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Intervally Distributed, Palindromic and Selfie Magic Squares: Genetic Table and Colored Pattern – Orders 11 to 20

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Abstract

This work brings magic squares contructed based on intervally distributed numbers. It has been done in different form, such as, using pairwise mutually orthogonal Latins quares, self-orthogonal diagonalized Latin squares, block-wise compositions, etc. Work is limited to magic square of orders 11 to 20. Restults are extended to palindromic and Selfie magic squqres. In case of order 16, bimagic square construction is presented. Genetic table based on magic square of order 16, and superimposed double colored pattern based on magic square of order 20 are presented. All results are obtained without use of any programming language.

1. Introduction

We shall work with different kinds of magic squares, such as, intervally distributed palindromic, Selfie, etc. These ideas are divided in following parts:

(i) Intervally distributed magic squares;
(ii) Palindromic representations;
(iii) Selfie magic squares;

Before starting, below are explanations of *intervally distributed magic squares*.

1.1. Intervally Distributed Magic Squares

*Let us consider in a sequence of n^2 numbers. Put these numbers in increasing order. Divide n^2 elements in n interval. If elements of each interval appears only once in each row, each column and in principal diagonals, then the we call it **intervally distributed**. If the distribution is a magic square, we call it a **intervally distributed magic squares.** If distribution is only in rows and columns, we call it **intervally semi-distributed**.*

For example, let us consider 100 (10^2) numbers in a sequence, i.e., from 1 to 100. Let us divide these numbers in the 10 intervals:

1st interval	01-10	1	2	3	4	5	6	7	8	9	10
2nd interval	11-20	11	12	13	14	15	16	17	18	19	20
3rd interval	21-30	21	22	23	24	25	26	27	28	29	30
4th interval	31-40	31	32	33	34	35	36	37	38	39	40
5th interval	41-50	41	42	43	44	45	46	47	48	49	50
6th interval	51-60	51	52	53	54	55	56	57	58	59	60
7th interval	61-70	61	62	63	64	65	66	67	68	69	70
8th interval	71-80	71	72	73	74	75	76	77	78	79	80
9th interval	81-90	81	82	83	84	85	86	87	88	89	90
10th interval	91-100	91	92	93	94	95	96	97	98	99	100

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Intervally distributed elements are understood as members of each interval should appear only once in *each row,* in *each column* and in *principal diagonals*, for example,

1							61		
		65		4					
	6				66				
			62			8			
	68								10
			5	63					
		7						64	
					9				69
70							2		
						67		3	

As explained above, if the distributions is only in rows and columns, it is understood as *intervally semi-distributed*. It is not necessary that all *intervally distributed* numbers always forms a magic square. Also there are magic squares *not necessarily intervally distributed*.

2. Procedures to Construct Intervally Distributed Magic Squares

In this section we shall present *intervally distributed magic squares* of orders 11 to 20 using different procedures based on:

(i) "*Mutually orthogonal diagonalized Latin squares*" (*MODLS*);
(ii) "*Self-orthogonal diagonalized Latin squares*" (*SODLS*);
(iii) *Block-wise compositions* (such as 3×4, 4×4, 4×5, etc.).

Later, the results are extended to *palindromic* and *Selfie magic squares*.

2.1. Magic Squares of Prime Order

We shall bring *intervally distributed magic squares* of prime orders 11, 13, 17 and 19. Two different ways are given. One is based on a pair of *MODLS*, and second is based on *SODLS*. In both the cases we always have *intervally distributed pan diagonal magic squares*. Since the procedure is same for all prime number order, we shall give only for order 11.

- **1st Procedure - MODLS**

In case of prime numbers it is easy to write a pair of *MODLS* just using *cyclic system*. See below:

1	2	3	4	5	6	7	8	9	10	11
10	11	1	2	3	4	5	6	7	8	9
8	9	10	11	1	2	3	4	5	6	7
6	7	8	9	10	11	1	2	3	4	5
4	5	6	7	8	9	10	11	1	2	3
2	3	4	5	6	7	8	9	10	11	1
11	1	2	3	4	5	6	7	8	9	10
9	10	11	1	2	3	4	5	6	7	8
7	8	9	10	11	1	2	3	4	5	6
5	6	7	8	9	10	11	1	2	3	4
3	4	5	6	7	8	9	10	11	1	2

A

1	2	3	4	5	6	7	8	9	10	11
9	10	11	1	2	3	4	5	6	7	8
6	7	8	9	10	11	1	2	3	4	5
3	4	5	6	7	8	9	10	11	1	2
11	1	2	3	4	5	6	7	8	9	10
8	9	10	11	1	2	3	4	5	6	7
5	6	7	8	9	10	11	1	2	3	4
2	3	4	5	6	7	8	9	10	11	1
10	11	1	2	3	4	5	6	7	8	9
7	8	9	10	11	1	2	3	4	5	6
4	5	6	7	8	9	10	11	1	2	3

B

… (1)

A and B are two *mutually orthogonal diagonalized Latin squares*. The construction is very simple. First line is kept with all the numbers in order. Second line starts with second element from last in grid A and third element from last in grid B and then proceeding in a cyclic way we get A and B.

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Example 1. Applying the procedure $11\times(A-1)+B$ in all the elements in (1) we get following *pan diagonal intervally distributed magic square* of order 11:

		671	671	671	671	671	671	671	671	671	671	671
	1	13	25	37	49	61	73	85	97	109	121	671
671	108	120	11	12	24	36	48	60	72	84	96	671
671	83	95	107	119	10	22	23	35	47	59	71	671
671	58	70	82	94	106	118	9	21	33	34	46	671
671	44	45	57	69	81	93	105	117	8	20	32	671
671	19	31	43	55	56	68	80	92	104	116	7	671
671	115	6	18	30	42	54	66	67	79	91	103	671
671	90	102	114	5	17	29	41	53	65	77	78	671
671	76	88	89	101	113	4	16	28	40	52	64	671
671	51	63	75	87	99	100	112	3	15	27	39	671
671	26	38	50	62	74	86	98	110	111	2	14	671
	671	671	671	671	671	671	671	671	671	671	671	671

- ## 2nd Procedure – SODLS

The grids A and B are also SODLS in each case. Let us consider A. Change the elements of row by columns in A we get B as transpose of A:

1	2	3	4	5	6	7	8	9	10	11
10	11	1	2	3	4	5	6	7	8	9
8	9	10	11	1	2	3	4	5	6	7
6	7	8	9	10	11	1	2	3	4	5
4	5	6	7	8	9	10	11	1	2	3
2	3	4	5	6	7	8	9	10	11	1
11	1	2	3	4	5	6	7	8	9	10
9	10	11	1	2	3	4	5	6	7	8
7	8	9	10	11	1	2	3	4	5	6
5	6	7	8	9	10	11	1	2	3	4
3	4	5	6	7	8	9	10	11	1	2

A

1	10	8	6	4	2	11	9	7	5	3
2	11	9	7	5	3	1	10	8	6	4
3	1	10	8	6	4	2	11	9	7	5
4	2	11	9	7	5	3	1	10	8	6
5	3	1	10	8	6	4	2	11	9	7
6	4	2	11	9	7	5	3	1	10	8
7	5	3	1	10	8	6	4	2	11	9
8	6	4	2	11	9	7	5	3	1	10
9	7	5	3	1	10	8	6	4	2	11
10	8	6	4	2	11	9	7	5	3	1
11	9	7	5	3	1	10	8	6	4	2

B

… (2)

Example 2. Applying the procedure $11\times(A-1)+B$ in all the elements in (2) we get following *pan diagonal intervally distributed magic square* of order 11:

		671	671	671	671	671	671	671	671	671	671	671
	1	21	30	39	48	57	77	86	95	104	113	671
671	101	121	9	18	27	36	45	65	74	83	92	671
671	80	89	109	118	6	15	24	44	53	62	71	671
671	59	68	88	97	106	115	3	12	32	41	50	671
671	38	47	56	76	85	94	103	112	11	20	29	671
671	17	26	35	55	64	73	82	91	100	120	8	671
671	117	5	14	23	43	52	61	70	79	99	108	671
671	96	105	114	2	22	31	40	49	58	67	87	671
671	75	84	93	102	111	10	19	28	37	46	66	671
671	54	63	72	81	90	110	119	7	16	25	34	671
671	33	42	51	60	69	78	98	107	116	4	13	671
	671	671	671	671	671	671	671	671	671	671	671	671

The same procedure applies to other prime order magic squares.

2.2. Magic Square of Order 12

In this subsection we shall present magic squares of order 12 constructed in two different form. One is based on a composition of magic squares of orders 4 and 3. The second is based on *MODLS*.

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• 1st Procedure – Block Wise

We shall bring intervally distributed magic square of order 12 by using composition of magic square of order 4 with of order 3. Let us consider *pan diagonal perfect magic square* of order 4 based on *SODLS*:

2	3	1	4
1	4	2	3
4	1	3	2
3	2	4	1

A

2	1	4	3
3	4	1	2
1	2	3	4
4	3	2	1

B

22	31	14	43
13	44	21	32
41	12	33	24
34	23	42	11

AB

		34	34	34	34
	6	9	4	15	34
34	3	16	5	10	34
34	13	2	11	8	34
34	12	7	14	1	34
	34	34	34	34	34

… (3)

Let us consider the a magic square of order 3:

1	3	2
3	2	1
2	1	3

A

2	3	1
1	2	3
3	1	2

B

12	33	21
31	22	13
23	11	32

AB

			15
2	9	4	15
7	5	3	15
6	1	8	15
15	15	15	15

… (4)

Let us write the numbers 1 to 12 in following 3 intervals:

1	6	7	12	26
2	5	8	11	26
3	4	9	10	26

… (5)

Apply values given in (5) over *A* and *B* in (3) and put according to *AB* in (4), we get 9 compositions, such as

12

6	7	1	12
1	12	6	7
12	1	7	6
7	6	12	1

5	2	11	8
8	11	2	5
2	5	8	11
11	8	5	2

		290	290	290	290
	65	74	11	140	290
290	8	143	62	77	290
290	134	5	80	71	290
290	83	68	137	2	290
	290	290	290	290	290

11

6	7	1	12
1	12	6	7
12	1	7	6
7	6	12	1

6	1	12	7
7	12	1	6
1	6	7	12
12	7	6	1

		290	290	290	290
	66	73	12	139	290
290	7	144	61	78	290
290	133	6	79	72	290
290	84	67	138	1	290
	290	290	290	290	290

Similarly we make another 7 compositions. Putting 9 compositions constructed above according to *AB* given (4), we get *pairwise mutually orthogonal (non diagonalized) Latin squares:*

6	7	1	12	4	9	3	10	5	8	2	**11**
1	12	6	7	3	10	4	9	2	11	**5**	8
12	1	7	6	10	3	9	4	11	**2**	8	5
7	6	12	1	9	4	10	3	**8**	5	11	2
4	9	3	10	5	8	2	**11**	6	7	1	12
3	10	4	9	2	11	**5**	8	1	12	6	7
10	3	9	4	11	**2**	8	5	12	1	7	6
9	4	10	3	**8**	5	11	2	7	6	12	1
5	8	2	**11**	6	7	1	12	4	9	3	10
2	11	**5**	8	1	12	6	7	3	10	4	9
11	**2**	8	5	12	1	7	6	10	3	9	4
8	5	11	2	7	6	12	1	9	4	10	3

A

5	2	11	8	4	3	10	9	6	1	12	7
8	**11**	2	5	9	10	3	4	7	12	1	6
2	5	**8**	11	3	4	9	10	1	6	7	12
11	8	5	**2**	10	9	4	3	12	7	6	1
6	1	12	7	5	2	11	8	4	3	10	9
7	12	1	6	8	**11**	2	5	9	10	3	4
1	6	7	12	2	5	8	11	3	4	9	10
12	7	6	1	11	8	5	**2**	10	9	4	3
4	3	10	9	6	1	12	7	**5**	2	11	8
9	10	3	4	7	12	1	6	8	**11**	2	5
3	4	9	10	1	6	7	12	2	5	**8**	11
10	9	4	3	12	7	6	1	11	8	5	**2**

B

… (6)

Example 3. Applying the procedure $12\times(A-1)+B$ in (6), we have *intervally semi-distributed magic square* of order 12:

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												870
65	74	11	140	40	99	34	117	54	85	24	127	870
8	143	62	77	33	118	39	100	19	132	49	90	870
134	5	80	71	111	28	105	46	121	18	91	60	870
83	68	137	2	106	45	112	27	96	55	126	13	870
42	97	36	115	53	86	23	128	64	75	10	141	870
31	120	37	102	20	131	50	89	9	142	63	76	870
109	30	103	48	122	17	92	59	135	4	81	70	870
108	43	114	25	95	56	125	14	82	69	136	3	870
52	87	22	129	66	73	12	139	41	98	35	116	870
21	130	51	88	7	144	61	78	32	119	38	101	870
123	16	93	58	133	6	79	72	110	29	104	47	870
94	57	124	15	84	67	138	1	107	44	113	26	870
870	870	870	870	870	870	870	870	870	870	870	870	870

Even though each block of order 4 is a *pan diagonal magic square* with sum $S_{4\times4} := 290$, but the magic square of order 12 is *not pan diagonal*. It is *intervally semi-distributed* as there are numbers in one of the diagonal belong to same intervals:

2nd interval	13	14	15	**16**	**17**	**18**	19	20	21	22	23	24
5th interval	**49**	**50**	**51**	52	53	54	55	56	57	58	59	60
8th interval	85	86	87	88	89	90	91	92	93	**94**	**95**	**96**
11th interval	121	122	123	124	125	126	**127**	**128**	**129**	130	131	132

- ## 2nd Procedure - MODLS

We shall present magic square of order 12 based on *MODLS* obtained long back by Wallis and Zhu [17]. In the paper [17] the authors gave four pairs of *MODLS*. We shall use only two of them to bring magic square of order 12:

1	12	8	3	7	2	4	6	10	5	11	9
6	11	7	2	12	1	3	5	9	4	10	8
5	10	12	1	11	6	2	4	8	3	9	7
4	9	11	6	10	5	1	3	7	2	8	12
3	8	10	5	9	4	6	2	12	1	7	11
2	7	9	4	8	3	5	1	11	6	12	10
7	2	4	9	3	8	10	12	6	11	1	5
8	3	5	10	4	9	11	7	1	12	2	6
9	4	6	11	5	10	12	8	2	7	3	1
10	5	1	12	6	11	7	9	3	8	4	2
11	6	2	7	1	12	8	10	4	9	5	3
12	1	3	8	2	7	9	11	5	10	6	4

A

1	8	2	11	6	9	5	12	7	3	10	4
6	7	1	10	5	8	4	11	12	2	9	3
5	12	6	9	4	7	3	10	11	1	8	2
4	11	5	8	3	12	2	9	10	6	7	1
3	10	4	7	2	11	1	8	9	5	12	6
2	9	3	12	1	10	6	7	8	4	11	5
7	4	8	1	12	5	11	2	3	9	6	10
8	5	9	2	7	6	12	3	4	10	1	11
9	6	10	3	8	1	7	4	5	11	2	12
10	1	11	4	9	2	8	5	6	12	3	7
11	2	12	5	10	3	9	6	1	7	4	8
12	3	7	6	11	4	10	1	2	8	5	9

B

… (7)

Example 4. Applying the procedure $12\times(A-1)+B$ in (7), we get *intervally distributed magic square* of order 12:

												870
1	140	86	35	78	21	41	72	115	51	130	100	870
66	127	73	22	137	8	28	59	108	38	117	87	870
53	120	138	9	124	67	15	46	95	25	104	74	870
40	107	125	68	111	60	2	33	82	18	91	133	870
27	94	112	55	98	47	61	20	141	5	84	126	870
14	81	99	48	85	34	54	7	128	64	143	113	870
79	16	44	97	36	89	119	134	63	129	6	58	870
92	29	57	110	43	102	132	75	4	142	13	71	870
105	42	70	123	56	109	139	88	17	83	26	12	870
118	49	11	136	69	122	80	101	30	96	39	19	870
131	62	24	77	10	135	93	114	37	103	52	32	870
144	3	31	90	23	76	106	121	50	116	65	45	870
870	870	870	870	870	870	870	870	870	870	870	870	870

Still we don't have magic square of order 12 based on *SODLS*. According to [1] it exists.

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2.3. Magic Square of Order 14

We shall present magic squares of order 14 based on *MODLS* and *SODLS* given respectively by Zhu [18] and Bennetta, Du and Zhang [1].

- **1st Procedures - MODLS**

1	4	11	5	8	2	9	3	7	14	12	13	6	10
14	2	5	11	6	3	10	4	8	12	13	7	1	9
12	14	3	6	11	4	1	5	9	13	8	2	10	7
13	12	14	4	7	5	2	6	10	9	3	1	8	11
10	13	12	14	5	6	3	7	1	4	2	9	11	8
7	8	9	10	1	11	13	14	12	6	5	4	3	2
9	10	1	2	3	14	12	11	13	8	7	6	5	4
6	7	8	9	10	12	14	13	11	5	4	3	2	1
8	9	10	1	2	13	11	12	14	7	6	5	4	3
3	11	4	7	9	1	8	2	6	10	14	12	13	5
11	3	6	8	4	10	7	1	5	2	9	14	12	13
2	5	7	3	13	9	6	10	4	11	1	8	14	12
4	6	2	13	12	8	5	9	3	1	11	10	7	14
5	1	13	12	14	7	4	8	2	3	10	11	9	6

A

1	13	10	14	11	6	7	9	8	3	4	2	12	5
4	2	13	1	14	7	8	10	9	5	3	12	6	11
6	5	3	13	2	8	9	1	10	4	12	7	11	14
5	7	6	4	13	9	10	2	1	12	8	11	14	3
12	6	8	7	5	10	1	3	2	9	11	14	4	13
9	10	1	2	3	11	14	12	13	8	7	6	5	4
2	3	4	5	6	13	12	14	11	1	10	9	8	7
3	4	5	6	7	14	11	13	12	2	1	10	9	8
7	8	9	10	1	12	13	11	14	6	5	4	3	2
13	9	14	11	4	5	6	8	7	10	2	3	1	12
8	14	11	3	12	4	5	7	6	13	9	1	2	10
14	11	2	12	9	3	4	6	5	7	13	8	10	1
11	1	12	8	10	2	3	5	4	14	6	13	7	9
10	12	7	9	8	1	2	4	3	11	14	5	13	6

B

… (8)

Example 5. Applying the procedure $14\times(A-1)+B$ in (8), we get *intervally distributed magic square* of order 14 with inner grid a magic square of order 4 with sum $S_{4\times4} := 694$:

														1379
1	55	150	70	109	20	119	37	92	185	158	170	82	131	1379
186	16	69	141	84	35	134	52	107	159	171	96	6	123	1379
160	187	31	83	142	50	9	57	122	172	110	21	137	98	1379
173	161	188	46	97	65	24	72	127	124	36	11	112	143	1379
138	174	162	189	61	80	29	87	2	51	25	126	144	111	1379
93	108	113	128	3	**151**	**182**	**194**	**167**	78	63	48	33	18	1379
114	129	4	19	34	**195**	**166**	**154**	**179**	99	94	79	64	49	1379
73	88	103	118	133	**168**	**193**	**181**	**152**	58	43	38	23	8	1379
105	120	135	10	15	**180**	**153**	**165**	**196**	90	75	60	45	30	1379
41	149	56	95	116	5	104	22	77	136	184	157	169	68	1379
148	42	81	101	54	130	89	7	62	27	121	183	156	178	1379
28	67	86	40	177	115	74	132	47	147	13	106	192	155	1379
53	71	26	176	164	100	59	117	32	14	146	139	91	191	1379
66	12	175	163	190	85	44	102	17	39	140	145	125	76	1379
1379	1379	1379	1379	1379	1379	1379	1379	1379	1379	1379	1379	1379	1379	1379

- **2nd Procedure - SODLS**

This procedure is bases on a work of Bennetta, Du and Zhang [1]. Here also middle square is a magic square of order 4. Below *is SODLS* and its transpose:

1	7	14	8	10	6	5	3	2	12	11	13	9	4
11	2	8	13	14	10	7	9	1	4	5	6	3	12
9	12	3	10	13	5	6	1	4	2	7	8	11	14
14	8	11	4	12	9	1	6	3	13	2	7	10	5
12	6	4	1	9	7	3	8	10	5	13	2	14	11
8	1	7	3	2	12	13	11	14	6	4	10	5	9
2	3	10	5	1	11	14	12	13	7	9	4	6	8
4	5	6	7	3	14	11	13	12	9	10	1	8	2
3	4	5	6	7	13	12	14	11	8	1	9	2	10
5	11	12	2	6	4	8	7	9	10	14	3	1	13
6	14	13	9	5	2	10	4	7	1	8	11	12	3
10	13	1	12	11	8	9	2	6	14	3	5	4	7
13	10	9	14	4	3	2	5	8	11	6	12	7	1
7	9	2	11	8	1	4	10	5	3	12	14	13	6

A

1	11	9	14	12	8	2	4	3	5	6	10	13	7
7	2	12	8	6	1	3	5	4	11	14	13	10	9
14	8	3	11	4	7	10	6	5	12	13	1	9	2
8	13	10	4	1	3	5	7	6	2	9	12	14	11
10	14	13	12	9	2	1	3	7	6	5	11	4	8
6	10	5	9	7	12	11	14	13	4	2	8	3	1
5	7	6	1	3	13	14	11	12	8	10	9	2	4
3	9	1	6	8	11	12	13	14	7	4	2	5	10
2	1	4	3	10	14	13	12	11	9	7	6	8	5
12	4	2	13	5	6	7	9	8	10	1	14	11	3
11	5	7	2	13	4	9	10	1	14	8	3	6	12
13	6	8	7	2	10	4	1	9	3	11	5	12	14
9	3	11	10	14	5	6	8	2	1	12	4	7	13
4	12	14	5	11	9	8	2	10	13	3	7	1	6

B

… (9)

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Example 6. Applying the procedure $14\times(A-1)+B$ in (9), we get *intervally distributed magic square* of order 14 with middle square as a magic square of order 4 with sum $S_{4\times4}:=694$:

														1379
1	95	191	112	138	78	58	32	17	159	146	178	125	49	1379
147	16	110	176	188	127	87	117	4	53	70	83	38	163	1379
126	162	31	137	172	63	80	6	47	26	97	99	149	184	1379
190	111	150	46	155	115	5	77	34	170	23	96	140	67	1379
164	84	55	12	121	86	29	101	133	62	173	25	186	148	1379
104	10	89	37	21	166	179	154	195	74	44	134	59	113	1379
19	35	132	57	3	153	196	165	180	92	122	51	72	102	1379
45	65	71	90	36	193	152	181	168	119	130	2	103	24	1379
30	43	60	73	94	182	167	194	151	107	7	118	22	131	1379
68	144	156	27	75	48	105	93	120	136	183	42	11	171	1379
81	187	175	114	69	18	135	52	85	14	106	143	160	40	1379
139	174	8	161	142	108	116	15	79	185	39	61	54	98	1379
177	129	123	192	56	33	20	64	100	141	82	158	91	13	1379
88	124	28	145	109	9	50	128	66	41	157	189	169	76	1379
1379	1379	1379	1379	1379	1379	1379	1379	1379	1379	1379	1379	1379	1379	1379

2.4. Magic Square of Order 15

We shall present magic squares of order 15 based on three procedures. The first is based on combinations of magic squares of orders 5 and 3. Second is based on *MODLS* is due to author and third is based on *SODLS* due to Cao and Li [3].

- **1st Procedure – Block Wise**

We shall bring *intervally semi-distributed magic square* of order 15 using composition of magic square of order 5 with order 3. Let us consider *pan diagonal* magic square of order 5 constructed based on *SODLS*:

1	2	3	4	5
4	5	1	2	3
2	3	4	5	1
5	1	2	3	4
3	4	5	1	2

A

1	4	2	5	3
2	5	3	1	4
3	1	4	2	5
4	2	5	3	1
5	3	1	4	2

B

11	24	32	45	53
42	55	13	21	34
23	31	44	52	15
54	12	25	33	41
35	43	51	14	22

AB

		65	65	65	65	65
	1	9	12	20	23	65
65	17	25	3	6	14	65
65	8	11	19	22	5	65
65	24	2	10	13	16	65
65	15	18	21	4	7	65
	65	65	65	65	65	65

…. (10)

Let us consider the a magic square of order 3:

1	3	2
3	2	1
2	1	3

A

2	3	1
1	2	3
3	1	2

B

12	33	21
31	22	13
23	11	32

AB

			15
2	9	4	15
7	5	3	15
6	1	8	15
15	15	15	15

…. (11)

Let us write the numbers 1 to 15 in following 3 intervals:

1	6	7	12	13	39
2	5	8	11	14	40
3	4	9	10	15	41

…. (12)

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Even though the sum of each interval is different, still we shall get a magic square of order 15. Apply values given in (12) over A and B in (10) and put according to AB in (11), we get 9 compositions, such as

31

3	4	9	10	15
10	15	3	4	9
4	9	10	15	3
15	3	4	9	10
9	10	15	3	4

A

1	12	6	13	7
6	13	7	1	12
7	1	12	6	13
12	6	13	7	1
13	7	1	12	6

B

		579	579	579	579	579
	31	57	126	148	217	579
579	141	223	37	46	132	579
579	52	121	147	216	43	579
579	222	36	58	127	136	579
579	133	142	211	42	51	579
	579	579	579	579	579	579

22

2	5	8	11	14
11	14	2	5	8
5	8	11	14	2
14	2	5	8	11
8	11	14	2	5

A

2	11	5	14	8
5	14	8	2	11
8	2	11	5	14
11	5	14	8	2
14	8	2	11	5

B

		565	565	565	565	565
	17	71	110	164	203	565
565	155	209	23	62	116	565
565	68	107	161	200	29	565
565	206	20	74	113	152	565
565	119	158	197	26	65	565
	565	565	565	565	565	565

Similarly we make another 7 compositions. Putting according to AB given (11), we get *pairwise mutually orthogonal (non diagonalized) Latin squares:*

1	6	7	12	13	3	4	9	10	15	2	5	8	11	**14**
12	13	1	6	7	10	15	3	4	9	11	14	2	**5**	8
6	7	12	13	1	4	9	10	15	3	5	8	**11**	14	2
13	1	6	7	12	15	3	4	9	10	14	**2**	5	8	11
7	12	13	1	6	9	10	15	3	4	**8**	11	14	2	5
3	4	9	10	15	2	5	8	11	**14**	1	6	7	12	13
10	15	3	4	9	11	14	2	**5**	8	12	13	1	6	7
4	9	10	15	3	5	8	**11**	14	2	6	7	12	13	1
15	3	4	9	10	14	**2**	5	8	11	13	1	6	7	12
9	10	15	3	4	**8**	11	14	2	5	7	12	13	1	6
2	5	8	11	**14**	1	6	7	12	13	3	4	9	10	15
11	14	2	**5**	8	12	13	1	6	7	10	15	3	4	9
5	8	**11**	14	2	6	7	12	13	1	4	9	10	15	3
14	**2**	5	8	11	13	1	6	7	12	15	3	4	9	10
8	11	14	2	5	7	12	13	1	6	9	10	15	3	4

A

2	11	5	14	8	3	10	4	15	9	1	12	6	13	7
5	**14**	8	2	11	4	15	9	3	10	6	13	7	1	12
8	2	**11**	5	14	9	3	10	4	15	7	1	12	6	13
11	5	14	**8**	2	10	4	15	9	3	12	6	13	7	1
14	8	2	11	**5**	15	9	3	10	4	13	7	1	12	6
1	12	6	13	7	**2**	11	5	14	8	3	10	4	15	9
6	13	7	1	12	5	**14**	8	2	11	4	15	9	3	10
7	1	12	6	13	8	2	**11**	5	14	9	3	10	4	15
12	6	13	7	1	11	5	14	**8**	2	10	4	15	9	3
13	7	1	12	6	14	8	2	11	**5**	15	9	3	10	4
3	10	4	15	9	1	12	6	13	7	**2**	11	5	14	8
4	15	9	3	10	6	13	7	1	12	5	**14**	8	2	11
9	3	10	4	15	7	1	12	6	13	8	2	**11**	5	14
10	4	15	9	3	12	6	13	7	1	11	5	14	**8**	2
15	9	3	10	4	13	7	1	12	6	14	8	2	11	**5**

B

… (13)

Example 7. Applying the procedure $15\times(A-1)+B$ in (13), we get *intervally semi-distributed magic square* of order 15:

															1695
2	86	95	179	188	33	55	124	150	219	16	72	111	163	202	1695
170	194	8	77	101	139	225	39	48	130	156	208	22	61	117	1695
83	92	176	185	14	54	123	145	214	45	67	106	162	201	28	1695
191	5	89	98	167	220	34	60	129	138	207	21	73	112	151	1695
104	173	182	11	80	135	144	213	40	49	118	157	196	27	66	1695
31	57	126	148	217	17	71	110	164	203	3	85	94	180	189	1695
141	223	37	46	132	155	209	23	62	116	169	195	9	78	100	1695
52	121	147	216	43	68	107	161	200	29	84	93	175	184	15	1695
222	36	58	127	136	206	20	74	113	152	190	4	90	99	168	1695
133	142	211	42	51	119	158	197	26	65	105	174	183	10	79	1695
18	70	109	165	204	1	87	96	178	187	32	56	125	149	218	1695
154	210	24	63	115	171	193	7	76	102	140	224	38	47	131	1695
69	108	160	199	30	82	91	177	186	13	53	122	146	215	44	1695
205	19	75	114	153	192	6	88	97	166	221	35	59	128	137	1695
120	159	198	25	64	103	172	181	12	81	134	143	212	41	50	1695
1695	1695	1695	1695	1695	1695	1695	1695	1695	1695	1695	1695	1695	1695	1695	1695

It is *intervally semi-distributed* as there are elements in diagonals belongs to same intervals of distribution:

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2nd interval	16	17	18	**19**	**20**	**21**	22	23	24	25	26	27	28	29	30
5th interval	**61**	**62**	**63**	64	65	66	67	68	69	70	71	72	73	74	75
8th interval	106	107	108	109	110	111	112	113	114	115	116	117	**118**	**119**	**120**
11th interval	151	152	153	154	155	156	157	158	159	**160**	**161**	**162**	163	164	165
14th interval	196	197	198	199	200	201	**202**	**203**	**204**	205	206	207	208	209	210

The above table show three values in each interval appearing in one of the diagonal. Moreover each block of 5 is a *pan diagonal magic square* of order 5 with different sums resulting in magic square of order 15 with magic sum 1695:

			1695
550	581	564	1695
579	565	551	1695
566	549	580	1695
1695	1695	1695	1695

Even though each block of order 5 is a *pan diagonal magic square* but the magic square of order 15 is *not pan diagonal*.

- ## 2nd Procedure - MODLS

This procedure is due to author, and is written without any rule. Here each block of order 3×5 has all the 15 numbers in A and B, still forming *MODLS*:

1	2	3	4	5	6	7	13	9	10	11	12	8	14	15
6	7	8	9	10	11	12	3	14	15	1	2	13	4	5
11	12	13	14	15	1	2	8	4	5	6	7	3	9	10
2	3	4	5	1	7	8	9	15	6	12	13	14	10	11
7	8	9	15	6	12	13	14	10	11	2	3	4	5	1
12	13	14	10	11	2	3	4	5	1	7	8	9	15	6
3	9	5	1	7	13	4	15	6	2	8	14	10	11	12
13	14	15	11	2	8	9	10	1	12	3	4	5	6	7
8	4	10	6	12	3	14	5	11	7	13	9	15	1	2
14	5	1	7	13	9	15	6	2	8	4	10	11	12	3
9	15	6	2	8	4	10	11	12	3	14	5	1	7	13
4	10	11	12	3	14	5	1	7	13	9	15	6	2	8
10	1	7	3	4	15	11	2	13	9	5	6	12	8	14
15	11	2	13	9	5	6	12	8	14	10	1	7	3	4
5	6	12	8	14	10	1	7	3	4	15	11	2	13	9

A

8	7	3	4	6	1	13	15	11	5	2	14	9	10	12
2	14	15	10	12	8	7	9	4	6	1	13	3	11	5
1	13	9	11	5	2	14	3	10	12	8	7	15	4	6
5	2	7	15	4	6	1	13	3	11	12	8	14	9	10
12	8	14	9	10	5	2	7	15	4	6	1	13	3	11
6	1	13	3	11	12	8	14	9	10	5	2	7	15	4
4	5	2	13	15	11	6	1	7	3	10	12	8	14	9
10	6	8	7	9	4	12	2	14	15	11	5	1	13	3
11	12	1	14	3	10	5	8	13	9	4	6	2	7	15
15	4	6	2	13	3	11	5	1	7	9	10	12	8	14
9	10	12	8	14	15	4	6	2	13	3	11	5	1	7
3	11	5	1	7	9	10	12	8	14	15	4	6	2	13
7	15	4	6	1	13	3	10	5	2	14	9	11	12	8
14	9	11	12	8	7	15	4	6	1	13	3	10	5	2
13	3	10	5	2	14	9	11	12	8	7	15	4	6	1

B

… (14)

Example 8. Applying the procedure $15 \times (A-1) + B$ in (14), we get *intervally distributed magic square* of order 15 with each block of order 3×5 having the same sum as of magic square:

															1695
8	22	33	49	66	76	103	195	131	140	152	179	114	205	222	1695
77	104	120	130	147	158	172	39	199	216	1	28	183	56	65	1695
151	178	189	206	215	2	29	108	55	72	83	97	45	124	141	1695
20	32	52	75	4	96	106	133	213	86	177	188	209	144	160	1695
102	113	134	219	85	170	182	202	150	154	21	31	58	63	11	1695
171	181	208	138	161	27	38	59	69	10	95	107	127	225	79	1695
34	125	62	13	105	191	51	211	82	18	115	207	143	164	174	1695
190	201	218	157	24	109	132	137	14	180	41	50	61	88	93	1695
116	57	136	89	168	40	200	68	163	99	184	126	212	7	30	1695
210	64	6	92	193	123	221	80	16	112	54	145	162	173	44	1695
129	220	87	23	119	60	139	156	167	43	198	71	5	91	187	1695
48	146	155	166	37	204	70	12	98	194	135	214	81	17	118	1695
142	15	94	36	46	223	153	25	185	122	74	84	176	117	203	1695
224	159	26	192	128	67	90	169	111	196	148	3	100	35	47	1695
73	78	175	110	197	149	9	101	42	53	217	165	19	186	121	1695
1695	1695	1695	1695	1695	1695	1695	1695	1695	1695	1695	1695	1695	1695	1695	1695

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- **3rd Procedure - SODLS**

This procedure is bases on a work of Cao and Li [3]. The A is SODLS and B is its transpose:

1	5	14	6	3	9	8	10	13	4	7	15	12	2	11
12	2	6	13	7	4	1	9	5	15	14	11	3	10	8
6	11	3	7	12	15	5	1	14	13	10	4	9	8	2
13	7	10	4	15	11	14	2	12	9	5	1	8	3	6
11	12	15	9	5	14	10	3	1	6	2	8	4	7	13
2	10	11	14	1	6	13	4	7	3	8	5	15	12	9
15	3	9	10	13	2	7	5	4	8	6	14	11	1	12
9	1	2	3	4	5	6	8	10	11	12	13	14	15	7
4	15	5	2	10	8	12	11	9	14	3	6	7	13	1
7	4	1	11	8	13	9	12	3	10	15	2	5	6	14
3	9	12	8	14	10	15	13	6	2	11	7	1	4	5
10	13	8	15	11	7	4	14	2	5	1	12	6	9	3
14	8	7	12	6	3	2	15	11	1	4	9	13	5	10
8	6	13	5	2	1	11	7	15	12	9	3	10	14	4
5	14	4	1	9	12	3	6	8	7	13	10	2	11	15

A

1	12	6	13	11	2	15	9	4	7	3	10	14	8	5
5	2	11	7	12	10	3	1	15	4	9	13	8	6	14
14	6	3	10	15	11	9	2	5	1	12	8	7	13	4
6	13	7	4	9	14	10	3	2	11	8	15	12	5	1
3	7	12	15	5	1	13	4	10	8	14	11	6	2	9
9	4	15	11	14	6	2	5	8	13	10	7	3	1	12
8	1	5	14	10	13	7	6	12	9	15	4	2	11	3
10	9	1	2	3	4	5	8	11	12	13	14	15	7	6
13	5	14	12	1	7	4	10	9	3	6	2	11	15	8
4	15	13	9	6	3	8	11	14	10	2	5	1	12	7
7	14	10	5	2	8	6	12	3	15	11	1	5	9	13
15	11	4	1	8	5	14	13	6	2	7	12	9	3	10
12	3	9	8	4	15	11	14	7	5	1	6	13	10	2
2	10	8	3	7	12	1	15	13	6	4	9	5	14	11
11	8	2	6	13	9	12	7	1	14	5	3	10	4	15

B

… (15)

Example 9. Applying the procedure $15\times(A-1)+B$ in (15), we get *intervally distributed magic square* of order 15:

															1695
1	72	201	88	41	122	120	144	184	52	93	220	179	23	155	1695
170	17	86	187	102	55	3	121	75	214	204	163	38	141	119	1695
89	156	33	100	180	221	69	2	200	181	147	53	127	118	19	1695
186	103	142	49	219	164	205	18	167	131	68	15	117	35	76	1695
153	172	222	135	65	196	148	34	10	83	29	116	51	92	189	1695
24	139	165	206	14	81	182	50	98	43	115	67	213	166	132	1695
218	31	125	149	190	28	97	66	57	114	90	199	152	11	168	1695
130	9	16	32	48	64	80	113	146	162	178	194	210	217	96	1695
58	215	74	27	136	112	169	160	129	198	36	77	101	195	8	1695
94	60	13	159	111	183	128	176	44	145	212	20	61	87	202	1695
37	134	175	110	197	143	216	192	78	30	161	91	5	54	73	1696
150	191	109	211	158	95	59	208	21	62	7	177	84	123	40	1695
207	108	99	173	79	45	26	224	157	5	46	126	193	70	137	1695
107	85	188	63	22	12	151	105	223	171	124	39	140	209	56	1695
71	203	47	6	133	174	42	82	106	104	185	138	25	154	225	1695
1695	1695	1695	1695	1695	1695	1695	1695	1695	1695	1695	1695	1696	1695	1695	1695

2.5. Magic and Bimagic Squares of Order 16

We shall bring *intervally distributed magic and bimagic squares* of order 16 using different approaches. These are based on *MODLS* and *SODLS*. All the cases we have used composition of order 4 magic squares or Latin squares. Mathematical studies on bimagic squares of order 16 can be seen in Boyer [2] and Derksen, Eggermont and Essen [4].

- **1st Procedure: Bimagic Square - MODLS**

Let us consider following two magic squares construced based on *MODLS*:

1	3	4	2
4	2	1	3
2	4	3	1
3	1	2	4

A

1	2	3	4
4	3	2	1
2	1	4	3
3	4	1	2

B

				34
1	10	15	8	34
16	7	2	9	34
6	13	12	3	34
11	4	5	14	34
34	34	34	34	34

... (16)

and

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1	2	3	4	
3	4	1	2	
4	3	2	1	
2	1	4	3	

A

1	2	3	4
4	3	2	1
2	1	4	3
3	4	1	2

B

11	22	33	44
34	43	12	21
42	31	24	13
23	14	41	32

AB

				34
1	6	11	16	34
12	15	2	5	34
14	9	8	3	34
7	4	13	10	34
34	34	34	34	34

... (17)

Magic squares appearing in (16) and (17) are *not pan diagonal*. Consider values of magic square given in (16) applying over A and B in (17) and putting according to AB in (17) we get 16 compositions, such as

$\boxed{11}$

1	10	15	8
15	8	1	10
8	15	10	1
10	1	8	15

1	10	15	8
8	15	10	1
10	1	8	15
15	8	1	10

				514
1	154	239	120	514
232	127	10	145	514
122	225	152	15	514
159	8	113	234	514
514	514	514	514	514

$\boxed{43}$

4	11	14	5
14	5	4	11
5	14	11	4
11	4	5	14

12	3	6	13
13	6	3	12
3	12	13	6
6	13	12	3

				514
60	163	214	77	514
221	70	51	172	514
67	220	173	54	514
166	61	76	211	514
514	514	514	514	514

Similarly, make another 14 compositions. Putting according to AB given (17), we get a pair of *MODLS*:

1	10	15	8	2	9	16	7	3	12	13	6	4	11	14	5
15	8	1	10	16	7	2	9	13	6	3	12	14	5	4	11
8	15	10	1	7	16	9	2	6	13	12	3	5	14	11	4
10	1	8	15	9	2	7	16	12	3	6	13	11	4	5	14
3	12	13	6	4	11	14	5	1	10	15	8	2	9	16	7
13	6	3	12	14	5	4	11	15	8	1	10	16	7	2	9
6	13	12	3	5	14	11	4	8	15	10	1	7	16	9	2
12	3	6	13	11	4	5	14	10	1	8	15	9	2	7	16
4	11	14	5	3	12	13	6	2	9	16	7	1	10	15	8
14	5	4	11	13	6	3	12	16	7	2	9	15	8	1	10
5	14	11	4	6	13	12	3	7	16	9	2	8	15	10	1
11	4	5	14	12	3	6	13	9	2	7	16	10	1	8	15
2	9	16	7	1	10	15	8	4	11	14	5	3	12	13	6
16	7	2	9	15	8	1	10	14	5	4	11	13	6	3	12
7	16	9	2	8	15	10	1	5	14	11	4	6	13	12	3
9	2	7	16	10	1	8	15	11	4	5	14	12	3	6	13

A

1	10	15	8	7	16	9	2	12	3	6	13	14	5	4	11
8	15	10	1	2	9	16	7	13	6	3	12	11	4	5	14
10	1	8	15	16	7	2	9	3	12	13	6	5	14	11	4
15	8	1	10	9	2	7	16	6	13	12	3	4	11	14	5
14	5	4	11	12	3	6	13	7	16	9	2	1	10	15	8
11	4	5	14	13	6	3	12	2	9	16	7	8	15	10	1
5	14	11	4	3	12	13	6	16	7	2	9	10	1	8	15
4	11	14	5	6	13	12	3	9	2	7	16	15	8	1	10
7	16	9	2	1	10	15	8	14	5	4	11	12	3	6	13
2	9	16	7	8	15	10	1	11	4	5	14	13	6	3	12
16	7	2	9	10	1	8	15	5	14	11	4	3	12	13	6
9	2	7	16	15	8	1	10	4	11	14	5	6	13	12	3
12	3	6	13	14	5	4	11	1	10	15	8	7	16	9	2
13	6	3	12	11	4	5	14	8	15	10	1	2	9	16	7
3	12	13	6	5	14	11	4	10	1	8	15	16	7	2	9
6	13	12	3	4	11	14	5	15	8	1	10	9	2	7	16

B

... (18)

Example 10. Applying $16\times(A-1)+B$ in (18), we get the *bimagic square* of order 16 with sums $S_{16\times16}:=2056$ and $Sb_{16\times16}:=351576$, each block of order 4 is a magic square with sum $S_{4\times4}:=512$:

																2056
1	154	239	120	23	144	249	98	44	179	198	93	62	165	212	75	2056
232	127	10	145	242	105	32	135	205	86	35	188	219	68	53	174	2056
122	225	152	15	112	247	130	25	83	204	189	38	69	222	171	52	2056
159	8	113	234	137	18	103	256	182	45	92	195	164	59	78	213	2056
46	181	196	91	60	163	214	77	7	160	233	114	17	138	255	104	2056
203	84	37	190	221	70	51	172	226	121	16	151	248	111	26	129	2056
85	206	187	36	67	220	173	54	128	231	146	9	106	241	136	31	2056
180	43	94	197	166	61	76	211	153	2	119	240	143	24	97	250	2056
55	176	217	66	33	186	207	88	30	133	244	107	12	147	230	125	2056
210	73	64	167	200	95	42	177	251	100	21	142	237	118	3	156	2056
80	215	162	57	90	193	184	47	101	254	139	20	115	236	157	6	2056
169	50	71	224	191	40	81	202	132	27	110	245	150	13	124	227	2056
28	131	246	109	14	149	228	123	49	170	223	72	39	192	201	82	2056
253	102	19	140	235	116	5	158	216	79	58	161	194	89	48	183	2056
99	252	141	22	117	238	155	4	74	209	168	63	96	199	178	41	2056
134	29	108	243	148	11	126	229	175	56	65	218	185	34	87	208	2056
2056	2056	2056	2056	2056	2056	2056	2056	2056	2056	2056	2056	2056	2056	2056	2056	2056

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• 2nd Procedure: Bimagic Square – MODLS

Each block of order 4 in example 10 is a magic square, where *MODLS* are made just of four numbers in each block. Below a construction of another *bimagic square* of order 16, where *MODLS* blocks of order 4 are made with 16 numbers in each block:

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
7	8	5	6	3	4	1	2	15	16	13	14	11	12	9	10
12	11	10	9	16	15	14	13	4	3	2	1	8	7	6	5
14	13	16	15	10	9	12	11	6	5	8	7	2	1	4	3
6	5	8	7	2	1	4	3	14	13	16	15	10	9	12	11
4	3	2	1	8	7	6	5	12	11	10	9	16	15	14	13
15	16	13	14	11	12	9	10	7	8	5	6	3	4	1	2
9	10	11	12	13	14	15	16	1	2	3	4	5	6	7	8
11	12	9	10	15	16	13	14	3	4	1	2	7	8	5	6
13	14	15	16	9	10	11	12	5	6	7	8	1	2	3	4
2	1	4	3	6	5	8	7	10	9	12	11	14	13	16	15
8	7	6	5	4	3	2	1	16	15	14	13	12	11	10	9
16	15	14	13	12	11	10	9	8	7	6	5	4	3	2	1
10	9	12	11	14	13	16	15	2	1	4	3	6	5	8	7
5	6	7	8	1	2	3	4	13	14	15	16	9	10	11	12
3	4	1	2	7	8	5	6	11	12	9	10	15	16	13	14

A

12	6	13	3	14	4	11	5	1	15	8	10	7	9	2	16
10	8	15	1	16	2	9	7	3	13	6	12	5	11	4	14
9	7	16	2	15	1	10	8	4	14	5	11	6	12	3	13
11	5	14	4	13	3	12	6	2	16	7	9	8	10	1	15
8	10	1	15	2	16	7	9	13	3	12	6	11	5	14	4
6	12	3	13	4	14	5	11	15	1	10	8	9	7	16	2
5	11	4	14	3	13	6	12	16	2	9	7	10	8	15	1
7	9	2	16	1	15	8	10	14	4	11	5	12	6	13	3
16	2	9	7	10	8	15	1	5	11	4	14	3	13	6	12
14	4	11	5	12	6	13	3	7	9	2	16	1	15	8	10
13	3	12	6	11	5	14	4	8	10	1	15	2	16	7	9
15	1	10	8	9	7	16	2	6	12	3	13	4	14	5	11
4	14	5	11	6	12	3	13	9	7	16	2	15	1	10	8
2	16	7	9	8	10	1	15	11	5	14	4	13	3	12	6
1	15	8	10	7	9	2	16	12	6	13	3	14	4	11	5
3	13	6	12	5	11	4	14	10	8	15	1	16	2	9	7

B

... (19)

Example 11. Applying $16\times(A-1)+B$ in (19), we get the *bimagic square* with sums $S_{16\times16} := 2056$ and $Sb_{16\times16} := 351576$:

																2056
12	22	45	51	78	84	107	117	129	159	168	186	199	217	226	256	2056
106	120	79	81	48	50	9	23	227	253	198	220	165	187	132	158	2056
185	167	160	130	255	225	218	200	52	46	21	11	118	108	83	77	2056
219	197	254	228	157	131	188	166	82	80	119	105	24	10	49	47	2056
88	74	113	111	18	16	55	41	221	195	252	230	155	133	190	164	2056
54	44	19	13	116	110	85	75	191	161	154	136	249	231	224	194	2056
229	251	196	222	163	189	134	156	112	114	73	87	42	56	15	17	2056
135	153	162	192	193	223	232	250	14	20	43	53	76	86	109	115	2056
176	178	137	151	234	248	207	209	37	59	4	30	99	125	70	92	2056
206	212	235	245	140	150	173	179	71	89	98	128	1	31	40	58	2056
29	3	60	38	91	69	126	100	152	138	177	175	210	208	247	233	2056
127	97	90	72	57	39	32	2	246	236	211	205	180	174	149	139	2056
244	238	213	203	182	172	147	141	121	103	96	66	63	33	26	8	2056
146	144	183	169	216	202	241	239	27	5	62	36	93	67	124	102	2056
65	95	104	122	7	25	34	64	204	214	237	243	142	148	171	181	2056
35	61	6	28	101	123	68	94	170	184	143	145	240	242	201	215	2056
2056	2056	2056	2056	2056	2056	2056	2056	2056	2056	2056	2056	2056	2056	2056	2056	2056

Both magic squares of examples 10 and 11 are *bimagic squares*. The difference among these two is that in example 10 each sub-block of order 4 is a *magic square*, while in example 11 sum of 16 elements in each sub-block of order 4 are same. *A* and *B* given in both examples are *MODLS*.

• 3rd Procedure – MODLS

We shall construct *pan diagonal magic square* of order 16 again using *MODLS*. Let us consider a magic square of order 4:

2	3	1	4
1	4	2	3
4	1	3	2
3	2	4	1

A

3	4	1	2
2	1	4	3
4	3	2	1
1	2	3	4

B

23	34	11	42
12	41	24	33
44	13	32	21
31	22	43	14

AB

		34	34	34	34
	7	12	1	14	34
34	2	13	8	11	34
34	16	3	10	5	34
34	9	6	15	4	34
	34	34	34	34	34

… (20)

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Let us write the numbers 1 to 16 in following 4 intervals:

1	8	9	16	34
2	7	10	15	34
3	6	11	14	34
4	5	12	13	34

… (21)

Apply the values given in (21) over A and B in (20) and put according to AB in (20) , we get 16 compositions, such as

31

6	11	3	14
3	14	6	11
14	3	11	6
11	6	14	3

9	16	1	8
8	1	16	9
16	9	8	1
1	8	9	16

		514	514	514	514
	89	176	33	216	514
514	40	209	96	169	514
514	224	41	168	81	514
514	161	88	217	48	514
	514	514	514	514	514

42

5	12	4	13
4	13	5	12
13	4	12	5
12	5	13	4

10	15	2	7
7	2	15	10
15	10	7	2
2	7	10	15

		514	514	514	514
	74	191	50	199	514
514	55	194	79	186	514
514	207	58	183	66	514
514	178	71	202	63	514
	514	514	514	514	514

Similarly we make another 14 compositions. Putting according to AB given (20), we get a pair of *MODLS*:

7	10	2	15	6	11	3	14	8	9	1	16	5	12	4	13
2	15	7	10	3	14	6	11	1	16	8	9	4	13	5	12
15	2	10	7	14	3	11	6	16	1	9	8	13	4	12	5
10	7	15	2	11	6	14	3	9	8	16	1	12	5	13	4
8	9	1	16	5	12	4	13	7	10	2	15	6	11	3	14
1	16	8	9	4	13	5	12	2	15	7	10	3	14	6	11
16	1	9	8	13	4	12	5	15	2	10	7	14	3	11	6
9	8	16	1	12	5	13	4	10	7	15	2	11	6	14	3
5	12	4	13	8	9	1	16	6	11	3	14	7	10	2	15
4	13	5	12	1	16	8	9	3	14	6	11	2	15	7	10
13	4	12	5	16	1	9	8	14	3	11	6	15	2	10	7
12	5	13	4	9	8	16	1	11	6	14	3	10	7	15	2
6	11	3	14	7	10	2	15	5	12	4	13	8	9	1	16
3	14	6	11	2	15	7	10	4	13	5	12	1	16	8	9
14	3	11	6	15	2	10	7	13	4	12	5	16	1	9	8
11	6	14	3	10	7	15	2	12	5	13	4	9	8	16	1

A

11	14	3	6	12	13	4	5	9	16	1	8	10	15	2	7
6	3	14	11	5	4	13	12	8	1	16	9	7	2	15	10
14	11	6	3	13	12	5	4	16	9	8	1	15	10	7	2
3	6	11	14	4	5	12	13	1	8	9	16	2	7	10	15
10	15	2	7	9	16	1	8	12	13	4	5	11	14	3	6
7	2	15	10	8	1	16	9	5	4	13	12	6	3	14	11
15	10	7	2	16	9	8	1	13	12	5	4	14	11	6	3
2	7	10	15	1	8	9	16	4	5	12	13	3	6	11	14
12	13	4	5	11	14	3	6	10	15	2	7	9	16	1	8
5	4	13	12	6	3	14	11	7	2	15	10	8	1	16	9
13	12	5	4	14	11	6	3	15	10	7	2	16	9	8	1
4	5	12	13	3	6	11	14	2	7	10	15	1	8	9	16
9	16	1	8	10	15	2	7	11	14	3	6	12	13	4	5
8	1	16	9	7	2	15	10	6	3	14	11	5	4	13	12
16	9	8	1	15	10	7	2	14	11	6	3	13	12	5	4
1	8	9	16	2	7	10	15	3	6	11	14	4	5	12	13

B

…. (22)

Example 12. Applying the procedure $16\times(A-1)+B$ in (22), we get *intervally distributed pan diagonal magic square* of order 16 with each block of order 4 a *pan diagonal magic square* with sum $S_{4\times4}:=504$:

		2056	2056	2056	2056	2056	2056	2056	2056	2056	2056	2056	2056	2056	2056	2056	2056
	107	158	19	230	92	173	36	213	121	144	1	248	74	191	50	199	2056
2056	22	227	110	155	37	212	93	172	8	241	128	137	55	194	79	186	2056
2056	238	27	150	99	221	44	165	84	256	9	136	113	207	58	183	66	2056
2056	147	102	235	30	164	85	220	45	129	120	249	16	178	71	202	63	2056
2056	122	143	2	247	73	192	49	200	108	157	20	229	91	174	35	214	2056
2056	7	242	127	138	56	193	80	185	21	228	109	156	38	211	94	171	2056
2056	255	10	135	114	208	57	184	65	237	28	149	100	222	43	166	83	2056
2056	130	119	250	15	177	72	201	64	148	101	236	29	163	86	219	46	2056
2056	76	189	52	197	123	142	3	246	90	175	34	215	105	160	17	232	2056
2056	53	196	77	188	6	243	126	139	39	210	95	170	24	225	112	153	2056
2056	205	60	181	68	254	11	134	115	223	42	167	82	240	25	152	97	2056
2056	180	69	204	61	131	118	251	14	162	87	218	47	145	104	233	32	2056
2056	89	176	33	216	106	159	18	231	75	190	51	198	124	141	4	245	2056
2056	40	209	96	169	23	226	111	154	54	195	78	187	5	244	125	140	2056
2056	224	41	168	81	239	26	151	98	206	59	182	67	253	12	133	116	2056
2056	161	88	217	48	146	103	234	31	179	70	203	62	132	117	252	13	2056
	2056	2056	2056	2056	2056	2056	2056	2056	2056	2056	2056	2056	2056	2056	2056	2056	2056

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- **4th Procedure - SODLS**

Proceeding on similar lines of above procedure we shall construct *pan diagonal magic square* of order 16 based on *SODLS*.

Let us consider the a *pan diagonal magic square* of order 4 constructed based on *SODLS*:

A

2	3	1	4
1	4	2	3
4	1	3	2
3	2	4	1

B

2	1	4	3
3	4	1	2
1	2	3	4
4	3	2	1

AB

22	31	14	43
13	44	21	32
41	12	33	24
34	23	42	11

		34	34	34	34
	6	9	4	15	34
34	3	16	5	10	34
34	13	2	11	8	34
34	12	7	14	1	34
	34	34	34	34	34

…. (23)

Let us organized numbers appearing in lines of magic square (23) in four interval given below:

1	7	12	14	34
2	8	11	13	34
3	5	10	16	34
4	6	9	15	34

…. (24)

Apply values given in (24) over A and B in (23), and put according to AB in (23) , we get 16 compositions, such as

$\boxed{13}$

7	12	1	14
1	14	7	12
14	1	12	7
12	7	14	1

5	3	16	10
10	16	3	5
3	5	10	16
16	10	5	3

				514
101	179	16	218	514
10	224	99	181	514
211	5	186	112	514
192	106	213	3	514
514	514	514	514	514

$\boxed{24}$

8	11	2	13
2	13	8	11
13	2	11	8
11	8	13	2

6	4	15	9
9	15	4	6
4	6	9	15
15	9	6	4

				514
118	164	31	201	514
25	207	116	166	514
196	22	169	127	514
175	121	198	20	514
514	514	514	514	514

Similarly we make another 14 compositions. Putting according to AB given (23), we get *SODLS* and its transpose of order 16*:*

8	11	2	13	5	10	3	16	7	12	1	14	6	9	4	15
2	13	8	11	3	16	5	10	1	14	7	12	4	15	6	9
13	2	11	8	16	3	10	5	14	1	12	7	15	4	9	6
11	8	13	2	10	5	16	3	12	7	14	1	9	6	15	4
7	12	1	14	6	9	4	15	8	11	2	13	5	10	3	16
1	14	7	12	4	15	6	9	2	13	8	11	3	16	5	10
14	1	12	7	15	4	9	6	13	2	11	8	16	3	10	5
12	7	14	1	9	6	15	4	11	8	13	2	10	5	16	3
6	9	4	15	7	12	1	14	5	10	3	16	8	11	2	13
4	15	6	9	1	14	7	12	3	16	5	10	2	13	8	11
15	4	9	6	14	1	12	7	16	3	10	5	13	2	11	8
9	6	15	4	12	7	14	1	10	5	16	3	11	8	13	2
5	10	3	16	8	11	2	13	6	9	4	15	7	12	1	14
3	16	5	10	2	13	8	11	4	15	6	9	1	14	7	12
16	3	10	5	13	2	11	8	15	4	9	6	14	1	12	7
10	5	16	3	11	8	13	2	9	6	15	4	12	7	14	1

A

8	2	13	11	7	1	14	12	6	4	15	9	5	3	16	10
11	13	2	8	12	14	1	7	9	15	4	6	10	16	3	5
2	8	11	13	1	7	12	14	4	6	9	15	3	5	10	16
13	11	8	2	14	12	7	1	15	9	6	4	16	10	5	3
5	3	16	10	6	4	15	9	7	1	14	12	8	2	13	11
10	16	3	5	9	15	4	6	12	14	1	7	11	13	2	8
3	5	10	16	4	6	9	15	1	7	12	14	2	8	11	13
16	10	5	3	15	9	6	4	14	12	7	1	13	11	8	2
7	1	14	12	8	2	13	11	5	3	16	10	6	4	15	9
12	14	1	7	11	13	2	8	10	16	3	5	9	15	4	6
1	7	12	14	2	8	11	13	3	5	10	16	4	6	9	15
14	12	7	1	13	11	8	2	16	10	5	3	15	9	6	4
6	4	15	9	5	3	16	10	8	2	13	11	7	1	14	12
9	15	4	6	10	16	3	5	11	13	2	8	12	14	1	7
4	6	9	15	3	5	10	16	2	8	11	13	1	7	12	14
15	9	6	4	16	10	5	3	13	11	8	2	14	12	7	1

B

…. (25)

Example 13. Applying the procedure $16 \times (A-1) + B$ in (25), we get *intervally distributed pan diagonal magic square* of order 16 with each block of order 4 a magic square with sum $S_{4\times 4} := 504$:

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	2056	2056	2056	2056	2056	2056	2056	2056	2056	2056	2056	2056	2056	2056	2056	2056	2056
	120	162	29	203	71	145	46	252	102	180	15	217	85	131	64	234	2056
2056	27	205	114	168	44	254	65	151	9	223	100	182	58	240	83	133	2056
2056	194	24	171	125	241	39	156	78	212	6	185	111	227	53	138	96	2056
2056	173	123	200	18	158	76	247	33	191	105	214	4	144	90	229	51	2056
2056	101	179	16	218	86	132	63	233	119	161	30	204	72	146	45	251	2056
2056	10	224	99	181	57	239	84	134	28	206	113	167	43	253	66	152	2056
2056	211	5	186	112	228	54	137	95	193	23	172	126	242	40	155	77	2056
2056	192	106	213	3	143	89	230	52	174	124	199	17	157	75	248	34	2056
2056	87	129	62	236	104	178	13	219	69	147	48	250	118	164	31	201	2056
2056	60	238	81	135	11	221	98	184	42	256	67	149	25	207	116	166	2056
2056	225	55	140	94	210	8	187	109	243	37	154	80	196	22	169	127	2056
2056	142	92	231	49	189	107	216	2	160	74	245	35	175	121	198	20	2056
2056	70	148	47	249	117	163	32	202	88	130	61	235	103	177	14	220	2056
2056	41	255	68	150	26	208	115	165	59	237	82	136	12	222	97	183	2056
2056	244	38	153	79	195	21	170	128	226	56	139	93	209	7	188	110	2056
2056	159	73	246	36	176	122	197	19	141	91	232	50	190	108	215	1	2056
	2056	2056	2056	2056	2056	2056	2056	2056	2056	2056	2056	2056	2056	2056	2056	2056	2056

Even though the magic square of order 16 is *pan diagonal*, but each sub-block of order 4 is *not a pan diagonal magic square*, for example,

$\boxed{22}$

		514	446	446	514
	120	162	29	203	514
454	27	205	114	168	514
514	194	24	171	125	514
574	173	123	200	18	514
	514	514	514	514	514

$\boxed{33}$

		514	582	582	514
	69	147	48	250	514
574	42	256	67	149	514
514	243	37	154	80	514
454	160	74	245	35	514
	514	514	514	514	514

Analyzing four magic squares of order 16 given in examples 10-13, we observe that two of them are *bimagic not pan diagonal* and two of them are *pan diagonal not bimagic*. The difference among two *bimagic* is that example 10 is better than 11, as in this case each block of order 4 is also magic square. The difference between two *pan diagonal magic squares* is that example 12 is better than 13, as in this case each block of order 4 is also a *pan diagonal magic square*. Summarizing, we have two examples 10 and 12 better than 11 and 13 respectively.

2.6. Magic Square of Order 18

Two ways are presented to bring *intervally distributed magic square* of order 18. The one is based on *SODLS* and second is a block-wise composition of magic squares of order 6 and 3. In the second case, the magic square obtained is *intervally semi-distributed*.

- **1st Procedure – Block Wise**

This process is based on composition of two magic squares of order 6 and 3. Let us consider *intervally distributed magic square* order 6 with Latin square decompositions:

2	1	4	5	6	3
1	3	6	4	5	2
3	6	5	2	4	1
4	2	1	6	3	5
5	4	2	3	1	6
6	5	3	1	2	4

A

5	2	3	4	2	5
1	3	5	5	1	6
1	3	3	2	6	6
2	4	3	4	6	2
6	4	3	2	5	1
6	5	4	4	1	1

B

						111
11	2	21	28	32	17	111
1	15	35	23	25	12	111
13	33	27	8	24	6	111
20	10	3	34	18	26	111
30	22	9	14	5	31	111
36	29	16	4	7	19	111
111	111	111	111	111	111	111

…. (26)

Let us consider the a magic square of order 3:

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1	3	2
3	2	1
2	1	3

A

2	3	1
1	2	3
3	1	2

B

12	33	21
31	22	13
23	11	32

AB

			15
2	9	4	15
7	5	3	15
6	1	8	15
15	15	15	15

…. (27)

Let us write the numbers 1-18 in following 3 intervals:

1	6	7	12	13	18	57
2	5	8	11	14	17	57
3	4	9	10	15	16	57

…. (28)

Apply value given in (28) over *A* and *B* in (26) and put according to AB in (27), we make 9 compositions, such as

$\boxed{12}$

6	1	12	13	18	7
1	7	18	12	13	6
7	18	13	6	12	1
12	6	1	18	7	13
13	12	6	7	1	18
18	13	7	1	6	12

14	5	8	11	5	14
2	8	14	14	2	17
2	8	8	5	17	17
5	11	8	11	17	5
17	11	8	5	14	2
17	14	11	11	2	2

						975
104	5	206	227	311	122	975
2	116	320	212	218	107	975
110	314	224	95	215	17	975
203	101	8	317	125	221	975
233	209	98	113	14	308	975
323	230	119	11	92	200	975
975	975	975	975	975	975	975

$\boxed{11}$

6	1	12	13	18	7
1	7	18	12	13	6
7	18	13	6	12	1
12	6	1	18	7	13
13	12	6	7	1	18
18	13	7	1	6	12

13	6	7	12	6	13
1	7	13	13	1	18
1	7	7	6	18	18
6	12	7	12	18	6
18	12	7	6	13	1
18	13	12	12	1	1

						975
103	6	205	228	312	121	975
1	115	319	211	217	108	971
109	313	223	96	216	18	975
204	102	7	318	126	222	979
234	210	97	114	13	307	975
324	229	120	12	91	199	975
975	975	971	979	975	975	971

In above two examples, first one is magic square, while second is not a magic square. Similarly we make another 7 compositions. Putting according to AB given (27), we get following two grids of order 18*:*

6	1	12	13	18	7	4	3	10	15	16	9	5	2	11	14	17	8
1	7	18	12	13	6	3	9	16	10	15	4	2	8	17	11	14	5
7	18	13	6	12	1	9	16	15	4	10	3	8	17	14	5	11	2
12	6	1	18	7	13	10	4	3	16	9	15	11	5	2	17	8	14
13	12	6	7	1	18	15	10	4	9	3	16	14	11	5	8	2	17
18	13	7	1	6	12	16	15	9	3	4	10	17	14	8	2	5	11
4	3	10	15	16	9	5	2	11	14	17	8	6	1	12	13	18	7
3	9	16	10	15	4	2	8	17	11	14	5	1	7	18	12	13	6
9	16	15	4	10	3	8	17	14	5	11	2	7	18	13	6	12	1
10	4	3	16	9	15	11	5	2	17	8	14	12	6	1	18	7	13
15	10	4	9	3	16	14	11	5	8	2	17	13	12	6	7	1	18
16	15	9	3	4	10	17	14	8	2	5	11	18	13	7	1	6	12
5	2	11	14	17	8	6	1	12	13	18	7	4	3	10	15	16	9
2	8	17	11	14	5	1	7	18	12	13	6	3	9	16	10	15	4
8	17	14	5	11	2	7	18	13	6	12	1	9	16	15	4	10	3
11	5	2	17	8	14	12	6	1	18	7	13	10	4	3	16	9	15
14	11	5	8	2	17	13	12	6	7	1	18	15	10	4	9	3	16
17	14	8	2	5	11	18	13	7	1	6	12	16	15	9	3	4	10

A

14	5	8	11	5	14	15	4	9	10	4	15	13	6	7	12	6	13
2	8	14	14	2	17	3	9	15	15	3	16	1	7	13	13	1	18
2	8	8	5	17	17	3	9	9	4	16	16	1	7	7	6	18	18
5	11	8	11	17	5	4	10	9	10	16	4	6	12	7	12	18	6
17	11	8	5	14	2	16	10	9	4	15	3	18	12	7	6	13	1
17	14	11	11	2	2	16	15	10	10	3	3	18	13	12	12	1	1
13	6	7	12	6	13	14	5	8	11	5	14	15	4	9	10	4	15
1	7	13	13	1	18	2	8	14	14	2	17	3	9	15	15	3	16
1	7	7	6	18	18	2	8	8	5	17	17	3	9	9	4	16	16
6	12	7	12	18	6	5	11	8	11	17	5	4	10	9	10	16	4
18	12	7	6	13	1	17	11	8	5	14	2	16	10	9	4	15	3
18	13	12	12	1	1	17	14	11	11	2	2	16	15	10	10	3	3
15	4	9	10	4	15	13	6	7	12	6	13	14	5	8	11	5	14
3	9	15	15	3	16	1	7	13	13	1	18	2	8	14	14	2	17
3	9	9	4	16	16	1	7	7	6	18	18	2	8	8	5	17	17
4	10	9	10	16	4	6	12	7	12	18	6	5	11	8	11	17	5
16	10	9	4	15	3	18	12	7	6	13	1	17	11	8	5	14	2
16	15	10	10	3	3	18	13	12	12	1	1	17	14	11	11	2	2

B

…. (29)

We observe that the grid *A* is *non diagonalized Latin square* and grid B is formed just of numbers and is not a Latin square.

Example 14. Applying the procedure $18\times(A-1)+B$ in (29), we get *intervally semi-distributed magic square* of order 18:

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																		2925
104	**5**	**206**	**227**	**311**	**122**	69	40	171	262	274	159	85	24	187	246	294	139	2925
2	**116**	**320**	**212**	**218**	**107**	39	153	285	177	255	70	19	133	301	193	235	90	2925
110	**314**	**224**	**95**	**215**	**17**	147	279	261	58	178	52	127	295	241	78	198	36	2925
203	**101**	**8**	**317**	**125**	**221**	166	64	45	280	160	256	186	84	25	300	144	240	2925
233	**209**	**98**	**113**	**14**	**308**	268	172	63	148	51	273	252	192	79	132	31	289	2925
323	**230**	**119**	**11**	**92**	**200**	286	267	154	46	57	165	306	247	138	30	73	181	2925
67	42	169	264	276	157	**86**	**23**	**188**	**245**	**293**	**140**	105	4	207	226	310	123	2925
37	151	283	175	253	72	**20**	**134**	**302**	**194**	**236**	**89**	3	117	321	213	219	106	2925
145	277	259	60	180	54	**128**	**296**	**242**	**77**	**197**	**35**	111	315	225	94	214	16	2925
168	66	43	282	162	258	**185**	**83**	**26**	**299**	**143**	**239**	202	100	9	316	124	220	2925
270	174	61	150	49	271	**251**	**191**	**80**	**131**	**32**	**290**	232	208	99	112	15	309	2925
288	265	156	48	55	163	**305**	**248**	**137**	**29**	**74**	**182**	322	231	118	10	93	201	2925
87	22	189	244	292	141	103	6	205	228	312	121	**68**	**41**	**170**	**263**	**275**	**158**	2925
21	135	303	195	237	88	1	115	319	211	217	108	**38**	**152**	**284**	**176**	**254**	**71**	2925
129	297	243	76	196	34	109	313	223	96	216	18	**146**	**278**	**260**	**59**	**179**	**53**	2925
184	82	27	298	142	238	204	102	7	318	126	222	**167**	**65**	**44**	**281**	**161**	**257**	2925
250	190	81	130	33	291	234	210	97	114	13	307	**269**	**173**	**62**	**149**	**50**	**272**	2925
304	249	136	28	75	183	324	229	120	12	91	199	**287**	**266**	**155**	**47**	**56**	**164**	2925
2925	2925	2925	2925	2925	2925	2925	2925	2925	2925	2925	2925	2925	2925	2925	2925	2925	2925	2925

The above magic square is *intervally semi-distributed* of order 18 as there are numbers in one of the diagonal not *intervally distributed*:

2	19	20	21	22	23	24	**25**	**26**	**27**	28	29	30	31	32	33	34	35	36
5	73	74	75	**76**	**77**	**78**	79	80	81	82	83	84	85	86	87	88	89	90
8	127	128	129	130	131	132	133	134	135	136	137	138	**139**	**140**	**141**	142	143	144
11	181	182	183	184	185	186	187	188	189	**190**	**191**	**192**	193	194	195	196	197	198
14	**235**	**236**	**237**	238	239	240	241	242	243	244	245	246	247	248	249	250	251	252
17	289	290	291	292	293	294	295	296	297	298	299	300	301	302	303	**304**	**305**	**306**

Out of 9 sub-blocks of order 6 appearing in above magic square, only three of them (in bold faces) are magic squares of order 6 with sum 975.

- **2nd Procedure - SODLS**

Let us consider the grid *A* and *B*, where *A* is *SODLS* and *B* is its transpose. The construction is based on the work of Bennetta, Du and Zhang [1].

1	12	13	2	14	9	8	3	16	5	4	17	11	10	18	6	7	15
8	2	18	12	3	13	9	4	15	6	5	11	10	17	7	1	14	16
9	8	3	17	18	4	12	5	14	7	6	10	16	1	2	13	15	11
18	9	8	4	16	17	5	6	13	1	7	15	2	3	12	14	11	10
6	17	9	8	5	15	16	7	12	2	1	3	4	18	13	11	10	14
15	7	16	9	8	6	14	1	18	3	2	5	17	12	11	10	13	4
13	14	1	15	9	8	7	2	17	4	3	16	18	11	10	12	5	6
7	1	2	3	4	5	6	8	11	9	10	13	14	15	16	17	18	12
17	16	15	14	13	12	18	10	9	11	8	2	1	7	6	5	4	3
12	18	17	16	15	14	13	11	8	10	9	7	6	5	4	3	2	1
2	3	4	5	6	7	1	9	10	8	11	18	12	13	14	15	16	17
4	13	14	1	10	11	2	17	3	15	16	12	8	9	5	18	6	7
14	15	7	10	11	1	3	18	2	16	17	6	13	8	9	4	12	5
16	6	10	11	7	2	15	12	1	17	18	4	5	14	8	9	3	13
5	10	11	6	1	16	17	13	7	18	12	14	3	4	15	8	9	2
10	11	5	7	17	18	4	14	6	12	13	1	15	2	3	16	8	9
11	4	6	18	12	3	10	15	5	13	14	9	7	16	1	2	17	8
3	5	12	13	2	10	11	16	4	14	15	8	9	6	17	7	1	18

A

1	8	9	18	6	15	13	7	17	12	2	4	14	16	5	10	11	3
12	2	8	9	17	7	14	1	16	18	3	13	15	6	10	11	4	5
13	18	3	8	9	16	1	2	15	17	4	14	7	10	11	5	6	12
2	12	17	4	8	9	15	3	14	16	5	1	10	11	6	7	18	13
14	3	18	16	5	8	9	4	13	15	6	10	11	7	1	17	12	2
9	13	4	17	15	6	8	5	12	14	7	11	1	2	16	18	3	10
8	9	12	5	16	14	7	6	18	13	1	2	3	15	17	4	10	11
3	4	5	6	7	1	2	8	10	11	9	17	18	12	13	14	15	16
16	15	14	13	12	18	17	11	9	8	10	3	2	1	7	6	5	4
5	6	7	1	2	3	4	9	11	10	8	15	16	17	18	12	13	14
4	5	6	7	1	2	3	10	8	9	11	16	17	18	12	13	14	15
17	11	10	15	3	5	16	13	2	7	18	12	6	4	14	1	9	8
11	10	16	2	4	17	18	14	1	6	12	8	13	5	3	15	7	9
10	17	1	3	18	12	11	15	7	5	13	9	8	14	4	2	16	6
18	7	2	12	13	11	10	16	6	4	14	5	9	8	15	3	1	17
6	1	13	14	11	10	12	17	5	3	15	18	4	9	8	16	2	7
7	14	15	11	10	13	5	18	4	2	16	6	12	3	9	8	17	1
15	16	11	10	14	4	6	12	3	1	17	7	5	13	2	9	8	18

B

… (30)

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Example 15. Applying the procedure $18\times(A-1)+B$ in (30) we get *intervally distributed magic square* of order 18:

																		2925
1	206	225	36	240	159	139	43	287	84	56	292	194	178	311	100	119	255	2925
138	20	314	207	53	223	158	55	268	108	75	193	177	294	118	11	238	275	2925
157	144	39	296	315	70	199	74	249	125	94	176	277	10	29	221	258	192	2925
308	156	143	58	278	297	87	93	230	16	113	253	28	47	204	241	198	175	2925
104	291	162	142	77	260	279	112	211	33	6	46	65	313	217	197	174	236	2925
261	121	274	161	141	96	242	5	318	50	25	83	289	200	196	180	219	64	2925
224	243	12	257	160	140	115	24	306	67	37	272	309	195	179	202	82	101	2925
111	4	23	42	61	73	92	**134**	**190**	**155**	**171**	233	252	264	283	302	321	214	2925
304	285	266	247	228	216	323	**173**	**153**	**188**	**136**	21	2	109	97	78	59	40	2925
203	312	295	271	254	237	220	**189**	**137**	**172**	**152**	123	106	89	72	48	31	14	2925
22	41	60	79	91	110	3	**154**	**170**	**135**	**191**	322	215	234	246	265	284	303	2925
71	227	244	15	165	185	34	301	38	259	288	210	132	148	86	307	99	116	2925
245	262	124	164	184	17	54	320	19	276	300	98	229	131	147	69	205	81	2925
280	107	163	183	126	30	263	213	7	293	319	63	80	248	130	146	52	222	2925
90	169	182	102	13	281	298	232	114	310	212	239	45	62	267	129	145	35	2925
168	181	85	122	299	316	66	251	95	201	231	18	256	27	44	286	128	151	2925
187	68	105	317	208	49	167	270	76	218	250	150	120	273	9	26	305	127	2925
51	88	209	226	32	166	186	282	57	235	269	133	149	103	290	117	8	324	2925
2925	2925	2925	2925	2925	2925	2925	2925	2925	2925	2925	2925	2925	2925	2925	2925	2925	2925	2925

Similar to order 14, here also we have inner grid of order 4 as a magic square with sum 650.

2.7. Magic Square of Order 20

We shall construct magic square of order 20 using two approaches. One as combinations of magic squares of order 4 (4×5) and second as combinations of magic squares of order 5 (5×4). In both the approaches the magic squares obtained are *pan diagonal* and *intervally distributed*.

- **1st Procedure – Block Wise and SODLS**

We shall construct magic square of order 20 using *SODLS* of orders 4 and 5 as below:

2	3	1	4
1	4	2	3
4	1	3	2
3	2	4	1

A

2	1	4	3
3	4	1	2
1	2	3	4
4	3	2	1

B

22	31	14	43
13	44	21	32
41	12	33	24
34	23	42	11

AB

		34	34	34	34
	6	9	4	15	34
34	3	16	5	10	34
34	13	2	11	8	34
34	12	7	14	1	34
	34	34	34	34	34

…. (31)

and

1	2	3	4	5
4	5	1	2	3
2	3	4	5	1
5	1	2	3	4
3	4	5	1	2

A

1	4	2	5	3
2	5	3	1	4
3	1	4	2	5
4	2	5	3	1
5	3	1	4	2

B

11	24	32	45	53
42	55	13	21	34
23	31	44	52	15
54	12	25	33	41
35	43	51	14	22

AB

		65	65	65	65	65
	1	9	12	20	23	65
65	17	25	3	6	14	65
65	8	11	19	22	5	65
65	24	2	10	13	16	65
65	15	18	21	4	7	65
	65	65	65	65	65	65

…. (32)

Both the magic squares of orders 4 and 5 are *pan diagonal*. Let us divide the numbers 1 to 20 in five intervals as follows:

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1	10	11	20	42
2	9	12	19	42
3	8	13	18	42
4	7	14	17	42
5	6	15	16	42

…. (33)

Let us make 20 magic squares of order 4 according to (31) using five intervals given in (33). Put these 20 magic squares according to *AB* given in (32). For example,

23				
	9	12	2	19
	2	19	9	12
	19	2	12	9
	12	9	19	2

A

8	3	18	13
13	18	3	8
3	8	13	18
18	13	8	3

B

		802	802	802	802
	168	223	38	373	802
802	33	378	163	228	802
802	363	28	233	178	802
802	238	173	368	23	802
	802	802	802	802	802

13				
	10	11	1	20
	1	20	10	11
	20	1	11	10
	11	10	20	1

A

8	3	18	13
13	18	3	8
3	8	13	18
18	13	8	3

B

		802	802	802	802
	188	203	18	393	802
802	13	398	183	208	802
802	383	8	213	198	802
802	218	193	388	3	802
	802	802	802	802	802

Combining these 20 sub-blocks according to (32), we get following *pan diagonal SODLS* and its transpose:

10	11	1	20	9	12	2	19	8	13	3	18	7	14	4	17	6	15	5	16
1	20	10	11	2	19	9	12	3	18	8	13	4	17	7	14	5	16	6	15
20	1	11	10	19	2	12	9	18	3	13	8	17	4	14	7	16	5	15	6
11	10	20	1	12	9	19	2	13	8	18	3	14	7	17	4	15	6	16	5
7	14	4	17	6	15	5	16	10	11	1	20	9	12	2	19	8	13	3	18
4	17	7	14	5	16	6	15	1	20	10	11	2	19	9	12	3	18	8	13
17	4	14	7	16	5	15	6	20	1	11	10	19	2	12	9	18	3	13	8
14	7	17	4	15	6	16	5	11	10	20	1	12	9	19	2	13	8	18	3
9	12	2	19	8	13	3	18	7	14	4	17	6	15	5	16	10	11	1	20
2	19	9	12	3	18	8	13	4	17	7	14	5	16	6	15	1	20	10	11
19	2	12	9	18	3	13	8	17	4	14	7	16	5	15	6	20	1	11	10
12	9	19	2	13	8	18	3	14	7	17	4	15	6	16	5	11	10	20	1
6	15	5	16	10	11	1	20	9	12	2	19	8	13	3	18	7	14	4	17
5	16	6	15	1	20	10	11	2	19	9	12	3	18	8	13	4	17	7	14
16	5	15	6	20	1	11	10	19	2	12	9	18	3	13	8	17	4	14	7
15	6	16	5	11	10	20	1	12	9	19	2	13	8	18	3	14	7	17	4
8	13	3	18	7	14	4	17	6	15	5	16	10	11	1	20	9	12	2	19
3	18	8	13	4	17	7	14	5	16	6	15	1	20	10	11	2	19	9	12
18	3	13	8	17	4	14	7	16	5	15	6	20	1	11	10	19	2	12	9
13	8	18	3	14	7	17	4	15	6	16	5	11	10	20	1	12	9	19	2

A

10	1	20	11	7	4	17	14	9	2	19	12	6	5	16	15	8	3	18	13
11	20	1	10	14	17	4	7	12	19	2	9	15	16	5	6	13	18	3	8
1	10	11	20	4	7	14	17	2	9	12	19	5	6	15	16	3	8	13	18
20	11	10	1	17	14	7	4	19	12	9	2	16	15	6	5	18	13	8	3
9	2	19	12	6	5	16	15	8	3	18	13	10	1	20	11	7	4	17	14
12	19	2	9	15	16	5	6	13	18	3	8	11	20	1	10	14	17	4	7
2	9	12	19	5	6	15	16	3	8	13	18	1	10	11	20	4	7	14	17
19	12	9	2	16	15	6	5	18	13	8	3	20	11	10	1	17	14	7	4
8	3	18	13	10	1	20	11	7	4	17	14	9	2	19	12	6	5	16	15
13	18	3	8	11	20	1	10	14	17	4	7	12	19	2	9	15	16	5	6
3	8	13	18	1	10	11	20	4	7	14	17	2	9	12	19	5	6	15	16
18	13	8	3	20	11	10	1	17	14	7	4	19	12	9	2	16	15	6	5
7	4	17	14	9	2	19	12	6	5	16	15	8	3	18	13	10	1	20	11
14	17	4	7	12	19	2	9	15	16	5	6	13	18	3	8	11	20	1	10
4	7	14	17	2	9	12	19	5	6	15	16	3	8	13	18	1	10	11	20
17	14	7	4	19	12	9	2	16	15	6	5	18	13	8	3	20	11	10	1
6	5	16	15	8	3	18	13	10	1	20	11	7	4	17	14	9	2	19	12
15	16	5	6	13	18	3	8	11	20	1	10	14	17	4	7	12	19	2	9
5	6	15	16	3	8	13	18	1	10	11	20	4	7	14	17	2	9	12	19
16	15	6	5	18	13	8	3	20	11	10	1	17	14	7	4	19	12	9	2

B

…. (34)

Example 16. Applying $20\times(A-1)+B$ in (34), we get a *intervally distributed pan diagonal magic square* of order 20:

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		4010	4010	4010	4010	4010	4010	4010	4010	4010	4010	4010	4010	4010	4010	4010	4010	4010	4010	4010	4010
	190	201	20	391	167	224	37	374	149	242	59	352	126	265	76	335	108	283	98	313	4010
4010	11	400	181	210	34	377	164	227	52	359	142	249	75	336	125	266	93	318	103	288	4010
4010	381	10	211	200	364	27	234	177	342	49	252	159	325	66	275	136	303	88	293	118	4010
4010	220	191	390	1	237	174	367	24	259	152	349	42	276	135	326	65	298	113	308	83	4010
4010	129	262	79	332	106	285	96	315	188	203	18	393	170	221	40	371	147	244	57	354	4010
4010	72	339	122	269	95	316	105	286	13	398	183	208	31	380	161	230	54	357	144	247	4010
4010	322	69	272	139	305	86	295	116	383	8	213	198	361	30	231	180	344	47	254	157	4010
4010	279	132	329	62	296	115	306	85	218	193	388	3	240	171	370	21	257	154	347	44	4010
4010	168	223	38	373	150	241	60	351	127	264	77	334	109	282	99	312	186	205	16	395	4010
4010	33	378	163	228	51	360	141	250	74	337	124	267	92	319	102	289	15	396	185	206	4010
4010	363	28	233	178	341	50	251	160	324	67	274	137	302	89	292	119	385	6	215	196	4010
4010	238	173	368	23	260	151	350	41	277	134	327	64	299	112	309	82	216	195	386	5	4010
4010	107	284	97	314	189	202	19	392	166	225	36	375	148	243	58	353	130	261	80	331	4010
4010	94	317	104	287	12	399	182	209	35	376	165	226	53	358	143	248	71	340	121	270	4010
4010	304	87	294	117	382	9	212	199	365	26	235	176	343	48	253	158	321	70	271	140	4010
4010	297	114	307	84	219	192	389	2	236	175	366	25	258	153	348	43	280	131	330	61	4010
4010	146	245	56	355	128	263	78	333	110	281	100	311	187	204	17	394	169	222	39	372	4010
4010	55	356	145	246	73	338	123	268	91	320	101	290	14	397	184	207	32	379	162	229	4010
4010	345	46	255	156	323	68	273	138	301	90	291	120	384	7	214	197	362	29	232	179	4010
4010	256	155	346	45	278	133	328	63	300	111	310	81	217	194	387	4	239	172	369	22	4010
	4010	4010	4010	4010	4010	4010	4010	4010	4010	4010	4010	4010	4010	4010	4010	4010	4010	4010	4010	4010	4010

The above magic square of order 20 is *pan diagonal* with each sub-block of order 4 *perfect magic square* with sum $S_{4\times 4} := 802$. Another interesting property of above magic square is that it is *alternatively semi-bimagic square*. See below:

- **Alternatively Semi-bimagic Square of Order 20**

																				1084870
36100	40401	400	152881	27889	50176	1369	139876	22201	58564	3481	123904	15876	70225	5776	112225	11664	80089	9604	97969	1060670
121	160000	32761	44100	1156	142129	26896	51529	2704	128881	20164	62001	5625	112896	15625	70756	8649	101124	10609	82944	**1080670**
145161	100	44521	40000	132496	729	54756	31329	116964	2401	63504	25281	105625	4356	75625	18496	91809	7744	85849	13924	1060670
48400	36481	152100	1	56169	30276	134689	576	67081	23104	121801	1764	76176	18225	106276	4225	88804	12769	94864	6889	**1080670**
16641	68644	6241	110224	11236	81225	9216	99225	35344	41209	324	154449	28900	48841	1600	137641	21609	59536	3249	125316	1060670
5184	114921	14884	72361	9025	99856	11025	81796	169	158404	33489	43264	961	144400	25921	52900	2916	127449	20736	61009	**1080670**
103684	4761	73984	19321	93025	7396	87025	13456	146689	64	45369	39204	130321	900	53361	32400	118336	2209	64516	24649	1060670
77841	17424	108241	3844	87616	13225	93636	7225	47524	37249	150544	9	57600	29241	136900	441	66049	23716	120409	1936	**1080670**
28224	49729	1444	139129	22500	58081	3600	123201	16129	69696	5929	111556	11881	79524	9801	97344	34596	42025	256	156025	1060670
1089	142884	26569	51984	2601	129600	19881	62500	5476	113569	15376	71289	8464	101761	10404	83521	225	156816	34225	42436	**1080670**
131769	784	54289	31684	116281	2500	63001	25600	104976	4489	75076	18769	91204	7921	85264	14161	148225	36	46225	38416	1060670
56644	29929	135424	529	67600	22801	122500	1681	76729	17956	106929	4096	89401	12544	95481	6724	46656	38025	148996	25	**1080670**
11449	80656	9409	98596	35721	40804	361	153664	27556	50625	1296	140625	21904	59049	3364	124609	16900	68121	6400	109561	1060670
8836	100489	10816	82369	144	159201	33124	43681	1225	141376	27225	51076	2809	128164	20449	61504	5041	115600	14641	72900	**1080670**
92416	7569	86436	13689	145924	81	44944	39601	133225	676	55225	30976	117649	2304	64009	24964	103041	4900	73441	19600	1060670
88209	12996	94249	7056	47961	36864	151321	4	55696	30625	133956	625	66564	23409	121104	1849	78400	17161	108900	3721	**1080670**
21316	60025	3136	126025	16384	69169	6084	110889	12100	78961	10000	96721	34969	41616	289	155236	28561	49284	1521	138384	1060670
3025	126736	21025	60516	5329	114244	15129	71824	8281	102400	10201	84100	196	157609	33856	42849	1024	143641	26244	52441	**1080670**
119025	2116	65025	24336	104329	4624	74529	19044	90601	8100	84681	14400	147456	49	45796	38809	131044	841	53824	32041	1060670
65536	24025	119716	2025	77284	17689	107584	3969	90000	12321	96100	6561	47089	37636	149769	16	57121	29584	136161	484	**1080670**
1060670	**1080670**	1060670	**1080670**	1060670	**1080670**	1060670	**1080670**	1060670	**1080670**	1060670	**1080670**	1060670	**1080670**	1060670	**1080670**	1060670	**1080670**	1060670	**1080670**	**1097270**

It is *alternatively semi-bimagic* with sums of rows and columns bimagic sums $Sb_{20\times 20} := 1060670$ and $Sb_{20\times 20} := 1080670$. Diagonals bimagic sums are $Sd_1 := 1084870$ and $Sd_2 := 1097270$.

- **2nd Procedure – Block-Wise and SODLS**

In the previous approach we consider the compositions as 4×5 resulting magic square of order 20, where each block of order 4 is a magic square. In this approach we shall bring magic square of order 20 making the compositions as 5×4 where each block of order 5 is a magic square. We shall construct magic square of order 20 using *SODLS* of orders 5 and 4 as below:

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RGMIA Collection, **18**(2015), pp. 1-43, http://rgmia.org/v18.php

1	2	3	4	5
4	5	1	2	3
2	3	4	5	1
5	1	2	3	4
3	4	5	1	2

A

1	4	2	5	3
2	5	3	1	4
3	1	4	2	5
4	2	5	3	1
5	3	1	4	2

B

11	24	32	45	53
42	55	13	21	34
23	31	44	52	15
54	12	25	33	41
35	43	51	14	22

AB

		65	65	65	65	65
	1	9	12	20	23	65
65	17	25	3	6	14	65
65	8	11	19	22	5	65
65	24	2	10	13	16	65
65	15	18	21	4	7	65
	65	65	65	65	65	65

…. (36)

and

2	3	1	4
1	4	2	3
4	1	3	2
3	2	4	1

A

2	1	4	3
3	4	1	2
1	2	3	4
4	3	2	1

B

22	31	14	43
13	44	21	32
41	12	33	24
34	23	42	11

AB

		34	34	34	34
	6	9	4	15	34
34	3	16	5	10	34
34	13	2	11	8	34
34	12	7	14	1	34
	34	34	34	34	34

…. (37)

Both the magic squares of orders 5 and 4 are *pan diagonal*. Let us divide the numbers 1 to 20 in four intervals as follows:

1	8	9	16	17	51
2	7	10	15	18	52
3	6	11	14	19	53
4	5	12	13	20	54

…. (38)

Apply values given in (38) over *A* and *B* in (36) and put according to *AB* in (37) we get 16 compositions of order 5, such as

13

1	8	9	16	17
16	17	1	8	9
8	9	16	17	1
17	1	8	9	16
9	16	17	1	8

A

3	14	6	19	11
6	19	11	3	14
11	3	14	6	19
14	6	19	11	3
19	11	3	14	6

B

		973	973	973	973	973
	3	154	166	319	331	973
973	306	339	11	143	174	973
973	151	163	314	326	19	973
973	334	6	159	171	303	973
973	179	311	323	14	146	973
	973	973	973	973	973	973

42

4	5	12	13	20
13	20	4	5	12
5	12	13	20	4
20	4	5	12	13
12	13	20	4	5

A

2	15	7	18	10
7	18	10	2	15
10	2	15	7	18
15	7	18	10	2
18	10	2	15	7

B

		1032	1032	1032	1032	1032
	62	95	227	258	390	1032
1032	247	398	70	82	235	1032
1032	90	222	255	387	78	1032
1032	395	67	98	230	242	1032
1032	238	250	382	75	87	1032
	1032	1032	1032	1032	1032	1032

Proceeding on similar lines, we have following *pan diagonal SODLS* and it transpose:

2	7	10	15	18	3	6	11	14	19	1	8	9	16	17	4	5	12	13	20
15	18	2	7	10	14	19	3	6	11	16	17	1	8	9	13	20	4	5	12
7	10	15	18	2	6	11	14	19	3	8	9	16	17	1	5	12	13	20	4
18	2	7	10	15	19	3	6	11	14	17	1	8	9	16	20	4	5	12	13
10	15	18	2	7	11	14	19	3	6	9	16	17	1	8	12	13	20	4	5
1	8	9	16	17	4	5	12	13	20	2	7	10	15	18	3	6	11	14	19
16	17	1	8	9	13	20	4	5	12	15	18	2	7	10	14	19	3	6	11
8	9	16	17	1	5	12	13	20	4	7	10	15	18	2	6	11	14	19	3
17	1	8	9	16	20	4	5	12	13	18	2	7	10	15	19	3	6	11	14
9	16	17	1	8	12	13	20	4	5	10	15	18	2	7	11	14	19	3	6
4	5	12	13	20	1	8	9	16	17	3	6	11	14	19	2	7	10	15	18
13	20	4	5	12	16	17	1	8	9	14	19	3	6	11	15	18	2	7	10
5	12	13	20	4	8	9	16	17	1	6	11	14	19	3	7	10	15	18	2
20	4	5	12	13	17	1	8	9	16	19	3	6	11	14	18	2	7	10	15
12	13	20	4	5	9	16	17	1	8	11	14	19	3	6	10	15	18	2	7
3	6	11	14	19	2	7	10	15	18	4	5	12	13	20	1	8	9	16	17
14	19	3	6	11	15	18	2	7	10	13	20	4	5	12	16	17	1	8	9
6	11	14	19	3	7	10	15	18	2	5	12	13	20	4	8	9	16	17	1
19	3	6	11	14	18	2	7	10	15	20	4	5	12	13	17	1	8	9	16
11	14	19	3	6	10	15	18	2	7	12	13	20	4	5	9	16	17	1	8

A

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2	15	7	18	10	1	16	8	17	9	4	13	5	20	12	3	14	6	19	11
7	18	10	2	15	8	17	9	1	16	5	20	12	4	13	6	19	11	3	14
10	2	15	7	18	9	1	16	8	17	12	4	13	5	20	11	3	14	6	19
15	7	18	10	2	16	8	17	9	1	13	5	20	12	4	14	6	19	11	3
18	10	2	15	7	17	9	1	16	8	20	12	4	13	5	19	11	3	14	6
3	14	6	19	11	4	13	5	20	12	1	16	8	17	9	2	15	7	18	10
6	19	11	3	14	5	20	12	4	13	8	17	9	1	16	7	18	10	2	15
11	3	14	6	19	12	4	13	5	20	9	1	16	8	17	10	2	15	7	18
14	6	19	11	3	13	5	20	12	4	16	8	17	9	1	15	7	18	10	2
19	11	3	14	6	20	12	4	13	5	17	9	1	16	8	18	10	2	15	7
1	16	8	17	9	2	15	7	18	10	3	14	6	19	11	4	13	5	20	12
8	17	9	1	16	7	18	10	2	15	6	19	11	3	14	5	20	12	4	13
9	1	16	8	17	10	2	15	7	18	11	3	14	6	19	12	4	13	5	20
16	8	17	9	1	15	7	18	10	2	14	6	19	11	3	13	5	20	12	4
17	9	1	16	8	18	10	2	15	7	19	11	3	14	6	20	12	4	13	5
4	13	5	20	12	3	14	6	19	11	2	15	7	18	10	1	16	8	17	9
5	20	12	4	13	6	19	11	3	14	7	18	10	2	15	8	17	9	1	16
12	4	13	5	20	11	3	14	6	19	10	2	15	7	18	9	1	16	8	17
13	5	20	12	4	14	6	19	11	3	15	7	18	10	2	16	8	17	9	1
20	12	4	13	5	19	11	3	14	6	18	10	2	15	7	17	9	1	16	8

B

…. (39)

Example 17. Applying $20\times(A-1)+B$ over A and B given in (39), we get a *intervally distributed pan diagonal magic square* of order 20:

		4010	4010	4010	4010	4010	4010	4010	4010	4010	4010	4010	4010	4010	4010	4010	4010	4010	4010	4010	4010
	22	135	187	298	350	41	116	208	277	369	4	153	165	320	332	63	94	226	259	391	4010
4010	287	358	30	122	195	268	377	49	101	216	305	340	12	144	173	246	399	71	83	234	4010
4010	130	182	295	347	38	109	201	276	368	57	152	164	313	325	20	91	223	254	386	79	4010
4010	355	27	138	190	282	376	48	117	209	261	333	5	160	172	304	394	66	99	231	243	4010
4010	198	290	342	35	127	217	269	361	56	108	180	312	324	13	145	239	251	383	74	86	4010
4010	3	154	166	319	331	64	93	225	260	392	21	136	188	297	349	42	115	207	278	370	4010
4010	306	339	11	143	174	245	400	72	84	233	288	357	29	121	196	267	378	50	102	215	4010
4010	151	163	314	326	19	92	224	253	385	80	129	181	296	348	37	110	202	275	367	58	4010
4010	334	6	159	171	303	393	65	100	232	244	356	28	137	189	281	375	47	118	210	262	4010
4010	179	311	323	14	146	240	252	384	73	85	197	289	341	36	128	218	270	362	55	107	4010
4010	61	96	228	257	389	2	155	167	318	330	43	114	206	279	371	24	133	185	300	352	4010
4010	248	397	69	81	236	307	338	10	142	175	266	379	51	103	214	285	360	32	124	193	4010
4010	89	221	256	388	77	150	162	315	327	18	111	203	274	366	59	132	184	293	345	40	4010
4010	396	68	97	229	241	335	7	158	170	302	374	46	119	211	263	353	25	140	192	284	4010
4010	237	249	381	76	88	178	310	322	15	147	219	271	363	54	106	200	292	344	33	125	4010
4010	44	113	205	280	372	23	134	186	299	351	62	95	227	258	390	1	156	168	317	329	4010
4010	265	380	52	104	213	286	359	31	123	194	247	398	70	82	235	308	337	9	141	176	4010
4010	112	204	273	365	60	131	183	294	346	39	90	222	255	387	78	149	161	316	328	17	4010
4010	373	45	120	212	264	354	26	139	191	283	395	67	98	230	242	336	8	157	169	301	4010
4010	220	272	364	53	105	199	291	343	34	126	238	250	382	75	87	177	309	321	16	148	4010
	4010	4010	4010	4010	4010	4010	4010	4010	4010	4010	4010	4010	4010	4010	4010	4010	4010	4010	4010	4010	4010

The above magic square of order 20 is *pan diagonal* with each sub-block of order 5 *pan diagonal* magic squares with different sums. The magic sums of each block of order 5 forms again a *pan diagonal perfect magic square* of order 4:

		4010	4010	4010	4010
	992	1011	974	1033	4010
4010	973	1034	991	1012	4010
4010	1031	972	1013	994	4010
4010	1014	993	1032	971	4010
	4010	4010	4010	4010	4010

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3. Palindromic Magic Squares

Palindromic numbers. *Numbers that read the same backwards and forwards, i.e., remains the same when its digits are reversed are famous as **palindromic numbers**, for example, 121, 3883, 15951, etc.* Table below give the quantity of *palindromes* for each number of digit:

Digits	Palindromes	Quantity
1-digit	*a*	9
2-digits	*aa*	9
3-digits	*aba*	90
4-digits	*abba*	90
5-digits	*abcba*	900
6-digits	*abccba*	900
7-digits	*abcdcba*	9000
8-digits	*abcddcba*	9000
…..	…..	…..

From table above, we observe that there are only 90 palindromes of 3-digits, while 5-digits are 900. In this work we shall use *5-digits palindromes* to bring magic squares. Since, we have 900 palindromes of 5-digits, it means that we can write *palindromic magic squares* up to order 30. This work is only up to order 20. *Palindromic magic squares* of orders 21 to 30 are in next work [16]. According to examples 3, 7 and 13, the magic squares of order 12, 15 and 18 are constructed on same procedure i.e., 4×3, 5×3 and 6×3. In these cases we have used magic squares of order 4, 5 and 6 and then making combinations with magic square of order 3. Magic square of order 16 is constructed applying composition 4×4. Also we have given *palindromic magic squares* constructed based on *MODLS* and/or *SODLS*.

3.1. Palindromic Magic Squares of Prime Order

In section 2, we have given in only one example of prime order magic square. Others are constructed based on same procedure. But here below we have given *palindromic magic square* of prime orders 11, 13, 17 and 19. In all the cases the magic square is pan diagonal. In each case we have given only one example, but we can have much more examples.

- ### Palindromic Magic Squares of Order 11

Example 18. Below is a *pan diagonal palindromic magic square* of order 11.

		225928	225928	225928	225928	225928	225928	225928	225928	225928	225928	225928
	10001	12921	14741	16561	18381	20102	23032	24842	26662	28482	30203	225928
225928	28182	31013	10801	12621	14441	16261	18081	20902	22722	24542	26362	225928
225928	24242	26062	28982	30703	10501	12321	14141	17071	18881	20602	22422	225928
225928	20302	22122	25052	26862	28682	30403	10201	12021	14941	16761	18581	225928
225928	16461	18281	20002	22922	24742	26562	28382	30103	11011	12821	14641	225928
225928	12521	14341	16161	19091	20802	22622	24442	26262	28082	30903	10701	225928
225928	30603	10401	12221	14041	16961	18781	20502	22322	24142	27072	28882	225928
225928	26762	28582	30303	10101	13031	14841	16661	18481	20202	22022	24942	225928
225928	22822	24642	26462	28282	30003	10901	12721	14541	16361	18181	21012	225928
225928	18981	20702	22522	24342	26162	29092	30803	10601	12421	14241	16061	225928
225928	15051	16861	18681	20402	22222	24042	26962	28782	30503	10301	12121	225928
	225928	225928	225928	225928	225928	225928	225928	225928	225928	225928	225928	225928

- ### Palindromic Magic Squares of Order 13

Example 19. Below is a *pan diagonal palindromic magic square* of order 13.

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		294314	294314	294314	294314	294314	294314	294314	294314	294314	294314	294314	294314	294314
	10001	13131	14941	16761	18581	20302	22122	25252	27072	28882	30603	32423	34243	294314
294314	32123	35253	11011	12821	14641	16461	18281	20002	23132	24942	26762	28582	30303	294314
294314	28282	30003	33133	34943	10701	12521	14341	16161	19291	21012	22822	24642	26462	294314
294314	24342	26162	29292	31013	32823	34643	10401	12221	14041	17171	18981	20702	22522	294314
294314	20402	22222	24042	27172	28982	30703	32523	34343	10101	13231	15051	16861	18681	294314
294314	16561	18381	20102	23232	25052	26862	28682	30403	32223	34043	11111	12921	14741	294314
294314	12621	14441	16261	18081	21112	22922	24742	26562	28382	30103	33233	35053	10801	294314
294314	34743	10501	12321	14141	17271	19091	20802	22622	24442	26262	28082	31113	32923	294314
294314	30803	32623	34443	10201	12021	15151	16961	18781	20502	22322	24142	27272	29092	294314
294314	26962	28782	30503	32323	34143	11211	13031	14841	16661	18481	20202	22022	25152	294314
294314	23032	24842	26662	28482	30203	32023	35153	10901	12721	14541	16361	18181	21212	294314
294314	19191	20902	22722	24542	26362	28182	31213	33033	34843	10601	12421	14241	16061	294314
294314	15251	17071	18881	20602	22422	24242	26062	29192	30903	32723	34543	10301	12121	294314
	294314	294314	294314	294314	294314	294314	294314	294314	294314	294314	294314	294314	294314	294314

- **Palindromic Magic Squares of Order 17**

Example 20. Below is a *pan diagonal palindromic magic square* of order 17.

		456328	456328	456328	456328	456328	456328	456328	456328	456328	456328	456328	456328	456328	456328	456328	456328	456328
	10001	13531	15351	17171	18981	20702	22522	24342	26162	29692	31413	33233	35053	36863	38683	40404	42224	456328
456328	40104	43634	11411	13231	15051	16861	18681	20402	22222	24042	27572	29392	31113	32923	34743	36563	38383	456328
456328	36263	38083	41514	43334	11111	12921	14741	16561	18381	20102	23632	25452	27272	29092	30803	32623	34443	456328
456328	32323	34143	37673	39493	41214	43034	10801	12621	14441	16261	18081	21512	23332	25152	26962	28782	30503	456328
456328	28482	30203	32023	35553	37373	39193	40904	42724	10501	12321	14141	17671	19491	21212	23032	24842	26662	456328
456328	24542	26362	28182	31613	33433	35253	37073	38883	40604	42424	10201	12021	15551	17371	19191	20902	22722	456328
456328	20602	22422	24242	26062	29592	31313	33133	34943	36763	38583	40304	42124	11611	13431	15251	17071	18881	456328
456328	16761	18581	20302	22122	25652	27472	29292	31013	32823	34643	36463	38283	40004	43534	11311	13131	14941	456328
456328	12821	14641	16461	18281	20002	23532	25352	27172	28982	30703	32523	34343	36163	39693	41414	43234	11011	456328
456328	42924	10701	12521	14341	16161	19691	21412	23232	25052	26862	28682	30403	32223	34043	37573	39393	41114	456328
456328	39093	40804	42624	10401	12221	14041	17571	19391	21112	22922	24742	26562	28382	30103	33633	35453	37273	456328
456328	35153	36963	38783	40504	42324	10101	13631	15451	17271	19091	20802	22622	24442	26262	28082	31513	33333	456328
456328	31213	33033	34843	36663	38483	40204	42024	11511	13331	15151	16961	18781	20502	22322	24142	27672	29492	456328
456328	27372	29192	30903	32723	34543	36363	38183	41614	43434	11211	13031	14841	16661	18481	20202	22022	25552	456328
456328	23432	25252	27072	28882	30603	32423	34243	36063	39593	41314	43134	10901	12721	14541	16361	18181	21612	456328
456328	19591	21312	23132	24942	26762	28582	30303	32123	35653	37473	39293	41014	42824	10601	12421	14241	16061	456328
456328	15651	17471	19291	21012	22822	24642	26462	28282	30003	33533	35353	37173	38983	40704	42524	10301	12121	456328
	456328	456328	456328	456328	456328	456328	456328	456328	456328	456328	456328	456328	456328	456328	456328	456328	456328	456328

- **Palindromic Magic Squares of Order 19**

Example 21. Below is a *pan diagonal palindromic magic square* of order 19.

		549956	549956	549956	549956	549956	549956	549956	549956	549956	549956	549956	549956	549956	549956	549956	549956	549956	549956	549956
	10001	13731	15551	17371	19191	20902	22722	24542	26362	28182	31813	33633	35453	37273	39093	40804	42624	44444	46264	549956
549956	44144	47874	11611	13431	15251	17071	18881	20602	22422	24242	26062	29792	31513	33333	35153	36963	38783	40504	42324	549956
549956	40204	42024	45754	47574	11311	13131	14941	16761	18581	20302	22122	25852	27672	29492	31213	33033	34843	36663	38483	549956
549956	36363	38183	41814	43634	45454	47274	11011	12821	14641	16461	18281	20002	23732	25552	27372	29192	30903	32723	34543	549956
549956	32423	34243	36063	39793	41514	43334	45154	46964	10701	12521	14341	16161	19891	21612	23432	25252	27072	28882	30603	549956
549956	28582	30303	32123	35853	37673	39493	41214	43034	44844	46664	10401	12221	14041	17771	19591	21312	23132	24942	26762	549956
549956	24642	26462	28282	30003	33733	35553	37373	39193	40904	42724	44544	46364	10101	13831	15651	17471	19291	21012	22822	549956
549956	20702	22522	24342	26162	29892	31613	33433	35253	37073	38883	40604	42424	44244	46064	11711	13531	15351	17171	18981	549956
549956	16861	18681	20402	22222	24042	27772	29592	31313	33133	34943	36763	38583	40304	42124	45854	47674	11411	13231	15051	549956
549956	12921	14741	16561	18381	20102	23832	25652	27472	29292	31013	32823	34643	36463	38283	40004	43734	45554	47374	11111	549956
549956	47074	10801	12621	14441	16261	18081	21712	23532	25352	27172	28982	30703	32523	34343	36163	39893	41614	43434	45254	549956
549956	43134	44944	46764	10501	12321	14141	17871	19691	21412	23232	25052	26862	28682	30403	32223	34043	37773	39593	41314	549956
549956	39293	41014	42824	44644	46464	10201	12021	15751	17571	19391	21112	22922	24742	26562	28382	30103	33833	35653	37473	549956
549956	35353	37173	38983	40704	42524	44344	46164	11811	13631	15451	17271	19091	20802	22622	24442	26262	28082	31713	33533	549956
549956	31413	33233	35053	36863	38683	40404	42224	44044	47774	11511	13331	15151	16961	18781	20502	22322	24142	27872	29692	549956
549956	27572	29392	31113	32923	34743	36563	38383	40104	43834	45654	47474	11211	13031	14841	16661	18481	20202	22022	25752	549956
549956	23632	25452	27272	29092	30803	32623	34443	36263	38083	41714	43534	45354	47174	10901	12721	14541	16361	18181	21812	549956
549956	19791	21512	23332	25152	26962	28782	30503	32323	34143	37873	39693	41414	43234	45054	46864	10601	12421	14241	16061	549956
549956	15851	17671	19491	21212	23032	24842	26662	28482	30203	32023	35753	37573	39393	41114	42924	44744	46564	10301	12121	549956
	549956	549956	549956	549956	549956	549956	549956	549956	549956	549956	549956	549956	549956	549956	549956	549956	549956	549956	549956	549956

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3.2. Palindromic Magic Square of Order 12

Below are two *palindromic magic squares* of order 12 having different palindromes. First one is based on a composition 4×3, where each block of order 4 a magic square.

Example 22. The example below give palindromic magic square of order 12 with magic sum $S_{12\times12} := 336650$, where all the nine grids of order 4 are *pan diagonal magic squares* with sum $S_{4\times4} := 115550$:

												346650
26262	32523	11211	45554	22322	36463	15351	41414	24142	34643	13131	43634	346650
11511	45254	26562	32223	15451	41314	22422	36363	13631	43134	24642	34143	346650
46564	12221	31513	25252	42424	16361	35453	21312	44644	14141	33633	23132	346650
31213	25552	46264	12521	35353	21412	42324	16461	33133	23632	44144	14641	346650
22122	36663	15151	41614	24242	34543	13231	43534	26362	32423	11311	45454	346650
15651	41114	22622	36163	13531	43234	24542	34243	11411	45354	26462	32323	346650
42624	16161	35653	21112	44544	14241	33533	23232	46464	12321	31413	25352	346650
35153	21612	42124	16661	33233	23532	44244	14541	31313	25452	46364	12421	346650
24342	34443	13331	43434	26162	32623	11111	45654	22222	36563	15251	41514	346650
13431	43334	24442	34343	11611	45154	26662	32123	15551	41214	22522	36263	346650
44444	14341	33433	23332	46664	12121	31613	25152	42524	16261	35553	21212	346650
33333	23432	44344	14441	31113	25652	46164	12621	35253	21512	42224	16561	346650
346650	346650	346650	346650	346650	346650	346650	346650	346650	346650	346650	346650	346650

Below is a magic square of order 12 with same sum based on *MODLS*:

Example 23. *Palindromic magic square* of order 12 based on *MODLS* is given by

												346650
11111	46264	33233	16561	31613	14341	21512	26662	42124	23332	44444	35453	346650
25652	44144	31113	14441	45554	12221	15451	24542	36663	21212	42324	33333	346650
23532	42624	45654	12321	43434	26162	13331	22422	34543	15151	36263	31213	346650
21412	36563	43534	26262	41314	24642	11211	16361	32423	13631	34143	45154	346650
15351	34443	41414	24142	35253	22522	25152	14241	46364	11511	32623	43634	346650
13231	32323	35353	22622	33133	16461	23632	12121	44244	25452	46564	41514	346650
32123	13431	22222	35153	16661	33533	42524	45254	25352	44344	11611	24442	346650
34243	15551	24342	41214	22122	35653	44644	31313	11411	46464	13131	26562	346650
36363	21612	26462	43334	24242	41114	46164	33433	13531	32523	15251	12621	346650
42424	23132	12521	45454	26362	43234	32223	35553	15651	34643	21312	14141	346650
44544	25252	14641	31513	12421	45354	34343	41614	21112	36163	23432	16261	346650
46664	11311	16161	33633	14541	31413	36463	43134	23232	42224	25552	22322	346650
346650	346650	346650	346650	346650	346650	346650	346650	346650	346650	346650	346650	346650

Example 22 is based on the example 3 and example 23 is based on example 4

3.3. Palindromic Magic Square of Order 14

Below is a *palindromic magic square* of order 14 based on *MODLS* given in example 5.

Example 24. *Palindromic magic square* of order 14 based on *MODLS* is given by

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														601006
11111	42424	74247	44744	54545	34143	31213	21412	13331	63536	61616	72327	52625	23732	601006
61716	13231	44544	72127	73637	53135	41314	51515	11411	24442	32723	34643	22322	64246	601006
52725	64146	21312	54445	71417	31713	34343	11611	23532	14541	42624	43134	62226	73237	601006
74147	44644	62326	23432	63136	51315	11511	33733	21612	71217	14241	42524	54745	32423	601006
64346	34743	24642	12521	52225	41214	21112	43334	53735	31613	71517	14441	73437	62126	601006
43634	12321	41514	22222	13731	**64546**	**72427**	**62726**	**74647**	33433	23232	54145	31313	51115	601006
13531	21712	53635	31113	11311	**62626**	**74747**	**64446**	**72527**	42124	52325	24242	33233	43434	601006
23332	32223	33133	41614	22122	**74447**	**62526**	**72627**	**64746**	51715	53435	11211	43534	14341	601006
21212	23132	31413	33333	42324	**72727**	**64646**	**74547**	**62426**	44244	11711	51615	14141	53535	601006
32523	61416	63236	14641	33533	23632	43734	42224	52125	54345	73137	22722	12421	71317	601006
34443	73537	71717	51215	32623	13431	54245	24342	41114	12721	44144	61316	63636	22522	601006
54645	71617	12121	63736	61216	44344	51415	13131	34243	73337	22422	31513	24542	42724	601006
72227	53335	52425	74347	24742	21512	13631	32123	43234	61116	34543	63436	41714	12621	601006
41414	52525	14741	61516	44444	12221	24142	53235	32323	22622	63336	73737	71117	33633	601006
601006	601006	601006	601006	601006	601006	601006	601006	601006	601006	601006	601006	601006	601006	601006

Inner grid is a magic square of order 4 with magic sum 274346.

3.4. Palindromic Magic Square of Order 15

This subsection deals with *palindromic magic square* of order 15 constructed based on composition of magic square of orders 5 and 3 given in example 7. The second procedure is based on *SODLS*.

Example 25. *Palindromic magic square* of order 15 is given by

															800025
41114	42224	43334	44444	45554	91119	92229	93339	94449	95559	21112	22222	23332	24442	25552	800025
44344	45454	41514	42124	43234	94349	95459	91519	92129	93239	24342	25452	21512	22122	23232	800025
42524	43134	44244	45354	41414	92529	93139	94249	95359	91419	22522	23132	24242	25352	21412	800025
45254	41314	42424	43534	44144	95259	91319	92429	93539	94149	25252	21312	22422	23532	24142	800025
43434	44544	45154	41214	42324	93439	94549	95159	91219	92329	23432	24542	25152	21212	22322	800025
31113	32223	33333	34443	35553	51115	52225	53335	54445	55555	71117	72227	73337	74447	75557	800025
34343	35453	31513	32123	33233	54345	55455	51515	52125	53235	74347	75457	71517	72127	73237	800025
32523	33133	34243	35353	31413	52525	53135	54245	55355	51415	72527	73137	74247	75357	71417	800025
35253	31313	32423	33533	34143	55255	51315	52425	53535	54145	75257	71317	72427	73537	74147	800025
33433	34543	35153	31213	32323	53435	54545	55155	51215	52325	73437	74547	75157	71217	72327	800025
81118	82228	83338	84448	85558	11111	12221	13331	14441	15551	61116	62226	63336	64446	65556	800025
84348	85458	81518	82128	83238	14341	15451	11511	12121	13231	64346	65456	61516	62126	63236	800025
82528	83138	84248	85358	81418	12521	13131	14241	15351	11411	62526	63136	64246	65356	61416	800025
85258	81318	82428	83538	84148	15251	11311	12421	13531	14141	65256	61316	62426	63536	64146	800025
83438	84548	85158	81218	82328	13431	14541	15151	11211	12321	63436	64546	65156	61216	62326	800025
800025	800025	800025	800025	800025	800025	800025	800025	800025	800025	800025	800025	800025	800025	800025	800025

Each block of order 5 is a magic square with different sums given by magic square of order 3:

			800025
216670	466695	116660	800025
166665	266675	366685	800025
416690	66655	316680	800025
800025	800025	800025	800025

Palindromic magic square of order 15 of above example is a simplified way where we have used *palindromic magic square* of order 5 with 3-digits and then putting necessary number in front and back according to magic square of order 3. Below is another example having the same idea but with different palindromes.

Example 26. *Palindromic magic square* of order 15 is given by

[1]Formerly, Professor of Mathematics, Federal University of Santa Catarina, 88040-400 Florianópolis, SC, Brazil. Email: ijtaneja@gmail.com

															502995
11411	22422	33433	44444	55455	11911	22922	33933	44944	55955	11211	22222	33233	44244	55255	502995
43434	54445	15451	21412	32423	43934	54945	15951	21912	32923	43234	54245	15251	21212	32223	502995
25452	31413	42424	53435	14441	25952	31913	42924	53935	14941	25252	31213	42224	53235	14241	502995
52425	13431	24442	35453	41414	52925	13931	24942	35953	41914	52225	13231	24242	35253	41214	502995
34443	45454	51415	12421	23432	34943	45954	51915	12921	23932	34243	45254	51215	12221	23232	502995
11311	22322	33333	44344	55355	11511	22522	33533	44544	55555	11711	22722	33733	44744	55755	502995
43334	54345	15351	21312	32323	43534	54545	15551	21512	32523	43734	54745	15751	21712	32723	502995
25352	31313	42324	53335	14341	25552	31513	42524	53535	14541	25752	31713	42724	53735	14741	502995
52325	13331	24342	35353	41314	52525	13531	24542	35553	41514	52725	13731	24742	35753	41714	502995
34343	45354	51315	12321	23332	34543	45554	51515	12521	23532	34743	45754	51715	12721	23732	502995
11811	22822	33833	44844	55855	11111	22122	33133	44144	55155	11611	22622	33633	44644	55655	502995
43834	54845	15851	21812	32823	43134	54145	15151	21112	32123	43634	54645	15651	21612	32623	502995
25852	31813	42824	53835	14841	25152	31113	42124	53135	14141	25652	31613	42624	53635	14641	502995
52825	13831	24842	35853	41814	52125	13131	24142	35153	41114	52625	13631	24642	35653	41614	502995
34843	45854	51815	12821	23832	34143	45154	51115	12121	23132	34643	45654	51615	12621	23632	502995
502995	502995	502995	502995	502995	502995	502995	502995	502995	502995	502995	502995	502995	502995	502995	502995

Each block of order 5 is a magic square with different sums given by magic square of order 3:

			502995
167165	169665	166165	502995
166665	167665	168665	502995
169165	165665	168165	502995
502995	502995	502995	502995

Below an example of *palindromic magic square* constructed according to *MODLS* given in example 8.

Example 27. *Palindromic magic square* of order 15 is given by

															371180
10701	12621	14241	16361	18581	20002	23232	35453	27072	28482	30103	33333	24842	36963	39193	371180
20102	23332	25452	26962	29192	30703	32623	14841	36363	38583	10001	13231	34243	17071	18481	371180
30003	33233	34843	37073	38483	10101	13331	24242	16961	19191	20702	22622	15451	26362	28582	371180
12421	14141	16661	19491	10301	22522	24042	27272	38283	21012	33133	34743	37373	28882	30903	371180
23132	24742	27372	38883	20902	32423	34143	36663	29492	30303	12521	14041	17271	18281	11011	371180
32523	34043	37273	28282	31013	13131	14741	17371	18881	10901	22422	24142	26662	39493	20302	371180
14341	26462	18181	11211	23432	35053	16561	38083	20602	12221	24942	37173	28782	31313	32823	371180
34943	36563	38783	30603	12821	24342	27172	28182	11311	33433	15051	16461	18081	21212	22222	371180
25052	17171	28082	21312	32223	14941	36463	18781	31213	22822	34343	26562	38183	10601	13431	371180
37473	18381	10501	22122	35253	26262	39093	20402	12021	24642	16861	28982	31113	32723	15351	371180
26862	38983	21112	12721	25352	17471	28382	30503	32123	15251	36263	19091	10401	22022	34643	371180
16261	29092	30403	32023	14641	36863	18981	11111	22722	35353	27472	38383	20502	12121	25252	371180
28682	11411	22322	14541	16061	39293	30203	12921	34443	26162	19391	20802	33033	25152	36763	371180
39393	30803	13031	35153	26762	18681	21412	32323	24542	36063	29292	10201	22922	14441	16161	371180
19291	20202	32923	24442	36163	29392	10801	23032	15151	16761	38683	31413	12321	34543	26062	371180
371180	371180	371180	371180	371180	371180	371180	371180	371180	371180	371180	371180	371180	371180	371180	371180

Each group of order 3×5 has the same sum as of magic square 371180. Using the same distribution of elements as of example 27, below an example of *palindromic magic square* constructed according to *SODLS* given in example 9.

Example 28. *Palindromic magic square* of order 15 is given by

[1]Formerly, Professor of Mathematics, Federal University of Santa Catarina, 88040-400 Florianópolis, SC, Brazil. Email: ijtaneja@gmail.com

															371180
10001	19191	36563	21212	15051	26162	25452	28882	34343	16661	22222	38983	33333	12721	30403	371180
32423	12121	21012	34643	23132	16961	10201	26062	19491	38383	36863	31213	14741	28582	25352	371180
21312	30503	14241	22922	33433	39093	18881	10101	36463	34043	29192	16761	26662	25252	12321	371180
34543	23232	28682	16361	38883	31313	36963	12221	32123	27072	18781	11411	25152	14441	20002	371180
30203	32623	39193	27472	18481	36063	29292	14341	10901	20702	13331	25052	16561	22122	34843	371180
12821	28382	31413	37073	11311	20502	34143	16461	22722	15251	24942	18681	38283	32023	27172	371180
38783	14041	26462	29392	34943	13231	22622	18581	17171	24842	21412	36363	30103	11011	32223	371180
26962	10801	12021	14141	16261	18381	20402	24742	29092	31113	33233	35353	37473	38683	22522	371180
17271	38483	19391	13131	28082	24642	32323	30903	26862	36263	14541	20102	23032	35453	10701	371180
22322	17471	11211	30803	24542	34243	26762	33033	15351	28982	38183	12421	18081	21112	36663	371180
14641	27372	32923	24442	36163	28782	38583	35153	20202	13431	31013	22022	10301	16861	19291	371180
29492	35053	24342	38083	30703	22422	17371	37273	12521	18181	10601	33133	20802	26262	14941	371180
37173	24242	22822	32723	20302	15451	13031	39393	30603	10401	16061	26562	35253	18981	28182	371180
24142	20902	34743	18281	12621	11111	30003	23432	39293	32523	26362	14841	28482	37373	17071	371180
19091	36763	16161	10501	27272	32823	15151	20602	24042	23332	34443	28282	12921	30303	39493	371180
371180	371180	371180	371180	371180	371180	371180	371180	371180	371180	371180	371180	371180	371180	371180	371180

3.5. Palindromic Magic and Bimagic Squares of Order 16

This subsection deals with different kinds of palindromic magic squares of order 16. We have given two different forms. One is bimagic and other is just magic square but pan diagonal.

Example 29. *Palindromic bimagic square* of order 16 is given by

																479960
11111	34243	46764	27872	13731	32823	48184	25252	16461	37373	41614	24542	18681	35553	43434	22322	479960
45854	28782	12221	33133	47274	26162	14841	31713	42524	23632	15351	38483	44344	21412	17571	36663	479960
28282	45154	33833	12721	26862	47774	31213	14141	23332	42424	38583	15651	21512	44644	36363	17471	479960
34743	11811	27172	46264	32123	13231	25752	48884	37673	16561	24442	41314	35453	18381	22622	43534	479960
16661	37573	41414	24342	18481	35353	43634	22522	11711	34843	46164	27272	13131	32223	48784	25852	479960
42324	23432	15551	38683	44544	21612	17371	36463	45254	28182	12821	33733	47874	26762	14241	31113	479960
23532	42624	38383	15451	21312	44444	36563	17671	28882	45754	33233	12121	26262	47174	31813	14741	479960
37473	16361	24642	41514	35653	18581	22422	43334	34143	11211	27772	46864	32723	13831	25152	48284	479960
17771	36863	44144	21212	15151	38283	42724	23832	14641	31513	47474	26362	12421	33333	45654	28582	479960
43234	22122	18881	35753	41814	24742	16261	37173	48384	25452	13531	32623	46564	27672	11311	34443	479960
22822	43734	35253	18181	24242	41114	37873	16761	25552	48684	32323	13431	27372	46464	34543	11611	479960
36163	17271	21712	44844	38783	15851	23132	42224	31413	14341	26662	47574	33633	12521	28482	45354	479960
14441	31313	47674	26562	12621	33533	45454	28382	17171	36263	44744	21812	15751	38883	42124	23232	479960
48584	25652	13331	32423	46364	27472	11511	34643	43834	22722	18281	35153	41214	24142	16861	37773	479960
25352	48484	32523	13631	27572	46664	34343	11411	22222	43134	35853	18781	24842	41714	37273	16161	479960
31613	14541	26462	47374	33433	12321	28682	45554	36763	17871	21112	44244	38183	15251	23732	42824	479960
479960	479960	479960	479960	479960	479960	479960	479960	479960	479960	479960	479960	479960	479960	479960	479960	479960

Above magic square is *bimagic* with *bimagic* sum $Sb_{16\times16} := 16484528520$. Moreover, each block of order 4 is a magic square with sum $S_{4\times4} := 19990$. Above magic square is *bimagic*, but *not pan diagonal*. Reorganizing the same elements, we can get *pan diagonal magic square*, but *not bimagic*.

Example 30. *Palindromic pan diagonal magic square* of order 16 is given by

[1]Formerly, Professor of Mathematics, Federal University of Santa Catarina, 88040-400 Florianópolis, SC, Brazil. Email: ijtaneja@gmail.com

		479960	479960	479960	479960	479960	479960	479960	479960	479960	479960	479960	479960	479960	479960	479960	479960
	26362	34643	13331	45654	24442	36563	15451	43534	28182	32823	11111	47874	22222	38783	17271	41714	479960
479960	13631	45354	26662	34343	15551	43434	24542	36463	11811	47174	28882	32123	17771	41214	22722	38283	479960
479960	46664	14341	33633	25352	44544	16461	35553	23432	48884	12121	31813	27172	42724	18281	37773	21212	479960
479960	33333	25652	46364	14641	35453	23532	44444	16561	31113	27872	48184	12821	37273	21712	42224	18781	479960
479960	28282	32723	11211	47774	22122	38883	17171	41814	26462	34543	13431	45554	24342	36663	15351	43634	479960
479960	11711	47274	28782	32223	17871	41114	22822	38183	13531	45454	26562	34443	15651	43334	24642	36363	479960
479960	48784	12221	31713	27272	42824	18181	37873	21112	46564	14441	33533	25452	44644	16361	35653	23332	479960
479960	31213	27772	48284	12721	37173	21812	42124	18881	33433	25552	46464	14541	35353	23632	44344	16661	479960
479960	22422	38583	17471	41514	28382	32623	11311	47674	24242	36763	15251	43734	26162	34843	13131	45854	479960
479960	17571	41414	22522	38483	11611	47374	28682	32323	15751	43234	24742	36263	13831	45154	26862	34143	479960
479960	42524	18481	37573	21412	48684	12321	31613	27372	44744	16261	35753	23232	46864	14141	33833	25152	479960
479960	37473	21512	42424	18581	31313	27672	48384	12621	35253	23732	44244	16761	33133	25852	46164	14841	479960
479960	24142	36863	15151	43834	26262	34743	13231	45754	22322	38683	17371	41614	28482	32523	11411	47574	479960
479960	15851	43134	24842	36163	13731	45254	26762	34243	17671	41314	22622	38383	11511	47474	28582	32423	479960
479960	44844	16161	35853	23132	46764	14241	33733	25252	42624	18381	37673	21312	48584	12421	31513	27472	479960
479960	35153	23832	44144	16861	33233	25752	46264	14741	37373	21612	42324	18681	31413	27572	48484	12521	479960
	479960	479960	479960	479960	479960	479960	479960	479960	479960	479960	479960	479960	479960	479960	479960	479960	479960

Above magic square is *not bimagic* but is *pan diagonal*. Each block of order 4 is also a magic square of order 4 with sum $S_{4\times 4} := 19990$.

3.6. Palindromic Magic Squares of Order 18

Example 31. *Palindromic magic square* of order 18 is given by

																		970020
42524	41214	44344	45454	46264	43534	92529	91219	94349	95459	96269	93539	22522	21212	24342	25452	26262	23532	970020
41114	43334	46564	44544	45154	42624	91119	93339	96569	94549	95159	92629	21112	23332	26562	24542	25152	22622	970020
43134	46364	45354	42224	44644	41614	93139	96369	95359	92229	94649	91619	23132	26362	25352	22222	24642	21612	970020
44244	42424	41314	46464	43634	45254	94249	92429	91319	96469	93639	95259	24242	22422	21312	26462	23632	25252	970020
45654	44444	42324	43234	41514	46164	95659	94449	92329	93239	91519	96169	25652	24442	22322	23232	21512	26162	970020
46664	45554	43434	41414	42124	44144	96669	95559	93439	91419	92129	94149	26662	25552	23432	21412	22122	24142	970020
32523	31213	34343	35453	36263	33533	52525	51215	54345	55455	56265	53535	72527	71217	74347	75457	76267	73537	970020
31113	33333	36563	34543	35153	32623	51115	53335	56565	54545	55155	52625	71117	73337	76567	74547	75157	72627	970020
33133	36363	35353	32223	34643	31613	53135	56365	55355	52225	54645	51615	73137	76367	75357	72227	74647	71617	970020
34243	32423	31313	36463	33633	35253	54245	52425	51315	56465	53635	55255	74247	72427	71317	76467	73637	75257	970020
35653	34443	32323	33233	31513	36163	55655	54445	52325	53235	51515	56165	75657	74447	72327	73237	71517	76167	970020
36663	35553	33433	31413	32123	34143	56665	55555	53435	51415	52125	54145	76667	75557	73437	71417	72127	74147	970020
82528	81218	84348	85458	86268	83538	12521	11211	14341	15451	16261	13531	62526	61216	64346	65456	66266	63536	970020
81118	83338	86568	84548	85158	82628	11111	13331	16561	14541	15151	12621	61116	63336	66566	64546	65156	62626	970020
83138	86368	85358	82228	84648	81618	13131	16361	15351	12221	14641	11611	63136	66366	65356	62226	64646	61616	970020
84248	82428	81318	86468	83638	85258	14241	12421	11311	16461	13631	15251	64246	62426	61316	66466	63636	65256	970020
85658	84448	82328	83238	81518	86168	15651	14441	12321	13231	11511	16161	65656	64446	62326	63236	61516	66166	970020
86668	85558	83438	81418	82128	84148	16661	15551	13431	11411	12121	14141	66666	65556	63436	61416	62126	64146	970020
970020	970020	970020	970020	970020	970020	970020	970020	970020	970020	970020	970020	970020	970020	970020	970020	970020	970020	970020

Each block of order 6 is also a magic square but with different sums forming a magic square of order 3:

			970020
263334	563364	143322	970020
203328	323340	443352	970020
503358	83316	383346	970020
970020	970020	970020	970020

Using the same procedure, still there is an alternative way of writing *palindromic magic square* of order 18 given in example below.

Example 32. *Palindromic magic square* of order 18 is given by

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																		702693
25452	12421	43434	54445	62426	35453	25952	12921	43934	54945	62926	35953	25252	12221	43234	54245	62226	35253	702693
11411	33433	65456	45454	51415	26462	11911	33933	65956	45954	51915	26962	11211	33233	65256	45254	51215	26262	702693
31413	63436	53435	22422	46464	16461	31913	63936	53935	22922	46964	16961	31213	63236	53235	22222	46264	16261	702693
42424	24442	13431	64446	36463	52425	42924	24942	13931	64946	36963	52925	42224	24242	13231	64246	36263	52225	702693
56465	44444	23432	32423	15451	61416	56965	44944	23932	32923	15951	61916	56265	44244	23232	32223	15251	61216	702693
66466	55455	34443	14441	21412	41414	66966	55955	34943	14941	21912	41914	66266	55255	34243	14241	21212	41214	702693
25352	12321	43334	54345	62326	35353	25552	12521	43534	54545	62526	35553	25752	12721	43734	54745	62726	35753	702693
11311	33333	65356	45354	51315	26362	11511	33533	65556	45554	51515	26562	11711	33733	65756	45754	51715	26762	702693
31313	63336	53335	22322	46364	16361	31513	63536	53535	22522	46564	16561	31713	63736	53735	22722	46764	16761	702693
42324	24342	13331	64346	36363	52325	42524	24542	13531	64546	36563	52525	42724	24742	13731	64746	36763	52725	702693
56365	44344	23332	32323	15351	61316	56565	44544	23532	32523	15551	61516	56765	44744	23732	32723	15751	61716	702693
66366	55355	34343	14341	21312	41314	66566	55555	34543	14541	21512	41514	66766	55755	34743	14741	21712	41714	702693
25852	12821	43834	54845	62826	35853	25152	12121	43134	54145	62126	35153	25652	12621	43634	54645	62626	35653	702693
11811	33833	65856	45854	51815	26862	11111	33133	65156	45154	51115	26162	11611	33633	65656	45654	51615	26662	702693
31813	63836	53835	22822	46864	16861	31113	63136	53135	22122	46164	16161	31613	63636	53635	22622	46664	16661	702693
42824	24842	13831	64846	36863	52825	42124	24142	13131	64146	36163	52125	42624	24642	13631	64646	36663	52625	702693
56865	44844	23832	32823	15851	61816	56165	44144	23132	32123	15151	61116	56665	44644	23632	32623	15651	61616	702693
66866	55855	34843	14841	21812	41814	66166	55155	34143	14141	21112	41114	66666	55655	34643	14641	21612	41614	702693
702693	702693	702693	702693	702693	702693	702693	702693	702693	702693	702693	702693	702693	702693	702693	702693	702693	702693	702693

Each block of order 6 is also a magic square but with different sums forming a magic square of order 3:

			702693
233631	236631	232431	702693
233031	234231	235431	702693
236031	231831	234831	702693
702693	702693	702693	702693

Following the example 15 based on *SODLS* and using the same distributions of example 32, we can write *a palindromic magic square* of order 18 given in example below.

Example 33. *Palindromic magic square* of order 18 based on *SODLS* is given by

																		702693
11111	45854	51915	14941	53635	36663	34443	15751	62826	24342	21212	63436	44544	42724	65556	26162	32223	55355	702693
34343	13231	65856	45954	16861	51715	36563	21112	56765	26962	23332	44444	42624	63636	32123	12221	53435	61516	702693
36463	34943	15351	63836	65956	22722	45154	23232	54645	32823	25452	42524	61716	12121	14241	51515	55655	44344	702693
65256	36363	34843	21412	61816	63936	24642	25352	52525	12721	31513	55155	14141	16261	45654	53735	44944	42424	702693
26562	63336	36963	34743	23532	55855	61916	31413	46464	14641	11611	16161	22222	65756	51115	44844	42324	53235	702693
55955	32423	61416	36863	34643	25652	53835	11511	66366	16561	13731	24242	63136	45254	44744	42924	51315	22122	702693
51815	53935	12321	55555	36763	34543	31713	13631	64946	22422	15151	61216	65356	44644	42824	45454	24142	26262	702693
31313	11411	13531	15651	21712	23132	25252	**33833**	**44144**	**36263**	**41914**	52825	54945	56365	62426	64546	66666	46764	702693
64746	62626	56565	54445	52325	46964	66866	**42224**	**35953**	**43834**	**34143**	13331	11211	31113	25752	23632	21512	15451	702693
45554	65656	63736	61116	55255	53335	51415	**43934**	**34243**	**42124**	**35853**	32623	26762	24842	22922	16361	14441	12521	702693
13431	15551	21612	23732	25152	31213	11311	**36163**	**41814**	**33933**	**44244**	66766	46864	52925	54345	56465	62526	64646	702693
22822	52225	54145	12621	41314	43534	14741	64446	15251	55755	62926	46364	33633	35453	24542	65156	25952	31813	702693
54245	56165	32723	41214	43434	12821	16961	66566	13131	61616	64346	25852	52425	33533	35353	22622	45754	23932	702693
62126	26862	41114	43334	32923	14341	56265	46664	11711	63536	66466	21912	23832	54545	33433	35253	16761	51615	702693
24942	41714	43234	26362	12421	62226	64146	52725	31613	65456	46564	53535	15951	21812	56665	33333	35153	14841	702693
41614	43134	24442	32523	64246	66166	22322	54845	25552	45354	52625	12921	55455	13931	15851	62726	33233	35753	702693
43734	22522	26662	66266	46164	16461	41514	56965	23432	51215	54745	35653	32323	61316	11911	13831	64846	33133	702693
16661	24742	46264	52125	14541	41414	43634	62326	21312	53135	56865	33733	35553	26462	63236	31913	11811	66966	702693
702693	702693	702693	702693	702693	702693	702693	702693	702693	702693	702693	702693	702693	702693	702693	702693	702693	702693	702693

Inner grid of order 4 is a magic square with magic sum $S_{4\times4} := 156154$.

3.7. Palindromic Magic Squares of Order 20

In this subsection we shall give *palindromic magic squares* of order 20 in two different forms. One is based on magic squares of order 4 combined with magic square of order 5. The second is

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based on magic squares of order 4 combined with magic square of order 4. In both the cased the magic square is *pan diagonal* and sub blocks are *pan diagonal magic squares* of order 4 and 5 respectively.

Example 34. *Pan diagonal palindromic magic square* of order 20 with each block of order 4 a *pan diagonal* with magic sum $S_{4\times 4} := 119990$ is given by

		599950	599950	599950	599950	599950	599950	599950	599950	599950	599950	599950	599950	599950	599950	599950	599950	599950	599950	599950	599950
	28982	30003	11911	49094	26662	32323	13631	47374	24842	34143	15851	45154	22522	36463	17571	43434	20702	38283	19791	41214	599950
599950	11011	49994	28082	30903	13331	47674	26362	32623	15151	45854	24142	34843	17471	43534	22422	36563	19291	41714	20202	38783	599950
599950	48084	10901	31013	29992	46364	12621	33333	27672	44144	14841	35153	25852	42424	16561	37473	23532	40204	18781	39293	21712	599950
599950	31913	29092	48984	10001	33633	27372	46664	12321	35853	25152	44844	14141	37573	23432	42524	16461	39793	21212	40704	18281	599950
599950	22822	36163	17871	43134	20502	38483	19591	41414	28782	30203	11711	49294	26962	32023	13931	47074	24642	34343	15651	45354	599950
599950	17171	43834	22122	36863	19491	41514	20402	38583	11211	49794	28282	30703	13031	47974	26062	32923	15351	45654	24342	34643	599950
599950	42124	16861	37173	23832	40404	18581	39493	21512	48284	10701	31213	29792	46064	12921	33033	27972	44344	14641	35353	25652	599950
599950	37873	23132	42824	16161	39593	21412	40504	18481	31713	29292	48784	10201	33933	27072	46964	12021	35653	25352	44644	14341	599950
599950	26762	32223	13731	47274	24942	34043	15951	45054	22622	36363	17671	43334	20802	38183	19891	41114	28582	30403	11511	49494	599950
599950	13231	47774	26262	32723	15051	45954	24042	34943	17371	43634	22322	36663	19191	41814	20102	38883	11411	49594	28482	30503	599950
599950	46264	12721	33233	27772	44044	14941	35053	25952	42324	16661	37373	23632	40104	18881	39193	21812	48484	10501	31413	29592	599950
599950	33733	27272	46764	12221	35953	25052	44944	14041	37673	23332	42624	16361	39893	21112	40804	18181	31513	29492	48584	10401	599950
599950	20602	38383	19691	41314	28882	30103	11811	49194	26562	32423	13531	47474	24742	34243	15751	45254	22922	36063	17971	43034	599950
599950	19391	41614	20302	38683	11111	49894	28182	30803	13431	47574	26462	32523	15251	45754	24242	34743	17071	43934	22022	36963	599950
599950	40304	18681	39393	21612	48184	10801	31113	29892	46464	12521	33433	27572	44244	14741	35253	25752	42024	16961	37073	23932	599950
599950	39693	21312	40604	18381	31813	29192	48884	10101	33533	27472	46564	12421	35753	25252	44744	14241	37973	23032	42924	16061	599950
599950	24542	34443	15551	45454	22722	36263	17771	43234	20902	38083	19991	41014	28682	30303	11611	49394	26862	32123	13831	47174	599950
599950	15451	45554	24442	34543	17271	43734	22222	36763	19091	41914	20002	38983	11311	49694	28382	30603	13131	47874	26162	32823	599950
599950	44444	14541	35453	25552	42224	16761	37273	23732	40004	18981	39093	21912	48384	10601	31313	29692	46164	12821	33133	27872	599950
599950	35553	25452	44544	14441	37773	23232	42724	16261	39993	21012	40904	18081	31613	29392	48684	10301	33833	27172	46864	12121	599950
	599950	599950	599950	599950	599950	599950	599950	599950	599950	599950	599950	599950	599950	599950	599950	599950	599950	599950	599950	599950	599950

Example 35. *Pan diagonal palindromic magic square* of order 20 is given by

		699960	699960	699960	699960	699960	699960	699960	699960	699960	699960	699960	699960	699960	699960	699960	699960	699960	699960	699960	699960
	13131	26462	33633	46764	53935	15051	24542	35753	44644	55855	11311	28282	31413	48984	52125	17271	22322	37573	42824	58085	699960
699960	45654	54745	13931	25152	34443	43734	56665	15851	23032	36563	47474	52925	12121	27372	32223	41514	58885	18081	21212	38383	699960
699960	25952	33133	46464	53635	14741	23832	35053	44544	55755	16661	28182	31313	48284	51415	12921	22022	37273	42324	57575	18881	699960
699960	54445	13631	26762	33933	45154	56565	15751	24642	35853	43034	52225	11411	28982	32123	47374	58385	17571	22822	38083	41214	699960
699960	34743	45954	53135	14441	25652	36663	43834	55055	16561	23732	32923	48184	51315	12221	27472	38883	42024	57275	18381	21512	699960
699960	11211	28382	31513	48884	52025	17371	22222	37473	42924	58185	13031	26562	33733	46664	53835	15151	24442	35653	44744	55955	699960
699960	47574	52825	12021	27272	32323	41414	58985	18181	21312	38283	45754	54645	13831	25052	34543	43634	56765	15951	23132	36463	699960
699960	28082	31213	48384	51515	12821	22122	37373	42224	57475	18981	25852	33033	46564	53735	14641	23932	35153	44444	55655	16761	699960
699960	52325	11511	28882	32023	47274	58285	17471	22922	38183	41314	54545	13731	26662	33833	45054	56465	15651	24742	35953	43134	699960
699960	32823	48084	51215	12321	27572	38983	42124	57375	18281	21412	34643	45854	53035	14541	25752	36763	43934	55155	16461	23632	699960
699960	17071	22522	37773	42624	57875	11111	28482	31613	48784	51915	15251	24342	35553	44844	56065	13331	26262	33433	46964	54145	699960
699960	41714	58685	17871	21012	38583	47674	52725	11911	27172	32423	43534	56865	16061	23232	36363	45454	54945	14141	25352	34243	699960
699960	21812	37073	42524	57775	18681	27972	31113	48484	51615	12721	24042	35253	44344	55555	16861	26162	33333	46264	53435	14941	699960
699960	58585	17771	22622	37873	41014	52425	11611	28782	31913	47174	56365	15551	24842	36063	43234	54245	13431	26962	34143	45354	699960
699960	38683	41814	57075	18581	21712	32723	47974	51115	12421	27672	36863	44044	55255	16361	23532	34943	46164	53335	14241	25452	699960
699960	15351	24242	35453	44944	56165	13231	26362	33533	46864	54045	17171	22422	37673	42724	57975	11011	28582	31713	48684	51815	699960
699960	43434	56965	16161	23332	36263	45554	54845	14041	25252	34343	41614	58785	17971	21112	38483	47774	52625	11811	27072	32523	699960
699960	24142	35353	44244	55455	16961	26062	33233	46364	53535	14841	21912	37173	42424	57675	18781	27872	31013	48584	51715	12621	699960
699960	56265	15451	24942	36163	43334	54345	13531	26862	34043	45254	58485	17671	22722	37973	41114	52525	11711	28682	31813	47074	699960
699960	36963	44144	55355	16261	23432	34843	46064	53235	14341	25552	38783	41914	57175	18481	21612	32623	47874	51015	12521	27772	699960
	699960	699960	699960	699960	699960	699960	699960	699960	699960	699960	699960	699960	699960	699960	699960	699960	699960	699960	699960	699960	699960

Each block of order 5 is a magic square with different magic sums forming pan diagonal magic square of order 4 given by

		699960	699960	699960	699960
	173925	175845	172115	178075	699960
699960	172015	178175	173825	175945	699960
699960	177865	171905	176055	174135	699960
699960	176155	174035	177965	171805	699960
	699960	699960	699960	699960	699960

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4. Selfie Magic Squares

Selfie magic square. *It is a magic square that remains the same after making the rotation of* 180^0 *and looking through the mirror. Most of the mirror looking magic squares are reversible, but there are* rotatable *magic square not mirror looking. In this case we call as* ***rotatable or upside down magic squares****. Selfie magic squares are also known by* ***universal magic squares****.*

More precisely, a *Selfie magic square* should have the following three properties:
(i) Upside down (180^0 rotatable);
(ii) Mirror looking;
(iii) Having the same sum (sum of rows, columns and of principal diagonals).

By *rotatable or upside down magic square*, it is understood that when we make a rotation of 180^0, it remains the same. *Mirror looking* magic square are when seen through mirror or their reflection in water, again remains as magic square. It may happen that when rotating to 180^0 or looking through mirror, they are magic squares, but with different sums. In this case, they are not *Selfie magic square*. *Selfie magic square* with letters are famous as IXOHOXI magic squares.

Working with numbers, there are 7 *upside down digits*, i.e., 0, 1, 2, 5, 6, 8 and 9. Digits 0, 1 and 8 are always *upside down* and *mirror looking*. After making rotation of 180^0, 6 becomes as 9 and 9 as 6, but these two are not mirror looking. Also, 2 and 5 are not rotatable. When we write them in digital form, like 2 and 5, they becomes rotatable and mirror looking. When we see them through mirror, 2 becomes as 5, and 5 as 2. Thus, there are only five digits 0, 1, 2, 5 and 8 written in digital form, give us conditions to bring *Selfie magic square*. In this work we shall give only *Selfie magic squares* of orders 11 to 20.

4.1. Selfie Magic Squares of Prime Order

Still, we don't have palindromic magic squares for prime numbers but we can write Selfie magic squares for prime order magic squares. As we have seen there are five digits 0, 1, 2, 5 and 8 that can be made mirror looking. In order to make Selfie magic squares of order 11 to 16 we shall use only four digits 1, 2, 5 and 8. From 17 order onward we shall use fifth digit 0. It will help us up to order 25. All these magic squares are pan diagonal.

- **Selfie Magic Square of Order 11**

Example 36. *Pan diagonal Selfie magic square* of order 11 with magic sum $S_{11\times 11} := 52217$ is given by

1212	1585	2158	2251	2822	5115	5588	5882	8255	8528	8821
8515	8888	1282	1555	2128	2221	2812	5185	5558	5851	8222
5821	8212	8585	8858	1251	1522	2115	2288	2882	5155	5528
5122	5515	5888	8282	8555	8828	1221	1512	2185	2258	2851
2228	2821	5112	5585	5858	8251	8522	8815	1288	1582	2155
1551	2122	2215	2888	5182	5555	5828	8221	8512	8885	1258
8855	1228	1521	2112	2285	2858	5151	5522	5815	8288	8582
8258	8551	8822	1215	1588	2182	2255	2828	5121	5512	5885
5582	5855	8228	8521	8812	1285	1558	2151	2222	2815	5188
2885	5158	5551	5822	8215	8588	8882	1255	1528	2121	2212
2188	2282	2855	5128	5521	5812	8285	8558	8851	1222	1515

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- ## Selfie Magic Square of Order 13

Example 37. *Pan diagonal Selfie magic square* of order 13 with magic sum $S_{13\times 13} := 62216$ is given by

1212	1585	1881	2155	2228	2821	5115	5588	5882	8158	8251	8522	8818
8515	8888	1282	1558	1851	2122	2218	2812	5185	5581	5855	8128	8221
8118	8212	8585	8881	1255	1528	1821	2115	2288	2882	5158	5551	5822
5521	5815	8188	8282	8558	8851	1222	1518	1812	2185	2281	2855	5128
2822	5118	5512	5885	8181	8255	8528	8821	1215	1588	1882	2158	2251
2128	2221	2815	5188	5582	5858	8151	8222	8518	8812	1285	1581	1855
1551	1822	2118	2212	2885	5181	5555	5828	8121	8215	8588	8882	1258
8855	1228	1521	1815	2188	2282	2858	5151	5522	5818	8112	8285	8581
8258	8551	8822	1218	1512	1885	2181	2255	2828	5121	5515	5888	8182
5881	8155	8228	8521	8815	1288	1582	1858	2151	2222	2818	5112	5585
5182	5558	5851	8122	8218	8512	8885	1281	1555	1828	2121	2215	2888
2285	2881	5155	5528	5821	8115	8288	8582	8858	1251	1522	1818	2112
1888	2182	2258	2851	5122	5518	5812	8185	8281	8555	8828	1221	1515

- ## Selfie Magic Square of Order 17

Example 38. *Pan diagonal Selfie magic square* of order 17 with magic sum $S_{17\times 17} := 71104$ is given by

0000	1185	1281	1555	1851	2125	2221	2515	2811	5188	5282	5558	5852	8128	8222	8518	8812
8511	8888	0082	1158	1252	1528	1822	2118	2212	2500	2885	5181	5255	5551	5825	8121	8215
8112	8200	8585	8881	0055	1151	1225	1521	1815	2111	2288	2582	2858	5152	5228	5522	5818
5515	5811	8188	8282	8558	8852	0028	1122	1218	1512	1800	2185	2281	2555	2851	5125	5221
5118	5212	5500	5885	8181	8255	8551	8825	0021	1115	1211	1588	1882	2158	2252	2528	2822
2521	2815	5111	5288	5582	5858	8152	8228	8522	8818	0012	1100	1285	1581	1855	2151	2225
2122	2218	2512	2800	5185	5281	5555	5851	8125	8221	8515	8811	0088	1182	1258	1552	1828
1525	1821	2115	2211	2588	2882	5158	5252	5528	5822	8118	8212	8500	8885	0081	1155	1251
1128	1222	1518	1812	2100	2285	2581	2855	5151	5225	5521	5815	8111	8288	8582	8858	0052
8851	0025	1121	1215	1511	1888	2182	2258	2552	2828	5122	5218	5512	5800	8185	8281	8555
8252	8528	8822	0018	1112	1200	1585	1881	2155	2251	2525	2821	5115	5211	5588	5882	8158
5855	8151	8225	8521	8815	0011	1188	1282	1558	1852	2128	2222	2518	2812	5100	5285	5581
5258	5552	5828	8122	8218	8512	8800	0085	1181	1255	1551	1825	2121	2215	2511	2888	5182
2881	5155	5251	5525	5821	8115	8211	8588	8882	0058	1152	1228	1522	1818	2112	2200	2585
2282	2558	2852	5128	5222	5518	5812	8100	8285	8581	8855	0051	1125	1221	1515	1811	2188
1885	2181	2255	2551	2825	5121	5215	5511	5888	8182	8258	8552	8828	0022	1118	1212	1500
1288	1582	1858	2152	2228	2522	2818	5112	5200	5585	5881	8155	8251	8525	8821	0015	1111

- ## Selfie Magic Square of Order 19

Example 39. *Pan diagonal Selfie magic square* of order 19 with magic sum $S_{19\times 19} := 72215$ is given by

0000	0185	1081	1155	1251	1525	1821	2115	2211	2501	2888	5182	5258	5552	5828	8122	8218	8512	8810
8501	8888	0082	0158	1052	1128	1222	1518	1812	2110	2200	2585	2881	5155	5251	5525	5821	8115	8211
8110	8200	8585	8881	0055	0151	1025	1121	1215	1511	1801	2188	2282	2558	2852	5128	5222	5518	5812
5511	5801	8188	8282	8558	8852	0028	0122	1018	1112	1210	1500	1885	2181	2255	2551	2825	5121	5215
5112	5210	5500	5885	8181	8255	8551	8825	0021	0115	1011	1101	1288	1582	1858	2152	2228	2522	2818
2515	2811	5101	5288	5582	5858	8152	8228	8522	8818	0012	0110	1000	1185	1281	1555	1851	2125	2221
2118	2212	2510	2800	5185	5281	5555	5851	8125	8221	8515	8811	0001	0188	1082	1158	1252	1528	1822
1521	1815	2111	2201	2588	2882	5158	5252	5528	5822	8118	8212	8510	8800	0085	0181	1055	1151	1225
1122	1218	1512	1810	2100	2285	2581	2855	5151	5225	5521	5815	8111	8201	8588	8882	0058	0152	1028
0125	1021	1115	1211	1501	1888	2182	2258	2552	2828	5122	5218	5512	5810	8100	8285	8581	8855	0051
8828	0022	1118	1012	1110	1200	1585	1881	2155	2251	2525	2821	5115	5211	5501	5888	8182	8258	8552
8251	8525	8821	0015	0111	1001	1188	1282	1558	1852	2128	2222	2518	2812	5110	5200	5585	5881	8155
5852	8128	8222	8518	8812	0010	0100	1085	1181	1255	1551	1825	2121	2215	2511	2801	5188	5282	5558
5255	5551	5825	8121	8215	8511	8801	0088	0182	1058	1152	1228	1522	1818	2112	2210	2500	2885	5181
2858	5152	5228	5522	5818	8112	8210	8500	8885	0081	0155	1051	1125	1221	1515	1811	2101	2288	2582
2281	2555	2851	5125	5221	5515	5811	8101	8288	8582	8858	0052	0128	1022	1118	1212	1510	1800	2185
1882	2158	2252	2528	2822	5118	5212	5510	5800	8185	8281	8555	8851	0025	0121	1015	1111	1201	1588
1285	1581	1855	2151	2225	2521	2815	5111	5201	5588	5882	8158	8252	8528	8822	0018	0112	1010	1100
1088	1182	1258	1552	1828	2122	2218	2512	2810	5100	5285	5581	5855	8151	8225	8521	8815	0011	0101

4.2. Selfie Magic Squares of Non-Prime Order

In this subsection, we shall give *Selfie magic squares* of orders 12, 14, 15, 16, 18 and 20.

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• Selfie Magic Square of Order 12

Example 40. *Selfie semi-magic square* of order 12 based on the procedure of example 3 is given by

5228	8251	1512	2585	5821	8158	1818	2181	5125	8552	1215	2882
2512	1585	8228	5251	2118	1881	8121	5858	2815	1282	8525	5152
8285	5212	2551	1528	8181	5818	2158	1821	8582	5115	2852	1225
1551	2528	5285	8212	1858	2121	5881	8118	1252	2825	5182	8515
5121	8558	1218	2881	5225	8252	1515	2582	5828	8151	1812	2185
2818	1281	8521	5158	2515	1582	8225	5252	2112	1885	8128	5851
8581	5118	2858	1221	8282	5215	2552	1525	8185	5812	2151	1828
1258	2821	5181	8518	1552	2525	5282	8215	1851	2128	5885	8112
5825	8152	1815	2182	5128	8551	1212	2885	5221	8258	1518	2581
2115	1882	8125	5852	2812	1285	8528	5151	2518	1581	8221	5258
8182	5815	2152	1825	8585	5112	2851	1228	8281	5218	2558	1521
1852	2125	5882	8115	1251	2828	5185	8512	1558	2521	5281	8218

Each block of order 4 is a magic square with different magic sums forming magic square of order 3:

			53322
17576	17978	17774	53328
17778	17574	17976	53328
17974	17776	17578	53328
53328	53328	53328	**52728**

It is *Selfie semi-magic square* with sum of rows and columns as 53328 and diagonals sum as 53322 and 52728. Using the procedure of *MODLS* given in example 4, we can get *Selfie magic square* of order 12. See the example below.

Example 41. *Selfie magic square* of order 12 based on the procedure example 3 with magic sum $S_{12\times 12} := 53328$ is given by

1212	5285	1552	8218	2851	5815	2521	8528	5181	1825	8182	2158
2828	5182	1251	8115	2585	5212	2118	8225	8558	1521	5881	1852
2525	8581	2885	5812	2182	5128	1815	8121	8252	1218	5258	1551
2121	8258	2582	5228	1881	8525	1512	5818	8151	2815	5152	1285
1818	8152	2181	5125	1558	8221	1228	5215	5885	2512	8551	2882
1515	5851	1858	8521	1252	8118	2825	5112	5282	2128	8285	2581
5151	2115	5221	1258	8518	2552	8281	1585	1828	5882	2812	8125
5252	2518	5825	1581	5121	2858	8582	1851	2112	8185	1215	8228
5858	2821	8128	1882	5225	1281	5185	2152	2515	8251	1518	8512
8181	1225	8212	2185	5828	1582	5251	2558	2818	8552	1821	5115
8282	1528	8515	2551	8112	1885	5852	2881	1221	5158	2125	5218
8585	1812	5118	2852	8215	2151	8158	1282	1525	5281	2528	5821

• Selfie Magic Square of Order 14

Example 42. *Selfie magic square* of order 14 with magic sum $S_{14\times 14} := 63327$ is given by

1111	2881	8855	5188	5882	2251	2112	1518	1215	8221	8122	8558	5585	1828
8128	1212	5182	8551	8822	5811	2815	5521	1118	1881	2188	2285	1558	8255
5588	8251	1515	5881	8518	2128	2258	1122	1821	1282	2885	5111	8155	8812
8851	5185	8158	1818	8211	5515	1121	2228	1522	8512	1255	2882	5888	2181
8258	2288	1885	1182	5555	2812	1511	5115	5828	2122	8521	1281	8818	8151
5122	1158	2821	1555	1228	8282	8581	8188	8885	2218	1812	5851	2115	5511
1221	1528	5822	2111	1115	8185	8888	8281	8582	2851	5558	1855	2212	5118
1815	2155	2211	2822	1551	8881	8182	8585	8288	5528	5818	1112	5121	1258
1512	1811	2118	2215	2858	8588	8285	8882	8181	5155	1128	5522	1251	5821
2182	8118	8212	1285	2221	1822	5128	2855	5551	5858	8811	1588	1181	8515
2281	8821	8528	5512	2185	1218	5855	1858	2811	1188	5151	8115	8222	1582
5885	8522	1151	8228	8112	5158	5518	1211	2255	8815	1581	2121	1882	2888
8555	5815	5581	8858	1888	1521	1222	2151	5112	8111	2282	8218	2828	1185
2818	5582	1288	8121	5181	1155	1851	5812	2158	1585	8215	8828	8511	2222

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Inner grid of order 4 is a magic square with sum $S_{4\times4} := 33936$. It is based on procedure of example 5.

- ## Selfie Magic Square of Order 15

We shall present two different ways of calculating Selfie magic square of 15. One is based composition procedure 5×3 and another on *SODLS*.

Example 43. *Selfie magic square* of order 15 based on the example 7 with magic sum $S_{15\times15} := 69993$ is given by

1215	2558	2822	8185	8251	1818	2155	5221	5588	8852	1512	2281	5125	5882	8528
8122	8285	1251	2515	2858	5521	8888	1852	2118	5255	5825	8582	1528	2212	5181
2551	2815	8158	8222	1285	2152	5218	5555	8821	1888	2228	5112	5881	8525	1582
8258	1222	2585	2851	8115	8855	1821	2188	5252	5518	8581	1525	2282	5128	5812
2885	8151	8215	1258	2522	5288	5552	8818	1855	2121	5182	5828	8512	1581	2225
1812	2181	5225	5582	8828	1515	2258	5122	5885	8551	1218	2555	2821	8188	8252
5525	8882	1828	2112	5281	5822	8585	1551	2215	5158	8121	8288	1252	2518	2855
2128	5212	5581	8825	1882	2251	5115	5858	8522	1585	2552	2818	8155	8221	1288
8881	1825	2182	5228	5512	8558	1522	2285	5151	5815	8255	1221	2588	2852	8118
5282	5528	8812	1881	2125	5185	5851	8515	1558	2222	2888	8152	8218	1255	2521
1518	2255	5121	5888	8552	1212	2581	2825	8182	8228	1815	2158	5222	5585	8851
5821	8588	1552	2218	5155	8125	8282	1228	2512	2881	5522	8885	1851	2115	5258
2252	5118	5855	8521	1588	2528	2812	8181	8225	1282	2151	5215	5558	8822	1885
8555	1521	2288	5152	5818	8281	1225	2582	2828	8112	8858	1822	2185	5251	5515
5188	5852	8518	1555	2221	2882	8128	8212	1281	2525	5285	5551	8815	1858	2122

Each block of order 5 is a *pan diagonal magic square* with different magic sums forming magic square of order 3:

			69993
23031	23634	23328	69993
23628	23331	23034	69993
23334	23028	23631	69993
69993	69993	69993	69993

Still we can have alternative way to write Selfie magic square of order 15 based on *SODLS* procedure. See example below.

Example 44. *Selfie magic square* of order 15 with magic sum $S_{15\times15} := 69993$ is given by

1212	2281	8525	2582	1858	5215	5188	5552	8221	2128	2818	8855	8185	1551	5822
8122	1515	2558	8228	2881	2155	1218	5212	2288	8821	8552	5882	1851	5525	5185
2585	5825	1818	2855	8188	8858	2252	1215	8522	8212	5581	2151	5228	5182	1521
8225	2882	5528	2121	8852	5885	8555	1518	8115	5258	2251	1288	5181	1822	2512
5818	8128	8881	5288	2222	8512	5582	1821	1255	2551	1585	5158	2125	2815	8252
1552	5521	5888	8558	1285	2525	8215	2122	2851	1882	5155	2228	8818	8112	5281
8851	1812	5222	5585	8255	1582	2828	2225	2181	5152	2588	8521	5815	1258	8118
5255	1252	1512	1815	2118	2221	2522	5151	5558	5881	8182	8285	8588	8828	2825
2182	8822	2285	1581	5512	5128	8121	5855	5252	8518	1825	2515	2858	8288	1251
2821	2188	1282	5852	5125	8218	5251	8158	1885	5555	8815	1522	2212	2581	8528
1828	5285	8155	5122	8515	5551	8825	8281	2518	1588	5858	2812	1221	2152	2282
5588	8258	5121	8812	5851	2822	2185	8582	1525	2215	1228	8181	2552	5218	1855
8581	5118	2852	8151	2521	1888	1558	8885	5828	1222	2112	5225	8282	2255	5515
5115	2555	8251	2218	1528	1281	5812	2888	8882	8125	5221	1852	5522	8585	2158
2258	8551	2115	1225	5282	8152	1881	2528	5112	2885	8222	5518	1555	5821	8888

It is based on procedure *SODLS* of example 8.

- ## Selfie and Bimagic Squares of Order 16

We shall give different forms of *Selfie magic square* of order 16, such as bimagic and pan diagonal. Just with two digits are also given.

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Example 45. *Selfie bimagic square* of order 16 with magic sums $S_{16\times16} := 71104$ and $Sb_{16\times16} := 437198296$, each block of order 4 as magic square with sum $S_{4\times4} := 17776$:

1111	5252	8585	2828	1225	5188	8851	2512	1558	5815	8122	2281	1882	5521	8218	2155
8528	2885	1152	5211	8812	2551	1288	5125	8181	2222	1515	5858	8255	2118	1821	5582
2852	8511	5228	1185	2588	8825	5112	1251	2215	8158	5881	1522	2121	8282	5555	1818
5285	1128	2811	8552	5151	1212	2525	8888	5822	1581	2258	8115	5518	1855	2182	8221
1582	5821	8118	2255	1858	5515	8222	2181	1125	5288	8551	2812	1211	5152	8885	2528
8155	2218	1521	5882	8281	2122	1815	5558	8512	2851	1188	5225	8828	2585	1252	5111
2221	8182	5855	1518	2115	8258	5581	1822	2888	8525	5212	1151	2552	8811	5128	1285
5818	1555	2282	8121	5522	1881	2158	8215	5251	1112	2825	8588	5185	1228	2511	8852
1825	5588	8251	2112	1511	5852	8185	2228	1282	5121	8818	2555	1158	5215	8522	2881
8212	2151	1888	5525	8128	2285	1552	5811	8855	2518	1221	5182	8581	2822	1115	5258
2188	8225	5512	1851	2252	8111	5828	1585	2521	8882	5155	1218	2815	8558	5281	1122
5551	1812	2125	8288	5885	1528	2211	8152	5118	1255	2582	8821	5222	1181	2858	8515
1258	5115	8822	2581	1182	5221	8518	2855	1811	5552	8285	2128	1525	5888	8151	2212
8881	2522	1215	5158	8555	2818	1121	5282	8228	2185	1852	5511	8112	2251	1588	5825
2515	8858	5181	1222	2821	8582	5255	1118	2152	8211	5528	1885	2288	8125	5812	1551
5122	1281	2558	8815	5218	1155	2882	8521	5585	1828	2111	8252	5851	1512	2225	8188

With the same distribution of example above, we don't have *pan diagonal Selfie magic square* of order 16. Instead using four digits 1, 2, 5 and 8 we shall consider only two digits 1 and 8 to bring *bimagic* and *pan diagonal* magic squares of order 16.

Example 46. *Selfie magic square* of order 16 with magic sums $S_{16\times16} := 799999992$ and $Sb_{16\times16} := 59797978997979800$, each block of order 4 as magic square with sum $S_{4\times4} := 199999998$:

11111111	81188118	88818881	18881888	11181881	81118888	88888111	18811118	11818188	81881181	88111818	18188811	11888818	81811811	88181188	18118181
88811888	18888881	11118118	81181111	88881118	18818111	11188888	81111881	88118811	18181818	11811181	81888188	88188181	18111188	11881811	81818818
18888118	88811111	81181888	11118881	18818888	88881881	81111118	11188111	18181181	88118188	81888811	11811818	18111811	88188818	81818181	11881188
81188881	11111888	18881111	88818118	81118111	11181118	18811881	88888888	81881818	11818811	18188188	88111181	81811188	11888181	18118818	88181811
11818818	81881811	88111188	18188181	11888188	81811181	88181818	18118811	11111881	81188888	88818111	18881118	11181111	81118118	88888881	18811888
88118181	18181188	11811811	81888818	88188811	18111818	11881181	81818188	88811118	18888111	11118888	81181881	88881888	18818881	11188118	81111111
18181811	88118818	81888181	11811188	18111181	88188188	81818811	11881818	18888888	88811881	81181118	11118111	18818118	88881111	81111888	11188881
81881188	11818181	18188818	88111811	81811818	11888811	18118188	88181181	81188111	11111118	18881881	88818888	81118881	11181888	18811111	88888118
11881881	81818888	88188111	18111118	11811111	81888118	88118881	18181888	11188818	81111811	88881188	18818181	11118188	81181181	88811818	18888811
88181118	18118111	11888888	81811881	88111888	18188881	11818118	81881111	88888181	18811188	11181811	81118818	88818811	18881818	11111181	81188188
18118888	88181881	81811118	11888111	18188118	88111111	81881888	11818881	18811811	88888818	81118181	11181188	18881181	88818188	81188811	11111818
81818111	11881118	18111881	88188888	81888881	11811888	18181111	88118118	81111188	11188181	18818818	88881811	81181818	11118811	18888188	88811181
11188188	81111181	88881818	18818811	11118818	81181811	88811188	18888181	11881111	81818118	88188881	18111888	11811881	81888888	88118111	18181118
88888811	18811818	11181181	81118188	88818181	18881188	11111811	81188818	88181888	18118881	11888118	81811111	88111118	18188111	11818888	81881881
18811181	88888188	81118811	11181818	18881811	88818818	81188181	11111188	18118118	88181111	81811888	11888881	18188888	88111881	81881118	11818111
81111818	11188811	18818188	88881181	81181188	11118181	18888818	88811811	81818881	11881888	18111111	88188118	81888111	11811118	18181881	88118888

Above distribution can be reorganized to make a *pan diagonal magic square* of order 16. See example below.

Example 47. *Pan diagonal Selfie magic square* of order 16 with magic sums $S_{16\times16} := 799999992$ with each block of order 4 as magic square with sum $S_{4\times4} := 199999998$:

18111811	81181181	11818888	88888118	18881818	81811188	11188881	88118111	18181881	81111111	11888818	88818188	18811888	81881118	11118811	88188181
11818118	88888888	18111181	81181811	11188111	88118881	18881188	81811818	11888188	88818818	18181111	81111881	11118181	88188811	18811118	81881888
88881181	11811811	81188118	18118888	88111188	11181818	81818111	18888881	88811111	11881881	81118188	18188818	88181118	11111888	81888181	18818811
81188888	18118118	88881811	11811181	81818881	18888111	88111818	11181188	81118818	18188188	88811881	11881111	81888811	18818181	88181888	11111118
18181888	81111118	11888811	88818181	18811881	81881111	11118818	88188188	18111818	81181188	11818881	88888111	18881811	81811181	11188888	88118118
11888181	88818811	18181118	81111888	11118188	88188818	18811111	81881881	11818111	88888881	18111188	81181818	11188118	88118888	18881181	81811811
88811118	11881888	81118181	18188811	88181111	11111881	81888188	18818818	88881188	11811818	81188111	18118881	88111181	11181811	81818118	18888888
81118811	18188181	88811888	11881118	81888818	18818188	88181881	11111111	81188881	18118111	88881818	11811188	81818888	18888118	88111811	11181181
18811818	81881188	11118881	88188111	18181811	81111181	11888888	88818118	18881888	81811118	11188811	88118181	18111881	81181111	11818818	88888188
11118111	88188881	18811188	81881818	11888118	88818888	18181181	81111811	11188181	88118811	18881118	81811888	11818188	88888818	18111111	81181881
88181188	11111818	81888111	18818881	88811181	11881811	81118118	18188888	88111118	11181888	81818181	18888811	88881111	11811881	81188188	18118818
81888881	18818111	88181818	11111188	81118888	18188118	88811811	11881181	81818811	18888181	88111888	11181118	81188818	18118188	88881881	11811111
18881881	81811111	11188818	88118188	18111888	81181118	11818811	88888181	18811811	81881181	11118888	88188118	18181818	81111188	11888881	88818111
11188188	88118818	18881111	81811881	11818181	88888811	18111118	81181888	11118118	88188888	18811181	81881811	11888111	88818881	18181188	81111818
88111111	11181881	81818188	18888818	88881118	11811888	81188181	18118811	88181181	11111811	81888118	18818888	88811188	11881818	81118111	18188881
81818818	18888188	88111881	11181111	81188811	18118181	88881888	11811118	81888888	18818118	88181811	11111181	81118881	18188111	88811818	11881188

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Based on similar lines we can bring *bimagic* and *pan diagonal magic squares* of order 16 just using two digits 6 and 9. In this case, it is known as *upside down*. See examples below:

Example 48. *Upside down magic square* of order 16 with magic sums $S_{16\times16} := 1333333320$ and $Sb_{16\times16} := 114747472525252536$, each block of order 4 as magic square with sum $S_{4\times4} := 33333330$:

66666666	96699669	99969996	69996999	66696996	96669999	99999666	69966669	66969699	96996696	99666969	69699966	66999969	96966966	99696699	69669696
99966999	69999996	66669669	96696666	99996669	69969666	66699999	96666996	99669966	69696969	66966696	96999699	99699696	69666699	66996966	96969969
69999669	99966666	96696999	66669996	69969999	99996996	96666669	66699666	69696696	99669699	96999966	66966969	69666966	99699969	96969696	66996699
96699996	66666999	69996666	99969669	96669666	66696669	69966996	99999999	96996969	66969966	69699699	99666696	96966699	66999696	69669969	99696966
66969969	96996966	99666699	69699696	66999699	96966696	99696969	69669966	66666996	96699999	99969666	69996669	66696666	96669669	99999996	69966999
99669696	69696699	66966966	96999969	99699966	69666969	66996696	96969699	99966669	69999666	66669999	96696996	99996999	69969996	66699669	96666666
69696966	99669969	96999696	66966699	69666696	99699699	96969966	66996969	69999999	99966996	96696669	66669666	69969669	99996666	96666999	66699996
96996699	66969696	69699969	99666966	96966969	66999966	69669699	99696696	96699666	66666669	69996996	99969999	96669996	66696999	69966666	99999669
66996996	96969999	99699666	69666669	66966666	96999669	99669996	69696999	66699969	96666966	99996699	69969696	66669699	96696696	99966969	69999966
99696669	69669666	66999999	96966996	99666999	69699996	66969669	96996666	99999696	69966699	66696966	96669969	99969966	69996969	66666696	96699699
69669999	99696996	96966669	66999666	69699669	99666666	96996999	66969996	69966966	99999969	96669696	66696699	69996696	99969699	96699966	66666969
96969666	66996669	69666996	99699999	96999996	66966999	69696666	99669669	96666699	66699696	69969969	99996966	96696969	66669966	69999699	99966696
66699699	96666696	99996969	69969966	66669969	96696966	99966699	69999696	66996666	96969669	99699996	69666999	66966996	96999999	99669666	69696669
99999966	69966969	66696696	96669699	99969696	69996699	66666966	96699969	99696999	69669996	66999669	96966666	99666669	69699666	66969999	96996996
69966696	99999699	96669966	66696969	69996966	99969969	96699696	66666699	69669669	99696666	96966999	66999996	69699999	99666996	96996669	66969666
96666969	66699966	69969699	99996696	96696699	66669696	69999969	99966966	96969996	66996999	69666666	99699669	96999666	66966669	69696996	99669999

Above distribution can be reorganized to make a *pan diagonal magic square* of order 16. See example below.

Example 49. *Pan diagonal upside down magic square* of order 16 with magic sums $S_{16\times16} := 1333333320$ with each block of order 4 as magic square with sum $S_{4\times4} := 33333330$:

69666966	96696696	66969999	99999669	69996969	96966699	66699996	99669666	69696996	96666666	66999969	99969699	69966999	96996669	66669966	99699696
66969669	99999999	69666696	96696966	66699666	99669996	69996699	96966969	66999699	99969969	69696666	96666996	66669696	99699966	69966669	96996999
99996696	66966966	96699669	69669999	99666699	66696969	96969666	69999996	99966666	66996996	96669699	69699969	99696669	66666999	96999696	69969966
96699999	69669669	99996966	66966696	96969996	69999666	99666969	66696699	96669969	69699699	99966996	66996666	96999966	69969696	99696999	66666669
69696999	96666669	66999966	99969696	69966996	96996666	66669969	99699699	69666969	96696699	66969996	99999666	69996966	96966696	66699999	99669669
66999696	99969966	69696669	96666999	66669699	99699969	69966666	96996996	66969666	99999996	69666699	96696969	66699669	99669999	69996696	96966966
99966669	66996999	96669696	69699966	99696666	66666996	96999699	69969969	99996699	66966969	96699666	69669996	99666696	66696966	96969669	69999999
96669966	69699696	99966999	66996669	96999969	69969699	99696996	66666666	96699996	69669666	99996969	66966699	96969999	69999669	99666966	66696696
69966969	96996699	66669996	99699666	69696966	96666696	66999999	99969669	69996999	96966669	66699966	99669696	69666996	96696666	66969969	99999699
66669666	99699996	69966699	96996969	66999669	99969999	69696696	96666966	66699696	99669966	69996669	96966999	66969699	99999969	69666666	96696996
99696699	66666969	96999666	69969996	99966696	66996966	96669669	69699999	99666669	66696999	96969696	69999966	99996666	66966996	96699699	69669969
96999996	69969666	99696969	66666699	96669999	69699669	99966966	66996696	96969966	69999696	99666999	66696669	96699969	69669699	99996996	66966666
69996996	96966666	66699969	99669699	69666999	96696669	66969966	99999696	69966966	96996696	66669999	99699669	69696969	96666699	66999996	99969666
66699699	99669969	69996666	96966996	66969696	99999966	69666669	96696999	66669669	99699999	69966696	96996966	66999666	99969996	69696699	96666969
99666666	66696996	96969699	69999969	99996669	66966999	96699696	69669966	99696696	66666966	96999669	69969999	99966699	66996969	96669666	69699996
96969969	69999699	99666996	66696666	96699966	69669696	99996999	66966669	96999999	69969669	99696966	66666696	96669996	69699666	99966969	66996699

- ## Selfie Magic Squares of Order 18

We shall give *Selfie magic square* of order 18 based on the SODLS according to example 15.

Example 50. *Selfie magic square* of order 18 with magic sum $S_{18\times18} := 72205$ is given by

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0101	5222	5525	1088	5818	2581	2255	1121	8285	1552	1210	8512	5158	2882	8815	1828	2151	8111
2252	1010	8822	5225	1185	5521	2558	1201	8182	1888	1511	5155	2881	8518	2128	0151	5812	8215
2555	2288	1111	8522	8825	1282	5201	1510	5881	2185	1812	2858	8221	0128	1051	5515	8118	5152
8810	2552	2285	1212	8222	8525	1581	1811	5558	0182	2115	8101	1028	1151	5218	5821	5188	2855
1858	8511	2588	2282	1515	8122	8225	2112	5255	1081	0118	1128	1251	8821	5501	5185	2852	5810
8125	2155	8212	2585	2281	1818	5822	0115	8852	1158	1021	1551	8501	5210	5182	2888	5511	1228
5522	5825	0152	8115	2582	2258	2121	1018	8588	1255	1101	8210	8811	5181	2885	5212	1528	1851
2111	0112	1015	1118	1221	1501	1810	2222	5128	2551	2825	5585	5888	8152	8255	8558	8881	5282
8582	8281	8158	5855	5552	5288	8885	2851	2525	5122	2228	1011	0110	2101	1821	1518	1215	1112
5215	8818	8521	8201	8110	5811	5512	5125	2251	2828	2522	2181	1882	1585	1288	1152	1055	0158
1012	1115	1218	1521	1801	2110	0111	2528	2822	2225	5151	8882	5285	5588	5852	8155	8258	8581
1285	5551	5828	0181	2811	5115	1082	8555	1110	8121	8288	5252	2218	2512	1558	8801	1825	2122
5851	8128	2182	2810	5112	0185	1188	8858	1001	8218	8552	1822	5555	2215	2511	1281	5221	1525
8228	1885	2801	5111	2188	1052	8151	5281	0121	8515	8855	1225	1522	5858	2212	2510	1182	5518
1588	2821	5110	1852	0155	8251	8528	5582	2118	8812	5258	5815	1125	1222	8181	2211	2501	1085
2818	5101	1555	2158	8551	8828	1252	5885	1815	5211	5581	0188	8112	1025	1122	8282	2210	2521
5121	1258	1881	8851	5228	1155	2815	8188	1512	5510	5882	2518	2152	8211	0125	1022	8585	2201
1181	1582	5251	5528	1058	2812	5118	8252	1211	5801	8185	2221	2515	1855	8510	2125	0122	8888

Inner grid of order 4 is a magic square with sum $S_{4\times 4} := 12726$. It is based on procedure of example 15.

- **Selfie Magic Squares of Order 20**

We shall give two different forms of *Selfie magic square* of order 20 based on the composition of magic squares of orders 4 and 5 one as 4×5 and another as 5×4.

Example 51. *Pan diagonal Selfie magic square* of order 20 with magic sums $S_{20\times 20} := 81103$ is given by

2525	2801	0188	8828	2218	5111	0881	8555	2122	5208	1085	8251	1815	5512	1180	8158	1521	5810	1282	8052
0128	8888	2501	2825	0855	8581	2211	5118	1051	8285	2108	5222	1158	8180	1812	5515	1252	8082	1510	5821
8801	0125	2828	2588	8511	0818	5155	2281	8208	1022	5251	2185	8112	1115	5558	1880	8010	1221	5852	1582
2888	2528	8825	0101	5181	2255	8518	0811	5285	2151	8222	1008	5580	1858	8115	1112	5882	1552	8021	1210
1822	5508	1185	8151	1515	5812	1280	8058	2521	2810	0182	8852	2225	5101	0888	8528	2118	5211	1081	8255
1151	8185	1808	5522	1258	8080	1512	5815	0152	8882	2510	2821	0828	8588	2201	5125	1055	8281	2111	5218
8108	1122	5551	1885	8012	1215	5858	1580	8810	0121	2852	2582	8501	0825	5128	2288	8211	1018	5255	2181
5585	1851	8122	1108	5880	1558	8015	1212	2882	2552	8821	0110	5188	2228	8525	0801	5281	2155	8218	1011
2221	5110	0882	8552	2125	5201	1088	8228	1818	5511	1181	8155	1522	5808	1285	8051	2515	2812	0180	8858
0852	8582	2210	5121	1028	8288	2101	5225	1155	8181	1811	5518	1251	8085	1508	5822	0158	8880	2512	2815
8510	0821	5152	2282	8201	1025	5228	2188	8111	1118	5555	1881	8008	1222	5851	1585	8812	0115	2858	2580
5182	2252	8521	0810	5288	2128	8225	1001	5581	1855	8118	1111	5885	1551	8022	1208	2880	2558	8815	0112
1518	5811	1281	8055	2522	2808	0185	8851	2215	5112	0880	8558	2121	5210	1082	8252	1825	5501	1188	8128
1255	8081	1511	5818	0151	8885	2508	2822	0858	8580	2212	5115	1052	8282	2110	5221	1128	8188	1801	5525
8011	1218	5855	1581	8808	0122	2851	2585	8512	0815	5158	2280	8210	1021	5252	2182	8101	1125	5528	1888
5881	1555	8018	1211	2885	2551	8822	0108	5180	2258	8515	0812	5282	2152	8221	1010	5588	1828	8125	1101
2115	5212	1080	8258	1821	5510	1182	8152	1525	5801	1288	8028	2518	2811	0181	8855	2222	5108	0885	8551
1058	8280	2112	5215	1152	8182	1810	5521	1228	8088	1501	5825	0155	8881	2511	2818	0851	8585	2208	5122
8212	1015	5258	2180	8110	1121	5552	1882	8001	1225	5828	1588	8811	0118	2855	2581	8508	0822	5151	2285
5280	2158	8215	1012	5582	1852	8121	1110	5888	1528	8025	1201	2881	2555	8818	0111	5185	2251	8522	0808

Each block of order 4 is a magic square (*not pan diagonal*) with different magic sums forming *pan diagonal magic square* of order 5:

		81103	81103	81103	81103	81103
	14342	16765	16666	16665	16665	81103
81103	16666	16665	14365	16742	16665	81103
81103	16765	16642	16665	16666	14365	81103
81103	16665	14366	16765	16665	16642	81103
81103	16665	16665	16642	14365	16766	81103
	81103	81103	81103	81103	81103	81103

Above magic square of order 5 formed by 20 blocks of magic squares of order 4 is *pan diagonal*, but the magic square of 20 is *not pan diagonal*. This example is based on example 16, where each group of order 4 is a pan diagonal magic square of same sum, while here each block of order 4 is a magic square not pan diagonal and with different sums.

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Example 52. *Selfie magic square* of order 20 with magic sums $S_{20\times 20} := 81103$ is given by

0808	1858	2518	5882	8225	1001	1580	2821	5581	8522	0111	2152	2212	8088	8151	1110	1255	5115	5285	8828
5818	8282	0825	1808	2558	5521	8581	1022	1501	2880	8012	8188	0151	2111	2252	5215	8885	1128	1210	5155
1825	2508	5858	8218	0882	1522	2801	5580	8521	1081	2151	2211	8052	8112	0188	1228	5110	5255	8815	1185
8258	0818	1882	2525	5808	8580	1021	1581	2822	5501	8152	0112	2188	2251	8011	8855	1115	1285	5128	5210
2582	5825	8208	0858	1818	2881	5522	8501	1080	1521	2288	8051	8111	0152	2112	5185	5228	8810	1155	1215
0110	2155	2215	8085	8128	1111	1252	5112	5288	8851	0801	1880	2521	5881	8222	1008	1558	2818	5582	8525
8015	8185	0128	2110	2255	5212	8888	1151	1211	5152	5821	8281	0822	1801	2580	5518	8582	1025	1508	2858
2128	2210	8055	8115	0185	1251	5111	5252	8812	1188	1822	2501	5880	8221	0881	1525	2808	5558	8518	1082
8155	0115	2185	2228	8010	8852	1112	1288	5151	5211	8280	0821	1881	2522	5801	8558	1018	1582	2825	5508
2285	8028	8110	0155	2115	5188	5251	8811	1152	1212	2581	5822	8201	0880	1821	2882	5525	8508	1058	1518
1101	1280	5121	5281	8822	0108	2158	2218	8082	8125	1010	1555	2815	5585	8528	0811	1852	2512	5888	8251
5221	8881	1122	1201	5180	8018	8182	0125	2108	2258	5515	8585	1028	1510	2855	5812	8288	0851	1811	2552
1222	5101	5280	8821	1181	2125	2208	8058	8118	0182	1528	2810	5555	8515	1085	1851	2511	5852	8212	0888
8880	1121	1281	5122	5201	8158	0118	2182	2225	8008	8555	1015	1585	2828	5510	8252	0812	1888	2551	5811
5181	5222	8801	1180	1221	2282	8025	8108	0158	2118	2885	5528	8510	1055	1515	2588	5851	8211	0852	1812
1011	1552	2812	5588	8551	0810	1855	2515	5885	8228	1108	1258	5118	5282	8825	0101	2180	2221	8081	8122
5512	8588	1051	1511	2852	5815	8285	0828	1810	2555	5218	8882	1125	1208	5158	8021	8181	0122	2101	2280
1551	2811	5552	8512	1088	1828	2510	5855	8215	0885	1225	5108	5258	8818	1182	2122	2201	8080	8121	0181
8552	1012	1588	2851	5511	8255	0815	1885	2528	5810	8858	1118	1282	5125	5208	8180	0121	2181	2222	8001
2888	5551	8511	1052	1512	2585	5828	8210	0855	1815	5182	5225	8808	1158	1218	2281	8022	8101	0180	2121

Each block of order 5 is a *pan diagonal* magic square with different magic sums forming magic square of order 4:

				81103
19291	19505	20714	21593	81103
20693	21614	19305	19491	81103
21605	20691	19493	19314	81103
19514	19293	21591	20705	81103
81103	81103	81103	81103	81103

Even though, we have magic squares of order 5 formed by 16 blocks of magic squares are *pan diagonals*, but the magic square of order 20 is *not pan diagonal*.

5. Genetic Table

For writing all the combinations of 4 digits with 1, 2, 3 and 4, we have exactly $4^3 = 256$ possibilities. Based on these 256 values we can have two magic squares of order 16. One is bimagic and another is pan diagonal. In both the cases each block of order 4 is a magic square with same sum. See the examples below:

Example 52. *Bimagic square* of order 16 formed by digits 1, 2, 3 and 4 is given by

																44440
1111	3232	4343	2424	1223	3144	4431	2312	1334	3413	4122	2241	1442	3321	4214	2133	44440
4324	2443	1132	3211	4412	2331	1244	3123	4141	2222	1313	3434	4233	2114	1421	3342	44440
2432	4311	3224	1143	2344	4423	3112	1231	2213	4134	3441	1322	2121	4242	3333	1414	44440
3243	1124	2411	4332	3131	1212	2323	4444	3422	1341	2234	4113	3314	1433	2142	4221	44440
1342	3421	4114	2233	1434	3313	4222	2141	1123	3244	4331	2412	1211	3132	4443	2324	44440
4133	2214	1321	3442	4241	2122	1413	3334	4312	2431	1144	3223	4424	2343	1232	3111	44440
2221	4142	3433	1314	2113	4234	3341	1422	2444	4323	3212	1131	2332	4411	3124	1243	44440
3414	1333	2242	4121	3322	1441	2134	4213	3231	1112	2423	4344	3143	1224	2311	4432	44440
1423	3344	4231	2112	1311	3432	4143	2224	1242	3121	4414	2333	1134	3213	4322	2441	44440
4212	2131	1444	3323	4124	2243	1332	3411	4433	2314	1221	3142	4341	2422	1113	3234	44440
2144	4223	3312	1431	2232	4111	3424	1343	2321	4442	3133	1214	2413	4334	3241	1122	44440
3331	1412	2123	4244	3443	1324	2211	4132	3114	1233	2342	4421	3222	1141	2434	4313	44440
1234	3113	4422	2341	1142	3221	4314	2433	1411	3332	4243	2124	1323	3444	4131	2212	44440
4441	2322	1213	3134	4333	2414	1121	3242	4224	2143	1432	3311	4112	2231	1344	3423	44440
2313	4434	3141	1222	2421	4342	3233	1114	2132	4211	3324	1443	2244	4123	3412	1331	44440
3122	1241	2334	4413	3214	1133	2442	4321	3343	1424	2111	4232	3431	1312	2223	4144	44440
44440	44440	44440	44440	44440	44440	44440	44440	44440	44440	44440	44440	44440	44440	44440	44440	44440

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Magic sums are $S_{16\times16} := 44440$ and $Sb_{16\times16} := 143634120$, each block of order 4 as magic square with sum $S_{4\times4} := 11110$.

Example 52. *Pan diagonal magic square* of order 16 formed by digits 1, 2, 3 and 4 with each block of order 4 also *a pan diagonal* magic square with sums $S_{16\times16} := 44440$ and $S_{4\times4} := 11110$ is given by

		44440	44440	44440	44440	44440	44440	44440	44440	44440	44440	44440	44440	44440	44440	44440	44440
	2333	3242	1213	4322	2234	3341	1314	4221	2431	3144	1111	4424	2132	3443	1412	4123	44440
44440	1222	4313	2342	3233	1321	4214	2241	3334	1124	4411	2444	3131	1423	4112	2143	3432	44440
44440	4342	1233	3222	2313	4241	1334	3321	2214	4444	1131	3124	2411	4143	1432	3423	2112	44440
44440	3213	2322	4333	1242	3314	2221	4234	1341	3111	2424	4431	1144	3412	2123	4132	1443	44440
44440	2432	3143	1112	4423	2131	3444	1411	4124	2334	3241	1214	4321	2233	3342	1313	4222	44440
44440	1123	4412	2443	3132	1424	4111	2144	3431	1221	4314	2341	3234	1322	4213	2242	3333	44440
44440	4443	1132	3123	2412	4144	1431	3424	2111	4341	1234	3221	2314	4242	1333	3322	2213	44440
44440	3112	2423	4432	1143	3411	2124	4131	1444	3214	2321	4334	1241	3313	2222	4233	1342	44440
44440	2134	3441	1414	4121	2433	3142	1113	4422	2232	3343	1312	4223	2331	3244	1211	4324	44440
44440	1421	4114	2141	3434	1122	4413	2442	3133	1323	4212	2243	3332	1224	4311	2344	3231	44440
44440	4141	1434	3421	2114	4442	1133	3122	2413	4243	1332	3323	2212	4344	1231	3224	2311	44440
44440	3414	2121	4134	1441	3113	2422	4433	1142	3312	2223	4232	1343	3211	2324	4331	1244	44440
44440	2231	3344	1311	4224	2332	3243	1212	4323	2133	3442	1413	4122	2434	3141	1114	4421	44440
44440	1324	4211	2244	3331	1223	4312	2343	3232	1422	4113	2142	3433	1121	4414	2441	3134	44440
44440	4244	1331	3324	2211	4343	1232	3223	2312	4142	1433	3422	2113	4441	1134	3121	2414	44440
44440	3311	2224	4231	1344	3212	2323	4332	1243	3413	2122	4133	1442	3114	2421	4434	1141	44440
	44440	44440	44440	44440	44440	44440	44440	44440	44440	44440	44440	44440	44440	44440	44440	44440	44440

Following the notation of Taneja [15] , let us replace $1=C$, $2=A$, $3=T$ and $4=G$ in example 52, we get following *genetic table*:

Example 53. Genetic table based on example 52 is given by

ATTT	TAGA	CACT	GTAA	AATG	TTGC	CTCG	GAAC	AGTC	TCGG	CCCC	GGAG	ACTA	TGGT	CGCA	GCAT
CAAA	GTCT	ATGA	TATT	CTAC	GACG	AAGC	TTTG	CCAG	GGCC	AGGG	TCTC	CGAT	GCCA	ACGT	TGTA
GTGA	CATT	TAAA	ATCT	GAGC	CTTG	TTAC	AACG	GGGG	CCTC	TCAG	AGCC	GCGT	CGTA	TGAT	ACCA
TACT	ATAA	GTTT	CAGA	TTCG	AAAC	GATG	CTGC	TCCC	AGAG	GGTC	CCGG	TGCA	ACAT	GCTA	CGGT
AGTA	TCGT	CCCA	GGAT	ACTC	TGGG	CGCC	GCAG	ATTG	TAGC	CACG	GTAC	AATT	TTGA	CTCT	GAAA
CCAT	GGCA	AGGT	TCTA	CGAG	GCCC	ACGG	TGTC	CAAC	GTCG	ATGC	TATG	CTAA	GACT	AAGA	TTTT
GGGT	CCTA	TCAT	AGCA	GCGG	CGTC	TGAG	ACCC	GTGC	CATG	TAAC	ATCG	GAGA	CTTT	TTAA	AACT
TCCA	AGAT	GGTA	CCGT	TGCC	ACAG	GCTC	CGGG	TACG	ATAC	GTTG	CAGC	TTCT	AAAA	GATT	CTGA
ACTG	TGGC	CGCG	GCAC	AGTT	TCGA	CCCT	GGAA	AATA	TTGT	CTCA	GAAT	ATTC	TAGG	CACC	GTAG
CGAC	GCCG	ACGC	TGTG	CCAA	GGCT	AGGA	TCTT	CTAT	GACA	AAGT	TTTA	CAAG	GTCC	ATGG	TATC
GCGC	CGTG	TGAC	ACCG	GGGA	CCTT	TCAA	AGCT	GAGT	CTTA	TTAT	AACA	GTGG	CATC	TAAG	ATCC
TGCG	ACAC	GCTG	CGGC	TCCT	AGAA	GGTT	CCGA	TTCA	AAAT	GATA	CTGT	TACC	ATAG	GTTC	CAGG
AATC	TTGG	CTCC	GAAG	ATTA	TAGT	CACA	GTAT	ACTT	TGGA	CGCT	GCAA	AGTG	TCGC	CCCG	GGAC
CTAG	GACC	AAGG	TTTC	CAAT	GTCA	ATGT	TATA	CGAA	GCCT	ACGA	TGTT	CCAC	GGCG	AGGC	TCTG
GAGG	CTTC	TTAG	AACC	GTGT	CATA	TAAT	ATCA	GCGA	CGTT	TGAA	ACCT	GGGC	CCTG	TCAC	AGCG
TTCC	AAAG	GATC	CTGG	TACA	ATAT	GTTA	CAGT	TGCT	ACAA	GCTT	CGGA	TCCG	AGAC	GGTG	CCGC

Similarly we can write genetic table based on example 51. For more studies on genetic table and applications to Shannon's entropy refer Taneja [15].

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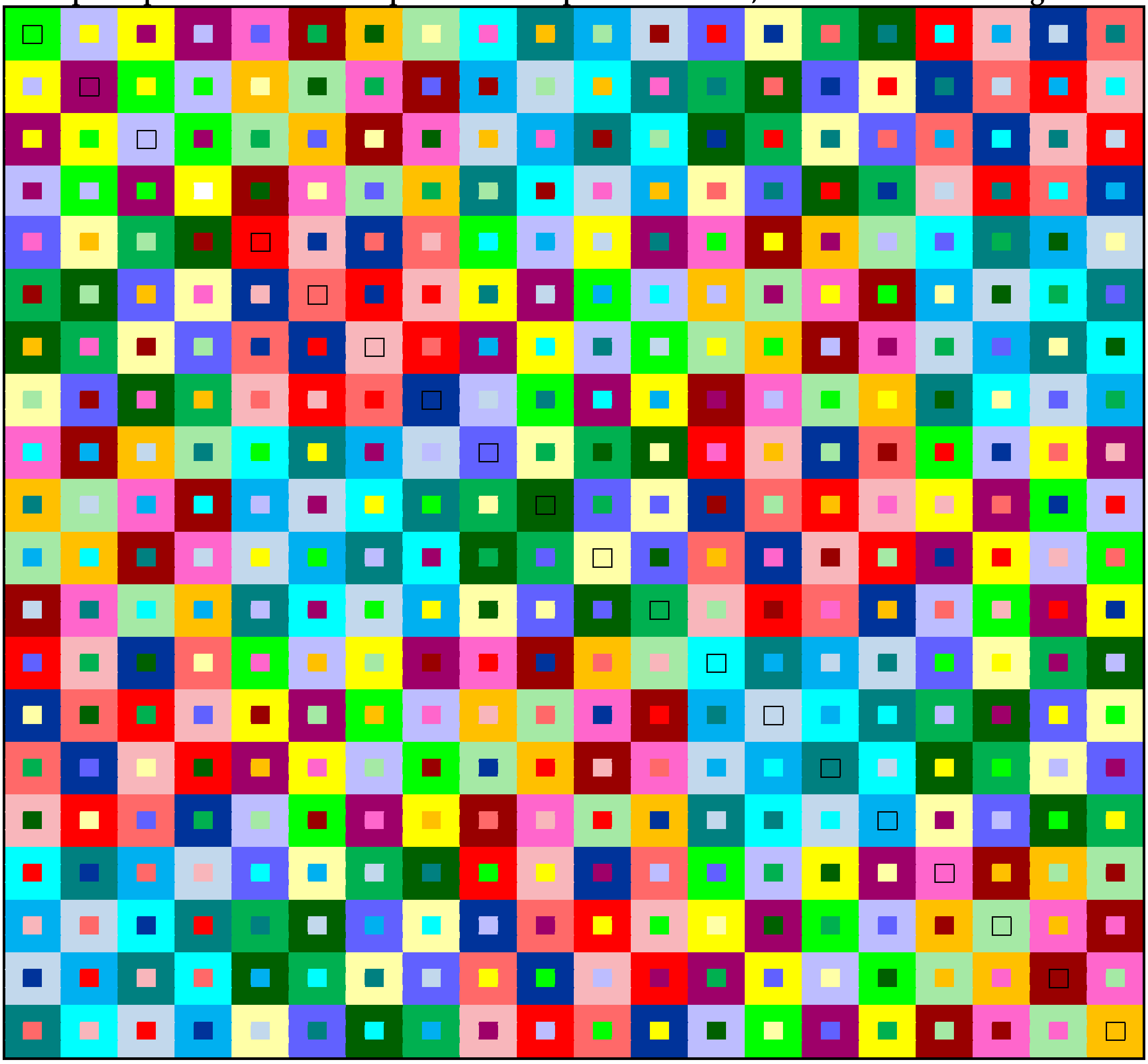
6. Superimposed Double Colors Pattern

Following the same procedure as used in [14], we can write double colored pattern for each magic square of orders 11 to 20. Since order 20 is the last magic square, let us write a doubled colored pattern based on example 16.

Superimposed double colors pattern - No repetition in rows, columns and main diagonals



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Above is double colored pattern is based on example 20, using SODLS and its transpose, where each block of order 4 is also a SODLS resulting in magic *pan diagonal magic squares* of order 4 and 20. Based on same procedure we can make similar grids for other magic squares constructed using MODLS/SODLS. This we call "*superimposed double colors pattern*".

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Final Comments

We constructed magic squares of order 11 to 20 based on *MODLS*, *SODLS*, and using composition relations. The magic square constructed are named "*intervally distributed magic squares*". These kind of magic squares helped us in extending to *palindromic* and *Selfie magic squares*. The construction of prime order magic square either by *MODLS* or *SODLS* are similar. In both the situations the magic squares are *pan diagonal intervally distributed*. In case of magic square of order 16, we have given four different ways of constructions. Two are *bimagic* and another two are *pan diagonal*. The construction of magic square of order 20 is based on composition relations either 4×5 or 5×4. In both the cases the magic squares are *pan diagonal*. Working with compositions, in three situations we get *intervally semi-distributed magic squares*. These magic squares are of orders 12, 15 and 18 given in examples 3, 7 and 14 respectively. Here the compositions are with magic square of order 3, formed by two *MOLS* (non diagonalized), resulting in *intervally semi-distributed magic square*. The compositions made for magic squares of orders 16 and 20 are with *MODLS* resulting in *intervally distributed magic squares*. The construction of *intervally distributed* or *semi-distributed magic squares* helped us bring *palindromic* and *Selfie magic squares*. Finally, genetic table connected with magic square of order 16 and superimposed double colored pattern are presented in sections 5 and 6 respectively. More studies on magic squares can be seen in [2, 5, 6-12].

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