*	*baabbabbaab*	*aaaaabaaaaa*	*aababababaa*	*babbababbab*
babababab	*aabbababbaa*	*aaabbabbaaa*	*baaaabaaaab*	*bbbbbbbbbbb*	*abbaaaaabba*	*abaabbbaaba*	*bbabaaababb*
bbbbaaabbbb	*abbabbbabba*	*abaaaaaaaba*	*bbabbbbbabb*	*babaabaabab*	*aabbbabbbaa*	*aaabababaaa*	*baaababaaab*

…. (34)

For all $a,b\in\{0,1,2,3,4,5,6,7,8,9\}$, above grid represent a *palindromic magic square* with sum

$$S_{8\times8}(a,b) = 4\times(a+b)\times 11111111111 \text{ or } \frac{S_{8\times8}(a,b)}{a+b} = 44444444444.$$

We observe that there are exactly 64 palindromes of 11-digits with only two letters *a* and *b*.

Example 32. For $a=2$ and $b=5$ in (34), we have 11-digits *Selfie palindromic pan diagonal magic square* of order 8:

[1]Formerly, Professor of Mathematics, Federal University of Santa Catarina, 88040-400 Florianópolis, SC, Brazil. Email: ijtaneja@gmail.com

22555555522	52522222525	52225552225	22252225222	25525252552	55552525555	55255255255	25222522252
25522522552	55555255555	55252525255	25225252252	22552225522	52525552525	52222222225	22255555222
22222222222	52255555225	52552225525	22525552522	25252525252	55225252255	55522522555	25555255552
25255255252	55222522255	55525252555	25552525552	22225552222	52252225225	52555555525	22522222522
52252525225	22225252222	22522522522	52555255525	55222222255	25255555252	25552225552	55525552555
55225552255	25252225252	25555555552	55522222555	52255255225	22222522222	22525252522	52552525525
52525252525	22552525522	22255255222	52222522225	55555555555	25522222552	25225552252	55252225255
55552225555	25525552552	25222222252	55255555255	52522522525	22555255522	22252525222	52225252225

It is a magic square but semi-bimagic. Its sums are

$$S_{8\times8}(2,5) = 4\times(2+5)\times11111111111 = 311111111108,$$
$$Sb_{8\times8}(2,5) = 1391694725183860830527 6 \text{ (rows and columns)}$$

and

diagonals bimagic sum: 13916947255474608305276.

Instead 2 and 5, we can work with 1 and 8, and still have *Selfie magic square*. If we use 6 and 9, it becomes *upside down palindromic magic square*.

Example 33. For $a=6$ and $b=9$ in (34), we have 11-digits *Selfie palindromic pan diagonal magic square* of order 8:

66999999966	96966666969	96669996669	66696669666	69969696996	99996969999	99699699699	69666966696
69966966996	99999699999	99696969699	69669696696	66996669966	96969996969	96666666669	66699999666
66666666666	96699999669	96996669969	66969996966	69696969696	99669696699	99966966999	69999699996
69699699696	99666966699	99969696999	69996969996	66669996666	96696669669	96999999969	66966666966
96696969669	66669696666	66966966966	96999699969	99666666699	69699999696	69996669996	99969996999
99669996699	69696669696	69999999996	99966666999	96699699669	66666966666	66969696966	96996969969
96969696969	66996969966	66699699666	96666966669	99999999999	69966666996	69669996696	99696669699
99996669999	69969996996	69666666696	99699999699	96966966969	66999699966	66696969666	96669696669

It is a magic square and semi-bimagic. Its sums are

$$S_{8\times8}(2,5) = 4\times(6+9)\times11111111111 = 666666666660,$$
$$Sb_{8\times8}(2,5) = 57373737374426262626268 \text{ (rows and columns)}$$

and

Diagonals bimagic sum: 57373737378062262626268.

2.6.4. Double Colored Pattern

Removing the last term in each cell in (30), and then decomposing lead us to following *mutually orthogonal diagonized Latin squares*:

[1]Formerly, Professor of Mathematics, Federal University of Santa Catarina, 88040-400 Florianópolis, SC, Brazil. Email: ijtaneja@gmail.com

b	*f*	*e*	*a*	*d*	*h*	*g*	*c*
d	*h*	*g*	*c*	*b*	*f*	*e*	*a*
a	*e*	*f*	*b*	*c*	*g*	*h*	*d*
c	*g*	*h*	*d*	*a*	*e*	*f*	*b*
e	*a*	*b*	*f*	*g*	*c*	*d*	*h*
g	*c*	*d*	*h*	*e*	*a*	*b*	*f*
f	*b*	*a*	*e*	*h*	*d*	*c*	*g*
h	*d*	*c*	*g*	*f*	*b*	*a*	*e*

A

h	*a*	*d*	*e*	*c*	*f*	*g*	*b*
b	*g*	*f*	*c*	*e*	*d*	*a*	*h*
a	*h*	*e*	*d*	*f*	*c*	*b*	*g*
g	*b*	*c*	*f*	*d*	*e*	*h*	*a*
f	*c*	*b*	*g*	*a*	*h*	*e*	*d*
d	*e*	*h*	*a*	*g*	*b*	*c*	*f*
c	*f*	*g*	*b*	*h*	*a*	*d*	*e*
e	*d*	*a*	*h*	*b*	*g*	*f*	*c*

B

bh	*fa*	*ed*	*ae*	*dc*	*hf*	*gg*	*cb*
db	*hg*	*gf*	*cc*	*be*	*fd*	*ea*	*ah*
aa	*eh*	*fe*	*bd*	*cf*	*gc*	*hb*	*dg*
cg	*gb*	*hc*	*df*	*ad*	*ee*	*fh*	*ba*
ef	*ac*	*bb*	*fg*	*ga*	*ch*	*de*	*hd*
gd	*ce*	*dh*	*ha*	*eg*	*ab*	*bc*	*ff*
fc	*bf*	*ag*	*eb*	*hh*	*da*	*cd*	*ge*
he	*dd*	*ca*	*gh*	*fb*	*bg*	*af*	*ec*

AB

In particular for $a=1, b=2, c=3, d=4, e=5, f=6, g=7$ and $h=8$, we get

2	6	5	1	4	8	7	3
4	8	7	3	2	6	5	1
1	5	6	2	3	7	8	4
3	7	8	4	1	5	6	2
5	1	2	6	7	3	4	8
7	3	4	8	5	1	2	6
6	2	1	5	8	4	3	7
8	4	3	7	6	2	1	5

A

8	1	4	5	3	6	7	2
2	7	6	3	5	4	1	8
1	8	5	4	6	3	2	7
7	2	3	6	4	5	8	1
6	3	2	7	1	8	5	4
4	5	8	1	7	2	3	6
3	6	7	2	8	1	4	5
5	4	1	8	2	7	6	3

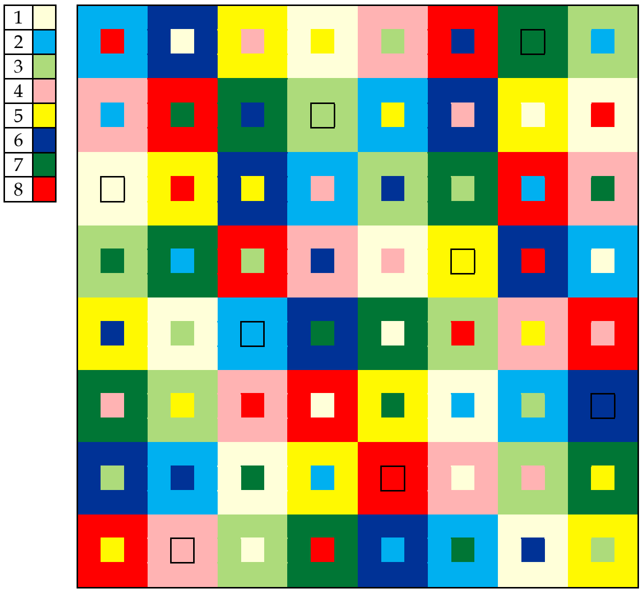
B

28	61	54	15	43	86	77	32
42	87	76	33	25	64	51	18
11	58	65	24	36	73	82	47
37	72	83	46	14	55	68	21
56	13	22	67	71	38	45	84
74	35	48	81	57	12	23	66
63	26	17	52	88	41	34	75
85	44	31	78	62	27	16	53

AB

…. (35)

Applying the procedure $8\times(A-1)+B$, we get magic square of order 8 given in (35). Moreover, A and B are *"pairwise mutually orthogonal diagonalized Latin squares"*. Based on (35), below is a *double colored pattern*.



We observed in above examples, that the magic squares obtained have the propery that each block of order 2×4 is of same sum as of magic square.

2.6.5. Self-orthogonal Diagonalized Latin Square of Order 8

The grids A and B appearing in (35) are self-orthogonal. For simplicity, let us consider grid A and write B as its transpose. Accordingly, we have following pan diagonal magic square of order 8:

[1]Formerly, Professor of Mathematics, Federal University of Santa Catarina, 88040-400 Florianópolis, SC, Brazil. Email: ijtaneja@gmail.com

Example 34. The following magic square of order 8 is due to self-orthogonality of grid A.

A

2	6	5	1	4	8	7	3
4	8	7	3	2	6	5	1
1	5	6	2	3	7	8	4
3	7	8	4	1	5	6	2
5	1	2	6	7	3	4	8
7	3	4	8	5	1	2	6
6	2	1	5	8	4	3	7
8	4	3	7	6	2	1	5

B

2	4	1	3	5	7	6	8
6	8	5	7	1	3	2	4
5	7	6	8	2	4	1	3
1	3	2	4	6	8	5	7
4	2	3	1	7	5	8	6
8	6	7	5	3	1	4	2
7	5	8	6	4	2	3	1
3	1	4	2	8	6	7	5

		260	260	260	260	260	260	260	260
	10	44	33	3	29	63	54	24	260
260	30	64	53	23	9	43	34	4	260
260	5	39	46	16	18	52	57	27	260
260	17	51	58	28	6	40	45	15	260
260	36	2	11	41	55	21	32	62	260
260	56	22	31	61	35	1	12	42	260
260	47	13	8	38	60	26	19	49	260
260	59	25	20	50	48	14	7	37	260
	260	260	260	260	260	260	260	260	260

….. (36)

The grid B is transpose of A. We get the magic square of order 8 applying the procedure $8\times(A-1)+B$. The magic square (36) is *pan-diagonal and compact* but not bimagic. The grid B is also *self-orthogonal diagonalized Latin square*, and lead us to a similar kind of magic square as of (36).

2.6.6. Non Palindromic Selfie Magic Squares of Order 8

Below are two examples of *Selfie magic squares* using only two numbers.

Example 35. The *Selfie magic square* given below is *pan-diagonal, compact of order 4* and *bimagic* with sums: $S_{8\times8}(2,5)=3111108$ and $Sb_{8\times8}(2,5)=1391692305276$:

225555	525222	522255	222522	255252	555525	552552	252225
255225	555552	552525	252252	225522	525255	522222	222555
222222	522555	525522	225255	252525	552252	555225	255552
252552	552225	555252	255525	222255	522522	525555	225222
522525	222252	225225	525552	552222	252555	255522	555255
552255	252522	255555	555222	522552	222225	225252	525525
525252	225525	222552	522225	555555	255222	252255	552522
555522	255255	252222	552555	525225	225552	222525	522252

Example 36. The *Selfie magic square* given below is *pan-diagonal, compact of order 4* and *bimagic* with sums $S_{8\times8}(1,8)=3999996$ and $Sb_{8\times8}(1,8)=2989894989900$:

118888	818111	811188	111811	188181	888818	881881	181118
188118	888881	881818	181181	118811	818188	811111	111888
111111	811888	818811	118188	181818	881181	888118	188881
181881	881118	888181	188818	111188	811811	818888	118111
811818	111181	118118	818881	881111	181888	188811	888188
881188	181811	188888	888111	811881	111118	118181	818818
818181	118818	111881	811118	888888	188111	181188	881811
888811	188188	181111	881888	818118	118881	111818	811181

The magic squares given in examples 35 and 36 are constructed based on the compostion (35). Above two magic square have 6 digits in each cell. Still, we can make *Selfie magic squares* of order 8 with less numbers in each cell, but are not bimagic.

[1]Formerly, Professor of Mathematics, Federal University of Santa Catarina, 88040-400 Florianópolis, SC, Brazil. Email: ijtaneja@gmail.com

Example 37. The example below is a *Selfie magic square* of order 8 with magic sum $S_{8\times8}(2,5,8):=41107$:

2585	5822	5552	2255	5228	8558	8282	2825
5225	8582	8258	2828	2555	5852	5522	2285
2222	5585	5855	2552	2858	8228	8525	5282
2882	8225	8528	5258	2252	5555	5885	2522
5558	2228	2525	5882	8222	2885	5255	8552
8252	2855	5285	8522	5582	2225	2528	5858
5828	2558	2282	5525	8585	5222	2852	8255
8555	5252	2822	8285	5825	2582	2258	5528

Example 38. The example below is a *upside magic square* of order 8 with magic sum $S_{8\times8}(1,6,9):=52217$:

1999	9116	6966	1669	6661	9991	9696	6119
6619	9996	9691	6161	1969	9166	6916	1699
1616	6999	9169	1966	6191	9661	9919	6696
6196	9619	9961	6691	1666	6969	9199	1916
6991	1661	1919	9196	9616	6199	6669	9966
9666	6169	6699	9916	6996	1619	1961	9191
9161	1991	1696	6919	9999	6616	6166	9669
9969	6666	6116	9699	9119	1996	1691	6961

Examples 37 and 38 are neither *pan diagonal* nor *compact of order 4*.

2.7. Magic Squares of Order 9

Let consider following well know Pfeffermann's [2,3,5] *intervally distributed bimagic square* of order 9:

									369
1	18	23	35	40	48	60	65	79	369
33	38	52	55	72	77	8	13	21	369
62	67	75	6	11	25	28	45	50	369
27	5	10	49	30	44	74	61	69	369
47	34	42	81	59	64	22	3	17	369
76	57	71	20	7	15	54	32	37	369
14	19	9	39	53	31	70	78	56	369
43	51	29	68	73	63	12	26	4	369
66	80	58	16	24	2	41	46	36	369
369	369	369	369	369	369	369	369	369	369

									20049
1	324	529	1225	1600	2304	3600	4225	6241	20049
1089	1444	2704	3025	5184	5929	64	169	441	20049
3844	4489	5625	36	121	625	784	2025	2500	20049
729	25	100	2401	900	1936	5476	3721	4761	20049
2209	1156	1764	6561	3481	4096	484	9	289	20049
5776	3249	5041	400	49	225	2916	1024	1369	20049
196	361	81	1521	2809	961	4900	6084	3136	20049
1849	2601	841	4624	5329	3969	144	676	16	20049
4356	6400	3364	256	576	4	1681	2116	1296	20049
20049	20049	20049	20049	20049	20049	20049	20049	20049	20049

…. (37)

This is *not a pan diagonal magic square*. Each block of order 3 has the same sum as of magic square.

2.7.1. Palindromic Magic Squares of Order 9

We have exactly 81 palindromes of 3-digits with nine letters a,b,c,d,e,f,g,h and k where $a,b,c,d,e,f,g,h,k\in\{0,1,2,3,4,5,6,7,8,9\}$:

[1]Formerly, Professor of Mathematics, Federal University of Santa Catarina, 88040-400 Florianópolis, SC, Brazil. Email: ijtaneja@gmail.com

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27
aaa	*aba*	*aca*	*ada*	*aea*	*afa*	*aga*	*aha*	*aka*	*bab*	*bbb*	*bcb*	*bdb*	*beb*	*bfb*	*bgb*	*bhb*	*bkb*	*cac*	*cbc*	*ccc*	*cdc*	*cec*	*cfc*	*cgc*	*chc*	*ckc*
28	29	30	31	32	33	34	35	36	37	38	39	40	41	42	43	44	45	46	47	48	49	50	51	52	53	54
dad	*dbd*	*dcd*	*ddd*	*ded*	*dfd*	*dgd*	*dhd*	*dkd*	*eae*	*ebe*	*ece*	*ede*	*eee*	*efe*	*ege*	*ehe*	*eke*	*faf*	*fbf*	*fcf*	*fdf*	*fef*	*fff*	*fgf*	*fhf*	*fkf*
55	56	57	58	59	60	61	62	63	64	65	66	67	68	69	70	71	72	73	74	75	76	77	78	79	80	81
gag	*gbg*	*gcg*	*gdg*	*geg*	*gfg*	*ggg*	*ghg*	*gkg*	*hah*	*hbh*	*hch*	*hdh*	*heh*	*hfh*	*hgh*	*hhh*	*hkh*	*kak*	*kbk*	*kck*	*kdk*	*kek*	*kfk*	*kgk*	*khk*	*kkk*

Replacing the above values with their respective palindromes in (37), we get the following grid of order 9 of nine letters *a, b, c, d, e, f, g, h* and *k*:

aaa	*bkb*	*cec*	*dhd*	*ede*	*fcf*	*gfg*	*hbh*	*kgk*
dfd	*ebe*	*fgf*	*gag*	*hkh*	*kek*	*aha*	*bdb*	*ccc*
ghg	*hdh*	*kck*	*afa*	*bbb*	*cgc*	*dad*	*eke*	*fef*
ckc	*aea*	*bab*	*fdf*	*dcd*	*ehe*	*kbk*	*ggg*	*hfh*
fbf	*dgd*	*efe*	*kkk*	*geg*	*hah*	*cdc*	*aca*	*bhb*
kdk	*gcg*	*hhh*	*cbc*	*aga*	*bfb*	*fkf*	*ded*	*eae*
beb	*cac*	*aka*	*ece*	*fhf*	*ddd*	*hgh*	*kfk*	*gbg*
ege	*fff*	*dbd*	*heh*	*kak*	*gkg*	*bcb*	*chc*	*ada*
hch	*khk*	*gdg*	*bgb*	*cfc*	*aba*	*eee*	*faf*	*dkd*

… (38)

For all $a,b,c,d,e,f,g,h,k \in \{0,1,2,3,4,5,6,7,8,9\}$, above grid represent a *palindromic magic square of order* 9. Its magic sum is

$$S_{9\times 9}(a,b,c,d,e,f,g,h,k) = (a+b+c+d+e+f+g+h+k)\times 111.$$

As a consequence of (38), we have following 4 and 5-digits *palindromic grids* with nine digits a, b, c, d, e, f, g, h and k:

aaaa	*bkkb*	*ceec*	*dhhd*	*edde*	*fccf*	*gffg*	*hbbh*	*kggk*
dffd	*ebbe*	*fggf*	*gaag*	*hkkh*	*keek*	*ahha*	*bddb*	*cccc*
ghhg	*hddh*	*kcck*	*affa*	*bbbb*	*cggc*	*daad*	*ekke*	*feef*
ckkc	*aeea*	*baab*	*fddf*	*dccd*	*ehhe*	*kbbk*	*gggg*	*hffh*
fbbf	*dggd*	*effe*	*kkkk*	*geeg*	*haah*	*cddc*	*acca*	*bhhb*
kddk	*gccg*	*hhhh*	*cbbc*	*agga*	*bffb*	*fkkf*	*deed*	*eaae*
beeb	*caac*	*akka*	*ecce*	*fhhf*	*dddd*	*hggh*	*kffk*	*gbbg*
egge	*ffff*	*dbbd*	*heeh*	*kaak*	*gkkg*	*bccb*	*chhc*	*adda*
hcch	*khhk*	*gddg*	*bggb*	*cffc*	*abba*	*eeee*	*faaf*	*dkkd*

aaaaa	*abkba*	*aceca*	*adhda*	*aedea*	*afcfa*	*agfga*	*ahbha*	*akgka*
adfda	*aebea*	*afgfa*	*agaga*	*ahkha*	*akeka*	*aahaa*	*abdba*	*accca*
aghga	*ahdha*	*akcka*	*aafaa*	*abbba*	*acgca*	*adada*	*aekea*	*afefa*
ackca	*aaeaa*	*ababa*	*afdfa*	*adcda*	*aehea*	*akbka*	*aggga*	*ahfha*
afbfa	*adgda*	*aefea*	*akkka*	*agega*	*ahaha*	*acdca*	*aacaa*	*abhba*
akdka	*agcga*	*ahhha*	*acbca*	*aagaa*	*abfba*	*afkfa*	*adeda*	*aeaea*
abeba	*acaca*	*aakaa*	*aecea*	*afhfa*	*addda*	*ahgha*	*akfka*	*agbga*
aegea	*afffa*	*adbda*	*aheha*	*akaka*	*agkga*	*abcba*	*achca*	*aadaa*
ahcha	*akhka*	*agdga*	*abgba*	*acfca*	*aabaa*	*aeeea*	*afafa*	*adkda*

aaaaa	*bkakb*	*ceaec*	*dhahd*	*edade*	*fcacf*	*gfafg*	*hbabh*	*kgagk*
dfafd	*ebabe*	*fgagf*	*gaaag*	*hkakh*	*keaek*	*ahaha*	*bdadb*	*ccacc*
ghahg	*hdadh*	*kcack*	*afafa*	*bbabb*	*cgagc*	*daaad*	*ekake*	*feaef*
ckakc	*aeaea*	*baaab*	*fdadf*	*dcacd*	*ehahe*	*kbabk*	*ggagg*	*hfafh*
fbabf	*dgagd*	*efafe*	*kkakk*	*geaeg*	*haaah*	*cdadc*	*acaca*	*bhahb*
kdadk	*gcacg*	*hhahh*	*cbabc*	*agaga*	*bfafb*	*fkakf*	*deaed*	*eaaae*
beaeb	*caaac*	*akaka*	*ecace*	*fhahf*	*ddadd*	*hgagh*	*kfafk*	*gbabg*
egage	*ffaff*	*dbabd*	*heaeh*	*kaaak*	*gkakg*	*bcacb*	*chahc*	*adada*
hcach	*khahk*	*gdadg*	*bgagb*	*cfafc*	*ababa*	*eeaee*	*faaaf*	*dkakd*

[1]Formerly, Professor of Mathematics, Federal University of Santa Catarina, 88040-400 Florianópolis, SC, Brazil. Email: ijtaneja@gmail.com

Below are some examples of *bimagic squares* of order 9:

Example 39. For $a = 1, b = 2, c = 3, d = 4, e = 5, f = 6, g = 7$, $h = 8$ and $k = 9$ in (38), we have 3-digits *palindromic magic square* of order 9:

111	292	353	484	545	636	767	828	979
464	525	676	717	898	959	181	242	333
787	848	939	161	222	373	414	595	656
393	151	212	646	434	585	929	777	868
626	474	565	999	757	818	343	131	282
949	737	888	323	171	262	696	454	515
252	313	191	535	686	444	878	969	727
575	666	424	858	919	797	232	383	141
838	989	747	272	363	121	555	616	494

It is a *palindromic bimagic square*. Its sums are

$$S_{9\times 9}(1,2,3,4,5,6,7,8,9) = (1+2+3+4+5+6+7+8+9)\times 111 = 4995$$

and

$$Sb_{9\times 9}(1,2,3,4,5,6,7,8,9) = 3390285.$$

The grid (38) is valid for 10 digits ,i.e., $a, b, c, d, e, f, g, h, k \in \{0,1,2,3,4,5,6,7,8,9\}$, and we have used nine. There are very few possibilities to change. Let us consider 0 and exclude 6. See the following example.

Example 40. For $a = 0, b = 1, c = 2, d = 3, e = 4, f = 5, g = 7$, $h = 8$ and $k = 9$ in (38), we have 3-digits *palindromic magic square* of order 9:

000	191	242	383	434	525	757	818	979
353	414	575	707	898	949	080	131	222
787	838	929	050	111	272	303	494	545
292	040	101	535	323	484	919	777	858
515	373	454	999	747	808	232	020	181
939	727	888	212	070	151	595	343	404
141	202	090	424	585	333	878	959	717
474	555	313	848	909	797	121	282	030
828	989	737	171	252	010	444	505	393

It is a *palindromic bimagic square*. Its sums are

$$S_{9\times 9}(0,1,2,3,4,5,7,8,9) = (0+1+2+3+4+5+7+8+9)\times 111 = 4329$$

and

$$Sb_{9\times 9}(0,1,2,3,4,5,7,8,9) = 2906329.$$

[1]Formerly, Professor of Mathematics, Federal University of Santa Catarina, 88040-400 Florianópolis, SC, Brazil. Email: ijtaneja@gmail.com

2.7.2. 7-Digits Palindromic Bimagic Squares of Order 9

Above subsection is with 3-digits palindromes having nine letters. Let us consider the following 7-digits *palindromic grid* of order 9 only with three letters *a, b* and *c*:

aaaaaaa	*abcccba*	*acbbbca*	*bacbcab*	*bbbabbb*	*bcacacb*	*cabcbac*	*cbababc*	*cccaccc*
babcbab	*bbababb*	*bccaccb*	*caaaaac*	*cbcccbc*	*ccbbbcc*	*aacbcaa*	*abbabba*	*acacaca*
cacbcac	*cbbabbc*	*ccacacc*	*aabcbaa*	*abababa*	*accacca*	*baaaaab*	*bbcccbb*	*bcbbbcb*
accccca	*aabbbaa*	*abaaaba*	*bcbabcb*	*baacaab*	*bbcbcbb*	*ccabacc*	*cacacac*	*cbbcbbc*
bcabacb	*bacacab*	*bbbcbbb*	*ccccccc*	*cabbbac*	*cbaaabc*	*acbabca*	*aaacaaa*	*abcbcba*
ccbabcc	*caacaac*	*cbcbcbc*	*acabaca*	*aacacaa*	*abbcbba*	*bcccccb*	*babbbab*	*bbaaabb*
abbbbba	*acaaaca*	*aacccaa*	*bbacabb*	*bccbccb*	*bababab*	*cbcacbc*	*ccbcbcc*	*caabaac*
bbcacbb	*bcbcbcb*	*baabaab*	*cbbbbbc*	*ccaaacc*	*cacccac*	*abacaba*	*accbcca*	*aababaa*
cbacabc	*cccbccc*	*cababac*	*abcacba*	*acbcbca*	*aaabaaa*	*bbbbbbb*	*bcaaacb*	*bacccab*

… (39)

For all $a, b, c \in \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9\}$, above grid represent a *palindromic square* with sum

$$S_{4\times 4}(a,b) = 3\times(a+b+c)\times 1111111 \text{ or } \frac{S_{9\times 9}(a,b,c)}{a+b+c} = 3333333.$$

Below are examples of (39):

Example 41. For $a=2, b=5$ and $c=8$ in (39), we have 7-digits *Selfie palindromic magic square* of order 9:

2222222	2588852	2855582	5285825	5552555	5828285	8258528	8525258	8882888
5258525	5525255	5882885	8222228	8588858	8855588	2285822	2552552	2828282
8285828	8552558	8828288	2258522	2525252	2882882	5222225	5588855	5855585
2888882	2255522	2522252	5852585	5228225	5585855	8825288	8282828	8558558
5825285	5282825	5558555	8888888	8255528	8522258	2852582	2228222	2585852
8852588	8228228	8585858	2825282	2282822	2558552	5888885	5255525	5522255
2555552	2822282	2288822	5528255	5885885	5252525	8582858	8858588	8225228
5582855	5858585	5225225	8555558	8822288	8288828	2528252	2885882	2252522
8528258	8885888	8252528	2582852	2858582	2225222	5555555	5822285	5288825

It is a *Selfie palindromic bimagic square*. Its sums are

$$S_{9\times 9}(2,5,8) = 49999995 = (1+2+3+4+5+6+7+8+9)\times 1111111$$

and

$$Sb_{9\times 9}(2,5,8) = 332323500767679.$$

Example 42. For $a=1, b=6$ and $c=9$, we have 7-digits *upside down palindromic magic square* of order 9:

[1]Formerly, Professor of Mathematics, Federal University of Santa Catarina, 88040-400 Florianópolis, SC, Brazil. Email: ijtaneja@gmail.com

1111111	1699961	1966691	6196916	6661666	6919196	9169619	9616169	9991999
6169616	6616166	6991996	9111119	9699969	9966699	1196911	1661661	1919191
9196919	9661669	9919199	1169611	1616161	1991991	6111116	6699966	6966696
1999991	1166611	1611161	6961696	6119116	6696966	9916199	9191919	9669669
6916196	6191916	6669666	9999999	9166619	9611169	1961691	1119111	1696961
9961699	9119119	9696969	1916191	1191911	1669661	6999996	6166616	6611166
1666661	1911191	1199911	6619166	6996996	6161616	9691969	9969699	9116119
6691966	6969696	6116116	9666669	9911199	9199919	1619161	1996991	1161611
9619169	9996999	9161619	1691961	1969691	1116111	6666666	6911196	6199916

It is *upside down palindromic bimagic square*. Its sums are

$$S_{9\times 9}(1,6,9) = 53333328 = \left(1+2^3+4+5+6+7+8+9\right)\times 1111111$$

and

$$Sb_{9\times 9}(1,6,9) = 415039806496074.$$

Examples 41 and 42 give us following interesting relation

$$\frac{S_{9\times 9}(1,6,9)}{1+6+9} = \frac{S_{9\times 9}(2,5,8)}{2+5+8} = 3333333.$$

Example 43. Here is another of *palindromic bimagic square* of order 9:

111111111	222999222	333555333	444888444	555444555	666333666	777666777	888222888	999777999
444666444	555222555	666777666	777111777	888999888	999555999	111888111	222444222	333333333
777888777	888444888	999333999	111666111	222222222	333777333	444111444	555999555	666555666
333999333	111555111	222111222	666444666	444333444	555888555	999222999	777777777	888666888
666222666	444777444	555666555	999999999	777555777	888111888	333444333	111333111	222888222
999444999	777333777	888888888	333222333	111777111	222666222	666999666	444555444	555111555
222555222	333111333	111999111	555333555	666888666	444444444	888777888	999666999	777222777
555777555	666666666	444222444	888555888	999111999	777999777	222333222	333888333	111444111
888333888	999888999	777444777	222777222	333666333	111222111	555555555	666111666	444999444

Its sums are $S_{9\times 9} = 4999999995$ and $Sb_{9\times 9} = 3517039990002961485$. Moreover, sum of nine members in each block of order 3 has the same sum as $S_{9\times 9}$.

2.7.3. Double Colored Pattern of Order 9

As before removing the third value in the grid (38), and spitting the resulting in two Latin squares, we get

a	*b*	*c*	*d*	*e*	*f*	*g*	*h*	*k*
d	*e*	*f*	*g*	*h*	*k*	*a*	*b*	*c*
g	*h*	*k*	*a*	*b*	*c*	*d*	*e*	*f*
c	*a*	*b*	*f*	*d*	*e*	*k*	*g*	*h*
f	*d*	*e*	*k*	*g*	*h*	*c*	*a*	*b*
k	*g*	*h*	*c*	*a*	*b*	*f*	*d*	*e*
b	*c*	*a*	*e*	*f*	*d*	*h*	*k*	*g*
e	*f*	*d*	*h*	*k*	*g*	*b*	*c*	*a*
h	*k*	*g*	*b*	*c*	*a*	*e*	*f*	*d*

A

a	*k*	*e*	*h*	*d*	*c*	*f*	*b*	*g*
f	*b*	*g*	*a*	*k*	*e*	*h*	*d*	*c*
h	*d*	*c*	*f*	*b*	*g*	*a*	*k*	*e*
k	*e*	*a*	*d*	*c*	*h*	*b*	*g*	*f*
b	*g*	*f*	*k*	*e*	*a*	*d*	*c*	*h*
d	*c*	*h*	*b*	*g*	*f*	*k*	*e*	*a*
e	*a*	*k*	*c*	*h*	*d*	*g*	*f*	*b*
g	*f*	*b*	*e*	*a*	*k*	*c*	*h*	*d*
c	*h*	*d*	*g*	*f*	*b*	*e*	*a*	*k*

B

aa	*bk*	*ce*	*dh*	*ed*	*fc*	*gf*	*hb*	*kg*
df	*eb*	*fg*	*ga*	*hk*	*ke*	*ah*	*bd*	*cc*
gh	*hd*	*kc*	*af*	*bb*	*cg*	*da*	*ek*	*fe*
ck	*ae*	*ba*	*fd*	*dc*	*eh*	*kb*	*gg*	*hf*
fb	*dg*	*ef*	*kk*	*ge*	*há*	*cd*	*ac*	*bh*
kd	*gc*	*hh*	*cb*	*ag*	*bf*	*fk*	*de*	*ea*
be	*ca*	*ak*	*ec*	*fh*	*dd*	*hg*	*kf*	*gb*
eg	*ff*	*db*	*he*	*ka*	*gk*	*bc*	*ch*	*ad*
hc	*kh*	*gd*	*bg*	*cf*	*ab*	*ee*	*fa*	*dk*

AB

[1]Formerly, Professor of Mathematics, Federal University of Santa Catarina, 88040-400 Florianópolis, SC, Brazil. Email: ijtaneja@gmail.com

In particular $a=1, b=2, c=3, d=4, e=5, f=6, g=7, h=8$ and $k=9$, we get

A

1	2	3	4	5	6	7	8	9
4	5	6	7	8	9	1	2	3
7	8	9	1	2	3	4	5	6
3	1	2	6	4	5	9	7	8
6	4	5	9	7	8	3	1	2
9	7	8	3	1	2	6	4	5
2	3	1	5	6	4	8	9	7
5	6	4	8	9	7	2	3	1
8	9	7	2	3	1	5	6	4

B

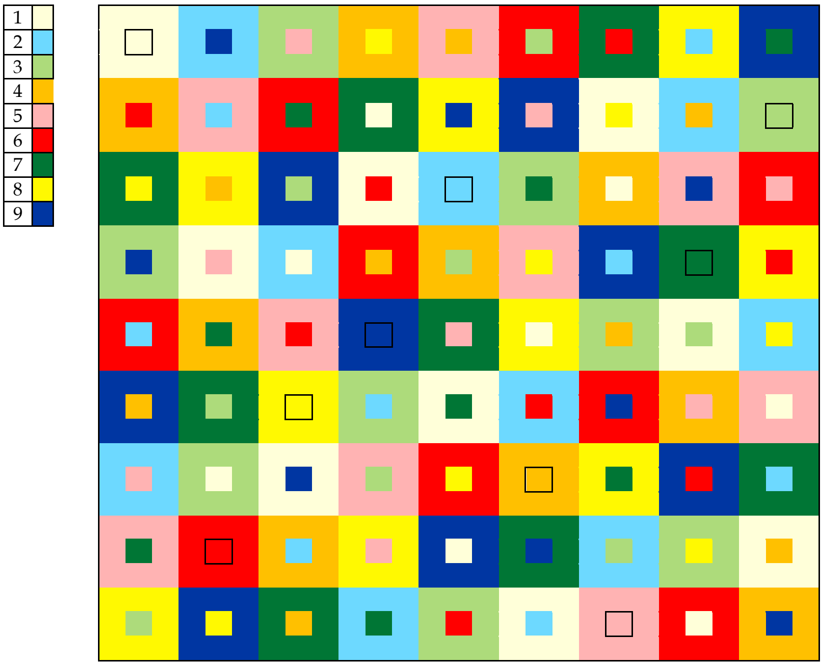
1	9	5	8	4	3	6	2	7
6	2	7	1	9	5	8	4	3
8	4	3	6	2	7	1	9	5
9	5	1	4	3	8	2	7	6
2	7	6	9	5	1	4	3	8
4	3	8	2	7	6	9	5	1
5	1	9	3	8	4	7	6	2
7	6	2	5	1	9	3	8	4
3	8	4	7	6	2	5	1	9

AB

11	29	35	48	54	63	76	82	97
46	52	67	71	89	95	18	24	33
78	84	93	16	22	37	41	59	65
39	15	21	64	43	58	92	77	86
62	47	56	99	75	81	34	13	28
94	73	88	32	17	26	69	45	51
25	31	19	53	68	44	87	96	72
57	66	42	85	91	79	23	38	14
83	98	74	27	36	12	55	61	49

…. (40)

Applying the procedure $9\times(A-1)+B$ in (40), we get magic square of order 9 given in (37). Moreover, A and B are *"pairwise mutually orthogonal diagonalized magic squares"*. Based on (40), here below is *double colored pattern*.



2.7.4. Self-Orthogonal Diagonalized Latin Square of Order 9

The grid A given in (40) is *self-orthogonal Latin square*. Considering B as transpose of A changing row by columns, and applying the procedure $9\times(A-1)+B$, we get a magic square of order 9.

Example 44.

A

1	2	3	4	5	6	7	8	9
4	5	6	7	8	9	1	2	3
7	8	9	1	2	3	4	5	6
3	1	2	6	4	5	9	7	8
6	4	5	9	7	8	3	1	2
9	7	8	3	1	2	6	4	5
2	3	1	5	6	4	8	9	7
5	6	4	8	9	7	2	3	1
8	9	7	2	3	1	5	6	4

B

1	4	7	3	6	9	2	5	8
2	5	8	1	4	7	3	6	9
3	6	9	2	5	8	1	4	7
4	7	1	6	9	3	5	8	2
5	8	2	4	7	1	6	9	3
6	9	3	5	8	2	4	7	1
7	1	4	9	3	6	8	2	5
8	2	5	7	1	4	9	3	6
9	3	6	8	2	5	7	1	4

		369	369	369	369	369	369	369	369	369
	1	13	25	30	42	54	56	68	80	369
369	29	41	53	55	67	79	3	15	27	369
369	57	69	81	2	14	26	28	40	52	369
369	22	7	10	51	36	39	77	62	65	369
369	50	35	38	76	61	64	24	9	12	369
369	78	63	66	23	8	11	49	34	37	369
369	16	19	4	45	48	33	71	74	59	369
369	44	47	32	70	73	58	18	21	6	369
369	72	75	60	17	20	5	43	46	31	369
	369	369	369	369	369	369	369	369	369	369

…. (41)

[1]Formerly, Professor of Mathematics, Federal University of Santa Catarina, 88040-400 Florianópolis, SC, Brazil. Email: ijtaneja@gmail.com

The grid B is transpose of A. We get the magic square of order 9 applying the procedure $9\times(A-1)+B$. The magic square (41) is *pan diagonal compact magic squares* as each block of order 3 has the same sum 369.

369	369	369	369	369	369	369	369	369
369	369	369	369	369	369	369	369	369
369	369	369	369	369	369	369	369	369
369	369	369	369	369	369	369	369	369
369	369	369	369	369	369	369	369	369
369	369	369	369	369	369	369	369	369
369	369	369	369	369	369	369	369	369
369	369	369	369	369	369	369	369	369
369	369	369	369	369	369	369	369	369

The grid B is also self-orthogonal diagonalized Latin square, and lead us to a similar kind of magic square as of (41).

2.7.5. Non Palindromic Selfie Magic Squares of Order 9

Below are two examples of non *palindromic magic squares*. First one is *upside down* and second is *Selfie magic square*.

Example 45. The following example is *non palindromic upside down bimagic square* of order 9 with sums $S_{9\times9}(1,6,9):=53328$ and $Sb_{4\times4}(1,6,9):=414976074$:

1111	1699	1966	6196	6661	6919	9169	9616	9991
6169	6616	6991	9111	9699	9966	1196	1661	1919
9196	9661	9919	1169	1616	1991	6111	6699	6966
1999	1166	1611	6961	6119	6696	9916	9191	9669
6916	6191	6669	9999	9166	9611	1961	1119	1696
9961	9119	9696	1916	1191	1669	6999	6166	6611
1666	1911	1199	6619	6996	6161	9691	9969	9116
6691	6969	6116	9666	9911	9199	1619	1996	1161
9619	9996	9161	1691	1969	1116	6666	6911	6199

Example 46. The following example is *non palindromic Selfie bimagic square* of order 9 with sums $S_{9\times9}(1,6,9):=53328$ and $Sb_{4\times4}(2,5,8):=332267679$:

2222	2588	2855	5285	5552	5828	8258	8525	8882
5258	5525	5882	8222	8588	8855	2285	2552	2828
8285	8552	8828	2258	2525	2882	5222	5588	5855
2888	2255	2522	5852	5228	5585	8825	8282	8558
5825	5282	5558	8888	8255	8522	2852	2228	2585
8852	8228	8585	2825	2282	2558	5888	5255	5522
2555	2822	2288	5528	5885	5252	8582	8858	8225
5582	5858	5225	8555	8822	8288	2528	2885	2252
8528	8885	8252	2582	2858	2225	5555	5822	5288

[1]Formerly, Professor of Mathematics, Federal University of Santa Catarina, 88040-400 Florianópolis, SC, Brazil. Email: ijtaneja@gmail.com

2.8. Magic Squares of Order 10

Let consider following *intervally distributed magic square* of order 10.

										505
1	64	85	13	99	72	36	50	28	57	505
43	12	37	9	54	81	98	26	80	65	505
70	96	23	51	87	48	75	14	2	39	505
95	88	10	34	21	17	42	59	66	73	505
38	71	16	90	45	69	4	97	53	22	505
19	30	62	77	8	56	83	35	94	41	505
74	49	58	25	20	93	67	82	31	6	505
52	7	91	46	63	40	29	78	15	84	505
27	33	44	92	76	5	60	61	89	18	505
86	55	79	68	32	24	11	3	47	100	505
505	505	505	505	505	505	505	505	505	505	505

…. (42)

2.8.1. Palindromic Magic Squares of Order 10

We have exactly 81 palindromic numbers of 3-digits using nine letters a,b,c,d,e,f,g,h,k and l:

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25
aaa	*aba*	*aca*	*ada*	*aea*	*afa*	*aga*	*aha*	*aka*	*ala*	*bab*	*bbb*	*bcb*	*bdb*	*beb*	*bfb*	*bgb*	*bhb*	*bkb*	*blb*	*cac*	*cbc*	*ccc*	*cdc*	*cec*
26	27	28	29	30	31	32	33	34	35	36	37	38	39	40	41	42	43	44	45	46	47	48	49	50
cfc	*cgc*	*chc*	*ckc*	*clc*	*dad*	*dbd*	*dcd*	*ddd*	*ded*	*dfd*	*dgd*	*dhd*	*dkd*	*dld*	*eae*	*ebe*	*ece*	*ede*	*eee*	*efe*	*ege*	*ehe*	*eke*	*ele*
51	52	53	54	55	56	57	58	59	60	61	62	63	64	65	66	67	68	69	70	71	72	73	74	75
faf	*fbf*	*fcf*	*fdf*	*fef*	*fff*	*fgf*	*fhf*	*fkf*	*flf*	*gag*	*gbg*	*gcg*	*gdg*	*geg*	*gfg*	*ggg*	*ghg*	*gkg*	*glg*	*hah*	*hbh*	*hch*	*hdh*	*heh*
76	77	78	79	80	81	82	83	84	85	86	87	88	89	90	91	92	93	94	94	96	97	98	99	100
hfh	*hgh*	*hhh*	*hkh*	*hlh*	*kak*	*kbk*	*kck*	*kdk*	*kek*	*kfk*	*kgk*	*khk*	*kkk*	*klk*	*lal*	*lbl*	*lcl*	*ldl*	*lel*	*lfl*	*lgl*	*lhl*	*lkl*	*lll*

Replacing the above values with their respective palindromes in (42), we get the following grid of order 10:

aaa	*eke*	*bfb*	*lbl*	*khk*	*cdc*	*heh*	*dgd*	*flf*	*gcg*
dfd	*bbb*	*ghg*	*kck*	*clc*	*lkl*	*eae*	*fdf*	*aea*	*hgh*
gbg	*fhf*	*ccc*	*ele*	*lfl*	*aga*	*dkd*	*kak*	*hdh*	*beb*
blb	*kgk*	*fef*	*ddd*	*hah*	*ece*	*lhl*	*aba*	*gfg*	*ckc*
ldl	*cac*	*aka*	*fgf*	*eee*	*hbh*	*kfk*	*glg*	*bcb*	*dhd*
kek	*gdg*	*dad*	*hkh*	*bgb*	*fff*	*ala*	*lcl*	*chc*	*ebe*
ehe	*aca*	*hlh*	*cfc*	*fbf*	*ded*	*ggg*	*bkb*	*lal*	*kdk*
cgc	*lel*	*ede*	*bab*	*gkg*	*klk*	*fcf*	*hhh*	*dbd*	*afa*
hch	*dld*	*lgl*	*geg*	*ada*	*bhb*	*cbc*	*efe*	*kkk*	*faf*
fkf	*hfh*	*kbk*	*aha*	*dcd*	*gag*	*bdb*	*cec*	*ege*	*lll*

…. (43)

For all $a,b,c,d,e,f,g,h,k,l \in \{0,1,2,3,4,5,6,7,8,9\}$, above grid represent a *palindromic magic square of order* 10. It sum is

[1]Formerly, Professor of Mathematics, Federal University of Santa Catarina, 88040-400 Florianópolis, SC, Brazil. Email: ijtaneja@gmail.com

$$S_{9\times 9}(a,b,c,d,e,f,g,h,k,l)=(a+b+c+d+e+f+g+h+k+l)\times 111.$$

Example 47. For $a=0, b=1, c=2, d=3, e=4, f=5, g=6, h=7, k=8$ and $l=9$ in (43), we have 3-digits *palindromic magic square* of order 10 with magic sum $S_{10\times 10}=4995$:

000	484	151	919	878	232	747	363	595	626
353	111	676	828	292	989	404	535	040	767
616	575	222	494	959	060	383	808	737	141
191	868	545	333	707	424	979	010	656	282
939	202	080	565	444	717	858	696	121	373
848	636	303	787	161	555	090	929	272	414
474	020	797	252	515	343	666	181	909	838
262	949	434	101	686	898	525	777	313	050
727	393	969	646	030	171	212	454	888	505
585	757	818	070	323	606	131	242	464	999

Since we have only 90 palindromes of 3-digits, other 10 numbers are used as 000, 010, 020, …, 090 to complete the magic square. In this case, neither we have *upside down* nor *Selfie palindromic magic squares*.

2.8.2. Double Colored Pattern of Order 10

As before removing the third value in the grid (43), and spitting the resulting in two Latin squares, we get

a	*g*	*k*	*b*	*l*	*h*	*d*	*e*	*c*	*f*
e	*b*	*d*	*a*	*f*	*k*	*l*	*c*	*h*	*g*
g	*l*	*c*	*f*	*k*	*e*	*h*	*b*	*a*	*d*
l	*k*	*a*	*d*	*c*	*b*	*e*	*f*	*g*	*h*
d	*h*	*b*	*k*	*e*	*g*	*a*	*l*	*f*	*c*
b	*c*	*g*	*h*	*a*	*f*	*k*	*d*	*l*	*e*
h	*e*	*f*	*c*	*b*	*l*	*g*	*k*	*d*	*a*
f	*a*	*l*	*e*	*g*	*d*	*c*	*h*	*b*	*k*
c	*d*	*e*	*l*	*h*	*a*	*f*	*g*	*k*	*b*
k	*f*	*h*	*g*	*d*	*c*	*b*	*a*	*e*	*l*

A

a	*d*	*e*	*c*	*k*	*b*	*f*	*l*	*h*	*g*
c	*b*	*g*	*k*	*d*	*a*	*h*	*f*	*l*	*e*
l	*f*	*c*	*a*	*g*	*h*	*e*	*d*	*b*	*k*
e	*h*	*l*	*d*	*a*	*g*	*b*	*k*	*f*	*c*
h	*a*	*f*	*l*	*e*	*k*	*d*	*g*	*c*	*b*
k	*l*	*b*	*g*	*h*	*f*	*c*	*e*	*d*	*a*
d	*k*	*h*	*e*	*l*	*c*	*g*	*b*	*a*	*f*
b	*g*	*a*	*f*	*c*	*l*	*k*	*h*	*e*	*d*
g	*c*	*d*	*b*	*f*	*e*	*l*	*a*	*k*	*h*
f	*e*	*k*	*h*	*b*	*d*	*a*	*c*	*g*	*l*

B

aa	*gd*	*ke*	*bc*	*lk*	*hb*	*df*	*el*	*ch*	*fg*
ec	*bb*	*dg*	*ak*	*fd*	*ka*	*lh*	*cf*	*hl*	*ge*
gl	*lf*	*cc*	*fa*	*kg*	*eh*	*he*	*bd*	*ab*	*dk*
le	*kh*	*al*	*dd*	*ca*	*bg*	*eb*	*fk*	*gf*	*hc*
dh	*ha*	*bf*	*kl*	*ee*	*gk*	*ad*	*lg*	*fc*	*cb*
bk	*cl*	*gb*	*hg*	*ah*	*ff*	*kc*	*de*	*ld*	*ea*
hd	*ek*	*fh*	*ce*	*bl*	*lc*	*gg*	*kb*	*da*	*af*
fb	*ag*	*la*	*ef*	*gc*	*dl*	*ck*	*hh*	*be*	*kd*
cg	*dc*	*ed*	*lb*	*hf*	*ae*	*fl*	*ga*	*kk*	*bh*
kf	*fe*	*hk*	*gh*	*db*	*cd*	*ba*	*ac*	*eg*	*ll*

AB

In particular $a=0, b=1, c=2, d=3, e=4, f=5, g=6, h=7, k=8$ and $l=9$, we get

0	6	8	1	9	7	3	4	2	5
4	1	3	0	5	8	9	2	7	6
6	9	2	5	8	4	7	1	0	3
9	8	0	3	2	1	4	5	6	7
3	7	1	8	4	6	0	9	5	2
1	2	6	7	0	5	8	3	9	4
7	4	5	2	1	9	6	8	3	0
5	0	9	4	6	3	2	7	1	8
2	3	4	9	7	0	5	6	8	1
8	5	7	6	3	2	1	0	4	9

A

0	3	4	2	8	1	5	9	7	6
2	1	6	8	3	0	7	5	9	4
9	5	2	0	6	7	4	3	1	8
4	7	9	3	0	6	1	8	5	2
7	0	5	9	4	8	3	6	2	1
8	9	1	6	7	5	2	4	3	0
3	8	7	4	9	2	6	1	0	5
1	6	0	5	2	9	8	7	4	3
6	2	3	1	5	4	9	0	8	7
5	4	8	7	1	3	0	2	6	9

B

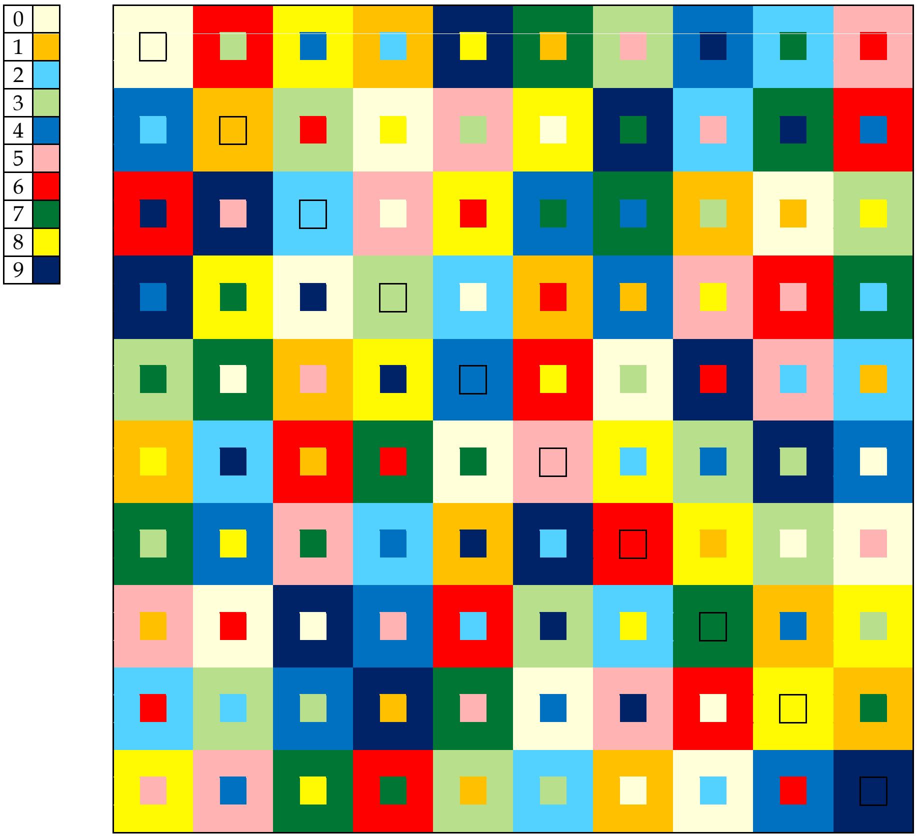
00	63	84	12	98	71	35	49	27	56
42	11	36	08	53	80	97	25	79	64
69	95	22	50	86	47	74	13	01	38
94	87	09	33	20	16	41	58	65	72
37	70	15	89	44	68	03	96	52	21
18	29	61	76	07	55	82	34	93	40
73	48	57	24	19	92	66	81	30	05
51	06	90	45	62	39	28	77	14	83
26	32	43	91	75	04	59	60	88	17
85	54	78	67	31	23	10	02	46	99

AB

…. (44)

[1]Formerly, Professor of Mathematics, Federal University of Santa Catarina, 88040-400 Florianópolis, SC, Brazil. Email: ijtaneja@gmail.com

Applying the procedure $10 \times A + B + 1$ in (44), we get magic square of order 10 given in (42). Moreover, A and B are *"pairwise mutually orthogonal diagonalized magic squares"*. The grids given in (44) are due to Wallis [16]. Due to (44) here below is *double colored pattern*.



2.8.3. Self-Orthogonal Latin Square of Order 10.

Any of above two diagonalized Latin squares A and B are not self-orthogoal. Here below is example of another *diagonalized self-orthogonal Latin square* [1].

Example 48.

0	7	6	9	3	2	4	8	5	1
9	1	0	6	8	7	5	3	4	2
4	8	2	7	1	3	9	5	6	0
6	5	8	3	0	9	2	1	7	4
8	9	5	1	4	6	7	0	2	3
1	3	4	0	7	5	8	2	9	6
7	4	3	8	2	1	6	9	0	5
5	2	9	4	6	0	1	7	3	8
2	0	1	5	9	4	3	6	8	7
3	6	7	2	5	8	0	4	1	9

A

0	9	4	6	8	1	7	5	2	3
7	1	8	5	9	3	4	2	0	6
6	0	2	8	5	4	3	9	1	7
9	6	7	3	1	0	8	4	5	2
3	8	1	0	4	7	2	6	9	5
2	7	3	9	6	5	1	0	4	8
4	5	9	2	7	8	6	1	3	0
8	3	5	1	0	2	9	7	6	4
5	4	6	7	2	9	0	3	8	1
1	2	0	4	3	6	5	8	7	9

B

										505
1	80	65	97	39	22	48	86	53	14	505
98	12	9	66	90	74	55	33	41	27	505
47	81	23	79	16	35	94	60	62	8	505
70	57	88	34	2	91	29	15	76	43	505
84	99	52	11	45	68	73	7	30	36	505
13	38	44	10	77	56	82	21	95	69	505
75	46	40	83	28	19	67	92	4	51	505
59	24	96	42	61	3	20	78	37	85	505
26	5	17	58	93	50	31	64	89	72	505
32	63	71	25	54	87	6	49	18	100	505
505	505	505	505	505	505	505	505	505	505	505

2.8.4. Non Palindromic Selfie Magic Squares of Order 10

Based on the procedure of pan diagonal magic square of order 5 given in (18) , below are three examples of *Selfie* and *upside down* magic squares. In each case the sub-blocks of

[1]Formerly, Professor of Mathematics, Federal University of Santa Catarina, 88040-400 Florianópolis, SC, Brazil. Email: ijtaneja@gmail.com

order 5 are also magic squares of same sum. The first and second are *Selfie magic squares*, while third example is of *upside down*.

Example 49. Following example is a *pan diagonal Selfie magic square* of order 10 with magic sum $S_{10\times10}(0,1,2,5)=17776$:

0000	0121	1501	2151	5115	500	1021	1101	1251	5015
2101	5151	0015	0100	1521	1201	5051	0515	1000	1121
0115	1500	2121	5101	0051	1015	1100	1221	5001	0551
5121	0001	0151	1515	2100	5021	0501	1051	1115	1200
1551	2115	5100	0021	0101	1151	1215	5000	0521	1001
0005	0112	1510	2150	5111	0505	1012	1110	1250	5011
2110	5150	0011	0105	1512	1210	5050	0511	1005	1112
0111	1505	2112	5110	0050	1011	1105	1212	05010	0550
5112	0010	0150	1511	2105	5012	0510	1050	1111	1205
1550	2111	5105	0012	0110	1150	1211	5005	0512	1010

Magic sum $S_{10\times10}(0,1,2,5)=8888+8888$ also turns Selfie. Here each block of order 5 is also a magic square with sum $S_{5\times5}(0,1,2,5)=8888.$ Magic square of order 10 and sub magic squares of order 5 are *pan diagonals.*

Example 50. Following example is a *pan diagonal Selfie magic square* of order 10 with magic sums $S_{10\times10}(1,2,5,8)=39996$:

1212	2255	2822	5581	8128	1812	2155	2522	5281	8228
5522	8181	1228	2212	2855	5222	8281	1828	2112	2555
2228	2812	5555	8122	1281	2128	2512	5255	8222	1881
8155	1222	2281	2828	5512	8255	1822	2181	2528	5212
2881	5528	8112	1255	2222	2581	5228	8212	1855	2122
1218	2252	2821	5582	8125	1818	2152	2521	5282	8225
5521	8182	1225	2218	2852	5221	8282	1825	2118	2552
2225	2818	5552	8121	1282	2125	2518	5252	8221	1882
8152	1221	2282	2825	5518	8252	1821	2182	2525	5218
2882	5525	8118	1252	2221	2582	5225	8218	1852	2121

Each block of order 5 is also a magic square with sum $S_{5\times5}(1,2,5,8)=19998.$ Magic square of order 10 and sub magic squares of order 5 are *pan diagonals.*

Example 51. Following example is a *pan diagonal upside down magic square* of order 10 with magic sum $S_{10\times10}(0,1,6,9)=53328$:

0000	1691	6116	9196	9661	1100	1991	6616	6996	9961
9116	9696	0061	1600	6191	6916	9996	1161	1900	6691
1661	6100	9191	9616	0096	1961	6600	6991	9916	1196
9691	0016	1696	6161	9100	9991	1116	1996	6661	6900
6196	9161	9600	0091	1616	6696	6961	9900	1191	1916
0011	1669	6119	9199	9666	1111	1969	6619	6999	9966
9119	9699	0066	1611	6169	6919	9999	1166	1911	6669
1666	6111	9169	9619	0099	1966	6611	6969	9919	1199
9669	0019	1699	6166	9111	9969	1119	1999	6666	6911
6199	9166	9611	0069	1619	6699	6966	9911	1169	1919

[1]Formerly, Professor of Mathematics, Federal University of Santa Catarina, 88040-400 Florianópolis, SC, Brazil. Email: ijtaneja@gmail.com

Each block of order 5 is also a magic square with sum $S_{5\times 5}(0,1,6,9) = 26664$. Magic square of order 10 and sub magic squares of order 5 are *pan diagonals.*

Still, we don't have *Selfie palindromic magic square* of order 10. The examples (49) and (50) can be made palindromic just increasing the values at the end if each number in reverse orders, such as, 58288285, 11855811, etc. This gives 8-digits *Selife palindromic magic square*. This becomes even order palindromic magic square. The idea is to bring odd order palindromic magic square.

Final Comments

In this work, we are able to bring some new ideas on magic squares. We brought *palindromic magic squares* of order 3 to 10. Idea of *Selfie* and *upside down magic square* is introduced. We are able to bring *Selfie palindromic magic squares* for the orders 3, 4, 5, 8 and 9. In case of order 7, we have just *upside down palindromic magic square*. Still, we don't have *upside down* or *Selfie palindromic magic squares* for the orders 6 and 10. *Non-palindromic magic square* are given for the order 7 and 10 too. We are talking about odd order palindromic magic square. Non-palindromic magic square of order 7 and 10 studied. The even order palindromic magic square are always possible. In order to bring *palindromic Selfie magic square of order 8* we worked with 11-digits palindromes having only two numbers (example 32 and 33). The non-dictionary words "*intervally distributed*" and "*Selfie magic squares*" are used with their explanations. This work includes all the material appeared in [13]. The idea of *pairwise mutually orthogonal Latins squares* and *self-orthogonal diagonalized Latins squares* is connected with *intervally distributed magic square*. The aim is to extend it for further magic squares. This shall be done elsewhere [15]. For more study on magic squares refer to author [7-14].

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[1]Formerly, Professor of Mathematics, Federal University of Santa Catarina, 88040-400 Florianópolis, SC, Brazil. Email: ijtaneja@gmail.com