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Intervally Distributed, Palindromic and Selfie Magic Squares – Orders 21 to 25

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Abstract

This work brings magic squares constructed based on intervally distributed numbers. It has been done in different form, such as, using pairwise mutually orthogonal diagonalized Latin squares, self-orthogonal diagonalized Latin squares, block-wise compositions, etc. Work is limited to magic square of orders 21 to 25. In case of magic square of order 21 and 24, the procedure is given in different forms. Results are extended to palindromic and Selfie magic squares. In case of order 25, magic square constructed is pan diagonal and bimagic. No programming language is used, except construction of magic squares of order 21 and 24 resulting in a pair of MOLS.

1. Introduction

We shall work with different kinds of magic squares, such as, intervally distributed palindromic, Selfie, etc. These ideas are divided in following parts:

(i) Intervally distributed magic squares;
(ii) Palindromic representations;
(iii) Selfie magic squares;

Before starting, below are explanations of *intervally distributed magic squares*.

1.1. Intervally Distributed Magic Squares

Let us consider in a sequence of n^2 numbers. Put these numbers in increasing order. Divide n^2 elements in n interval. If elements of each interval appears only once in each row, each column and in principal diagonals, then the we call it ***intervally distributed****. If the distribution is a magic square, we call it a* ***intervally distributed magic squares.*** *If distribution is only in rows and columns, we call it* ***intervally semi-distributed****.*

For example, let us consider 100 (10^2) numbers in a sequence, i.e., from 1 to 100. Let us divide these numbers in the 10 intervals:

1st interval	01-10	1	2	3	4	5	6	7	8	9	10
2nd interval	11-20	11	12	13	14	15	16	17	18	19	20
3rd interval	21-30	21	22	23	24	25	26	27	28	29	30
4th interval	31-40	31	32	33	34	35	36	37	38	39	40
5th interval	41-50	41	42	43	44	45	46	47	48	49	50
6th interval	51-60	51	52	53	54	55	56	57	58	59	60
7th interval	61-70	61	62	63	64	65	66	67	68	69	70
8th interval	71-80	71	72	73	74	75	76	77	78	79	80
9th interval	81-90	81	82	83	84	85	86	87	88	89	90
10th interval	91-100	91	92	93	94	95	96	97	98	99	100

Intervally distributed elements are understood as members of each interval should appear only once in *each row*, in *each column* and in *principal diagonals*, for example,

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1							61		
		65		4					
	6				66				
			62			8			
	68								10
			5	63					
		7						64	
					9				69
70							2		
						67		3	

As explained above, if the distribution is only in rows and columns, it is understood as *intervally semi-distributed*. It is not necessary that all *intervally distributed* numbers always forms a magic square. Also there are magic squares *not necessarily intervally distributed*.

2. Construction of Intervally Distributed Magic Squares

In this section we shall present *intervally distributed magic squares* of orders 21 to 30 using different procedures based on:

(i) "*Mutually orthogonal diagonalized Latin squares*" (*MODLS*). For non diagonalized, we write the pair as *MOLS*;

(ii) "*Self-orthogonal diagonalized Latin squares*" (*SODLS*);

(iii) *Block-wise compositions* (such as 7×3, 7×4, 5×5, etc.).

Later, the results are extended to *palindromic* and *Selfie magic squares*.

2.1. Magic Squares of Order 21

In this subsection, magic squares of order 21 constructed in three two different form. One is based on a composition of magic squares of orders 7 and 3. The second is based on *SODLS*. The third is way is based on *MOLS*.

- **1st Procedure – Block Wise**

We shall bring intervally distributed magic square of order 21 by using composition of magic square of order 7 with of order 3. Let us consider *pan diagonal magic square* of order 7 constructed based on *SODLS* and its transpose:

1	2	3	4	5	6	7
6	7	1	2	3	4	5
4	5	6	7	1	2	3
2	3	4	5	6	7	1
7	1	2	3	4	5	6
5	6	7	1	2	3	4
3	4	5	6	7	1	2

A

1	6	4	2	7	5	3
2	7	5	3	1	6	4
3	1	6	4	2	7	5
4	2	7	5	3	1	6
5	3	1	6	4	2	7
6	4	2	7	5	3	1
7	5	3	1	6	4	2

B

	175	175	175	175	175	175	175	
	1	13	18	23	35	40	45	175
175	37	49	5	10	15	27	32	175
175	24	29	41	46	2	14	19	175
175	11	16	28	33	38	43	6	175
175	47	3	8	20	25	30	42	175
175	34	39	44	7	12	17	22	175
175	21	26	31	36	48	4	9	175
	175	175	175	175	175	175	175	175

… (1)

Let us consider a magic square of order 3:

												15
1	3	2	2	3	1	12	33	21	2	9	4	15
3	2	1	1	2	3	31	22	13	7	5	3	15
2	1	3	3	1	2	23	11	32	6	1	8	15
A			B			AB			15	15	15	15

… (2)

Let us write the numbers 1-21 in following 3 intervals:

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1	6	7	12	13	18	19	76
2	5	8	11	14	17	20	77
3	4	9	10	15	16	21	78

… (3)

Apply values given in (3) over A and B in (1) and put according to AB in (2), we get 9 compositions, such as

13

1	6	7	12	13	18	19
18	19	1	6	7	12	13
12	13	18	19	1	6	7
6	7	12	13	18	19	1
19	1	6	7	12	13	18
13	18	19	1	6	7	12
7	12	13	18	19	1	6

A

3	16	10	4	21	15	9
4	21	15	9	3	16	10
9	3	16	10	4	21	15
10	4	21	15	9	3	16
15	9	3	16	10	4	21
16	10	4	21	15	9	3
21	15	9	3	16	10	4

B

	1527	1527	1527	1527	1527	1527	1527	
1527	3	121	136	235	273	372	387	1527
1527	361	399	15	114	129	247	262	1527
1527	240	255	373	388	4	126	141	1527
1527	115	130	252	267	366	381	16	1527
1527	393	9	108	142	241	256	378	1527
1527	268	367	382	21	120	135	234	1527
1527	147	246	261	360	394	10	109	1527
	1527	1527	1527	1527	1527	1527	1527	1527

23

2	5	8	11	14	17	20
17	20	2	5	8	11	14
11	14	17	20	2	5	8
5	8	11	14	17	20	2
20	2	5	8	11	14	17
14	17	20	2	5	8	11
8	11	14	17	20	2	5

A

3	16	10	4	21	15	9
4	21	15	9	3	16	10
9	3	16	10	4	21	15
10	4	21	15	9	3	16
15	9	3	16	10	4	21
16	10	4	21	15	9	3
21	15	9	3	16	10	4

B

	1548	1548	1548	1548	1548	1548	1548	
1548	24	100	157	214	294	351	408	1548
1548	340	420	36	93	150	226	283	1548
1548	219	276	352	409	25	105	162	1548
1548	94	151	231	288	345	402	37	1548
1548	414	30	87	163	220	277	357	1548
1548	289	346	403	42	99	156	213	1548
1548	168	225	282	339	415	31	88	1548
	1548	1548	1548	1548	1548	1548	1548	1548

Similarly we make another 7 compositions. Putting according to AB given (2), we get a pair of MOLS:

2	5	8	11	14	17	20	3	4	9	10	15	16	21	1	6	7	12	13	18	19
17	20	2	5	8	11	14	16	21	3	4	9	10	15	18	19	1	6	7	12	13
11	14	17	20	2	5	8	10	15	16	21	3	4	9	12	13	18	19	1	6	7
5	8	11	14	17	20	2	4	9	10	15	16	21	3	6	7	12	13	18	19	1
20	2	5	8	11	14	17	21	3	4	9	10	15	16	19	1	6	7	12	13	18
14	17	20	2	5	8	11	15	16	21	3	4	9	10	13	18	19	1	6	7	12
8	11	14	17	20	2	5	9	10	15	16	21	3	4	7	12	13	18	19	1	6
1	6	7	12	13	18	19	2	5	8	11	14	17	20	3	4	9	10	15	16	21
18	19	1	6	7	12	13	17	20	2	5	8	11	14	16	21	3	4	9	10	15
12	13	18	19	1	6	7	11	14	17	20	2	5	8	10	15	16	21	3	4	9
6	7	12	13	18	19	1	5	8	11	14	17	20	2	4	9	10	15	16	21	3
19	1	6	7	12	13	18	20	2	5	8	11	14	17	21	3	4	9	10	15	16
13	18	19	1	6	7	12	14	17	20	2	5	8	11	15	16	21	3	4	9	10
7	12	13	18	19	1	6	8	11	14	17	20	2	5	9	10	15	16	21	3	4
3	4	9	10	15	16	21	1	6	7	12	13	18	19	2	5	8	11	14	17	20
16	21	3	4	9	10	15	18	19	1	6	7	12	13	17	20	2	5	8	11	14
10	15	16	21	3	4	9	12	13	18	19	1	6	7	11	14	17	20	2	5	8
4	9	10	15	16	21	3	6	7	12	13	18	19	1	5	8	11	14	17	20	2
21	3	4	9	10	15	16	19	1	6	7	12	13	18	20	2	5	8	11	14	17
15	16	21	3	4	9	10	13	18	19	1	6	7	12	14	17	20	2	5	8	11
9	10	15	16	21	3	4	7	12	13	18	19	1	6	8	11	14	17	20	2	5

A

1	18	12	6	19	13	7	3	16	10	4	21	15	9	2	17	11	5	20	14	8
6	19	13	7	1	18	12	4	21	15	9	3	16	10	5	20	14	8	2	17	11
7	1	18	12	6	19	13	9	3	16	10	4	21	15	8	2	17	11	5	20	14
12	6	19	13	7	1	18	10	4	21	15	9	3	16	11	5	20	14	8	2	17
13	7	1	18	12	6	19	15	9	3	16	10	4	21	14	8	2	17	11	5	20
18	12	6	19	13	7	1	16	10	4	21	15	9	3	17	11	5	20	14	8	2
19	13	7	1	18	12	6	21	15	9	3	16	10	4	20	14	8	2	17	11	5
3	16	10	4	21	15	9	2	17	11	5	20	14	8	1	18	12	6	19	13	7
4	21	15	9	3	16	10	5	20	14	8	2	17	11	6	19	13	7	1	18	12
9	3	16	10	4	21	15	8	2	17	11	5	20	14	7	1	18	12	6	19	13
10	4	21	15	9	3	16	11	5	20	14	8	2	17	12	6	19	13	7	1	18
15	9	3	16	10	4	21	14	8	2	17	11	5	20	13	7	1	18	12	6	19
16	10	4	21	15	9	3	17	11	5	20	14	8	2	18	12	6	19	13	7	1
21	15	9	3	16	10	4	20	14	8	2	17	11	5	19	13	7	1	18	12	6
2	17	11	5	20	14	8	1	18	12	6	19	13	7	3	16	10	4	21	15	9
5	20	14	8	2	17	11	6	19	13	7	1	18	12	4	21	15	9	3	16	10
8	2	17	11	5	20	14	7	1	18	12	6	19	13	9	3	16	10	4	21	15
11	5	20	14	8	2	17	12	6	19	13	7	1	18	10	4	21	15	9	3	16
14	8	2	17	11	5	20	13	7	1	18	12	6	19	15	9	3	16	10	4	21
17	11	5	20	14	8	2	18	12	6	19	13	7	1	16	10	4	21	15	9	3
20	14	8	2	17	11	5	19	13	7	1	18	12	6	21	15	9	3	16	10	4

B

… (4)

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Example 1. Apply the procedure $21\times(A-1)+B$ in (4), we get *intervally semi-distributed magic square* of order 21:

																						4641
22	102	159	216	292	349	406	45	79	178	193	315	330	429	2	122	137	236	272	371	386		4641
342	418	34	91	148	228	285	319	441	57	72	171	205	304	362	398	14	113	128	248	263		4641
217	274	354	411	27	103	160	198	297	331	430	46	84	183	239	254	374	389	5	125	140		4641
96	153	229	286	343	400	39	73	172	210	309	324	423	58	116	131	251	266	365	380	17		4641
412	28	85	165	222	279	355	435	51	66	184	199	298	336	392	8	107	143	242	257	377		4641
291	348	405	40	97	154	211	310	325	424	63	78	177	192	269	368	383	20	119	134	233		4641
166	223	280	337	417	33	90	189	204	303	318	436	52	67	146	245	260	359	395	11	110		4641
3	121	136	235	273	372	387	23	101	158	215	293	350	407	43	81	180	195	313	328	427		4641
361	399	15	114	129	247	262	341	419	35	92	149	227	284	321	439	55	70	169	207	306		4641
240	255	373	388	4	126	141	218	275	353	410	26	104	161	196	295	333	432	48	82	181		4641
115	130	252	267	366	381	16	95	152	230	287	344	401	38	75	174	208	307	322	421	60		4641
393	9	108	142	241	256	378	413	29	86	164	221	278	356	433	49	64	186	201	300	334		4641
268	367	382	21	120	135	234	290	347	404	41	98	155	212	312	327	426	61	76	175	190		4641
147	246	261	360	394	10	109	167	224	281	338	416	32	89	187	202	301	316	438	54	69		4641
44	80	179	194	314	329	428	1	123	138	237	271	370	385	24	100	157	214	294	351	408		4641
320	440	56	71	170	206	305	363	397	13	112	127	249	264	340	420	36	93	150	226	283		4641
197	296	332	431	47	83	182	238	253	375	390	6	124	139	219	276	352	409	25	105	162		4641
74	173	209	308	323	422	59	117	132	250	265	364	379	18	94	151	231	288	345	402	37		4641
434	50	65	185	200	299	335	391	7	106	144	243	258	376	414	30	87	163	220	277	357		4641
311	326	425	62	77	176	191	270	369	384	19	118	133	232	289	346	403	42	99	156	213		4641
188	203	302	317	437	53	68	145	244	259	358	396	12	111	168	225	282	339	415	31	88		4641
4641	4641	4641	4641	4641	4641	4641	4641	4641	4641	4641	4641	4641	4641	4641	4641	4641	4641	4641	4641	4641	4641	

Moreover each block of 7 is a *pan diagonal magic square* with different sums resulting in *magic square* of order 21 with magic sum 4641:

			4641
1546	1569	1526	4641
1527	1547	1567	4641
1568	1525	1548	4641
4641	4641	4641	4641

It is *intervally semi-distributed* as there are elements in one of the diagonal belongs to same intervals of distribution:

2nd interval	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40	41	42
5th interval	85	86	87	88	89	90	91	92	93	94	95	96	97	98	99	100	101	102	103	104	105
8th interval	148	149	150	151	152	153	154	155	156	157	158	159	160	161	162	163	164	165	166	167	168
11th interval	211	212	213	214	215	216	217	218	219	220	221	222	223	224	225	226	227	228	229	230	231
14th interval	274	275	276	277	278	279	280	281	282	283	284	285	286	287	288	289	290	291	292	293	294
17th interval	337	338	339	340	341	342	343	344	345	346	347	348	349	350	351	352	353	354	355	356	357
20th interval	400	401	402	403	404	405	406	407	408	409	410	411	412	413	414	415	416	417	418	419	420

The above table show three values in each interval appearing in one of the diagonal.

- **2nd Procedure - SODLS**

Let us consider the grid A and B, where A is *SODLS* and B is its transpose. Its construction (M. Nakamura [7]) based on the work Bennetta, Du and Zhang [1].

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1	13	4	3	18	21	19	20	14	17	15	16	2	9	12	10	11	5	8	6	7
3	2	1	13	20	19	21	18	16	15	17	14	4	11	10	12	9	7	6	8	5
13	4	3	2	21	18	20	19	17	14	16	15	1	12	9	11	10	8	5	7	6
2	1	13	4	19	20	18	21	15	16	14	17	3	10	11	9	12	6	7	5	8
9	12	10	11	5	13	8	7	1	4	2	3	6	18	21	19	20	14	17	15	16
11	10	12	9	7	6	5	13	3	2	4	1	8	20	19	21	18	16	15	17	14
12	9	11	10	13	8	7	6	4	1	3	2	5	21	18	20	19	17	14	16	15
10	11	9	12	6	5	13	8	2	3	1	4	7	19	20	18	21	15	16	14	17
18	21	19	20	14	17	15	16	9	13	12	11	10	5	8	6	7	1	4	2	3
20	19	21	18	16	15	17	14	11	10	9	13	12	7	6	8	5	3	2	4	1
21	18	20	19	17	14	16	15	13	12	11	10	9	8	5	7	6	4	1	3	2
19	20	18	21	15	16	14	17	10	9	13	12	11	6	7	5	8	2	3	1	4
4	3	2	1	8	7	6	5	12	11	10	9	13	17	16	15	14	21	20	19	18
5	8	6	7	1	4	2	3	18	21	19	20	15	14	13	17	16	9	12	10	11
7	6	8	5	3	2	4	1	20	19	21	18	17	16	15	14	13	11	10	12	9
8	5	7	6	4	1	3	2	21	18	20	19	14	13	17	16	15	12	9	11	10
6	7	5	8	2	3	1	4	19	20	18	21	16	15	14	13	17	10	11	9	12
14	17	15	16	9	12	10	11	5	8	6	7	19	1	4	2	3	18	13	21	20
16	15	17	14	11	10	12	9	7	6	8	5	21	3	2	4	1	20	19	18	13
17	14	16	15	12	9	11	10	8	5	7	6	18	4	1	3	2	13	21	20	19
15	16	14	17	10	11	9	12	6	7	5	8	20	2	3	1	4	19	18	13	21

A

1	3	13	2	9	11	12	10	18	20	21	19	4	5	7	8	6	14	16	17	15
13	2	4	1	12	10	9	11	21	19	18	20	3	8	6	5	7	17	15	14	16
4	1	3	13	10	12	11	9	19	21	20	18	2	6	8	7	5	15	17	16	14
3	13	2	4	11	9	10	12	20	18	19	21	1	7	5	6	8	16	14	15	17
18	20	21	19	5	7	13	6	14	16	17	15	8	1	3	4	2	9	11	12	10
21	19	18	20	13	6	8	5	17	15	14	16	7	4	2	1	3	12	10	9	11
19	21	20	18	8	5	7	13	15	17	16	14	6	2	4	3	1	10	12	11	9
20	18	19	21	7	13	6	8	16	14	15	17	5	3	1	2	4	11	9	10	12
14	16	17	15	1	3	4	2	9	11	13	10	12	18	20	21	19	5	7	8	6
17	15	14	16	4	2	1	3	13	10	12	9	11	21	19	18	20	8	6	5	7
15	17	16	14	2	4	3	1	12	9	11	13	10	19	21	20	18	6	8	7	5
16	14	15	17	3	1	2	4	11	13	10	12	9	20	18	19	21	7	5	6	8
2	4	1	3	6	8	5	7	10	12	9	11	13	15	17	14	16	19	21	18	20
9	11	12	10	18	20	21	19	5	7	8	6	17	14	16	13	15	1	3	4	2
12	10	9	11	21	19	18	20	8	6	5	7	16	13	15	17	14	4	2	1	3
10	12	11	9	19	21	20	18	6	8	7	5	15	17	14	16	13	2	4	3	1
11	9	10	12	20	18	19	21	7	5	6	8	14	16	13	15	17	3	1	2	4
5	7	8	6	14	16	17	15	1	3	4	2	21	9	11	12	10	18	20	13	19
8	6	5	7	17	15	14	16	4	2	1	3	20	12	10	9	11	13	19	21	18
6	8	7	5	15	17	16	14	2	4	3	1	19	10	12	11	9	21	18	20	13
7	5	6	8	16	14	15	17	3	1	2	4	18	11	9	10	12	20	13	19	21

B

… (5)

Example 2. Apply the procedure $21\times(A-1)+B$ in (5), we get *intervally distributed magic square* of order 21:

																					4641
1	255	76	44	**366**	**431**	**390**	**409**	**291**	**356**	**315**	**334**	25	**173**	**238**	**197**	**216**	**98**	**163**	**122**	**141**	4641
55	23	4	253	**411**	**388**	**429**	**368**	**336**	**313**	**354**	**293**	66	**218**	**195**	**236**	**175**	**143**	**120**	**161**	**100**	4641
256	64	45	34	**430**	**369**	**410**	**387**	**355**	**294**	**335**	**312**	2	**237**	**176**	**217**	**194**	**162**	**101**	**142**	**119**	4641
24	13	254	67	**389**	**408**	**367**	**432**	**314**	**333**	**292**	**357**	43	**196**	**215**	**174**	**239**	**121**	**140**	**99**	**164**	4641
186	**251**	**210**	**229**	89	259	160	132	**14**	**79**	**38**	**57**	113	**358**	**423**	**382**	**401**	**282**	**347**	**306**	**325**	4641
231	**208**	**249**	**188**	139	111	92	257	**59**	**36**	**77**	**16**	154	**403**	**380**	**421**	**360**	**327**	**304**	**345**	**284**	4641
250	**189**	**230**	**207**	260	152	133	118	**78**	**17**	**58**	**35**	90	**422**	**361**	**402**	**379**	**346**	**285**	**326**	**303**	4641
209	**228**	**187**	**252**	112	97	258	155	**37**	**56**	**15**	**80**	131	**381**	**400**	**359**	**424**	**305**	**324**	**283**	**348**	4641
371	**436**	**395**	**414**	**274**	**339**	**298**	**317**	177	263	244	220	201	**102**	**167**	**126**	**145**	**5**	**70**	**29**	**48**	4641
416	**393**	**434**	**373**	**319**	**296**	**337**	**276**	223	199	180	261	242	**147**	**124**	**165**	**104**	**50**	**27**	**68**	**7**	4641
435	**374**	**415**	**392**	**338**	**277**	**318**	**295**	264	240	221	202	178	**166**	**105**	**146**	**123**	**69**	**8**	**49**	**26**	4641
394	**413**	**372**	**437**	**297**	**316**	**275**	**340**	200	181	262	243	219	**125**	**144**	**103**	**168**	**28**	**47**	**6**	**71**	4641
65	46	22	3	153	134	110	91	241	222	198	179	265	351	332	308	289	439	420	396	377	4641
93	**158**	**117**	**136**	**18**	**83**	**42**	**61**	**362**	**427**	**386**	**405**	311	287	268	349	330	**169**	**234**	**193**	**212**	4641
138	**115**	**156**	**95**	**63**	**40**	**81**	**20**	**407**	**384**	**425**	**364**	352	328	309	290	266	**214**	**191**	**232**	**171**	4641
157	**96**	**137**	**114**	**82**	**21**	**62**	**39**	**426**	**365**	**406**	**383**	288	269	350	331	307	**233**	**172**	**213**	**190**	4641
116	**135**	**94**	**159**	**41**	**60**	**19**	**84**	**385**	**404**	**363**	**428**	329	310	286	267	353	**192**	**211**	**170**	**235**	4641
278	**343**	**302**	**321**	**182**	**247**	**206**	**225**	**85**	**150**	**109**	**128**	399	**9**	**74**	**33**	**52**	375	272	433	418	4641
323	**300**	**341**	**280**	**227**	**204**	**245**	**184**	**130**	**107**	**148**	**87**	440	**54**	**31**	**72**	**11**	412	397	378	270	4641
342	**281**	**322**	**299**	**246**	**185**	**226**	**203**	**149**	**88**	**129**	**106**	376	**73**	**12**	**53**	**30**	273	438	419	391	4641
301	**320**	**279**	**344**	**205**	**224**	**183**	**248**	**108**	**127**	**86**	**151**	417	**32**	**51**	**10**	**75**	398	370	271	441	4641
4641	4641	4641	4641	4641	4641	4641	4641	4641	4641	4641	4641	4641	4641	4641	4641	4641	4641	4641	4641	4641	4641

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There are 20 broken magic squares of order 4 appearing in above magic square (bold face). See below,

				808
169	234	193	212	808
214	191	232	171	808
233	172	213	190	808
192	211	170	235	808
808	808	808	808	808

				168
9	74	33	52	168
54	31	72	11	168
73	12	53	30	168
32	51	10	75	168
168	168	168	168	168

				152
5	70	29	48	152
50	27	68	7	152
69	8	49	26	152
28	47	6	71	152
152	152	152	152	152

				524
98	163	122	141	524
143	120	161	100	524
162	101	142	119	524
121	140	99	164	524
524	524	524	524	524

				1260
282	347	306	325	1260
327	304	345	284	1260
346	285	326	303	1260
305	324	283	348	1260
1260	1260	1260	1260	1260

				824
173	238	197	216	824
218	195	236	175	824
237	176	217	194	824
196	215	174	239	824
824	824	824	824	824

				1564
358	423	382	401	1564
403	380	421	360	1564
422	361	402	379	1564
381	400	359	424	1564
1564	1564	1564	1564	1564

				540
102	167	126	145	540
147	124	165	104	540
166	105	146	123	540
125	144	103	168	540
540	540	540	540	540

				876
186	251	210	229	876
231	208	249	188	876
250	189	230	207	876
209	228	187	252	876
876	876	876	876	876

				1616
371	436	395	414	1616
416	393	434	373	1616
435	374	415	392	1616
394	413	372	437	1616
1616	1616	1616	1616	1616

				1244
278	343	302	321	1244
323	300	341	280	1244
342	281	322	299	1244
301	320	279	344	1244
1244	1244	1244	1244	1244

				1596
366	431	390	409	1596
411	388	429	368	1596
430	369	410	387	1596
389	408	367	432	1596
1596	1596	1596	1596	1596

				1228
274	339	298	317	1228
319	296	337	276	1228
338	277	318	295	1228
297	316	275	340	1228
1228	1228	1228	1228	1228

				1296
291	356	315	334	1296
336	313	354	293	1296
355	294	335	312	1296
314	333	292	357	1296
1296	1296	1296	1296	1296

				188
14	79	38	57	188
59	36	77	16	188
78	17	58	35	188
37	56	15	80	188
188	188	188	188	188

				472
85	150	109	128	472
130	107	148	87	472
149	88	129	106	472
108	127	86	151	472
472	472	472	472	472

				860
182	247	206	225	860
227	204	245	184	860
246	185	226	203	860
205	224	183	248	860
860	860	860	860	860

				1580
362	427	386	405	1580
407	384	425	364	1580
426	365	406	383	1580
385	404	363	428	1580
1580	1580	1580	1580	1580

				504
93	158	117	136	504
138	115	156	95	504
157	96	137	114	504
116	135	94	159	504
504	504	504	504	504

				204
18	83	42	61	204
63	40	81	20	204
82	21	62	39	204
41	60	19	84	204
204	204	204	204	204

- **3rd Procedure - MOLS**

Let us consider a pair of *MOLS* constructed by use of computer program (H. White [19]) given by

1	2	3	4	10	7	9	19	18	13	14	15	17	12	20	6	5	16	8	21	11
11	12	13	14	20	17	19	8	7	2	3	4	6	1	9	16	15	5	18	10	21
21	1	2	3	9	6	8	18	17	12	13	14	16	11	19	5	4	15	7	20	10
10	11	12	13	19	16	18	7	6	1	2	3	5	21	8	15	14	4	17	9	20
20	21	1	2	8	5	7	17	16	11	12	13	15	10	18	4	3	14	6	19	9
9	10	11	12	18	15	17	6	5	21	1	2	4	20	7	14	13	3	16	8	19
19	20	21	1	7	4	6	16	15	10	11	12	14	9	17	3	2	13	5	18	8
8	9	10	11	17	14	16	5	4	20	21	1	3	19	6	13	12	2	15	7	18
18	19	20	21	6	3	5	15	14	9	10	11	13	8	16	2	1	12	4	17	7
7	8	9	10	16	13	15	4	3	19	20	21	2	18	5	12	11	1	14	6	17
17	18	19	20	5	2	4	14	13	8	9	10	12	7	15	1	21	11	3	16	6
6	7	8	9	15	12	14	3	2	18	19	20	1	17	4	11	10	21	13	5	16
16	17	18	19	4	1	3	13	12	7	8	9	11	6	14	21	20	10	2	15	5
5	6	7	8	14	11	13	2	1	17	18	19	21	16	3	10	9	20	12	4	15
12	13	14	15	21	18	20	9	8	3	4	5	7	2	10	17	16	6	19	11	1
2	3	4	5	11	8	10	20	19	14	15	16	18	13	21	7	6	17	9	1	12
13	14	15	16	1	19	21	10	9	4	5	6	8	3	11	18	17	7	20	12	2
3	4	5	6	12	9	11	21	20	15	16	17	19	14	1	8	7	18	10	2	13
14	15	16	17	2	20	1	11	10	5	6	7	9	4	12	19	18	8	21	13	3
4	5	6	7	13	10	12	1	21	16	17	18	20	15	2	9	8	19	11	3	14
15	16	17	18	3	21	2	12	11	6	7	8	10	5	13	20	19	9	1	14	4

A

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1	2	3	4	10	7	9	19	18	13	14	15	17	12	20	6	5	16	8	21	11
12	13	14	15	21	18	20	9	8	3	4	5	7	2	10	17	16	6	19	11	1
2	3	4	5	11	8	10	20	19	14	15	16	18	13	21	7	6	17	9	1	12
13	14	15	16	1	19	21	10	9	4	5	6	8	3	11	18	17	7	20	12	2
3	4	5	6	12	9	11	21	20	15	16	17	19	14	1	8	7	18	10	2	13
14	15	16	17	2	20	1	11	10	5	6	7	9	4	12	19	18	8	21	13	3
4	5	6	7	13	10	12	1	21	16	17	18	20	15	2	9	8	19	11	3	14
15	16	17	18	3	21	2	12	11	6	7	8	10	5	13	20	19	9	1	14	4
5	6	7	8	14	11	13	2	1	17	18	19	21	16	3	10	9	20	12	4	15
16	17	18	19	4	1	3	13	12	7	8	9	11	6	14	21	20	10	2	15	5
6	7	8	9	15	12	14	3	2	18	19	20	1	17	4	11	10	21	13	5	16
17	18	19	20	5	2	4	14	13	8	9	10	12	7	15	1	21	11	3	16	6
7	8	9	10	16	13	15	4	3	19	20	21	2	18	5	12	11	1	14	6	17
18	19	20	21	6	3	5	15	14	9	10	11	13	8	16	2	1	12	4	17	7
11	12	13	14	20	17	19	8	7	2	3	4	6	1	9	16	15	5	18	10	21
21	1	2	3	9	6	8	18	17	12	13	14	16	11	19	5	4	15	7	20	10
10	11	12	13	19	16	18	7	6	1	2	3	5	21	8	15	14	4	17	9	20
20	21	1	2	8	5	7	17	16	11	12	13	15	10	18	4	3	14	6	19	9
9	10	11	12	18	15	17	6	5	21	1	2	4	20	7	14	13	3	16	8	19
19	20	21	1	7	4	6	16	15	10	11	12	14	9	17	3	2	13	5	18	8
8	9	10	11	17	14	16	5	4	20	21	1	3	19	6	13	12	2	15	7	18

B

… (6)

The grid A is *diagonalized Latin square*, while grid B *not diagonalized*. Even then resulting in a *intervally distributed magic square* of order 21.

Example 3. Apply the procedure $21\times(A-1)+B$ in (6), we get *intervally distributed magic square* of order 21:

																					4641
1	23	45	67	199	133	177	397	375	265	287	309	353	243	419	111	89	331	155	441	221	4641
222	244	266	288	420	354	398	156	134	24	46	68	112	2	178	332	310	90	376	200	421	4641
422	3	25	47	179	113	157	377	355	245	267	289	333	223	399	91	69	311	135	400	201	4641
202	224	246	268	379	334	378	136	114	4	26	48	92	423	158	312	290	70	356	180	401	4641
402	424	5	27	159	93	137	357	335	225	247	269	313	203	358	71	49	291	115	380	181	4641
182	204	226	248	359	314	337	116	94	425	6	28	72	403	138	292	270	50	336	160	381	4641
382	404	426	7	139	73	117	316	315	205	227	249	293	183	338	51	29	271	95	360	161	4641
162	184	206	228	339	294	317	96	74	405	427	8	52	383	118	272	250	30	295	140	361	4641
362	384	406	428	119	53	97	296	274	185	207	229	273	163	318	31	9	251	75	340	141	4641
142	164	186	208	319	253	297	76	54	385	407	429	32	363	98	252	230	10	275	120	341	4641
342	364	386	408	99	33	77	276	254	165	187	209	232	143	298	11	430	231	55	320	121	4641
122	144	166	188	299	233	277	56	34	365	387	409	12	343	78	211	210	431	255	100	321	4641
322	344	366	388	79	13	57	256	234	145	167	189	212	123	278	432	410	190	35	300	101	4641
102	124	146	168	279	213	257	36	14	345	367	389	433	323	58	191	169	411	235	80	301	4641
242	264	286	308	440	374	418	176	154	44	66	88	132	22	198	352	330	110	396	220	21	4641
42	43	65	87	219	153	197	417	395	285	307	329	373	263	439	131	109	351	175	20	241	4641
262	284	306	328	19	394	438	196	174	64	86	108	152	63	218	372	350	130	416	240	41	4641
62	84	85	107	239	173	217	437	415	305	327	349	393	283	18	151	129	371	195	40	261	4641
282	304	326	348	39	414	17	216	194	105	106	128	172	83	238	392	370	150	436	260	61	4641
82	104	126	127	259	193	237	16	435	325	347	369	413	303	38	171	149	391	215	60	281	4641
302	324	346	368	59	434	37	236	214	125	147	148	192	103	258	412	390	170	15	280	81	4641
4641	4641	4641	4641	4641	4641	4641	4641	4641	4641	4641	4641	4641	4641	4641	4641	4641	4641	4641	4641	4641	4641

2.2. Magic Squares of Order 22

Let us consider the grid A and B, where A is *SODLS* and B is its transpose constructed (M. Nakamura [7]) based on work of Bennetta, Du and Zhang [1].

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1	5	4	10	6	18	22	21	20	19	2	3	11	12	9	8	7	13	17	16	15	14
6	2	1	5	10	20	19	18	22	21	3	4	8	7	11	12	9	15	14	13	17	16
10	6	3	2	1	22	21	20	19	18	4	5	12	9	8	7	11	17	16	15	14	13
2	10	6	4	3	19	18	22	21	20	5	1	7	11	12	9	8	14	13	17	16	15
4	3	10	6	5	21	20	19	18	22	1	2	9	8	7	11	12	16	15	14	13	17
13	17	16	15	14	6	12	11	10	9	8	7	18	22	21	20	19	1	5	4	3	2
15	14	13	17	16	8	7	6	12	11	10	9	20	19	18	22	21	3	2	1	5	4
17	16	15	14	13	10	9	8	7	6	12	11	22	21	20	19	18	5	4	3	2	1
14	13	17	16	15	12	11	10	9	8	7	6	19	18	22	21	20	2	1	5	4	3
16	15	14	13	17	7	6	12	11	10	9	8	21	20	19	18	22	4	3	2	1	5
3	4	5	1	2	9	8	7	6	12	11	10	15	16	17	13	14	20	21	22	18	19
5	1	2	3	4	11	10	9	8	7	6	12	17	13	14	15	16	22	18	19	20	21
18	22	21	20	19	1	5	4	3	2	14	15	13	17	16	10	6	11	12	9	8	7
20	19	18	22	21	3	2	1	5	4	15	16	6	14	13	17	10	8	7	11	12	9
22	21	20	19	18	5	4	3	2	1	16	17	10	6	15	14	13	12	9	8	7	11
19	18	22	21	20	2	1	5	4	3	17	13	14	10	6	16	15	7	11	12	9	8
21	20	19	18	22	4	3	2	1	5	13	14	16	15	10	6	17	9	8	7	11	12
11	12	9	8	7	13	17	16	15	14	19	20	1	5	4	3	2	18	22	21	10	6
8	7	11	12	9	15	14	13	17	16	20	21	3	2	1	5	4	6	19	18	22	10
12	9	8	7	11	17	16	15	14	13	21	22	5	4	3	2	1	10	6	20	19	18
7	11	12	9	8	14	13	17	16	15	22	18	2	1	5	4	3	19	10	6	21	20
9	8	7	11	12	16	15	14	13	17	18	19	4	3	2	1	5	21	20	10	6	22

A

1	6	10	2	4	13	15	17	14	16	3	5	18	20	22	19	21	11	8	12	7	9
5	2	6	10	3	17	14	16	13	15	4	1	22	19	21	18	20	12	7	9	11	8
4	1	3	6	10	16	13	15	17	14	5	2	21	18	20	22	19	9	11	8	12	7
10	5	2	4	6	15	17	14	16	13	1	3	20	22	19	21	18	8	12	7	9	11
6	10	1	3	5	14	16	13	15	17	2	4	19	21	18	20	22	7	9	11	8	12
18	20	22	19	21	6	8	10	12	7	9	11	1	3	5	2	4	13	15	17	14	16
22	19	21	18	20	12	7	9	11	6	8	10	5	2	4	1	3	17	14	16	13	15
21	18	20	22	19	11	6	8	10	12	7	9	4	1	3	5	2	16	13	15	17	14
20	22	19	21	18	10	12	7	9	11	6	8	3	5	2	4	1	15	17	14	16	13
19	21	18	20	22	9	11	6	8	10	12	7	2	4	1	3	5	14	16	13	15	17
2	3	4	5	1	8	10	12	7	9	11	6	14	15	16	17	13	19	20	21	22	18
3	4	5	1	2	7	9	11	6	8	10	12	15	16	17	13	14	20	21	22	18	19
11	8	12	7	9	18	20	22	19	21	15	17	13	6	10	14	16	1	3	5	2	4
12	7	9	11	8	22	19	21	18	20	16	13	17	14	6	10	15	5	2	4	1	3
9	11	8	12	7	21	18	20	22	19	17	14	16	13	15	6	10	4	1	3	5	2
8	12	7	9	11	20	22	19	21	18	13	15	10	17	14	16	6	3	5	2	4	1
7	9	11	8	12	19	21	18	20	22	14	16	6	10	13	15	17	2	4	1	3	5
13	15	17	14	16	1	3	5	2	4	20	22	11	8	12	7	9	18	6	10	19	21
17	14	16	13	15	5	2	4	1	3	21	18	12	7	9	11	8	22	19	6	10	20
16	13	15	17	14	4	1	3	5	2	22	19	9	11	8	12	7	21	18	20	6	10
15	17	14	16	13	3	5	2	4	1	18	20	8	12	7	9	11	10	22	19	21	6
14	16	13	15	17	2	4	1	3	5	19	21	7	9	11	8	12	6	10	18	20	22

B

… (7)

Example 4. Apply the procedure $22\times(A-1)+B$ in (7), we get *intervally distributed magic square* of order 22:

																						5335
1	94	76	200	114	**387**	**477**	**457**	**432**	**412**	25	49	**238**	**262**	**198**	**173**	**153**	**275**	**360**	**342**	**315**	**295**	5335
115	24	6	98	201	**435**	**410**	**390**	**475**	**455**	48	67	**176**	**151**	**241**	**260**	**196**	**320**	**293**	**273**	**363**	**338**	5335
202	111	47	28	10	**478**	**453**	**433**	**413**	**388**	71	90	**263**	**194**	**174**	**154**	**239**	**361**	**341**	**316**	**298**	**271**	5335
32	203	112	70	50	**411**	**391**	**476**	**456**	**431**	89	3	**152**	**242**	**261**	**197**	**172**	**294**	**276**	**359**	**339**	**319**	5335
72	54	199	113	93	**454**	**434**	**409**	**389**	**479**	2	26	**195**	**175**	**150**	**240**	**264**	**337**	**317**	**297**	**272**	**364**	5335
282	**372**	**352**	**327**	**307**	**116**	**250**	**230**	**210**	**183**	**163**	**143**	**375**	**465**	**445**	**420**	**400**	**13**	**103**	**83**	**58**	**38**	5335
330	**305**	**285**	**370**	**350**	**166**	**139**	**119**	**253**	**226**	**206**	**186**	**423**	**398**	**378**	**463**	**443**	**61**	**36**	**16**	**101**	**81**	5335
373	**348**	**328**	**308**	**283**	**209**	**182**	**162**	**142**	**122**	**249**	**229**	**466**	**441**	**421**	**401**	**376**	**104**	**79**	**59**	**39**	**14**	5335
306	**286**	**371**	**351**	**326**	**252**	**232**	**205**	**185**	**165**	**138**	**118**	**399**	**379**	**464**	**444**	**419**	**37**	**17**	**102**	**82**	**57**	5335
349	**329**	**304**	**284**	**374**	**141**	**121**	**248**	**228**	**208**	**188**	**161**	**442**	**422**	**397**	**377**	**467**	**80**	**60**	**35**	**15**	**105**	5335
46	69	92	5	23	**184**	**164**	**144**	**117**	**251**	**231**	**204**	322	345	368	281	299	437	460	483	396	414	5335
91	4	27	45	68	**227**	**207**	**187**	**160**	**140**	**120**	**254**	367	280	303	321	344	482	395	418	436	459	5335
385	**470**	**452**	**425**	**405**	**18**	**108**	**88**	**63**	**43**	301	325	277	358	340	212	126	**221**	**245**	**181**	**156**	**136**	5335
430	**403**	**383**	**473**	**448**	**66**	**41**	**21**	**106**	**86**	324	343	127	300	270	362	213	**159**	**134**	**224**	**243**	**179**	5335
471	**451**	**426**	**408**	**381**	**109**	**84**	**64**	**44**	**19**	347	366	214	123	323	292	274	**246**	**177**	**157**	**137**	**222**	5335
404	**386**	**469**	**449**	**429**	**42**	**22**	**107**	**87**	**62**	365	279	296	215	124	346	314	**135**	**225**	**244**	**180**	**155**	5335
447	**427**	**407**	**382**	**474**	**85**	**65**	**40**	**20**	**110**	278	302	336	318	211	125	369	**178**	**158**	**133**	**223**	**247**	5335
233	**257**	**193**	**168**	**148**	**265**	**355**	**335**	**310**	**290**	416	440	**11**	**96**	**78**	**51**	**31**	392	468	450	217	131	5335
171	**146**	**236**	**255**	**191**	**313**	**288**	**268**	**353**	**333**	439	458	**56**	**29**	**9**	**99**	**74**	132	415	380	472	218	5335
258	**189**	**169**	**149**	**234**	**356**	**331**	**311**	**291**	**266**	462	481	**97**	**77**	**52**	**34**	**7**	219	128	438	402	384	5335
147	**237**	**256**	**192**	**167**	**289**	**269**	**354**	**334**	**309**	480	394	**30**	**12**	**95**	**75**	**55**	406	220	129	461	424	5335
190	**170**	**145**	**235**	**259**	**332**	**312**	**287**	**267**	**357**	393	417	**73**	**53**	**33**	**8**	**100**	446	428	216	130	484	5335
5335	5335	5335	5335	5335	5335	5335	5335	5335	5335	5335	5335	5335	5335	5335	5335	5335	5335	5335	5335	5335	5335	5335

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There are 13 broken magic squares, 12 of order 5 and 1 of order 7, see below,

282	372	352	327	307	1640
330	305	285	370	350	1640
373	348	328	308	283	1640
306	286	371	351	326	1640
349	329	304	284	374	1640
1640	1640	1640	1640	1640	1640

387	477	457	432	412	2165
435	410	390	475	455	2165
478	453	433	413	388	2165
411	391	476	456	431	2165
454	434	409	389	479	2165
2165	2165	2165	2165	2165	2165

233	257	193	168	148	999
171	146	236	255	191	999
258	189	169	149	234	999
147	237	256	192	167	999
190	170	145	235	259	999
999	999	999	999	999	999

265	355	335	310	290	1555
313	288	268	353	333	1555
356	331	311	291	266	1555
289	269	354	334	309	1555
332	312	287	267	357	1555
1555	1555	1555	1555	1555	1555

375	465	445	420	400	2105
423	398	378	463	443	2105
466	441	421	401	376	2105
399	379	464	444	419	2105
442	422	397	377	467	2105
2105	2105	2105	2105	2105	2105

238	262	198	173	153	1024
176	151	241	260	196	1024
263	194	174	154	239	1024
152	242	261	197	172	1024
195	175	150	240	264	1024
1024	1024	1024	1024	1024	1024

275	360	342	315	295	1587
320	293	273	363	338	1587
361	341	316	298	271	1587
294	276	359	339	319	1587
337	317	297	272	364	1587
1587	1587	1587	1587	1587	1587

385	470	452	425	405	2137
430	403	383	473	448	2137
471	451	426	408	381	2137
404	386	469	449	429	2137
447	427	407	382	474	2137
2137	2137	2137	2137	2137	2137

221	245	181	156	136	939
159	134	224	243	179	939
246	177	157	137	222	939
135	225	244	180	155	939
178	158	133	223	247	939
939	939	939	939	939	939

18	108	88	63	43	320
66	41	21	106	86	320
109	84	64	44	19	320
42	22	107	87	62	320
85	65	40	20	110	320
320	320	320	320	320	320

11	96	78	51	31	267
56	29	9	99	74	267
97	77	52	34	7	267
30	12	95	75	55	267
73	53	33	8	100	267
267	267	267	267	267	267

13	103	83	58	38	295
61	36	16	101	81	295
104	79	59	39	14	295
37	17	102	82	57	295
80	60	35	15	105	295
295	295	295	295	295	295

116	250	230	210	183	163	143	1295
166	139	119	253	226	206	186	1295
209	182	162	142	122	249	229	1295
252	232	205	185	165	138	118	1295
141	121	248	228	208	188	161	1295
184	164	144	117	251	231	204	1295
227	207	187	160	140	120	254	1295
1295	1295	1295	1295	1295	1295	1295	1295

2.3. Magic Squares of Order 23

Let us consider the grids A and B, where A is *SODLS* and B its transpose. The construction is made just in cyclic way as done in Taneja [17, 18] for prime numbers:

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23
22	23	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21
20	21	22	23	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19
18	19	20	21	22	23	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17
16	17	18	19	20	21	22	23	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
14	15	16	17	18	19	20	21	22	23	1	2	3	4	5	6	7	8	9	10	11	12	13
12	13	14	15	16	17	18	19	20	21	22	23	1	2	3	4	5	6	7	8	9	10	11
10	11	12	13	14	15	16	17	18	19	20	21	22	23	1	2	3	4	5	6	7	8	9
8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	1	2	3	4	5	6	7
6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	1	2	3	4	5
4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	1	2	3
2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	1
23	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22
21	22	23	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
19	20	21	22	23	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18
17	18	19	20	21	22	23	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
15	16	17	18	19	20	21	22	23	1	2	3	4	5	6	7	8	9	10	11	12	13	14
13	14	15	16	17	18	19	20	21	22	23	1	2	3	4	5	6	7	8	9	10	11	12
11	12	13	14	15	16	17	18	19	20	21	22	23	1	2	3	4	5	6	7	8	9	10
9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	1	2	3	4	5	6	7	8
7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	1	2	3	4	5	6
5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	1	2	3	4
3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	1	2

A

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1	22	20	18	16	14	12	10	8	6	4	2	23	21	19	17	15	13	11	9	7	5	3
2	23	21	19	17	15	13	11	9	7	5	3	1	22	20	18	16	14	12	10	8	6	4
3	1	22	20	18	16	14	12	10	8	6	4	2	23	21	19	17	15	13	11	9	7	5
4	2	23	21	19	17	15	13	11	9	7	5	3	1	22	20	18	16	14	12	10	8	6
5	3	1	22	20	18	16	14	12	10	8	6	4	2	23	21	19	17	15	13	11	9	7
6	4	2	23	21	19	17	15	13	11	9	7	5	3	1	22	20	18	16	14	12	10	8
7	5	3	1	22	20	18	16	14	12	10	8	6	4	2	23	21	19	17	15	13	11	9
8	6	4	2	23	21	19	17	15	13	11	9	7	5	3	1	22	20	18	16	14	12	10
9	7	5	3	1	22	20	18	16	14	12	10	8	6	4	2	23	21	19	17	15	13	11
10	8	6	4	2	23	21	19	17	15	13	11	9	7	5	3	1	22	20	18	16	14	12
11	9	7	5	3	1	22	20	18	16	14	12	10	8	6	4	2	23	21	19	17	15	13
12	10	8	6	4	2	23	21	19	17	15	13	11	9	7	5	3	1	22	20	18	16	14
13	11	9	7	5	3	1	22	20	18	16	14	12	10	8	6	4	2	23	21	19	17	15
14	12	10	8	6	4	2	23	21	19	17	15	13	11	9	7	5	3	1	22	20	18	16
15	13	11	9	7	5	3	1	22	20	18	16	14	12	10	8	6	4	2	23	21	19	17
16	14	12	10	8	6	4	2	23	21	19	17	15	13	11	9	7	5	3	1	22	20	18
17	15	13	11	9	7	5	3	1	22	20	18	16	14	12	10	8	6	4	2	23	21	19
18	16	14	12	10	8	6	4	2	23	21	19	17	15	13	11	9	7	5	3	1	22	20
19	17	15	13	11	9	7	5	3	1	22	20	18	16	14	12	10	8	6	4	2	23	21
20	18	16	14	12	10	8	6	4	2	23	21	19	17	15	13	11	9	7	5	3	1	22
21	19	17	15	13	11	9	7	5	3	1	22	20	18	16	14	12	10	8	6	4	2	23
22	20	18	16	14	12	10	8	6	4	2	23	21	19	17	15	13	11	9	7	5	3	1
23	21	19	17	15	13	11	9	7	5	3	1	22	20	18	16	14	12	10	8	6	4	2

B

… (8)

Example 5. Apply the procedure $23\times(A-1)+B$ in (8) we get *pan diagonal intervally distributed magic square* of order 23:

		6095	6095	6095	6095	6095	6095	6095	6095	6095	6095	6095	6095	6095	6095	6095	6095	6095	6095	6095	6095	6095	6095	6095
	1	45	66	87	108	129	150	171	192	213	234	255	299	320	341	362	383	404	425	446	467	488	509	6095
6095	485	529	21	42	63	84	105	126	147	168	189	210	231	275	296	317	338	359	380	401	422	443	464	6095
6095	440	461	505	526	18	39	60	81	102	123	144	165	186	230	251	272	293	314	335	356	377	398	419	6095
6095	395	416	460	481	502	523	15	36	57	78	99	120	141	162	206	227	248	269	290	311	332	353	374	6095
6095	350	371	392	436	457	478	499	520	12	33	54	75	96	117	161	182	203	224	245	266	287	308	329	6095
6095	305	326	347	391	412	433	454	475	496	517	9	30	51	72	93	137	158	179	200	221	242	263	284	6095
6095	260	281	302	323	367	388	409	430	451	472	493	514	6	27	48	92	113	134	155	176	197	218	239	6095
6095	215	236	257	278	322	343	364	385	406	427	448	469	490	511	3	24	68	89	110	131	152	173	194	6095
6095	170	191	212	233	254	298	319	340	361	382	403	424	445	466	487	508	23	44	65	86	107	128	149	6095
6095	125	146	167	188	209	253	274	295	316	337	358	379	400	421	442	463	484	528	20	41	62	83	104	6095
6095	80	101	122	143	164	185	229	250	271	292	313	334	355	376	397	418	439	483	504	525	17	38	59	6095
6095	35	56	77	98	119	140	184	205	226	247	268	289	310	331	352	373	394	415	459	480	501	522	14	6095
6095	519	11	32	53	74	95	116	160	181	202	223	244	265	286	307	328	349	370	414	435	456	477	498	6095
6095	474	495	516	8	29	50	71	115	136	157	178	199	220	241	262	283	304	325	346	390	411	432	453	6095
6095	429	450	471	492	513	5	26	47	91	112	133	154	175	196	217	238	259	280	301	345	366	387	408	6095
6095	384	405	426	447	468	489	510	2	46	67	88	109	130	151	172	193	214	235	256	277	321	342	363	6095
6095	339	360	381	402	423	444	465	486	507	22	43	64	85	106	127	148	169	190	211	232	276	297	318	6095
6095	294	315	336	357	378	399	420	441	462	506	527	19	40	61	82	103	124	145	166	187	208	252	273	6095
6095	249	270	291	312	333	354	375	396	417	438	482	503	524	16	37	58	79	100	121	142	163	207	228	6095
6095	204	225	246	267	288	309	330	351	372	393	437	458	479	500	521	13	34	55	76	97	118	139	183	6095
6095	159	180	201	222	243	264	285	306	327	348	369	413	434	455	476	497	518	10	31	52	73	94	138	6095
6095	114	135	156	177	198	219	240	261	282	303	324	368	389	410	431	452	473	494	515	7	28	49	70	6095
6095	69	90	111	132	153	174	195	216	237	258	279	300	344	365	386	407	428	449	470	491	512	4	25	6095
	6095	6095	6095	6095	6095	6095	6095	6095	6095	6095	6095	6095	6095	6095	6095	6095	6095	6095	6095	6095	6095	6095	6095	6095

2.4 Magic Square of Order 24

We shall construct *intervally distributed* or *semi-distributed magic squares* of order 24 in four different ways. Two are based on composition relations of magic squares of order 8 and 3, and 4 and 6. Third approach uses *SODLS* and its transpose. The forth is based on a pair of *MOLS*.

- **1st Procedure – Block Wise – 8x3**

We shall bring *intervally distributed magic square* of order 24 by using compositions of magic squares of order 8 with of order 3. Let us consider a *pan diagonal bimagic square* of order 8 [4, 6]:

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2	6	5	1	4	8	7	3
4	8	7	3	2	6	5	1
1	5	6	2	3	7	8	4
3	7	8	4	1	5	6	2
5	1	2	6	7	3	4	8
7	3	4	8	5	1	2	6
6	2	1	5	8	4	3	7
8	4	3	7	6	2	1	5

A

8	1	4	5	3	6	7	2
2	7	6	3	5	4	1	8
1	8	5	4	6	3	2	7
7	2	3	6	4	5	8	1
6	3	2	7	1	8	5	4
4	5	8	1	7	2	3	6
3	6	7	2	8	1	4	5
5	4	1	8	2	7	6	3

B

	260	260	260	260	260	260	260	260	
	16	41	36	5	27	62	55	18	260
260	26	63	54	19	13	44	33	8	260
260	1	40	45	12	22	51	58	31	260
260	23	50	59	30	4	37	48	9	260
260	38	3	10	47	49	24	29	60	260
260	52	21	32	57	39	2	11	46	260
260	43	14	7	34	64	25	20	53	260
260	61	28	17	56	42	15	6	35	260
	260	260	260	260	260	260	260	260	260

								11180
256	1681	1296	25	729	3844	3025	324	11180
676	3969	2916	361	169	1936	1089	64	11180
1	1600	2025	144	484	2601	3364	961	11180
529	2500	3481	900	16	1369	2304	81	11180
1444	9	100	2209	2401	576	841	3600	11180
2704	441	1024	3249	1521	4	121	2116	11180
1849	196	49	1156	4096	625	400	2809	11180
3721	784	289	3136	1764	225	36	1225	11180
11180	11180	11180	11180	11180	11180	11180	11180	11180

…. (9)

Let us consider a magic square of order 3:

											15
1	3	2	2	3	1	12	33	21	2	9	4	15
3	2	1	1	2	3	31	22	13	7	5	3	15
2	1	3	3	1	2	23	11	32	6	1	8	15
A			B			AB			15	15	15	15

… (10)

Let us divide the numbers 1-24 in three intervals as follows:

1	6	7	12	13	18	19	24	100
2	5	8	11	14	17	20	23	100
3	4	9	10	15	16	21	22	100

… (11)

Apply intervals given in (11) over (9), and make 9 magic squares of order 8 according *AB* given in (10), such as

31								
	4	16	15	3	10	22	21	9
	10	22	21	9	4	16	15	3
	3	15	16	4	9	21	22	10
	9	21	22	10	3	15	16	4
	15	3	4	16	21	9	10	22
	21	9	10	22	15	3	4	16
	16	4	3	15	22	10	9	21
	22	10	9	21	16	4	3	15

A

24	1	12	13	7	18	19	6
6	19	18	7	13	12	1	24
1	24	13	12	18	7	6	19
19	6	7	18	12	13	24	1
18	7	6	19	1	24	13	12
12	13	24	1	19	6	7	18
7	18	19	6	24	1	12	13
13	12	1	24	6	19	18	7

B

		2308	2308	2308	2308	2308	2308	2308	2308
	96	361	348	61	223	522	499	198	2308
2308	222	523	498	199	85	372	337	72	2308
2308	49	360	373	84	210	487	510	235	2308
2308	211	486	511	234	60	349	384	73	2308
2308	354	55	78	379	481	216	229	516	2308
2308	492	205	240	505	355	54	79	378	2308
2308	367	90	67	342	528	217	204	493	2308
2308	517	228	193	504	366	91	66	343	2308
	2308	2308	2308	2308	2308	2308	2308	2308	2308

21								
	5	17	14	2	11	23	20	8
	11	23	20	8	5	17	14	2
	2	14	17	5	8	20	23	11
	8	20	23	11	2	14	17	5
	14	2	5	17	20	8	11	23
	20	8	11	23	14	2	5	17
	17	5	2	14	23	11	8	20
	23	11	8	20	17	5	2	14

A

24	1	12	13	7	18	19	6
6	19	18	7	13	12	1	24
1	24	13	12	18	7	6	19
19	6	7	18	12	13	24	1
18	7	6	19	1	24	13	12
12	13	24	1	19	6	7	18
7	18	19	6	24	1	12	13
13	12	1	24	6	19	18	7

B

		2308	2308	2308	2308	2308	2308	2308	2308
	120	385	324	37	247	546	475	174	2308
2308	246	547	474	175	109	396	313	48	2308
2308	25	336	397	108	186	463	534	259	2308
2308	187	462	535	258	36	325	408	97	2308
2308	330	31	102	403	457	192	253	540	2308
2308	468	181	264	529	331	30	103	402	2308
2308	391	114	43	318	552	241	180	469	2308
2308	541	252	169	480	390	115	42	319	2308
	2308	2308	2308	2308	2308	2308	2308	2308	2308

Similarly, we have more 7 magic square of order 8. Keeping 9 magic squares of order 8 according to *AB* in (10), we get a pair of *MOLS* of order 24:

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6	18	13	1	12	24	19	7	4	16	15	3	10	22	21	9	5	17	14	2	11	23	20	8
12	24	19	7	6	18	13	1	10	22	21	9	4	16	15	3	11	23	20	8	5	17	14	2
1	13	18	6	7	19	24	12	3	15	16	4	9	21	22	10	2	14	17	5	8	20	23	11
7	19	24	12	1	13	18	6	9	21	22	10	3	15	16	4	8	20	23	11	2	14	17	5
13	1	6	18	19	7	12	24	15	3	4	16	21	9	10	22	14	2	5	17	20	8	11	23
19	7	12	24	13	1	6	18	21	9	10	22	15	3	4	16	20	8	11	23	14	2	5	17
18	6	1	13	24	12	7	19	16	4	3	15	22	10	9	21	17	5	2	14	23	11	8	20
24	12	7	19	18	6	1	13	22	10	9	21	16	4	3	15	23	11	8	20	17	5	2	14
4	16	15	3	10	22	21	9	5	17	14	2	11	23	20	8	6	18	13	1	12	24	19	7
10	22	21	9	4	16	15	3	11	23	20	8	5	17	14	2	12	24	19	7	6	18	13	1
3	15	16	4	9	21	22	10	2	14	17	5	8	20	23	11	1	13	18	6	7	19	24	12
9	21	22	10	3	15	16	4	8	20	23	11	2	14	17	5	7	19	24	12	1	13	18	6
15	3	4	16	21	9	10	22	14	2	5	17	20	8	11	23	13	1	6	18	19	7	12	24
21	9	10	22	15	3	4	16	20	8	11	23	14	2	5	17	19	7	12	24	13	1	6	18
16	4	3	15	22	10	9	21	17	5	2	14	23	11	8	20	18	6	1	13	24	12	7	19
22	10	9	21	16	4	3	15	23	11	8	20	17	5	2	14	24	12	7	19	18	6	1	13
5	17	14	2	11	23	20	8	6	18	13	1	12	24	19	7	4	16	15	3	10	22	21	9
11	23	20	8	5	17	14	2	12	24	19	7	6	18	13	1	10	22	21	9	4	16	15	3
2	14	17	5	8	20	23	11	1	13	18	6	7	19	24	12	3	15	16	4	9	21	22	10
8	20	23	11	2	14	17	5	7	19	24	12	1	13	18	6	9	21	22	10	3	15	16	4
14	2	5	17	20	8	11	23	13	1	6	18	19	7	12	24	15	3	4	16	21	9	10	22
20	8	11	23	14	2	5	17	19	7	12	24	13	1	6	18	21	9	10	22	15	3	4	16
17	5	2	14	23	11	8	20	18	6	1	13	24	12	7	19	16	4	3	15	22	10	9	21
23	11	8	20	17	5	2	14	24	12	7	19	18	6	1	13	22	10	9	21	16	4	3	15

A

23	2	11	14	8	17	20	5	22	3	10	15	9	16	21	4	24	1	12	13	7	18	19	6
5	20	17	8	14	11	2	23	4	21	16	9	15	10	3	22	6	19	18	7	13	12	1	24
2	23	14	11	17	8	5	20	3	22	15	10	16	9	4	21	1	24	13	12	18	7	6	19
20	5	8	17	11	14	23	2	21	4	9	16	10	15	22	3	19	6	7	18	12	13	24	1
17	8	5	20	2	23	14	11	16	9	4	21	3	22	15	10	18	7	6	19	1	24	13	12
11	14	23	2	20	5	8	17	10	15	22	3	21	4	9	16	12	13	24	1	19	6	7	18
8	17	20	5	23	2	11	14	9	16	21	4	22	3	10	15	7	18	19	6	24	1	12	13
14	11	2	23	5	20	17	8	15	10	3	22	4	21	16	9	13	12	1	24	6	19	18	7
24	1	12	13	7	18	19	6	23	2	11	14	8	17	20	5	22	3	10	15	9	16	21	4
6	19	18	7	13	12	1	24	5	20	17	8	14	11	2	23	4	21	16	9	15	10	3	22
1	24	13	12	18	7	6	19	2	23	14	11	17	8	5	20	3	22	15	10	16	9	4	21
19	6	7	18	12	13	24	1	20	5	8	17	11	14	23	2	21	4	9	16	10	15	22	3
18	7	6	19	1	24	13	12	17	8	5	20	2	23	14	11	16	9	4	21	3	22	15	10
12	13	24	1	19	6	7	18	11	14	23	2	20	5	8	17	10	15	22	3	21	4	9	16
7	18	19	6	24	1	12	13	8	17	20	5	23	2	11	14	9	16	21	4	22	3	10	15
13	12	1	24	6	19	18	7	14	11	2	23	5	20	17	8	15	10	3	22	4	21	16	9
22	3	10	15	9	16	21	4	24	1	12	13	7	18	19	6	23	2	11	14	8	17	20	5
4	21	16	9	15	10	3	22	6	19	18	7	13	12	1	24	5	20	17	8	14	11	2	23
3	22	15	10	16	9	4	21	1	24	13	12	18	7	6	19	2	23	14	11	17	8	5	20
21	4	9	16	10	15	22	3	19	6	7	18	12	13	24	1	20	5	8	17	11	14	23	2
16	9	4	21	3	22	15	10	18	7	6	19	1	24	13	12	17	8	5	20	2	23	14	11
10	15	22	3	21	4	9	16	12	13	24	1	19	6	7	18	11	14	23	2	20	5	8	17
9	16	21	4	22	3	10	15	7	18	19	6	24	1	12	13	8	17	20	5	23	2	11	14
15	10	3	22	4	21	16	9	13	12	1	24	6	19	18	7	14	11	2	23	5	20	17	8

B

… (12)

Example 6. Apply the procedure $24\times(A-1)+B$ in (12), we get *intervally semi-distributed semi-bimagic square* of order 24:

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																								6924
143	410	299	14	272	569	452	149	94	363	346	63	225	520	501	196	120	385	324	37	247	546	475	174	6924
269	572	449	152	134	419	290	23	220	525	496	201	87	370	339	70	246	547	474	175	109	396	313	48	6924
2	311	422	131	161	440	557	284	51	358	375	82	208	489	508	237	25	336	397	108	186	463	534	259	6924
164	437	560	281	11	302	431	122	213	484	513	232	58	351	382	75	187	462	535	258	36	325	408	97	6924
305	8	125	428	434	167	278	563	352	57	76	381	483	214	231	514	330	31	102	403	457	192	253	540	6924
443	158	287	554	308	5	128	425	490	207	238	507	357	52	81	376	468	181	264	529	331	30	103	402	6924
416	137	20	293	575	266	155	446	369	88	69	340	526	219	202	495	391	114	43	318	552	241	180	469	6924
566	275	146	455	413	140	17	296	519	226	195	502	364	93	64	345	541	252	169	480	390	115	42	319	6924
96	361	348	61	223	522	499	198	119	386	323	38	248	545	476	173	142	411	298	15	273	568	453	148	6924
222	523	498	199	85	372	337	72	245	548	473	176	110	395	314	47	268	573	448	153	135	418	291	22	6924
49	360	373	84	210	487	510	235	26	335	398	107	185	464	533	260	3	310	423	130	160	441	556	285	6924
211	486	511	234	60	349	384	73	188	461	536	257	35	326	407	98	165	436	561	280	10	303	430	123	6924
354	55	78	379	481	216	229	516	329	32	101	404	458	191	254	539	304	9	124	429	435	166	279	562	6924
492	205	240	505	355	54	79	378	467	182	263	530	332	29	104	401	442	159	286	555	309	4	129	424	6924
367	90	67	342	528	217	204	493	392	113	44	317	551	242	179	470	417	136	21	292	574	267	154	447	6924
517	228	193	504	366	91	66	343	542	251	170	479	389	116	41	320	567	274	147	454	412	141	16	297	6924
118	387	322	39	249	544	477	172	144	409	300	13	271	570	451	150	95	362	347	62	224	521	500	197	6924
244	549	472	177	111	394	315	46	270	571	450	151	133	420	289	24	221	524	497	200	86	371	338	71	6924
27	334	399	106	184	465	532	261	1	312	421	132	162	439	558	283	50	359	374	83	209	488	509	236	6924
189	460	537	256	34	327	406	99	163	438	559	282	12	301	432	121	212	485	512	233	59	350	383	74	6924
328	33	100	405	459	190	255	538	306	7	126	427	433	168	277	564	353	56	77	380	482	215	230	515	6924
466	183	262	531	333	28	105	400	444	157	288	553	307	6	127	426	491	206	239	506	356	53	80	377	6924
393	112	45	316	550	243	178	471	415	138	19	294	576	265	156	445	368	89	68	341	527	218	203	494	6924
543	250	171	478	388	117	40	321	565	276	145	456	414	139	18	295	518	227	194	503	365	92	65	344	6924
6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924

Interestingly, above magic square is formed by 9 *pan diagonal magic squares* of order 8 having the same magic sum $S_{8\times 8} := 2308$. Moreover, each block of order 8 is *semi-bimagic* with bimagic sums of rows and columns, and of diagonals given respectively by

			2651908
902396	874732	883996	2661124
874780	883964	902380	2661124
883948	902428	874748	2661124
2661124	2661124	2661124	**2661108**

and

			2651908
904700	874732	881692	2661124
870172	883964	906988	2661124
886252	902428	872444	2661124
2661124	2661124	2661124	**2661108**

Each block of order 8 is *pan diagonal*, but the magic square of order 24 is *not pan diagonal*. It is *intervally semi-distributed* as there are numbers in one the diagonal belongs to same intervals. See below:

2nd interval	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40	41	42	43	44	45	46	47	48
5th interval	97	98	99	100	101	102	103	104	105	106	107	108	109	110	111	112	113	114	115	116	117	118	119	120
8th interval	169	170	171	172	173	174	175	176	177	178	179	180	181	182	183	184	185	186	187	188	189	190	191	192
11th interval	241	242	243	244	245	246	247	248	249	250	251	252	253	254	255	256	257	258	259	260	261	262	263	264
14th interval	313	314	315	316	317	318	319	320	321	322	323	324	325	326	327	328	329	330	331	332	333	334	335	336
17th interval	385	386	387	388	389	390	391	392	393	394	395	396	397	398	399	400	401	402	403	404	405	406	407	408
20th interval	457	458	459	460	461	462	463	464	465	466	467	468	469	470	471	472	473	474	475	476	477	478	479	480
23rd interval	529	530	531	532	533	534	535	536	537	538	539	540	541	542	543	544	545	546	547	548	549	550	551	552

We observe that the above procedure lead us to *intervally semi-distributed magic square* of order 24. Below are more three procedures leading to *intervally distributed magic squares*.

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- **2nd Procedure – Block Wise – 4x6**

We shall construct magic square of order 24 using magic square of orders 6 and 4, both are *intervally distributed* given respectively by

2	1	4	5	6	3
1	3	6	4	5	2
3	6	5	2	4	1
4	2	1	6	3	5
5	4	2	3	1	6
6	5	3	1	2	4

A

5	2	3	4	2	5
1	3	5	5	1	6
1	3	3	2	6	6
2	4	3	4	6	2
6	4	3	2	5	1
6	5	4	4	1	1

B

25	12	43	54	62	35
11	33	65	45	51	26
31	63	53	22	46	16
42	24	13	64	36	52
56	44	23	32	15	61
66	55	34	14	21	41

AB

						111
1	23	28	34	17	8	111
29	7	35	14	21	5	111
12	6	13	27	31	22	111
32	16	4	24	10	25	111
19	33	11	3	30	15	111
18	26	20	9	2	36	111
111	111	111	111	111	111	111

…. (13)

and

2	3	1	4
1	4	2	3
4	1	3	2
3	2	4	1

A

2	1	4	3
3	4	1	2
1	2	3	4
4	3	2	1

B

22	31	14	43
13	44	21	32
41	12	33	24
34	23	42	11

AB

		34	34	34	34
	6	9	4	15	34
34	3	16	5	10	34
34	13	2	11	8	34
34	12	7	14	1	34
	34	34	34	34	34

…. (14)

Let us divide the numbers 1-24 in six intervals as follows:

1	12	13	24	50
2	11	14	23	50
3	10	15	22	50
4	9	16	21	50
5	8	17	20	50
6	7	18	19	50

… (15)

Let us make 20 magic squares of order 4 applying values given in (15) over (14), then put them according to *AB* given in (13), we get 24 magic squares of order 4, such as,

63				
	7	18	6	19
	6	19	7	18
	19	6	18	7
	18	7	19	6

A

10	3	22	15
15	22	3	10
3	10	15	22
22	15	10	3

B

		1154	1154	1154	1154
	154	411	142	447	1154
1154	135	454	147	418	1154
1154	435	130	423	166	1154
1154	430	159	442	123	1154
	1154	1154	1154	1154	1154

32				
	10	15	3	22
	3	22	10	15
	22	3	15	10
	15	10	22	3

A

11	2	23	14
14	23	2	11
2	11	14	23
23	14	11	2

B

		1154	1154	1154	1154
	227	338	71	518	1154
1154	62	527	218	347	1154
1154	506	59	350	239	1154
1154	359	230	515	50	1154
	1154	1154	1154	1154	1154

Similar way we have more 22 magic square of order 4, calculated with same procedure. This lead to two grids of order 24 given by

11	14	2	23	12	13	1	24	9	16	4	21	8	17	5	20	7	18	6	19	10	15	3	22
2	23	11	14	1	24	12	13	4	21	9	16	5	20	8	17	6	19	7	18	3	22	10	15
23	2	14	11	24	1	13	12	21	4	16	9	20	5	17	8	19	6	18	7	22	3	15	10
14	11	23	2	13	12	24	1	16	9	21	4	17	8	20	5	18	7	19	6	15	10	22	3
12	13	1	24	10	15	3	22	7	18	6	19	9	16	4	21	8	17	5	20	11	14	2	23
1	24	12	13	3	22	10	15	6	19	7	18	4	21	9	16	5	20	8	17	2	23	11	14
24	1	13	12	22	3	15	10	19	6	18	7	21	4	16	9	20	5	17	8	23	2	14	11
13	12	24	1	15	10	22	3	18	7	19	6	16	9	21	4	17	8	20	5	14	11	23	2
10	15	3	22	7	18	6	19	8	17	5	20	11	14	2	23	9	16	4	21	12	13	1	24
3	22	10	15	6	19	7	18	5	20	8	17	2	23	11	14	4	21	9	16	1	24	12	13
22	3	15	10	19	6	18	7	20	5	17	8	23	2	14	11	21	4	16	9	24	1	13	12
15	10	22	3	18	7	19	6	17	8	20	5	14	11	23	2	16	9	21	4	13	12	24	1
9	16	4	21	11	14	2	23	12	13	1	24	7	18	6	19	10	15	3	22	8	17	5	20
4	21	9	16	2	23	11	14	1	24	12	13	6	19	7	18	3	22	10	15	5	20	8	17
21	4	16	9	23	2	14	11	24	1	13	12	19	6	18	7	22	3	15	10	20	5	17	8
16	9	21	4	14	11	23	2	13	12	24	1	18	7	19	6	15	10	22	3	17	8	20	5
8	17	5	20	9	16	4	21	11	14	2	23	10	15	3	22	12	13	1	24	7	18	6	19
5	20	8	17	4	21	9	16	2	23	11	14	3	22	10	15	1	24	12	13	6	19	7	18
20	5	17	8	21	4	16	9	23	2	14	11	22	3	15	10	24	1	13	12	19	6	18	7
17	8	20	5	16	9	21	4	14	11	23	2	15	10	22	3	13	12	24	1	18	7	19	6
7	18	6	19	8	17	5	20	10	15	3	22	12	13	1	24	11	14	2	23	9	16	4	21
6	19	7	18	5	20	8	17	3	22	10	15	1	24	12	13	2	23	11	14	4	21	9	16
19	6	18	7	20	5	17	8	22	3	15	10	24	1	13	12	23	2	14	11	21	4	16	9
18	7	19	6	17	8	20	5	15	10	22	3	13	12	24	1	14	11	23	2	16	9	21	4

A

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8	5	20	17	11	2	23	14	10	3	22	15	9	4	21	16	11	2	23	14	8	5	20	17
17	20	5	8	14	23	2	11	15	22	3	10	16	21	4	9	14	23	2	11	17	20	5	8
5	8	17	20	2	11	14	23	3	10	15	22	4	9	16	21	2	11	14	23	5	8	17	20
20	17	8	5	23	14	11	2	22	15	10	3	21	16	9	4	23	14	11	2	20	17	8	5
12	1	24	13	10	3	22	15	8	5	20	17	8	5	20	17	12	1	24	13	7	6	19	18
13	24	1	12	15	22	3	10	17	20	5	8	17	20	5	8	13	24	1	12	18	19	6	7
1	12	13	24	3	10	15	22	5	8	17	20	5	8	17	20	1	12	13	24	6	7	18	19
24	13	12	1	22	15	10	3	20	17	8	5	20	17	8	5	24	13	12	1	19	18	7	6
12	1	24	13	10	3	22	15	10	3	22	15	11	2	23	14	7	6	19	18	7	6	19	18
13	24	1	12	15	22	3	10	15	22	3	10	14	23	2	11	18	19	6	7	18	19	6	7
1	12	13	24	3	10	15	22	3	10	15	22	2	11	14	23	6	7	18	19	6	7	18	19
24	13	12	1	22	15	10	3	22	15	10	3	23	14	11	2	19	18	7	6	19	18	7	6
11	2	23	14	9	4	21	16	10	3	22	15	9	4	21	16	7	6	19	18	11	2	23	14
14	23	2	11	16	21	4	9	15	22	3	10	16	21	4	9	18	19	6	7	14	23	2	11
2	11	14	23	4	9	16	21	3	10	15	22	4	9	16	21	6	7	18	19	2	11	14	23
23	14	11	2	21	16	9	4	22	15	10	3	21	16	9	4	19	18	7	6	23	14	11	2
12	1	24	13	9	4	21	16	10	3	22	15	11	2	23	14	8	5	20	17	12	1	24	13
13	24	1	12	16	21	4	9	15	22	3	10	14	23	2	11	17	20	5	8	13	24	1	12
1	12	13	24	4	9	16	21	3	10	15	22	2	11	14	23	5	8	17	20	1	12	13	24
24	13	12	1	21	16	9	4	22	15	10	3	23	14	11	2	20	17	8	5	24	13	12	1
7	6	19	18	8	5	20	17	9	4	21	16	9	4	21	16	12	1	24	13	12	1	24	13
18	19	6	7	17	20	5	8	16	21	4	9	16	21	4	9	13	24	1	12	13	24	1	12
6	7	18	19	5	8	17	20	4	9	16	21	4	9	16	21	1	12	13	24	1	12	13	24
19	18	7	6	20	17	8	5	21	16	9	4	21	16	9	4	24	13	12	1	24	13	12	1

B

… (16)

Grid A is *diagonalized Latin square* and grid B is just of numbers. A and B forms *intervally distributed magic square* of order 24.

Example 7. Apply the procedure $24\times(A-1)+B$ in (16), we get *intervally distributed magic square* of order 24:

																								6924
248	317	44	545	275	290	23	566	202	363	94	495	177	388	117	472	155	410	143	446	224	341	68	521	6924
41	548	245	320	14	575	266	299	87	502	195	370	112	477	172	393	134	455	146	419	65	524	221	344	6924
533	32	329	260	554	11	302	287	483	82	375	214	460	105	400	189	434	131	422	167	509	56	353	236	6924
332	257	536	29	311	278	563	2	382	207	490	75	405	184	465	100	431	158	443	122	356	233	512	53	6924
276	289	24	565	226	339	70	519	152	413	140	449	200	365	92	497	180	385	120	469	247	318	43	546	6924
13	576	265	300	63	526	219	346	137	452	149	416	89	500	197	368	109	480	169	396	42	547	246	319	6924
553	12	301	288	507	58	351	238	437	128	425	164	485	80	377	212	457	108	397	192	534	31	330	259	6924
312	277	564	1	358	231	514	51	428	161	440	125	380	209	488	77	408	181	468	97	331	258	535	30	6924
228	337	72	517	154	411	142	447	178	387	118	471	251	314	47	542	199	366	91	498	271	294	19	570	6924
61	528	217	348	135	454	147	418	111	478	171	394	38	551	242	323	90	499	198	367	18	571	270	295	6924
505	60	349	240	435	130	423	166	459	106	399	190	530	35	326	263	486	79	378	211	558	7	306	283	6924
360	229	516	49	430	159	442	123	406	183	466	99	335	254	539	26	379	210	487	78	307	282	559	6	6924
203	362	95	494	249	316	45	544	274	291	22	567	153	412	141	448	223	342	67	522	179	386	119	470	6924
86	503	194	371	40	549	244	321	15	574	267	298	136	453	148	417	66	523	222	343	110	479	170	395	6924
482	83	374	215	532	33	328	261	555	10	303	286	436	129	424	165	510	55	354	235	458	107	398	191	6924
383	206	491	74	333	256	537	28	310	279	562	3	429	160	441	124	355	234	511	54	407	182	467	98	6924
180	385	120	469	201	364	93	496	250	315	46	543	227	338	71	518	272	293	20	569	156	409	144	445	6924
109	480	169	396	88	501	196	369	39	550	243	322	62	527	218	347	17	572	269	296	133	456	145	420	6924
457	108	397	192	484	81	376	213	531	34	327	262	506	59	350	239	557	8	305	284	433	132	421	168	6924
408	181	468	97	381	208	489	76	334	255	538	27	359	230	515	50	308	281	560	5	432	157	444	121	6924
151	414	139	450	176	389	116	473	225	340	69	520	273	292	21	568	252	313	48	541	204	361	96	493	6924
138	451	150	415	113	476	173	392	64	525	220	345	16	573	268	297	37	552	241	324	85	504	193	372	6924
438	127	426	163	461	104	401	188	508	57	352	237	556	9	304	285	529	36	325	264	481	84	373	216	6924
427	162	439	126	404	185	464	101	357	232	513	52	309	280	561	4	336	253	540	25	384	205	492	73	6924
6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924

Interestingly, above magic square is formed by 24 *pan diagonal perfect magic squares* of order 4 having same magic sum $S_{4\times4}:=1154$. Moreover, each block of order 2×2 has the same sum as of magic square of order 4.

Based on similar lines we can also write *intervally distributed magic square* of order 24 using the composition 6×4, but in this case we don't have any magic square of order 6. Thus the procedure done above is much more better.

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- **3rd Procedure - SODLS**

Let us consider the grid A and B, where A is *SODLS* and B is its transpose. It is constructed by M. Nakamura [7] based on work of Bennetta, Du and Zhang [1].

1	5	4	12	11	20	24	23	14	13	2	3	21	22	6	10	9	8	7	15	19	18	17	16
11	2	1	5	12	13	21	20	24	14	3	4	22	23	8	7	6	10	9	17	16	15	19	18
12	11	3	2	1	14	13	22	21	20	4	5	23	24	10	9	8	7	6	19	18	17	16	15
2	12	11	4	3	21	14	13	23	22	5	1	24	20	7	6	10	9	8	16	15	19	18	17
4	3	12	11	5	23	22	14	13	24	1	2	20	21	9	8	7	6	10	18	17	16	15	19
15	19	18	14	13	6	10	9	12	11	7	8	16	17	20	24	23	22	21	1	5	4	3	2
13	16	15	19	14	11	7	6	10	12	8	9	17	18	22	21	20	24	23	3	2	1	5	4
14	13	17	16	15	12	11	8	7	6	9	10	18	19	24	23	22	21	20	5	4	3	2	1
16	14	13	18	17	7	12	11	9	8	10	6	19	15	21	20	24	23	22	2	1	5	4	3
18	17	14	13	19	9	8	12	11	10	6	7	15	16	23	22	21	20	24	4	3	2	1	5
3	4	5	1	2	8	9	10	6	7	11	14	12	13	17	18	19	15	16	22	23	24	20	21
5	1	2	3	4	10	6	7	8	9	13	12	14	11	19	15	16	17	18	24	20	21	22	23
17	18	19	15	16	22	23	24	20	21	14	11	13	12	3	4	5	1	2	8	9	10	6	7
19	15	16	17	18	24	20	21	22	23	12	13	11	14	5	1	2	3	4	10	6	7	8	9
20	24	23	22	21	1	5	4	3	2	16	17	7	8	15	19	18	12	11	6	10	9	14	13
22	21	20	24	23	3	2	1	5	4	17	18	8	9	11	16	15	19	12	13	7	6	10	14
24	23	22	21	20	5	4	3	2	1	18	19	9	10	12	11	17	16	15	14	13	8	7	6
21	20	24	23	22	2	1	5	4	3	19	15	10	6	16	12	11	18	17	7	14	13	9	8
23	22	21	20	24	4	3	2	1	5	15	16	6	7	18	17	12	11	19	9	8	14	13	10
6	10	9	8	7	15	19	18	17	16	21	22	2	3	1	5	4	14	13	20	24	23	12	11
8	7	6	10	9	17	16	15	19	18	22	23	3	4	13	2	1	5	14	11	21	20	24	12
10	9	8	7	6	19	18	17	16	15	23	24	4	5	14	13	3	2	1	12	11	22	21	20
7	6	10	9	8	16	15	19	18	17	24	20	5	1	2	14	13	4	3	21	12	11	23	22
9	8	7	6	10	18	17	16	15	19	20	21	1	2	4	3	14	13	5	23	22	12	11	24

A

1	11	12	2	4	15	13	14	16	18	3	5	17	19	20	22	24	21	23	6	8	10	7	9
5	2	11	12	3	19	16	13	14	17	4	1	18	15	24	21	23	20	22	10	7	9	6	8
4	1	3	11	12	18	15	17	13	14	5	2	19	16	23	20	22	24	21	9	6	8	10	7
12	5	2	4	11	14	19	16	18	13	1	3	15	17	22	24	21	23	20	8	10	7	9	6
11	12	1	3	5	13	14	15	17	19	2	4	16	18	21	23	20	22	24	7	9	6	8	10
20	13	14	21	23	6	11	12	7	9	8	10	22	24	1	3	5	2	4	15	17	19	16	18
24	21	13	14	22	10	7	11	12	8	9	6	23	20	5	2	4	1	3	19	16	18	15	17
23	20	22	13	14	9	6	8	11	12	10	7	24	21	4	1	3	5	2	18	15	17	19	16
14	24	21	23	13	12	10	7	9	11	6	8	20	22	3	5	2	4	1	17	19	16	18	15
13	14	20	22	24	11	12	6	8	10	7	9	21	23	2	4	1	3	5	16	18	15	17	19
2	3	4	5	1	7	8	9	10	6	11	13	14	12	16	17	18	19	15	21	22	23	24	20
3	4	5	1	2	8	9	10	6	7	14	12	11	13	17	18	19	15	16	22	23	24	20	21
21	22	23	24	20	16	17	18	19	15	12	14	13	11	7	8	9	10	6	2	3	4	5	1
22	23	24	20	21	17	18	19	15	16	13	11	12	14	8	9	10	6	7	3	4	5	1	2
6	8	10	7	9	20	22	24	21	23	17	19	3	5	15	11	12	16	18	1	13	14	2	4
10	7	9	6	8	24	21	23	20	22	18	15	4	1	19	16	11	12	17	5	2	13	14	3
9	6	8	10	7	23	20	22	24	21	19	16	5	2	18	15	17	11	12	4	1	3	13	14
8	10	7	9	6	22	24	21	23	20	15	17	1	3	12	19	16	18	11	14	5	2	4	13
7	9	6	8	10	21	23	20	22	24	16	18	2	4	11	12	15	17	19	13	14	1	3	5
15	17	19	16	18	1	3	5	2	4	22	24	8	10	6	13	14	7	9	20	11	12	21	23
19	16	18	15	17	5	2	4	1	3	23	20	9	6	10	7	13	14	8	24	21	11	12	22
18	15	17	19	16	4	1	3	5	2	24	21	10	7	9	6	8	13	14	23	20	22	11	12
17	19	16	18	15	3	5	2	4	1	20	22	6	8	14	10	7	9	13	12	24	21	23	11
16	18	15	17	19	2	4	1	3	5	21	23	7	9	13	14	6	8	10	11	12	20	22	24

B

… (17)

Example 8. Applying the procedure $24\times(A-1)+B$ in (17) we get *intervally distributed magic square* of order 24:

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																								6924
1	107	84	266	244	471	565	542	328	306	27	53	497	523	**140**	**238**	**216**	**189**	**167**	**342**	**440**	**418**	**391**	**369**	6924
245	26	11	108	267	307	496	469	566	329	52	73	522	543	**192**	**165**	**143**	**236**	**214**	**394**	**367**	**345**	**438**	**416**	6924
268	241	51	35	12	330	303	521	493	470	77	98	547	568	**239**	**212**	**190**	**168**	**141**	**441**	**414**	**392**	**370**	**343**	6924
36	269	242	76	59	494	331	304	546	517	97	3	567	473	**166**	**144**	**237**	**215**	**188**	**368**	**346**	**439**	**417**	**390**	6924
83	60	265	243	101	541	518	327	305	571	2	28	472	498	**213**	**191**	**164**	**142**	**240**	**415**	**393**	**366**	**344**	**442**	6924
356	445	422	333	311	126	227	204	271	249	152	178	382	408	**457**	**555**	**533**	**506**	**484**	**15**	**113**	**91**	**64**	**42**	6924
312	381	349	446	334	250	151	131	228	272	177	198	407	428	**509**	**482**	**460**	**553**	**531**	**67**	**40**	**18**	**111**	**89**	6924
335	308	406	373	350	273	246	176	155	132	202	223	432	453	**556**	**529**	**507**	**485**	**458**	**114**	**87**	**65**	**43**	**16**	6924
374	336	309	431	397	156	274	247	201	179	222	128	452	358	**483**	**461**	**554**	**532**	**505**	**41**	**19**	**112**	**90**	**63**	6924
421	398	332	310	456	203	180	270	248	226	127	153	357	383	**530**	**508**	**481**	**459**	**557**	**88**	**66**	**39**	**17**	**115**	6924
50	75	100	5	25	175	200	225	130	150	**251**	**325**	**278**	**300**	400	425	450	355	375	525	550	575	480	500	6924
99	4	29	49	74	224	129	154	174	199	**302**	**276**	**323**	**253**	449	354	379	399	424	574	479	504	524	549	6924
405	430	455	360	380	520	545	570	475	495	**324**	**254**	**301**	**275**	55	80	105	10	30	170	195	220	125	145	6924
454	359	384	404	429	569	474	499	519	544	**277**	**299**	**252**	**326**	104	9	34	54	79	219	124	149	169	194	6924
462	**560**	**538**	**511**	**489**	**20**	**118**	**96**	**69**	**47**	377	403	147	173	351	443	420	280	258	121	229	206	314	292	6924
514	**487**	**465**	**558**	**536**	**72**	**45**	**23**	**116**	**94**	402	423	172	193	259	376	347	444	281	293	146	133	230	315	6924
561	**534**	**512**	**490**	**463**	**119**	**92**	**70**	**48**	**21**	427	448	197	218	282	255	401	371	348	316	289	171	157	134	6924
488	**466**	**559**	**537**	**510**	**46**	**24**	**117**	**95**	**68**	447	353	217	123	372	283	256	426	395	158	317	290	196	181	6924
535	**513**	**486**	**464**	**562**	**93**	**71**	**44**	**22**	**120**	352	378	122	148	419	396	279	257	451	205	182	313	291	221	6924
135	**233**	**211**	**184**	**162**	**337**	**435**	**413**	**386**	**364**	502	528	32	58	6	109	86	319	297	476	563	540	285	263	6924
187	**160**	**138**	**231**	**209**	**389**	**362**	**340**	**433**	**411**	527	548	57	78	298	31	13	110	320	264	501	467	564	286	6924
234	**207**	**185**	**163**	**136**	**436**	**409**	**387**	**365**	**338**	552	573	82	103	321	294	56	37	14	287	260	526	491	468	6924
161	**139**	**232**	**210**	**183**	**363**	**341**	**434**	**412**	**385**	572	478	102	8	38	322	295	81	61	492	288	261	551	515	6924
208	**186**	**159**	**137**	**235**	**410**	**388**	**361**	**339**	**437**	477	503	7	33	85	62	318	296	106	539	516	284	262	576	6924
6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924

There are 9 broken magic squares, 8 of order 5 and 1 of order 4, see below,

					950
140	238	216	189	167	950
192	165	143	236	214	950
239	212	190	168	141	950
166	144	237	215	188	950
213	191	164	142	240	950
950	950	950	950	950	950

					1960
342	440	418	391	369	1960
394	367	345	438	416	1960
441	414	392	370	343	1960
368	346	439	417	390	1960
415	393	366	344	442	1960
1960	1960	1960	1960	1960	1960

					2535
457	555	533	506	484	2535
509	482	460	553	531	2535
556	529	507	485	458	2535
483	461	554	532	505	2535
530	508	481	459	557	2535
2535	2535	2535	2535	2535	2535

					325
15	113	91	64	42	325
67	40	18	111	89	325
114	87	65	43	16	325
41	19	112	90	63	325
88	66	39	17	115	325
325	325	325	325	325	325

					2560
462	560	538	511	489	2560
514	487	465	558	536	2560
561	534	512	490	463	2560
488	466	559	537	510	2560
535	513	486	464	562	2560
2560	2560	2560	2560	2560	2560

					350
20	118	96	69	47	350
72	45	23	116	94	350
119	92	70	48	21	350
46	24	117	95	68	350
93	71	44	22	120	350
350	350	350	350	350	350

					925
135	233	211	184	162	925
187	160	138	231	209	925
234	207	185	163	136	925
161	139	232	210	183	925
208	186	159	137	235	925
925	925	925	925	925	925

					1935
337	435	413	386	364	1935
389	362	340	433	411	1935
436	409	387	365	338	1935
363	341	434	412	385	1935
410	388	361	339	437	1935
1935	1935	1935	1935	1935	1935

				1154
251	325	278	300	1154
302	276	323	253	1154
324	254	301	275	1154
277	299	252	326	1154
1154	1154	1154	1154	1154

- **4th Procedure - MOLS**

Let us consider two *MOLS* grids *A* and *B* constructed based on computer program (H. White [19]):

1	2	3	4	5	6	8	7	11	24	23	9	14	18	17	10	12	20	13	22	16	15	21	19
8	9	10	11	12	13	3	14	6	19	18	4	21	1	24	5	7	15	20	17	23	22	16	2
24	1	2	3	4	5	7	6	10	11	22	8	13	17	16	9	23	19	12	21	15	14	20	18
9	10	11	12	13	14	16	15	7	20	19	5	22	2	1	6	8	4	21	18	24	23	17	3
23	24	1	2	3	4	6	5	21	10	9	7	12	16	15	8	22	18	11	20	14	13	19	17
10	11	12	13	14	15	17	16	8	21	20	18	23	3	2	7	9	5	22	19	1	24	6	4
22	23	24	1	2	3	5	4	20	9	8	6	11	15	14	19	21	17	10	7	13	12	18	16
11	12	13	14	15	16	18	17	9	22	21	19	24	4	3	20	10	6	23	8	2	1	7	5
21	22	23	24	1	2	4	3	19	8	7	17	10	14	13	18	20	16	9	6	12	11	5	15
12	13	14	15	16	17	19	18	22	23	10	20	1	5	4	21	11	7	24	9	3	2	8	6
20	21	22	23	24	1	15	2	18	7	6	16	9	13	12	17	19	3	8	5	11	10	4	14
13	14	15	16	17	18	20	19	23	12	11	21	2	6	5	22	24	8	1	10	4	3	9	7
19	20	21	22	23	24	14	13	17	6	5	15	8	12	11	16	18	2	7	4	10	9	3	1
2	15	16	17	18	19	21	20	24	13	12	22	3	7	6	23	1	9	14	11	5	4	10	8
18	19	20	21	22	11	13	12	16	5	4	14	7	23	10	15	17	1	6	3	9	8	2	24
3	4	17	18	19	20	22	21	1	14	13	23	16	8	7	24	2	10	15	12	6	5	11	9
17	18	19	20	9	10	12	11	15	4	3	13	6	22	21	14	16	24	5	2	8	7	1	23
4	5	6	19	20	21	23	22	2	15	14	24	17	9	8	1	3	11	16	13	7	18	12	10
16	17	18	7	8	9	11	10	14	3	2	12	5	21	20	13	15	23	4	1	19	6	24	22
5	6	7	8	21	22	24	23	3	16	15	1	18	10	9	2	4	12	17	14	20	19	13	11
15	16	5	6	7	8	10	9	13	2	1	11	4	20	19	12	14	22	3	24	18	17	23	21
6	7	8	9	10	23	1	24	4	17	16	2	19	11	22	3	5	13	18	15	21	20	14	12
14	3	4	5	6	7	9	8	12	1	24	10	15	19	18	11	13	21	2	23	17	16	22	20
7	8	9	10	11	12	2	1	5	18	17	3	20	24	23	4	6	14	19	16	22	21	15	13

A

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1	2	3	4	5	6	8	7	11	24	23	9	14	18	17	10	12	20	13	22	16	15	21	19
19	18	17	16	15	14	24	13	21	8	9	23	6	2	3	22	20	12	7	10	4	5	11	1
2	3	4	5	6	7	9	8	12	13	24	10	15	19	18	11	1	21	14	23	17	16	22	20
20	19	18	17	16	15	13	14	22	9	10	24	7	3	4	23	21	1	8	11	5	6	12	2
3	4	5	6	7	8	10	9	1	14	13	11	16	20	19	12	2	22	15	24	18	17	23	21
21	20	19	18	17	16	14	15	23	10	11	13	8	4	5	24	22	2	9	12	6	7	1	3
4	5	6	7	8	9	11	10	2	15	14	12	17	21	20	1	3	23	16	13	19	18	24	22
22	21	20	19	18	17	15	16	24	11	12	14	9	5	6	13	23	3	10	1	7	8	2	4
5	6	7	8	9	10	12	11	3	16	15	1	18	22	21	2	4	24	17	14	20	19	13	23
23	22	21	20	19	18	16	17	13	12	1	15	10	6	7	14	24	4	11	2	8	9	3	5
6	7	8	9	10	11	1	12	4	17	16	2	19	23	22	3	5	13	18	15	21	20	14	24
24	23	22	21	20	19	17	18	14	1	2	16	11	7	8	15	13	5	12	3	9	10	4	6
7	8	9	10	11	12	2	1	5	18	17	3	20	24	23	4	6	14	19	16	22	21	15	13
13	24	23	22	21	20	18	19	15	2	3	17	12	8	9	16	14	6	1	4	10	11	5	7
8	9	10	11	12	1	3	2	6	19	18	4	21	13	24	5	7	15	20	17	23	22	16	14
14	13	24	23	22	21	19	20	16	3	4	18	1	9	10	17	15	7	2	5	11	12	6	8
9	10	11	12	1	2	4	3	7	20	19	5	22	14	13	6	8	16	21	18	24	23	17	15
15	14	13	24	23	22	20	21	17	4	5	19	2	10	11	18	16	8	3	6	12	1	7	9
10	11	12	1	2	3	5	4	8	21	20	6	23	15	14	7	9	17	22	19	13	24	18	16
16	15	14	13	24	23	21	22	18	5	6	20	3	11	12	19	17	9	4	7	1	2	8	10
11	12	1	2	3	4	6	5	9	22	21	7	24	16	15	8	10	18	23	20	14	13	19	17
17	16	15	14	13	24	22	23	19	6	7	21	4	12	1	20	18	10	5	8	2	3	9	11
12	1	2	3	4	5	7	6	10	23	22	8	13	17	16	9	11	19	24	21	15	14	20	18
18	17	16	15	14	13	23	24	20	7	8	22	5	1	2	21	19	11	6	9	3	4	10	12

B

… (18)

We observe that the grid A is *diagonalized Latin square*, while grid B *not diagonalized*, resulting in *intervally distributed magic square* of order 24.

Example 9. Apply the procedure $24\times(A-1)+B$ in (18), we get *intervally distributed magic square* of order 24 given by

																								6924
1	26	51	76	101	126	176	151	251	576	551	201	326	426	401	226	276	476	301	526	376	351	501	451	6924
187	210	233	256	279	302	72	325	141	440	417	95	486	2	555	118	164	348	463	394	532	509	371	25	6924
554	3	28	53	78	103	153	128	228	253	528	178	303	403	378	203	529	453	278	503	353	328	478	428	6924
212	235	258	281	304	327	373	350	166	465	442	120	511	27	4	143	189	73	488	419	557	534	396	50	6924
531	556	5	30	55	80	130	105	481	230	205	155	280	380	355	180	506	430	255	480	330	305	455	405	6924
237	260	283	306	329	352	398	375	191	490	467	421	536	52	29	168	214	98	513	444	6	559	121	75	6924
508	533	558	7	32	57	107	82	458	207	182	132	257	357	332	433	483	407	232	157	307	282	432	382	6924
262	285	308	331	354	377	423	400	216	515	492	446	561	77	54	469	239	123	538	169	31	8	146	100	6924
485	510	535	560	9	34	84	59	435	184	159	385	234	334	309	410	460	384	209	134	284	259	109	359	6924
287	310	333	356	379	402	448	425	517	540	217	471	10	102	79	494	264	148	563	194	56	33	171	125	6924
462	487	512	537	562	11	337	36	412	161	136	362	211	311	286	387	437	61	186	111	261	236	86	336	6924
312	335	358	381	404	427	473	450	542	265	242	496	35	127	104	519	565	173	12	219	81	58	196	150	6924
439	464	489	514	539	564	314	289	389	138	113	339	188	288	263	364	414	38	163	88	238	213	63	13	6924
37	360	383	406	429	452	498	475	567	290	267	521	60	152	129	544	14	198	313	244	106	83	221	175	6924
416	441	466	491	516	241	291	266	366	115	90	316	165	541	240	341	391	15	140	65	215	190	40	566	6924
62	85	408	431	454	477	523	500	16	315	292	546	361	177	154	569	39	223	338	269	131	108	246	200	6924
393	418	443	468	193	218	268	243	343	92	67	293	142	518	493	318	368	568	117	42	192	167	17	543	6924
87	110	133	456	479	502	548	525	41	340	317	571	386	202	179	18	64	248	363	294	156	409	271	225	6924
370	395	420	145	170	195	245	220	320	69	44	270	119	495	470	295	345	545	94	19	445	144	570	520	6924
112	135	158	181	504	527	573	550	66	365	342	20	411	227	204	43	89	273	388	319	457	434	296	250	6924
347	372	97	122	147	172	222	197	297	46	21	247	96	472	447	272	322	522	71	572	422	397	547	497	6924
137	160	183	206	229	552	22	575	91	390	367	45	436	252	505	68	114	298	413	344	482	459	321	275	6924
324	49	74	99	124	149	199	174	274	23	574	224	349	449	424	249	299	499	48	549	399	374	524	474	6924
162	185	208	231	254	277	47	24	116	415	392	70	461	553	530	93	139	323	438	369	507	484	346	300	6924
6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924

2.5. Bimagic Square of Order 25

Let us consider a magic square constructed based on *MODLS*:

1	2	3	4	5
4	5	1	2	3
2	3	4	5	1
5	1	2	3	4
3	4	5	1	2

A

1	2	3	4	5
3	4	5	1	2
5	1	2	3	4
2	3	4	5	1
4	5	1	2	3

B

11	22	33	44	55
43	54	15	21	32
25	31	42	53	14
52	13	24	35	41
34	45	51	12	23

AB

	65	65	65	65	65	
	1	7	13	19	25	65
65	18	24	5	6	12	65
65	10	11	17	23	4	65
65	22	3	9	15	16	65
65	14	20	21	2	8	65
	65	65	65	65	65	65

... (19)

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We shall use lines of a magic square (19) as interval numbers, and apply over A and B in (19), bringing 25 magic squares of order 5 such as,

54				
14	20	21	2	8
2	8	14	20	21
20	21	2	8	14
8	14	20	21	2
21	2	8	14	20

A

22	15	3	16	9
3	16	9	22	15
9	22	15	3	16
15	3	16	9	22
16	9	22	15	3

B

		1565	1565	1565	1565	1565
	347	490	503	41	184	1565
1565	28	191	334	497	515	1565
1565	484	522	40	178	341	1565
1565	190	328	491	509	47	1565
1565	516	34	197	340	478	1565
	1565	1565	1565	1565	1565	1565

14				
1	7	13	19	25
19	25	1	7	13
7	13	19	25	1
25	1	7	13	19
13	19	25	1	7

A

22	15	3	16	9
3	16	9	22	15
9	22	15	3	16
15	3	16	9	22
16	9	22	15	3

B

		1565	1565	1565	1565	1565
	22	165	303	466	609	1565
1565	453	616	9	172	315	1565
1565	159	322	465	603	16	1565
1565	615	3	166	309	472	1565
1565	316	459	622	15	153	1565
	1565	1565	1565	1565	1565	1565

Making more 23 magic squares of same type and put them according to (19) resulting in two MODLS of order 25 given by

1	7	13	19	25	18	24	5	6	12	10	11	17	23	4	22	3	9	15	16	14	20	21	2	8
19	25	1	7	13	6	12	18	24	5	23	4	10	11	17	15	16	22	3	9	2	8	14	20	21
7	13	19	25	1	24	5	6	12	18	11	17	23	4	10	3	9	15	16	22	20	21	2	8	14
25	1	7	13	19	12	18	24	5	6	4	10	11	17	23	16	22	3	9	15	8	14	20	21	2
13	19	25	1	7	5	6	12	18	24	17	23	4	10	11	9	15	16	22	3	21	2	8	14	20
22	3	9	15	16	14	20	21	2	8	1	7	13	19	25	18	24	5	6	12	10	11	17	23	4
15	16	22	3	9	2	8	14	20	21	19	25	1	7	13	6	12	18	24	5	23	4	10	11	17
3	9	15	16	22	20	21	2	8	14	7	13	19	25	1	24	5	6	12	18	11	17	23	4	10
16	22	3	9	15	8	14	20	21	2	25	1	7	13	19	12	18	24	5	6	4	10	11	17	23
9	15	16	22	3	21	2	8	14	20	13	19	25	1	7	5	6	12	18	24	17	23	4	10	11
18	24	5	6	12	10	11	17	23	4	22	3	9	15	16	14	20	21	2	8	1	7	13	19	25
6	12	18	24	5	23	4	10	11	17	15	16	22	3	9	2	8	14	20	21	19	25	1	7	13
24	5	6	12	18	11	17	23	4	10	3	9	15	16	22	20	21	2	8	14	7	13	19	25	1
12	18	24	5	6	4	10	11	17	23	16	22	3	9	15	8	14	20	21	2	25	1	7	13	19
5	6	12	18	24	17	23	4	10	11	9	15	16	22	3	21	2	8	14	20	13	19	25	1	7
14	20	21	2	8	1	7	13	19	25	18	24	5	6	12	10	11	17	23	4	22	3	9	15	16
2	8	14	20	21	19	25	1	7	13	6	12	18	24	5	23	4	10	11	17	15	16	22	3	9
20	21	2	8	14	7	13	19	25	1	24	5	6	12	18	11	17	23	4	10	3	9	15	16	22
8	14	20	21	2	25	1	7	13	19	12	18	24	5	6	4	10	11	17	23	16	22	3	9	15
21	2	8	14	20	13	19	25	1	7	5	6	12	18	24	17	23	4	10	11	9	15	16	22	3
10	11	17	23	4	22	3	9	15	16	14	20	21	2	8	1	7	13	19	25	18	24	5	6	12
23	4	10	11	17	15	16	22	3	9	2	8	14	20	21	19	25	1	7	13	6	12	18	24	5
11	17	23	4	10	3	9	15	16	22	20	21	2	8	14	7	13	19	25	1	24	5	6	12	18
4	10	11	17	23	16	22	3	9	15	8	14	20	21	2	25	1	7	13	19	12	18	24	5	6
17	23	4	10	11	9	15	16	22	3	21	2	8	14	20	13	19	25	1	7	5	6	12	18	24

A

1	19	7	25	13	18	6	24	12	5	10	23	11	4	17	22	15	3	16	9	14	2	20	8	21
7	25	13	1	19	24	12	5	18	6	11	4	17	10	23	3	16	9	22	15	20	8	21	14	2
13	1	19	7	25	5	18	6	24	12	17	10	23	11	4	9	22	15	3	16	21	14	2	20	8
19	7	25	13	1	6	24	12	5	18	23	11	4	17	10	15	3	16	9	22	2	20	8	21	14
25	13	1	19	7	12	5	18	6	24	4	17	10	23	11	16	9	22	15	3	8	21	14	2	20
10	23	11	4	17	22	15	3	16	9	14	2	20	8	21	1	19	7	25	13	18	6	24	12	5
11	4	17	10	23	3	16	9	22	15	20	8	21	14	2	7	25	13	1	19	24	12	5	18	6
17	10	23	11	4	9	22	15	3	16	21	14	2	20	8	13	1	19	7	25	5	18	6	24	12
23	11	4	17	10	15	3	16	9	22	2	20	8	21	14	19	7	25	13	1	6	24	12	5	18
4	17	10	23	11	16	9	22	15	3	8	21	14	2	20	25	13	1	19	7	12	5	18	6	24
14	2	20	8	21	1	19	7	25	13	18	6	24	12	5	10	23	11	4	17	22	15	3	16	9
20	8	21	14	2	7	25	13	1	19	24	12	5	18	6	11	4	17	10	23	3	16	9	22	15
21	14	2	20	8	13	1	19	7	25	5	18	6	24	12	17	10	23	11	4	9	22	15	3	16
2	20	8	21	14	19	7	25	13	1	6	24	12	5	18	23	11	4	17	10	15	3	16	9	22
8	21	14	2	20	25	13	1	19	7	12	5	18	6	24	4	17	10	23	11	16	9	22	15	3
18	6	24	12	5	10	23	11	4	17	22	15	3	16	9	14	2	20	8	21	1	19	7	25	13
24	12	5	18	6	11	4	17	10	23	3	16	9	22	15	20	8	21	14	2	7	25	13	1	19
5	18	6	24	12	17	10	23	11	4	9	22	15	3	16	21	14	2	20	8	13	1	19	7	25
6	24	12	5	18	23	11	4	17	10	15	3	16	9	22	2	20	8	21	14	19	7	25	13	1
12	5	18	6	24	4	17	10	23	11	16	9	22	15	3	8	21	14	2	20	25	13	1	19	7
22	15	3	16	9	14	2	20	8	21	1	19	7	25	13	18	6	24	12	5	10	23	11	4	17
3	16	9	22	15	20	8	21	14	2	7	25	13	1	19	24	12	5	18	6	11	4	17	10	23
9	22	15	3	16	21	14	2	20	8	13	1	19	7	25	5	18	6	24	12	17	10	23	11	4
15	3	16	9	22	2	20	8	21	14	19	7	25	13	1	6	24	12	5	18	23	11	4	17	10
16	9	22	15	3	8	21	14	2	20	25	13	1	19	7	12	5	18	6	24	4	17	10	23	11

B

... (20)

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Example 10. Apply the procedure $25\times(A-1)+B$ in (20), we get *pan diagonal intervally distributed bimagic square* of order 25:

		7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825
	1	169	307	475	613	443	581	124	137	280	235	273	411	554	92	547	65	203	366	384	339	477	520	33	196	7825
7825	457	625	13	151	319	149	287	430	593	106	561	79	242	260	423	353	391	534	72	215	45	183	346	489	502	7825
7825	163	301	469	607	25	580	118	131	299	437	267	410	573	86	229	59	222	365	378	541	496	514	27	195	333	7825
7825	619	7	175	313	451	281	449	587	105	143	98	236	254	417	560	390	528	66	209	372	177	345	483	521	39	7825
7825	325	463	601	19	157	112	130	293	431	599	404	567	85	248	261	216	359	397	540	53	508	46	189	327	495	7825
7825	535	73	211	354	392	347	490	503	41	184	14	152	320	458	621	426	594	107	150	288	243	256	424	562	80	7825
7825	361	379	542	60	223	28	191	334	497	515	470	608	21	164	302	132	300	438	576	119	574	87	230	268	406	7825
7825	67	210	373	386	529	484	522	40	178	341	171	314	452	620	8	588	101	144	282	450	255	418	556	99	237	7825
7825	398	536	54	217	360	190	328	491	509	47	602	20	158	321	464	294	432	600	113	126	81	249	262	405	568	7825
7825	204	367	385	548	61	516	34	197	340	478	308	471	614	2	170	125	138	276	444	582	412	555	93	231	274	7825
7825	439	577	120	133	296	226	269	407	575	88	543	56	224	362	380	335	498	511	29	192	22	165	303	466	609	7825
7825	145	283	446	589	102	557	100	238	251	419	374	387	530	68	206	36	179	342	485	523	453	616	9	172	315	7825
7825	596	114	127	295	433	263	401	569	82	250	55	218	356	399	537	492	510	48	186	329	159	322	465	603	16	7825
7825	277	445	583	121	139	94	232	275	413	551	381	549	62	205	368	198	336	479	517	35	615	3	166	309	472	7825
7825	108	146	289	427	595	425	563	76	244	257	212	355	393	531	74	504	42	185	348	486	316	459	622	15	153	7825
7825	343	481	524	37	180	10	173	311	454	617	447	590	103	141	284	239	252	420	558	96	526	69	207	375	388	7825
7825	49	187	330	493	506	461	604	17	160	323	128	291	434	597	115	570	83	246	264	402	357	400	538	51	219	7825
7825	480	518	31	199	337	167	310	473	611	4	584	122	140	278	441	271	414	552	95	233	63	201	369	382	550	7825
7825	181	349	487	505	43	623	11	154	317	460	290	428	591	109	147	77	245	258	421	564	394	532	75	213	351	7825
7825	512	30	193	331	499	304	467	610	23	161	116	134	297	440	578	408	571	89	227	270	225	363	376	544	57	7825
7825	247	265	403	566	84	539	52	220	358	396	326	494	507	50	188	18	156	324	462	605	435	598	111	129	292	7825
7825	553	91	234	272	415	370	383	546	64	202	32	200	338	476	519	474	612	5	168	306	136	279	442	585	123	7825
7825	259	422	565	78	241	71	214	352	395	533	488	501	44	182	350	155	318	456	624	12	592	110	148	286	429	7825
7825	90	228	266	409	572	377	545	58	221	364	194	332	500	513	26	606	24	162	305	468	298	436	579	117	135	7825
7825	416	559	97	240	253	208	371	389	527	70	525	38	176	344	482	312	455	618	6	174	104	142	285	448	586	7825
	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825	7825

The above *intervally distributed magic square* is *pan diagonal* and *bimagic* with bimagic sum $Sb_{25\times25} := 3263025$. Each sub-block of order 5 is also a *pan diagonal magic square* with sum $S_{5\times5} := 1565$.

3. Palindromic Magic Squares

Palindromic numbers. *Numbers that read the same backwards and forwards, i.e., remains the same when its digits are reversed are famous as* ***palindromic numbers****, for example, 121, 3883, 15951, etc.* Table below give the quantity of *palindromes* for each number of digit:

Digits	Palindromes	Quantity
1-digit	*a*	9
2-digits	*aa*	9
3-digits	*aba*	90
4-digits	*abba*	90
5-digits	*abcba*	900
6-digits	*abccba*	900
7-digits	*abcdcba*	9000
8-digits	*abcddcba*	9000
…..	…..	…..

From table above, we observe that there are only 90 palindromes of 3-digits, while 5-digits are 900. In this work we shall work with *5-digits palindromes* to bring magic squares. Since, we have 900 palindromes of 5-digits, it means that we can write *palindromic magic squares* up to order 30. This work is only up to order 25. *Palindromic magic squares* of order 21 and 24 are constructed in more than one way. *Palindromic magic square* of order 25 is bimagic and is constructed applying composition 5×5.

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- **Palindromic Magic Squares of Order 21**

Below we shall give three different ways of *palindromic magic squares* of order 21. Two of them are based on composition of magic squares of order 7 with of order 3. The third one is based on *SODLS* given in example 2.

Example 11. *Palindromic magic square* of order 21 with magic sum $S_{21\times 21} := 935424$ is given by

11411	22422	33433	44444	55455	66466	77477	11911	22922	33933	44944	55955	66966	77977	11211	22222	33233	44244	55255	66266	77277
65456	76467	17471	21412	32423	43434	54445	65956	76967	17971	21912	32923	43934	54945	65256	76267	17271	21212	32223	43234	54245
42424	53435	64446	75457	16461	27472	31413	42924	53935	64946	75957	16961	27972	31913	42224	53235	64246	75257	16261	27272	31213
26462	37473	41414	52425	63436	74447	15451	26962	37973	41914	52925	63936	74947	15951	26262	37273	41214	52225	63236	74247	15251
73437	14441	25452	36463	47474	51415	62426	73937	14941	25952	36963	47974	51915	62926	73237	14241	25252	36263	47274	51215	62226
57475	61416	72427	13431	24442	35453	46464	57975	61916	72927	13931	24942	35953	46964	57275	61216	72227	13231	24242	35253	46264
34443	45454	56465	67476	71417	12421	23432	34943	45954	56965	67976	71917	12921	23932	34243	45254	56265	67276	71217	12221	23232
11311	22322	33333	44344	55355	66366	77377	11511	22522	33533	44544	55555	66566	77577	11711	22722	33733	44744	55755	66766	77777
65356	76367	17371	21312	32323	43334	54345	65556	76567	17571	21512	32523	43534	54545	65756	76767	17771	21712	32723	43734	54745
42324	53335	64346	75357	16361	27372	31313	42524	53535	64546	75557	16561	27572	31513	42724	53735	64746	75757	16761	27772	31713
26362	37373	41314	52325	63336	74347	15351	26562	37573	41514	52525	63536	74547	15551	26762	37773	41714	52725	63736	74747	15751
73337	14341	25352	36363	47374	51315	62326	73537	14541	25552	36563	47574	51515	62526	73737	14741	25752	36763	47774	51715	62726
57375	61316	72327	13331	24342	35353	46364	57575	61516	72527	13531	24542	35553	46564	57775	61716	72727	13731	24742	35753	46764
34343	45354	56365	67376	71317	12321	23332	34543	45554	56565	67576	71517	12521	23532	34743	45754	56765	67776	71717	12721	23732
11811	22822	33833	44844	55855	66866	77877	11111	22122	33133	44144	55155	66166	77177	11611	22622	33633	44644	55655	66666	77677
65856	76867	17871	21812	32823	43834	54845	65156	76167	17171	21112	32123	43134	54145	65656	76667	17671	21612	32623	43634	54645
42824	53835	64846	75857	16861	27872	31813	42124	53135	64146	75157	16161	27172	31113	42624	53635	64646	75657	16661	27672	31613
26862	37873	41814	52825	63836	74847	15851	26162	37173	41114	52125	63136	74147	15151	26662	37673	41614	52625	63636	74647	15651
73837	14841	25852	36863	47874	51815	62826	73137	14141	25152	36163	47174	51115	62126	73637	14641	25652	36663	47674	51615	62626
57875	61816	72827	13831	24842	35853	46864	57175	61116	72127	13131	24142	35153	46164	57675	61616	72627	13631	24642	35653	46664
34843	45854	56865	67876	71817	12821	23832	34143	45154	56165	67176	71117	12121	23132	34643	45654	56665	67676	71617	12621	23632

Above example is based on example 1, constructed in a simplified way using *palindromic magic square* of order 7 with 3 digits and then making it to 5 digits to bring palindromic magic square of order 21. Each block of order 7 is a magic square of different sum forming a magic square of order 3 given by

			935424
311108	314608	309708	935424
310408	311808	313208	935424
313908	309008	312508	935424
935424	935424	935424	935424

Below is another example of *palindromic magic square* of order 21 written in different way with different palindromes but with same composition 7×3.

Example 12. *Palindromic magic square* of order 21 with magic sum $S_{21\times 21} := 789969$ is given by

11011	44744	47174	20502	53835	56265	29692	12221	43534	18981	19391	84048	55455	30803	10101	45654	46064	21412	52925	57375	28782
26562	59895	41214	14641	17071	50705	53135	25352	90009	42424	13831	18281	49594	24942	27472	58985	40304	15751	16161	51615	52025
20602	23032	56765	59195	11511	44844	47274	19891	24242	55555	30903	12321	73037	48484	21712	22122	57675	58085	10401	45954	46364
44144	17571	50805	53235	26662	29092	41714	13931	18381	79097	54445	25852	30203	42524	45054	16461	51915	52325	27772	28182	40604
59295	11611	14041	47774	50105	23532	56865	60406	12821	13231	48584	19991	24342	85058	58385	10701	15151	46664	51015	22422	57975
53735	56165	29592	41814	44244	17671	20002	54545	25952	30303	72027	43434	18881	19291	52625	57075	28482	40904	45354	16761	21112
47874	50205	23632	26062	59795	41114	14541	78087	49494	24842	25252	60506	12921	13331	46964	51315	22722	27172	58685	40004	15451
10201	45554	16961	21312	82028	57475	28882	11111	44644	47074	20402	53935	56365	29792	12021	43734	48184	19591	54845	55255	30603
27372	88088	40404	15851	16261	51515	22922	26462	59995	41314	14741	17171	50605	53035	25552	60806	42224	13631	18081	49794	54145
21812	22222	57575	28982	10301	75057	46464	20702	23132	56665	59095	11411	44944	47374	19691	24042	55755	60106	12521	43834	48284
15951	16361	81018	52425	27872	28282	40504	44044	17471	50905	53335	26762	29192	41614	43134	18581	49894	54245	25652	30003	42724
58485	10801	15251	46564	21912	22322	87078	59395	11711	14141	47674	50005	23432	56965	60206	12621	13031	48784	49194	24542	55855
52525	27972	28382	70007	45454	16861	21212	53635	56065	29492	41914	44344	17771	20102	54745	55155	30503	42824	43234	18681	19091
76067	51415	22822	27272	58585	10901	15351	47974	50305	23732	26162	59695	41014	14441	48884	49294	24642	25052	60706	42124	13531
12121	43634	48084	19491	54945	55355	30703	10001	45754	46164	21512	52825	57275	28682	11211	44544	17971	20302	83038	56465	29892
25452	60906	42324	13731	18181	49694	54045	27572	58885	40204	15651	16061	51715	52125	26362	89098	41414	14841	17271	50505	23932
19791	24142	55655	60006	12421	43934	48384	21612	22022	57775	58185	10501	45854	46264	20802	23232	56565	29992	11311	74047	47474
43034	18481	49994	54345	25752	30103	42624	45154	16561	51815	52225	27672	28082	40704	14941	17371	80008	53435	26862	29292	41514
60306	12721	13131	48684	49094	24442	55955	58285	10601	15051	46764	51115	22522	57875	59495	11811	14241	47574	20902	23332	86068
54645	55055	30403	42924	43334	18781	19191	52725	57175	28582	40804	45254	16661	21012	53535	26962	29392	71017	44444	17871	20202
48984	49394	24742	25152	60606	42024	13431	46864	51215	22622	27072	58785	40104	15551	77077	50405	23832	26262	59595	11911	14341

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Here also, each block of order 7 is also a *palindromic magic square* of different sums forming again a magic square of order 3 given by

			789969
263223	264433	262313	789969
262413	263323	264233	789969
264333	262213	263423	789969
789969	789969	789969	789969

Below is an example of *palindromic magic square* of order 21 based on an example 2 with same distribution of palindromes as of example 12.

Example 13. *Palindromic magic square* of order 21 based on example 2 written according to *SODLS* with magic sum $S_{21\times 21} := 789969$ is given by

10001	22222	43234	12121	**27872**	**60006**	**58185**	**29992**	**53735**	**56965**	**84048**	**55855**	11311	**18481**	**21612**	**19791**	**20502**	**44344**	**47574**	**45654**	**46464**
42224	11111	10301	22022	**59195**	**28982**	**30803**	**57075**	**85058**	**54845**	**56765**	**53935**	13231	**20702**	**19591**	**21412**	**18681**	**46664**	**45454**	**47374**	**44544**
22322	13031	12221	41214	**30903**	**57175**	**59095**	**28882**	**56865**	**83038**	**55955**	**54745**	10101	**21512**	**18781**	**20602**	**19491**	**47474**	**44644**	**46564**	**45354**
11211	40204	22122	13331	**58085**	**29892**	**27972**	**60106**	**54945**	**55755**	**53835**	**86068**	12021	**19691**	**20402**	**18581**	**21712**	**45554**	**46364**	**44444**	**47674**
48784	**51915**	**79097**	**50805**	14441	22622	47274	16561	**40304**	**43534**	**41614**	**42424**	15751	**27072**	**30203**	**28382**	**29192**	**23832**	**56065**	**54145**	**25952**
80008	**49894**	**51715**	**48984**	46264	15551	14741	22422	**42624**	**41414**	**43334**	**40504**	17671	**29392**	**28182**	**30003**	**27272**	**55155**	**24942**	**26862**	**53035**
51815	**78087**	**50905**	**49794**	22722	17471	16661	45254	**43434**	**40604**	**42524**	**41314**	14541	**30103**	**27372**	**29292**	**28082**	**26962**	**53135**	**55055**	**24842**
49994	**50705**	**48884**	**81018**	15651	44244	22522	17771	**41514**	**42324**	**40404**	**43634**	16461	**28282**	**29092**	**27172**	**30303**	**54045**	**25852**	**23932**	**56165**
57375	**60506**	**58685**	**59495**	**23032**	**26262**	**24342**	**25152**	18881	52025	51215	20902	49194	**44744**	**47974**	**75057**	**46864**	**10401**	**13631**	**11711**	**12521**
59695	**58485**	**60306**	**57575**	**25352**	**24142**	**26062**	**23232**	50205	19991	48184	22822	51015	**76067**	**45854**	**47774**	**44944**	**12721**	**11511**	**13431**	**10601**
60406	**57675**	**59595**	**58385**	**26162**	**23332**	**25252**	**24042**	52125	21812	50005	49294	18981	**47874**	**74047**	**46964**	**45754**	**13531**	**10701**	**12621**	**11411**
58585	**59395**	**57475**	**60606**	**24242**	**25052**	**23132**	**26362**	49094	48284	22922	51115	20802	**45954**	**46764**	**44844**	**77077**	**11611**	**12421**	**10501**	**13731**
13131	12321	11011	10201	17571	16761	15451	14641	21912	50105	19891	48084	52225	56465	55655	54345	53535	60806	89098	58785	57975
14841	**47074**	**45154**	**16961**	**40704**	**43934**	**71017**	**42824**	**27472**	**30603**	**28782**	**29592**	54645	53335	52525	56265	55455	**18081**	**21212**	**19391**	**20102**
46164	**15951**	**17871**	**44044**	**72027**	**41814**	**43734**	**40904**	**29792**	**28582**	**30403**	**27672**	56565	55255	54445	53635	52325	**20302**	**19191**	**21012**	**18281**
17971	**44144**	**46064**	**15851**	**43834**	**70007**	**42924**	**41714**	**30503**	**27772**	**29692**	**28482**	53435	52625	56365	55555	54245	**21112**	**18381**	**20202**	**19091**
45054	**16861**	**14941**	**47174**	**41914**	**42724**	**40804**	**73037**	**28682**	**29492**	**27572**	**30703**	55355	54545	53235	52425	56665	**19291**	**20002**	**18181**	**21312**
23432	**26662**	**24742**	**25552**	**48384**	**51515**	**49694**	**50405**	**14041**	**17271**	**15351**	**16161**	88088	**10801**	**43034**	**41114**	**12921**	57775	52925	60206	59895
25752	**24542**	**26462**	**23632**	**50605**	**49494**	**51315**	**48584**	**16361**	**15151**	**17071**	**14241**	60906	**42124**	**11911**	**13831**	**40004**	59295	58885	87078	52725
26562	**23732**	**25652**	**24442**	**51415**	**48684**	**50505**	**49394**	**17171**	**14341**	**16261**	**15051**	57875	**13931**	**40104**	**42024**	**11811**	82028	60706	59995	58285
24642	**25452**	**23532**	**26762**	**49594**	**50305**	**48484**	**51615**	**15251**	**16061**	**14141**	**17371**	59795	**41014**	**12821**	**10901**	**43134**	58985	57275	52825	90009

Examples 12 and 13 have the same distributions of palindromes. In above example, there are 20 *magic squares* of order 4 given by

				176055
27872	60006	58185	29992	176055
59195	28982	30803	57075	176055
30903	57175	59095	28882	176055
58085	29892	27972	60106	176055
176055	176055	176055	176055	176055

				250603
53735	56965	84048	55855	250603
85058	54845	56765	53935	250603
56865	83038	55955	54745	250603
54945	55755	53835	86068	250603
250603	250603	250603	250603	250603

				230601
48784	51915	79097	50805	230601
80008	49894	51715	48984	230601
51815	78087	50905	49794	230601
49994	50705	48884	81018	230601
230601	230601	230601	230601	230601

				167876
40304	43534	41614	42424	167876
42624	41414	43334	40504	167876
43434	40604	42524	41314	167876
41514	42324	40404	43634	167876
167876	167876	167876	167876	167876

				80386
18481	21612	19791	20502	80386
20702	19591	21412	18681	80386
21512	18781	20602	19491	80386
19691	20402	18581	21712	80386
80386	80386	80386	80386	80386

				184036
44344	47574	45654	46464	184036
46664	45454	47374	44544	184036
47474	44644	46564	45354	184036
45554	46364	44444	47674	184036
184036	184036	184036	184036	184036

				114849
27072	30203	28382	29192	114849
29392	28182	30003	27272	114849
30103	27372	29292	28082	114849
28282	29092	27172	30303	114849
114849	114849	114849	114849	114849

				159994
23832	56065	54145	25952	159994
55155	24942	26862	53035	159994
26962	53135	55055	24842	159994
54045	25852	23932	56165	159994
159994	159994	159994	159994	159994

				236061
57375	60506	58685	59495	236061
59695	58485	60306	57575	236061
60406	57675	59595	58385	236061
58585	59395	57475	60606	236061
236061	236061	236061	236061	236061

				98788
23032	26262	24342	25152	98788
25352	24142	26062	23232	98788
26162	23332	25252	24042	98788
24242	25052	23132	26362	98788
98788	98788	98788	98788	98788

				124030
14841	47074	45154	16961	124030
46164	15951	17871	44044	124030
17971	44144	46064	15851	124030
45054	16861	14941	47174	124030
124030	124030	124030	124030	124030

				198479
40704	43934	71017	42824	198479
72027	41814	43734	40904	198479
43834	70007	42924	41714	198479
41914	42724	40804	73037	198479
198479	198479	198479	198479	198479

				116449
27472	30603	28782	29592	116449
29792	28582	30403	27672	116449
30503	27772	29692	28482	116449
28682	29492	27572	30703	116449
116449	116449	116449	116449	116449

				100388
23432	26662	24742	25552	100388
25752	24542	26462	23632	100388
26562	23732	25652	24442	100388
24642	25452	23532	26762	100388
100388	100388	100388	100388	100388

				199998
48384	51515	49694	50405	199998
50605	49494	51315	48584	199998
51415	48684	50505	49394	199998
49594	50305	48484	51615	199998
199998	199998	199998	199998	199998

				62824
14041	17271	15351	16161	62824
16361	15151	17071	14241	62824
17171	14341	16261	15051	62824
15251	16061	14141	17371	62824
62824	62824	62824	62824	62824

				214639
44744	47974	75057	46864	214639
76067	45854	47774	44944	214639
47874	74047	46964	45754	214639
45954	46764	44844	77077	214639
214639	214639	214639	214639	214639

				48264
10401	13631	11711	12521	48264
12721	11511	13431	10601	48264
13531	10701	12621	11411	48264
11611	12421	10501	13731	48264
48264	48264	48264	48264	48264

				78786
18081	21212	19391	20102	78786
20302	19191	21012	18281	78786
21112	18381	20202	19091	78786
19291	20002	18181	21312	78786
78786	78786	78786	78786	78786

				107870
10801	43034	41114	12921	107870
42124	11911	13831	40004	107870
13931	40104	42024	11811	107870
41014	12821	10901	43134	107870
107870	107870	107870	107870	107870

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- **Palindromic Magic Squares of Order 22**

Below is a *palindromic magic square* of order 22 constructed based on example 4 using *SODLS* procedure.

Example 14. *Palindromic magic square* of order 22 with magic sum $S_{22\times 22} := 881088$ is given by

10001	14541	13931	19191	15351	57275	61416	60606	59395	58585	11211	12421	50705	51915	78187	47874	76067	52025	26762	55155	24642	23832
15451	11111	10501	14941	19291	59695	58385	57575	61216	60406	12321	13031	77177	46864	80008	51715	48984	54145	23632	22822	56065	25752
19391	15051	12221	11511	10901	61516	60206	59495	58685	57375	13431	14141	81018	48784	47974	76167	50805	26862	55055	24742	53135	22622
11911	19491	15151	13331	12521	58485	57675	61316	60506	59295	14041	10201	46964	80108	51815	78087	47774	23732	52125	26662	25852	54045
13531	12921	19091	15251	14441	60306	59595	58285	57475	61616	10101	11311	48884	77077	46764	50905	81118	25652	24842	53035	22722	56165
52725	56965	85158	54845	83038	15551	21712	20902	49194	18681	17871	46064	27072	31213	30403	29192	28382	40204	44444	43634	42324	41514
84148	53835	82028	56765	55955	47174	16661	15851	51015	20502	19791	18981	29492	28182	27372	31013	30203	42624	41314	40504	44244	43434
86068	55755	54945	83138	52825	49094	18581	17771	16961	45154	21612	20802	31313	30003	29292	28482	27172	44544	43234	42424	41614	40304
53935	82128	56865	85058	54745	21912	50105	19691	18881	47074	16561	15751	28282	27472	31113	30303	29092	41414	40604	44344	43534	42224
55855	84048	53735	52925	86168	16861	45054	21512	20702	19991	48184	17671	30103	29392	28082	27272	31413	43334	42524	41214	40404	44644
12121	13231	14341	10401	11011	18781	17971	46164	15651	21812	50005	19591	54345	55455	56565	52625	53235	59895	60906	91019	87178	58785
14241	10301	11411	12021	13131	20602	19891	48084	17571	16761	15951	51115	56465	52525	53635	54245	55355	61916	87078	88188	59795	60806
57075	31713	60106	29692	28882	40704	44944	73137	42824	71017	53435	54645	52225	26562	25952	49394	45554	20002	21212	18481	17171	16361
59195	28682	27872	61016	30703	72127	41814	70007	44744	43934	54545	55255	45654	53335	22522	26962	49494	17471	16161	20302	21012	18281
31813	60006	29792	58185	27672	74047	43734	42924	71117	40804	55655	56365	49594	45254	54445	23532	22922	21312	18081	17271	16461	20102
28782	57175	31613	30803	59095	41914	70107	44844	73037	42724	56265	52425	23932	49694	45354	55555	24542	16261	20402	21112	18381	17071
30603	29892	58085	27772	61116	43834	72027	41714	40904	74147	52325	53535	25552	24942	49294	45454	56665	18181	17371	16061	20202	21412
50205	51415	48684	47374	46564	22022	26262	25452	24142	23332	58985	89198	40004	14741	43134	12621	11811	57775	31513	30903	49894	75057
47674	46364	50505	51215	48484	24442	23132	22322	26062	25252	89098	60706	42124	11611	10801	44044	13731	75157	58885	27572	31913	49994
51515	48284	47474	46664	50305	26362	25052	24242	23432	22122	90109	61816	14841	43034	12721	41114	10601	79097	45754	59995	28582	27972
46464	50605	51315	48584	47274	23232	22422	26162	25352	24042	61716	57975	11711	40104	14641	13831	42024	28982	79197	45854	90009	29592
48384	47574	46264	50405	51615	25152	24342	23032	22222	26462	57875	88088	13631	12821	41014	10701	44144	30503	29992	49794	45954	91119

Here also we have 14 magic squares, out of them 13 are of order 5 and 1 is of order 7:

					297277
57275	61416	60606	59395	58585	297277
59695	58385	57575	61216	60406	297277
61516	60206	59495	58685	57375	297277
58485	57675	61316	60506	59295	297277
60306	59595	58285	57475	61616	297277
297277	297277	297277	297277	297277	297277

					304748
50705	51915	78187	47874	76067	304748
77177	46864	80008	51715	48984	304748
81018	48784	47974	76167	50805	304748
46964	80108	51815	78087	47774	304748
48884	77077	46764	50905	81118	304748
304748	304748	304748	304748	304748	304748

					182416
52025	26762	55155	24642	23832	182416
54145	23632	22822	56065	25752	182416
26862	55055	24742	53135	22622	182416
23732	52125	26662	25852	54045	182416
25652	24842	53035	22722	56165	182416
182416	182416	182416	182416	182416	182416

					332731
52725	56965	85158	54845	83038	332731
84148	53835	82028	56765	55955	332731
86068	55755	54945	83138	52825	332731
53935	82128	56865	85058	54745	332731
55855	84048	53735	52925	86168	332731
332731	332731	332731	332731	332731	332731

					297277
57275	61416	60606	59395	58585	297277
59695	58385	57575	61216	60406	297277
61516	60206	59495	58685	57375	297277
58485	57675	61316	60506	59295	297277
60306	59595	58285	57475	61616	297277
297277	297277	297277	297277	297277	297277

					146262
27072	31213	30403	29192	28382	146262
29492	28182	27372	31013	30203	146262
31313	30003	29292	28482	27172	146262
28282	27472	31113	30303	29092	146262
30103	29392	28082	27272	31413	146262
146262	146262	146262	146262	146262	146262

					212120
40204	44444	43634	42324	41514	212120
42624	41314	40504	44244	43434	212120
44544	43234	42424	41614	40304	212120
41414	40604	44344	43534	42224	212120
43334	42524	41214	40404	44644	212120
212120	212120	212120	212120	212120	212120

					207468
57075	31713	60106	29692	28882	207468
59195	28682	27872	61016	30703	207468
31813	60006	29792	58185	27672	207468
28782	57175	31613	30803	59095	207468
30603	29892	58085	27772	61116	207468
207468	207468	207468	207468	207468	207468

					272626
40704	44944	73137	42824	71017	272626
72127	41814	70007	44744	43934	272626
74047	43734	42924	71117	40804	272626
41914	70107	44844	73037	42724	272626
43834	72027	41714	40904	74147	272626
272626	272626	272626	272626	272626	272626

					244242
50205	51415	48684	47374	46564	244242
47674	46364	50505	51215	48484	244242
51515	48284	47474	46664	50305	244242
46464	50605	51315	48584	47274	244242
48384	47574	46264	50405	51615	244242
244242	244242	244242	244242	244242	244242

					121210
22022	26262	25452	24142	23332	121210
24442	23132	22322	26062	25252	121210
26362	25052	24242	23432	22122	121210
23232	22422	26162	25352	24042	121210
25152	24342	23032	22222	26462	121210
121210	121210	121210	121210	121210	121210

					93227
20002	21212	18481	17171	16361	93227
17471	16161	20302	21012	18281	93227
21312	18081	17271	16461	20102	93227
16261	20402	21112	18381	17071	93227
18181	17371	16061	20202	21412	93227
93227	93227	93227	93227	93227	93227

					122311
40004	14741	43134	12621	11811	122311
42124	11611	10801	44044	13731	122311
14841	43034	12721	41114	10601	122311
11711	40104	14641	13831	42024	122311
13631	12821	41014	10701	44144	122311
122311	122311	122311	122311	122311	122311

							189975
15551	21712	20902	49194	18681	17871	46064	189975
47174	16661	15851	51015	20502	19791	18981	189975
49094	18581	17771	16961	45154	21612	20802	189975
21912	50105	19691	18881	47074	16561	15751	189975
16861	45054	21512	20702	19991	48184	17671	189975
18781	17971	46164	15651	21812	50005	19591	189975
20602	19891	48084	17571	16761	15951	51115	189975
189975	189975	189975	189975	189975	189975	189975	189975

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- **Palindromic Magic Squares of Order 23**

As we know, 23 is a prime number. Its construction is similar to other prime numbers done before [17,18]. Here it is based on example on example 5.

Example 15. *Pan diagonal palindromic magic square* of order 23 with magic sum $S_{23\times 23} := 973317$ constructed based on example 5 is given by

10001	71117	42924	43734	44544	45354	46164	17971	18781	19591	20302	21112	82228	83038	54845	55655	56465	57275	58085	29892	30603	31413	32223
31113	92229	70007	41814	42624	43434	44244	45054	16861	17671	18481	19291	20002	81118	52925	53735	54545	55355	56165	27972	28782	29592	30303
29292	30003	91119	62926	40704	41514	42324	43134	14941	15751	16561	17371	18181	79297	80008	51815	52625	53435	54245	55055	26862	27672	28482
27372	28182	89298	90009	61816	62626	40404	41214	42024	13831	14641	15451	16261	17071	78187	49994	50705	51515	52325	53135	24942	25752	26562
25452	26262	27072	88188	59995	60706	61516	62326	40104	11911	12721	13531	14341	15151	76267	77077	48884	49694	50405	51215	52025	23832	24642
23532	24342	25152	86268	87078	58885	59695	60406	61216	62026	10801	11611	12421	13231	14041	75157	46964	47774	48584	49394	50105	21912	22722
21612	22422	23232	24042	85158	56965	57775	58585	59395	60106	31913	32723	10501	11311	12121	73237	74047	45854	46664	47474	48284	49094	20802
19791	20502	21312	22122	83238	84048	55855	56665	57475	58285	59095	30803	31613	32423	10201	11011	72127	43934	44744	45554	46364	47174	18981
17871	18681	19491	20202	21012	82128	53935	54745	55555	56365	57175	28982	29792	30503	31313	32123	70207	71017	42824	43634	44444	45254	46064
15951	16761	17571	18381	19191	80208	81018	52825	53635	54445	55255	56065	27872	28682	29492	30203	31013	92129	40904	41714	42524	43334	44144
43034	14841	15651	16461	17271	18081	79197	50905	51715	52525	53335	54145	25952	26762	27572	28382	29192	90209	91019	62826	40604	41414	42224
41114	12921	13731	14541	15351	16161	77277	78087	49894	50605	51415	52225	53035	24842	25652	26462	27272	28082	89198	60906	61716	62526	40304
62226	40004	11811	12621	13431	14241	15051	76167	47974	48784	49594	50305	51115	22922	23732	24542	25352	26162	87278	88088	59895	60606	61416
60306	61116	32923	10701	11511	12321	13131	74247	75057	46864	47674	48484	49294	50005	21812	22622	23432	24242	25052	86168	57975	58785	59595
58485	59295	60006	31813	32623	10401	11211	12021	73137	44944	45754	46564	47374	48184	19991	20702	21512	22322	23132	84248	85058	56865	57675
56565	57375	58185	29992	30703	31513	32323	10101	71217	72027	43834	44644	45454	46264	47074	18881	19691	20402	21212	22022	83138	54945	55755
54645	55455	56265	57075	28882	29692	30403	31213	32023	70107	41914	42724	43534	44344	45154	16961	17771	18581	19391	20102	81218	82028	53835
52725	53535	54345	55155	26962	27772	28582	29392	30103	91219	92029	40804	41614	42424	43234	44044	15851	16661	17471	18281	19091	80108	51915
50805	51615	52425	53235	54045	25852	26662	27472	28282	29092	90109	61916	62726	40504	41314	42124	13931	14741	15551	16361	17171	78287	79097
48984	49794	50505	51315	52125	23932	24742	25552	26362	27172	88288	89098	60806	61616	62426	40204	41014	12821	13631	14441	15251	16061	77177
76067	47874	48684	49494	50205	51015	22822	23632	24442	25252	26062	87178	58985	59795	60506	61316	62126	10901	11711	12521	13331	14141	75257
74147	45954	46764	47574	48384	49194	20902	21712	22522	23332	24142	85258	86068	57875	58685	59495	60206	61016	32823	10601	11411	12221	13031
72227	73037	44844	45654	46464	47274	48084	19891	20602	21412	22222	23032	84148	55955	56765	57575	58385	59195	30903	31713	32523	10301	11111

- **Palindromic Magic Squares of Order 24**

Below we shall give four different ways of constructing *palindromic magic square* of order 24 in three different ways. Two of them are based on composition of magic square of order 8 with of order 3. The third one is based on *SODLS* given in example 8.

Example 16. *Palindromic magic square* of order 24 with magic sum $S_{24\times 24} := 1201188$ is given by

28482	61416	54445	15451	43434	86468	77477	32423	28982	61916	54945	15951	43934	86968	77977	32923	28282	61216	54245	15251	43234	86268	77277	32223
42424	87478	76467	33433	25452	64446	51415	18481	42924	87978	76967	33933	25952	64946	51915	18981	42224	87278	76267	33233	25252	64246	51215	18281
11411	58485	65456	24442	36463	73437	82428	47474	11911	58985	65956	24942	36963	73937	82928	47974	11211	58285	65256	24242	36263	73237	82228	47274
37473	72427	83438	46464	14441	55455	68486	21412	37973	72927	83938	46964	14941	55955	68986	21912	37273	72227	83238	46264	14241	55255	68286	21212
56465	13431	22422	67476	71417	38483	45454	84448	56965	13931	22922	67976	71917	38983	45954	84948	56265	13231	22222	67276	71217	38283	45254	84248
74447	35453	48484	81418	57475	12421	23432	66466	74947	35953	48984	81918	57975	12921	23932	66966	74247	35253	48284	81218	57275	12221	23232	66266
63436	26462	17471	52425	88488	41414	34443	75457	63936	26962	17971	52925	88988	41914	34943	75957	63236	26262	17271	52225	88288	41214	34243	75257
85458	44444	31413	78487	62426	27472	16461	53435	85958	44944	31913	78987	62926	27972	16961	53935	85258	44244	31213	78287	62226	27272	16261	53235
28382	61316	54345	15351	43334	86368	77377	32323	28582	61516	54545	15551	43534	86568	77577	32523	28782	61716	54745	15751	43734	86768	77777	32723
42324	87378	76367	33333	25352	64346	51315	18381	42524	87578	76567	33533	25552	64546	51515	18581	42724	87778	76767	33733	25752	64746	51715	18781
11311	58385	65356	24342	36363	73337	82328	47374	11511	58585	65556	24542	36563	73537	82528	47574	11711	58785	65756	24742	36763	73737	82728	47774
37373	72327	83338	46364	14341	55355	68386	21312	37573	72527	83538	46564	14541	55555	68586	21512	37773	72727	83738	46764	14741	55755	68786	21712
56365	13331	22322	67376	71317	38383	45354	84348	56565	13531	22522	67576	71517	38583	45554	84548	56765	13731	22722	67776	71717	38783	45754	84748
74347	35353	48384	81318	57375	12321	23332	66366	74547	35553	48584	81518	57575	12521	23532	66566	74747	35753	48784	81718	57775	12721	23732	66766
63336	26362	17371	52325	88388	41314	34343	75357	63536	26562	17571	52525	88588	41514	34543	75557	63736	26762	17771	52725	88788	41714	34743	75757
85358	44344	31313	78387	62326	27372	16361	53335	85558	44544	31513	78587	62526	27572	16561	53535	85758	44744	31713	78787	62726	27772	16761	53735
28882	61816	54845	15851	43834	86868	77877	32823	28182	61116	54145	15151	43134	86168	77177	32123	28682	61616	54645	15651	43634	86668	77677	32623
42824	87878	76867	33833	25852	64846	51815	18881	42124	87178	76167	33133	25152	64146	51115	18181	42624	87678	76667	33633	25652	64646	51615	18681
11811	58885	65856	24842	36863	73837	82828	47874	11111	58185	65156	24142	36163	73137	82128	47174	11611	58685	65656	24642	36663	73637	82628	47674
37873	72827	83838	46864	14841	55855	68886	21812	37173	72127	83138	46164	14141	55155	68186	21112	37673	72627	83638	46664	14641	55655	68686	21612
56865	13831	22822	67876	71817	38883	45854	84848	56165	13131	22122	67176	71117	38183	45154	84148	56665	13631	22622	67676	71617	38683	45654	84648
74847	35853	48884	81818	57875	12821	23832	66866	74147	35153	48184	81118	57175	12121	23132	66166	74647	35653	48684	81618	57675	12621	23632	66666
63836	26862	17871	52825	88888	41814	34843	75857	63136	26162	17171	52125	88188	41114	34143	75157	63636	26662	17671	52625	88688	41614	34643	75657
85858	44844	31813	78887	62826	27872	16861	53835	85158	44144	31113	78187	62126	27172	16161	53135	85658	44644	31613	78687	62626	27672	16661	53635

Each block of order 8 is a *magic square* with different sums forming a magic square of order 3 given by

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			1201188
399596	403596	397996	1201188
398796	400396	401996	1201188
402796	397196	401196	1201188
1201188	1201188	1201188	1201188

In above example there are 9 magic squares of order 8 with different magic sums. Below is an example of same type having all the nine magic squares of order 8 same magic sum.

Example 17. *Palindromic magic square* of order 24 with magic sum $S_{24\times 24} := 1066656$ with each block of order 8 a magic square with magic sum $S_{8\times 8} := 355552$ is given by

75257	27172	52025	40304	21712	63636	58985	16461	73137	25252	24942	42424	19891	61516	90009	18381	74347	26062	53135	41214	20602	62726	59895	17571
21412	63936	58685	16761	45354	57075	22122	70207	19391	91019	60506	18881	43434	25952	24242	72127	20502	62826	59795	17671	44244	56165	23032	71317
10101	82228	57375	45054	46664	28782	33433	51915	12221	84148	55455	13931	48584	30803	31313	79097	11011	83338	56265	44144	47774	29692	32523	50805
46964	28482	33733	51615	40004	52325	87278	15151	78087	30303	31813	49594	12921	54445	85158	13231	47874	29592	32623	50705	41114	53235	86368	14041
52625	10701	15451	57975	28182	76267	51315	63036	54545	12821	13331	85058	30203	78187	49494	31913	53735	11611	14541	56865	29092	77377	50205	62126
58085	46364	81218	33133	52925	10401	15751	57675	30903	48484	79197	31213	84048	12321	13831	55555	59195	47274	80308	32023	53835	11511	14641	56765
27772	45654	40904	22422	93239	21112	46064	58385	25852	43534	72027	24342	91119	19291	18981	60406	26662	44744	41814	23532	92329	20002	47174	59295
63336	51015	16161	88288	27472	45954	40604	22722	61416	19991	18281	90109	25352	73037	42524	24842	62226	50105	17071	89398	26562	44844	41714	23632
73337	25052	54145	42224	19691	61716	60806	18581	74247	26162	53035	41314	20702	62626	59995	17471	75157	27272	22922	40404	21812	63536	88088	16361
19591	61816	60706	18681	43234	55155	24042	72327	20402	62926	59695	17771	44344	56065	23132	71217	21312	93039	58585	16861	45454	27972	22222	70107
12021	84348	55255	43134	48784	30603	31513	49894	11111	83238	56365	44044	47674	29792	32423	50905	10201	82128	57475	15951	46564	28882	33333	81018
48884	30503	31613	49794	42124	54245	85358	13031	47974	29492	32723	50605	41014	53335	86268	14141	76067	28382	33833	51515	10901	52425	87178	15251
54745	12621	13531	55855	30003	78387	49294	61116	53635	11711	14441	56965	29192	77277	50305	62026	52525	10801	15351	87078	28282	76167	51415	33933
60106	48284	79397	31013	54845	12521	13631	55755	59095	47374	80208	32123	53935	11411	14741	56665	28982	46464	81118	33233	82028	10301	15851	57575
25652	43734	42824	24542	91319	19091	48184	60206	26762	44644	41914	23432	92229	20102	47074	59395	27872	45554	70007	22322	93139	21212	16961	58485
61216	49194	18081	90309	25552	43834	42724	24642	62326	50005	17171	89298	26462	44944	41614	23732	63436	21912	16261	88188	27372	75057	40504	22822
74147	26262	23932	41414	20802	62526	89098	17371	75357	27072	52125	40204	21612	63736	58885	16561	73237	25152	54045	42324	19791	61616	60906	18481
20302	92029	59595	17871	44444	26962	23232	71117	21512	63836	58785	16661	45254	57175	22022	70307	19491	61916	60606	18781	43334	55055	24142	72227
11211	83138	56465	14941	47574	29892	32323	80008	10001	82328	57275	45154	46764	28682	33533	51815	12121	84248	55355	43034	48684	30703	31413	49994
77077	29392	32823	50505	11911	53435	86168	14241	46864	28582	33633	51715	40104	52225	87378	15051	48984	30403	31713	49694	42024	54345	85258	13131
53535	11811	14341	86068	29292	77177	50405	32923	52725	10601	15551	57875	28082	76367	51215	63136	54645	12721	13431	55955	30103	78287	49394	61016
29992	47474	80108	32223	83038	11311	14841	56565	58185	46264	81318	33033	52825	10501	15651	57775	60006	48384	79297	31113	54945	12421	13731	55655
26862	44544	71017	23332	92129	20202	17971	59495	27672	45754	40804	22522	93339	21012	46164	58285	25752	43634	42924	24442	91219	19191	48084	60306
62426	20902	17271	89198	26362	74047	41514	23832	63236	51115	16061	88388	27572	45854	40704	22622	61316	49094	18181	90209	25452	43934	42624	24742

Using the same distribution of palindromes as of above example, below is a *palindromic magic square* of order 24 based on example 8 using *SODLS* and its transpose.

Example 18. *Palindromic magic square* of order 24 based on example 8 with magic sum $S_{24\times 24} := 1066656$ is given by

10001	44044	43134	21112	20302	59495	63236	62326	53535	52725	11211	12421	60606	61816	**45954**	**79197**	**78387**	**77077**	**76267**	**24542**	**28782**	**27972**	**26662**	**25852**
20402	11111	40004	44144	21212	52825	60506	59295	63336	53635	12321	13031	61716	62426	**77377**	**76067**	**75257**	**49994**	**78187**	**26962**	**25652**	**24842**	**28582**	**27772**
21312	20002	12221	41014	40104	53735	52425	61616	60206	59395	13431	14141	62826	63536	**79297**	**48984**	**77177**	**76367**	**75057**	**28882**	**27572**	**26762**	**25952**	**24642**
41114	21412	20102	13331	42024	60306	53835	52525	62726	61216	14041	10201	63436	59695	**76167**	**75357**	**79097**	**78287**	**47974**	**25752**	**24942**	**28682**	**27872**	**26562**
43034	42124	21012	20202	14441	62226	61316	53435	52625	63836	10101	11311	59595	60706	**78087**	**77277**	**46964**	**75157**	**79397**	**27672**	**26862**	**25552**	**24742**	**28982**
54945	58285	57375	83038	82228	15551	49094	48184	21612	20802	16761	17971	85158	86368	**29092**	**33233**	**32423**	**31113**	**30303**	**40404**	**44644**	**43834**	**42524**	**41714**
82328	85058	54245	58385	83138	20902	16661	45054	49194	21712	17871	18581	86268	57975	**31413**	**30103**	**29392**	**33033**	**32223**	**42824**	**41514**	**40704**	**44444**	**43634**
83238	52925	86168	55255	54345	21812	20502	17771	46064	45154	18981	19691	87378	88088	**33333**	**32023**	**31213**	**30403**	**29192**	**44744**	**43434**	**42624**	**41814**	**40504**
55355	83338	82028	87278	56265	46164	21912	20602	18881	47074	19591	15751	58985	84148	**30203**	**29492**	**33133**	**32323**	**31013**	**41614**	**40804**	**44544**	**43734**	**42424**
57275	56365	53935	82128	88388	48084	47174	21512	20702	19991	15651	16861	84048	85258	**32123**	**31313**	**30003**	**29292**	**33433**	**43534**	**42724**	**41414**	**40604**	**44844**
12121	13231	14341	10401	11011	17671	18781	19891	15951	16561	**50005**	**53235**	**51315**	**52125**	56565	57675	58785	54845	55455	91019	92129	93239	89398	60906
14241	10301	11411	12021	13131	19791	15851	16961	17571	18681	**52325**	**51115**	**53035**	**50205**	58685	54745	55855	56465	57575	93139	89298	90309	61916	92029
86068	87178	88288	84348	55955	61516	62626	63736	59895	60406	**53135**	**50305**	**52225**	**51015**	12621	13731	14841	10901	11511	17171	18281	19391	15451	16061
88188	84248	85358	56965	87078	63636	59795	60806	61416	62526	**51215**	**52025**	**50105**	**53335**	14741	10801	11911	12521	13631	19291	15351	16461	17071	18181
29592	**33733**	**32923**	**31613**	**30803**	**40904**	**74147**	**73337**	**72027**	**71217**	55655	56865	16261	17471	54445	58085	57175	51515	50705	15051	49294	48384	23132	22322
31913	**30603**	**29892**	**33533**	**32723**	**72327**	**71017**	**70207**	**44944**	**73137**	56765	57475	17371	18081	50805	55555	54045	58185	51615	22422	16161	45254	49394	23232
33833	**32523**	**31713**	**30903**	**29692**	**74247**	**43934**	**72127**	**71317**	**70007**	57875	58585	18481	19191	51715	50405	56665	55055	54145	23332	22022	17271	46264	45354
30703	**29992**	**33633**	**32823**	**31513**	**71117**	**70307**	**74047**	**73237**	**42924**	58485	54645	19091	15251	55155	51815	50505	57775	56065	46364	23432	22122	18381	47274
32623	**31813**	**30503**	**29792**	**33933**	**73037**	**72227**	**41914**	**70107**	**74347**	54545	55755	15151	16361	57075	56165	51415	50605	58885	48284	47374	23032	22222	19491
45454	**49694**	**48884**	**47574**	**46764**	**24042**	**28282**	**27472**	**26162**	**25352**	90109	91319	11711	12921	10501	44244	43334	23632	22822	59995	63036	62126	81018	80208
47874	**46564**	**45754**	**49494**	**48684**	**26462**	**25152**	**24342**	**28082**	**27272**	91219	62926	12821	13531	22922	11611	40204	44344	23732	80308	90009	59095	63136	81118
49794	**48484**	**47674**	**46864**	**45554**	**28382**	**27072**	**26262**	**25452**	**24142**	92329	93039	13931	14641	23832	22522	12721	41214	40304	81218	50905	91119	60006	59195
46664	**45854**	**49594**	**48784**	**47474**	**25252**	**24442**	**28182**	**27372**	**26062**	63936	89198	14541	10701	41314	23932	22622	13831	42224	60106	81318	80008	92229	61016
48584	**47774**	**46464**	**45654**	**49894**	**27172**	**26362**	**25052**	**24242**	**28482**	89098	90209	10601	11811	43234	42324	23532	22722	14941	62026	61116	51915	80108	93339

Here we have 9 magic square, out of them 8 are of order 5 and 1 is of order 4. See below:

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45954	79197	78387	77077	76267	356882
77377	76067	75257	49994	78187	356882
79297	48984	77177	76367	75057	356882
76167	75357	79097	78287	47974	356882
78087	77277	46964	75157	79397	356882
356882	356882	356882	356882	356882	356882

24542	28782	27972	26662	25852	133810
26962	25652	24842	28582	27772	133810
28882	27572	26762	25952	24642	133810
25752	24942	28682	27872	26562	133810
27672	26862	25552	24742	28982	133810
133810	133810	133810	133810	133810	133810

29092	33233	32423	31113	30303	156164
31413	30103	29392	33033	32223	156164
33333	32023	31213	30403	29192	156164
30203	29492	33133	32323	31013	156164
32123	31313	30003	29292	33433	156164
156164	156164	156164	156164	156164	156164

40404	44644	43834	42524	41714	213120
42824	41514	40704	44444	43634	213120
44744	43434	42624	41814	40504	213120
41614	40804	44544	43734	42424	213120
43534	42724	41414	40604	44844	213120
213120	213120	213120	213120	213120	213120

29592	33733	32923	31613	30803	158664
31913	30603	29892	33533	32723	158664
33833	32523	31713	30903	29692	158664
30703	29992	33633	32823	31513	158664
32623	31813	30503	29792	33933	158664
158664	158664	158664	158664	158664	158664

40904	74147	73337	72027	71217	331632
72327	71017	70207	44944	73137	331632
74247	43934	72127	71317	70007	331632
71117	70307	74047	73237	42924	331632
73037	72227	41914	70107	74347	331632
331632	331632	331632	331632	331632	331632

45454	49694	48884	47574	46764	238370
47874	46564	45754	49494	48684	238370
49794	48484	47674	46864	45554	238370
46664	45854	49594	48784	47474	238370
48584	47774	46464	45654	49894	238370
238370	238370	238370	238370	238370	238370

24042	28282	27472	26162	25352	131310
26462	25152	24342	28082	27272	131310
28382	27072	26262	25452	24142	131310
25252	24442	28182	27372	26062	131310
27172	26362	25052	24242	28482	131310
131310	131310	131310	131310	131310	131310

50005	53235	51315	52125	206680
52325	51115	53035	50205	206680
53135	50305	52225	51015	206680
51215	52025	50105	53335	206680
206680	206680	206680	206680	206680

Example 7 is a magic square of order 24 where each block of order 4 is a *pan diagonal magic square*. Question is, whether we can write a *palindromic magic square* of similar kind. We are unable to do using palindromes as of examples 17 or 18. But, we can write *palindromic magic square* of order 24 based on example 7 considering different distribution of palindromes.

Example 19. *Palindromic magic square* of order 24 with magic sum $S_{24\times 24} := 1320000$ with each block of order 4 a *pan diagonal magic* square with sum $S_{4\times 4} := 220000$ is given by

53035	26762	74247	65956	54345	25452	73537	66666	51215	28582	76467	63736	50105	29692	77377	62826	49394	30403	78587	61616	52025	27772	75257	64946
44944	95259	23732	56065	43634	96569	24442	55355	46764	93439	21512	58285	47874	92329	20602	59195	48684	91519	19491	60306	45954	94249	22722	57075
35753	44044	56965	83238	36463	43334	55655	84548	33533	46264	58785	81418	32623	47174	59895	80308	31413	48384	60606	79597	34743	45054	57975	82228
86268	53935	65056	14741	85558	54645	66366	13431	88488	51715	63236	16561	89398	50805	62126	17671	90509	49694	61316	18481	87278	52925	64046	15751
54445	25352	73637	66566	52225	27572	75457	64746	49094	30703	78287	61916	51015	28782	76267	63936	50405	29392	77677	62526	23932	26862	74147	95059
43534	96669	24342	55455	45754	94449	22522	57275	48984	91219	19791	60006	46964	93239	21712	58085	47574	92629	20302	59495	74047	95159	23832	26962
36363	43434	55555	84648	34543	45254	57775	82428	31713	48084	60906	79297	33733	46064	58985	81218	32323	47474	59595	80608	35853	14941	86068	83138
85658	54545	66466	13331	87478	52725	64246	15551	90209	49994	61016	18781	88288	51915	63036	16761	89698	50505	62426	17371	86168	83038	35953	14841
52425	27372	75657	64546	49294	30503	78487	61716	50205	29592	77477	62726	53335	26462	74547	65656	21912	28882	76167	93039	24942	25852	73137	96069
45554	94649	22322	57475	48784	91419	19591	60206	47774	92429	20502	59295	44644	95559	23432	56365	76067	93139	21812	28982	73037	96169	24842	25952
34343	45454	57575	82628	31513	48284	60706	79497	32523	47274	59795	80408	35453	44344	56665	83538	33833	16961	88088	81118	36863	13931	85058	84148
87678	52525	64446	15351	90409	49794	61216	18581	89498	50705	62226	17571	86568	53635	65356	14441	88188	81018	33933	16861	85158	84048	36963	13831
51315	28482	76567	63636	53135	26662	74347	65856	54245	25552	73437	66766	49194	30603	78387	61816	22922	27872	75157	94049	50305	29492	77577	62626
46664	93539	21412	58385	44844	95359	23632	56165	43734	96469	24542	55255	48884	91319	19691	60106	75057	94149	22822	27972	47674	92529	20402	59395
33433	46364	58685	81518	35653	44144	56865	83338	36563	43234	55755	84448	31613	48184	60806	79397	34843	15951	87078	82128	32423	47374	59695	80508
88588	51615	63336	16461	86368	53835	65156	14641	85458	54745	66266	13531	90309	49894	61116	18681	87178	82028	34943	15851	89598	50605	62326	17471
50405	29392	77677	62526	51115	28682	76367	63836	53235	26562	74447	65756	52325	27472	75557	64646	54045	25752	73237	66966	49494	30303	78687	61516
47574	92629	20302	59495	46864	93339	21612	58185	44744	95459	23532	56265	45654	94549	22422	57375	43934	96269	24742	55055	48584	91619	19391	60406
32323	47474	59595	80608	33633	46164	58885	81318	35553	44244	56765	83438	34443	45354	57675	82528	36763	43034	55955	84248	31313	48484	60506	79697
89698	50505	62426	17371	88388	51815	63136	16661	86468	53735	65256	14541	87578	52625	64346	15451	85258	54945	66066	13731	90609	49594	61416	18381
19991	30803	78187	91019	50005	29792	77277	62926	52125	27672	75357	64846	54145	25652	73337	66866	53435	26362	74647	65556	51415	28382	76667	63536
78087	91119	19891	30903	47974	92229	20702	59095	45854	94349	22622	57175	43834	96369	24642	55155	44544	95659	23332	56465	46564	93639	21312	58485
31813	18981	90009	79197	32723	47074	59995	80208	34643	45154	57875	82328	36663	43134	55855	84348	35353	44444	56565	83638	33333	46464	58585	81618
90109	79097	31913	18881	89298	50905	62026	17771	87378	52825	64146	15651	85358	54845	66166	13631	86668	53535	65456	14341	88688	51515	63436	16361

The above magic square also admits 9 magic squares of order 8 with equal magic sums. Writing according to example 17, we can have some blocks of order 8 as *semi-bimagic squares*. See example below.

Example 20. *Palindromic magic square* of order 24 with magic sum $S_{24\times 24} := 1320000$ with written according to example 17 with each block of order 8 a *pan diagonal magic square* of magic sum $S_{8\times 8} := 440000$ is given by

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78587	30403	55355	43634	54045	66966	91219	19791	76467	28582	57275	45754	52125	64846	93339	21612	77677	29392	56465	44544	23932	95059	92129	20802
24742	96269	61916	49094	48684	60306	25452	73537	22622	94349	63836	51115	46764	58285	27572	75457	23832	95159	92029	20902	47574	59495	26362	74647
13431	85558	60606	48384	49994	61016	36763	84248	15551	87478	58785	46264	51815	63136	34643	82328	14341	86668	59595	47474	80008	32923	35853	83138
79297	31713	66066	54945	43334	55655	90509	18481	81318	33633	64146	52825	45254	57775	88488	16561	80108	32823	35953	83038	44444	56565	89698	17371
55955	43034	18781	90209	31413	79597	54645	66366	57875	45154	16661	88388	33533	81418	52725	64246	86068	14941	17871	89198	32323	80608	53535	65456
61316	49694	84548	36463	85258	13731	48084	60906	63236	51715	82428	34543	87378	15651	46164	58885	62426	50505	83638	35353	86168	14841	17971	89098
60006	48984	73237	25752	96569	24442	49394	61616	58185	46864	75357	27672	94449	22522	51215	63736	29992	77077	74147	26862	95659	23332	50405	62526
66666	54345	19491	91519	30703	78287	43934	55055	64746	52225	21512	93439	28682	76367	45854	57175	65556	53435	20302	92629	29892	77177	74047	26962
76667	28382	57475	45554	22922	94049	93139	21812	**77577**	**29492**	**56365**	**44644**	**53035**	**65956**	**92229**	**20702**	**78487**	**30503**	**55255**	**43734**	**54145**	**66866**	**91319**	**19691**
22822	94149	93039	21912	46564	58485	27372	75657	**23732**	**95259**	**62926**	**50005**	**47674**	**59395**	**26462**	**74547**	**24642**	**96369**	**61816**	**49194**	**48784**	**60206**	**25552**	**73437**
15351	87678	58585	46464	81018	33933	34843	82128	**14441**	**86568**	**59695**	**47374**	**50905**	**62026**	**35753**	**83238**	**13531**	**85458**	**60706**	**48284**	**49894**	**61116**	**36663**	**84348**
81118	33833	34943	82028	45454	57575	88688	16361	**80208**	**32723**	**65056**	**53935**	**44344**	**56665**	**89598**	**17471**	**79397**	**31613**	**66166**	**54845**	**43234**	**55755**	**90409**	**18581**
87078	15951	16861	88188	33333	81618	52525	64446	**56965**	**44044**	**17771**	**89298**	**32423**	**80508**	**53635**	**65356**	**55855**	**43134**	**18681**	**90309**	**31513**	**79497**	**54745**	**66266**
63436	51515	82628	34343	87178	15851	16961	88088	**62326**	**50605**	**83538**	**35453**	**86268**	**14741**	**47074**	**59995**	**61216**	**49794**	**84448**	**36563**	**85358**	**13631**	**48184**	**60806**
28982	76067	75157	27872	94649	22322	51415	63536	**59095**	**47974**	**74247**	**26762**	**95559**	**23432**	**50305**	**62626**	**60106**	**48884**	**73337**	**25652**	**96469**	**24542**	**49294**	**61716**
64546	52425	21312	93639	28882	76167	75057	27972	**65656**	**53335**	**20402**	**92529**	**29792**	**77277**	**44944**	**56065**	**66766**	**54245**	**19591**	**91419**	**30603**	**78387**	**43834**	**55155**
77477	**29592**	**56265**	**44744**	**53135**	**65856**	**92329**	**20602**	78687	30303	55455	43534	24942	96069	91119	19891	**76567**	**28482**	**57375**	**45654**	**52025**	**64946**	**93239**	**21712**
23632	**95359**	**62826**	**50105**	**47774**	**59295**	**26562**	**74447**	24842	96169	91019	19991	48584	60406	25352	73637	**22722**	**94249**	**63936**	**51015**	**46664**	**58385**	**27472**	**75557**
14541	**86468**	**59795**	**47274**	**50805**	**62126**	**35653**	**83338**	13331	85658	60506	48484	79097	31913	36863	84148	**15451**	**87578**	**58685**	**46364**	**51915**	**63036**	**34743**	**82228**
80308	**32623**	**65156**	**53835**	**44244**	**56765**	**89498**	**17571**	79197	31813	36963	84048	43434	55555	90609	18381	**81218**	**33733**	**64046**	**52925**	**45354**	**57675**	**88588**	**16461**
56865	**44144**	**17671**	**89398**	**32523**	**80408**	**53735**	**65256**	85058	13931	18881	90109	31313	79697	54545	66466	**57975**	**45054**	**16761**	**88288**	**33433**	**81518**	**52625**	**64346**
62226	**50705**	**83438**	**35553**	**86368**	**14641**	**47174**	**59895**	61416	49594	84648	36363	85158	13831	18981	90009	**63336**	**51615**	**82528**	**34443**	**87278**	**15751**	**46064**	**58985**
59195	**47874**	**74347**	**26662**	**95459**	**23532**	**50205**	**62726**	30903	78087	73137	25852	96669	24342	49494	61516	**58085**	**46964**	**75257**	**27772**	**94549**	**22422**	**51315**	**63636**
65756	**53235**	**20502**	**92429**	**29692**	**77377**	**44844**	**56165**	66566	54445	19391	91619	30803	78187	73037	25952	**64646**	**52325**	**21412**	**93539**	**28782**	**76267**	**45954**	**57075**

Out of 9 blocks of order 8, there are 4 blocks *semi-bimagic squares*, such as

		440000	440000	440000	440000	440000	440000	440000	440000
	77477	29592	56265	44744	53135	65856	92329	20602	440000
440000	23632	95359	62826	50105	47774	59295	26562	74447	440000
440000	14541	86468	59795	47274	50805	62126	35653	83338	440000
440000	80308	32623	65156	53835	44244	56765	89498	17571	440000
440000	56865	44144	17671	89398	32523	80408	53735	65256	440000
440000	62226	50705	83438	35553	86368	14641	47174	59895	440000
440000	59195	47874	74347	26662	95459	23532	50205	62726	440000
440000	65756	53235	20502	92429	29692	77377	44844	56165	440000
	440000	440000	440000	440000	440000	440000	440000	440000	440000

								28516827320
6002685529	875686464	3165750225	2002025536	2823328225	4337012736	8524644241	424442404	28155575360
558471424	9093338881	3947106276	2510511025	2282355076	3515897025	705539844	5542355809	28155575360
211440681	7476715024	3575442025	2234831076	2581148025	3859639876	1271136409	6945222244	28155575360
6449374864	1064260129	4245304336	2898207225	1957531536	3222265225	8009892004	308740041	28155575360
3233628225	1948692736	312264241	7992002404	1057745529	6465446464	2887450225	4258345536	28155575360
3872075076	2570997025	6961899844	1264015809	7459431424	214358881	2225386276	3587411025	28155575360
3504048025	2291919876	5527476409	710862244	9112420681	553755024	2520542025	3934551076	28155575360
4323851536	2833965225	420332004	8543120041	881614864	5987200129	2010984336	3154507225	28155575360
28155575360	28155575360	28155575360	28155575360	28155575360	28155575360	28155575360	28155575360	**28516827320**

Comparing examples 17 and 20, we observe that example 20 is better as its sub-blocks of order 8 are pan diagonal magic squares, and out of 9 sub-blocks, there are 4 blocks *semi-bimagic* as shown above. Also, it is interesting to observe that the distributions given in examples 17 or 18 is not good to write examples 19 and 20. But, we can write examples 17 and 18 using the distributions of example 19 or 20. See below example 18 written with distribution of palindromes of example 19 or 20.

Example 21. *Palindromic magic square* of order 24 based on example 8 with magic sum $S_{24\times 24} := 1320000$ is given by

13331	47374	46464	24442	23632	62726	66566	65656	56865	85058	14541	15751	63936	94149	**78287**	**82428**	**81618**	**80308**	**79597**	**27872**	**61016**	**60206**	**29992**	**58185**
23732	14441	43334	47474	24542	85158	63836	62526	66666	56965	15651	16361	94049	65756	**80608**	**79397**	**78587**	**82228**	**81418**	**59295**	**28982**	**57175**	**31813**	**60006**
24642	23332	15551	44344	43434	86068	55755	64946	63536	62626	16761	17471	95159	66866	**82528**	**81218**	**80408**	**79697**	**78387**	**61116**	**30803**	**59095**	**58285**	**27972**
44444	24742	23432	16661	45354	63636	86168	55855	95059	64546	17371	13531	66766	62926	**79497**	**78687**	**82328**	**81518**	**80208**	**58085**	**57275**	**31913**	**60106**	**29892**
46364	45454	24342	23532	17771	65556	64646	56765	55955	96169	13431	14641	62826	93039	**81318**	**80508**	**79297**	**78487**	**82628**	**30903**	**59195**	**28882**	**57075**	**61216**
87278	61516	60606	86368	85558	18881	52325	51415	24942	53135	49094	50205	88488	89698	**32323**	**36563**	**35753**	**34443**	**33633**	**43734**	**47974**	**76167**	**45854**	**74047**
85658	88388	57575	61616	86468	53235	19991	48384	52425	54045	50105	21812	89598	90209	**34743**	**33433**	**32623**	**36363**	**35553**	**75157**	**44844**	**73037**	**47774**	**46964**
86568	85258	89498	58585	57675	54145	23832	50005	49394	48484	51215	22922	90609	91319	**36663**	**35353**	**34543**	**33733**	**32423**	**77077**	**46764**	**45954**	**74147**	**43834**
58685	86668	85358	90509	59595	49494	54245	23932	51115	50305	22822	48084	91219	87478	**33533**	**32723**	**36463**	**35653**	**34343**	**44944**	**73137**	**47874**	**76067**	**45754**
60506	59695	86268	85458	91619	51315	50405	24842	53035	52225	18981	49194	87378	88588	**35453**	**34643**	**33333**	**32523**	**36763**	**46864**	**75057**	**44744**	**43934**	**77177**
15451	16561	17671	13731	14341	20902	51015	52125	48284	19891	**53335**	**56565**	**54645**	**55455**	59895	60906	91019	87178	58785	94349	95459	96569	92629	93239
17571	13631	14741	15351	16461	52025	48184	49294	20802	21912	**55655**	**54445**	**56365**	**53535**	61916	87078	88188	59795	60806	96469	92529	93639	94249	95359
89398	90409	91519	87678	88288	64846	65956	96069	92129	63736	**56465**	**53635**	**55555**	**54345**	15951	46064	47174	43234	14841	20402	21512	22622	18781	19391
91419	87578	88688	89298	90309	66966	92029	93139	64746	65856	**54545**	**55355**	**53435**	**56665**	47074	43134	44244	15851	16961	22522	18681	19791	20302	21412
32823	**66066**	**65256**	**34943**	**63136**	**73237**	**77477**	**76667**	**75357**	**74547**	58985	89198	19591	20702	57775	61316	60406	54845	83038	18381	52525	51615	26462	25652
64246	**33933**	**62126**	**36863**	**65056**	**75657**	**74347**	**73537**	**77277**	**76467**	89098	60706	20602	21312	83138	58885	57375	61416	54945	25752	19491	48584	52625	26562
66166	**35853**	**64046**	**63236**	**32923**	**77577**	**76267**	**75457**	**74647**	**73337**	90109	61816	21712	22422	84048	53735	59995	58385	57475	26662	25352	20502	49594	48684
63036	**62226**	**36963**	**65156**	**34843**	**74447**	**73637**	**77377**	**76567**	**75257**	61716	57975	22322	18581	58485	84148	53835	90009	59395	49694	26762	25452	21612	50505
35953	**64146**	**33833**	**62026**	**66266**	**76367**	**75557**	**74247**	**73437**	**77677**	57875	88088	18481	19691	60306	59495	54745	53935	91119	51515	50605	26362	25552	22722
48784	**52925**	**81118**	**50805**	**79097**	**27372**	**31513**	**30703**	**29492**	**28682**	93439	94649	44044	45254	13831	47574	46664	26962	55155	92229	66366	65456	84348	83538
80108	**49894**	**78087**	**52725**	**51915**	**29792**	**28482**	**27672**	**31313**	**30503**	94549	95259	45154	16861	55255	14941	43534	47674	56065	83638	93339	62326	66466	84448
82028	**51715**	**50905**	**79197**	**48884**	**31613**	**30303**	**29592**	**28782**	**27472**	95659	96369	46264	17971	56165	25852	45054	44544	43634	84548	83238	94449	63336	62426
49994	**78187**	**52825**	**81018**	**50705**	**28582**	**27772**	**31413**	**30603**	**29392**	96269	92429	17871	43034	44644	56265	25952	46164	45554	63436	84648	83338	95559	64346
51815	**80008**	**49794**	**48984**	**82128**	**30403**	**29692**	**28382**	**27572**	**31713**	92329	93539	13931	44144	46564	45654	26862	55055	47274	65356	64446	84248	83438	96669

Here also we have 9 magic squares, where 8 are of order 5 and 1 is of order 4.

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- **Palindromic Bimagic Square of Order 25**

Below is an example of *palindromic bimagic square* of order 25 written based on the approach of example 10.

Example 22. *Palindromic magic square* of order 25 with magic sum $S_{25\times 25} := 1161105$, where each block of order 5 a *pan diagonal magic square* with magic sum $S_{5\times 5} := 232221$ is given by

10001	46864	22622	88488	64246	57775	33533	74347	45154	21412	19991	80208	56065	32323	43634	91119	42424	18281	54545	25852	53335	29192	60906	11711	77077
28682	94449	40204	16061	52825	75357	51115	27472	63736	14541	62026	13331	49694	20902	86268	24242	55555	31813	72127	48484	41914	17771	83038	59395	30103
46264	22022	58885	34643	70407	33433	44744	15551	81318	57175	50605	26962	92229	43034	19391	12821	78187	54445	25252	61516	89098	60306	11111	47974	23732
64846	10601	76467	52225	28082	21512	87378	63136	14441	45754	73237	49094	20302	56665	32923	55455	31213	42524	18881	84148	17171	53935	29792	90009	41314
82428	58285	34043	40804	16661	44144	15451	51715	27572	93339	26362	62626	13931	79297	50005	48584	24842	85158	61416	12221	30703	71017	47374	23132	59995
31913	72227	48084	24342	55655	83138	59495	30203	41514	17871	40304	16161	52925	28782	94049	27072	63836	14641	75457	51215	49794	20502	86368	62126	13431
54045	25352	61616	12921	78287	11211	47574	23832	89198	60406	58985	34743	70007	46364	22122	15651	81418	57275	33033	44844	92329	43134	19491	50705	26562
42624	18981	84248	55055	31313	29892	90109	41414	17271	53535	76067	52325	28182	64946	10701	63236	14041	45854	21612	87478	20402	56765	32523	73337	49194
85258	61016	12321	48684	24942	47474	23232	59595	30803	71117	34143	40904	16761	82028	58385	51815	27672	93439	44244	15051	13531	79397	50105	26462	62726
18381	54645	25952	91219	42024	60506	11811	77177	53435	29292	22722	88088	64346	10101	46964	74447	45254	21012	57875	33633	56165	32423	43734	19591	80308
57375	33133	44944	15751	81018	19091	50805	26662	92429	43234	61716	12521	78387	54145	25452	23932	89298	60006	11311	47674	70107	46464	22222	58585	34843
45954	21712	87078	63336	14141	32623	73437	49294	20002	56865	84348	55155	31413	42724	18581	41014	17371	53635	29992	90209	28282	64546	10801	76167	52425
93039	44344	15151	51915	27772	50205	26062	62826	13631	79497	12421	48784	24542	85358	61116	59695	30903	71217	47074	23332	16861	82128	58485	34243	40504
21112	57975	33733	74047	45354	43834	19691	80408	56265	32023	25552	91319	42124	18481	54745	77277	53035	29392	60606	11911	64446	10201	46564	22822	88188
14741	75057	51315	27172	63936	86468	62226	13031	49894	20602	48184	24442	55755	31513	72327	30303	41614	17971	83238	59095	52525	28882	94149	40404	16261
53735	29592	90309	41114	17471	10901	76267	52025	28382	64646	87178	63436	14241	45554	21812	49394	20102	56965	32723	73037	31013	42824	18681	84448	55255
71317	47174	23432	59795	30503	58085	34343	40604	16961	82228	15251	51515	27872	93139	44444	62926	13731	79097	50305	26162	24642	85458	61216	12021	48884
29492	60706	11511	77377	53135	46664	22922	88288	64046	10301	33833	74147	45454	21212	57575	80008	56365	32123	43934	19791	42224	18081	54845	25652	91419
17571	83338	59195	30403	41714	94249	40004	16361	52625	28982	51415	27272	63536	14841	75157	13131	49994	20702	86068	62326	55855	31613	72427	48284	24042
60106	11411	47774	23532	89398	22322	58685	34943	70207	46064	44544	15851	81118	57475	33233	26762	92029	43334	19191	50905	78487	54245	25052	61816	12621
79197	50405	26262	62526	13831	61316	12121	48984	24742	85058	23032	59895	30603	71417	47274	40704	16561	82328	58185	34443	27972	93239	44044	15351	51615
32223	43534	19891	80108	56465	54945	25752	91019	42324	18181	11611	77477	53235	29092	60806	88388	64146	10401	46764	22522	45054	21312	57675	33933	74247
20802	86168	62426	13231	49594	72027	48384	24142	55955	31713	59295	30003	41814	17671	83438	16461	52725	28582	94349	40104	63636	14941	75257	51015	27372
43434	19291	50505	26862	92129	25152	61916	12721	78087	54345	47874	23632	89498	60206	11011	34543	70307	46164	22422	58785	81218	57075	33333	44644	15951
56565	32823	73137	49494	20202	18781	84048	55355	31113	42924	90409	41214	17071	53835	29692	52125	28482	64746	10501	76367	14341	45654	21912	87278	63036

4. Selfie Magic Squares

Selfie magic square. *It is a magic square that remains the same after making the rotation of 180⁰ and looking through the mirror. Most of the mirror looking magic squares are reversible, but there are* rotatable *magic square not mirror looking. In this case we call as* ***rotatable or upside down magic squares****. Selfie magic squares are also known by* ***universal magic squares****.*

More precisely, a *Selfie magic square* should have the following three properties:
(i) Upside down (180^0 rotatable);
(ii) Mirror looking;
(iii) Having the same sum (sum of rows, columns and of principal diagonals).

By *rotatable or upside down magic square*, it is understood that when we make a rotation of 180^0, it remains the same. *Mirror looking* magic square are when seen through mirror or their reflection in water, again remains as magic square. It may happen that when rotating to 180^0 or looking through mirror, they are magic squares, but with different sums. In this case, they are not *Selfie magic square*. *Selfie magic square* with letters are famous as IXOHOXI magic squares.

Working with numbers, there are 7 *upside down digits*, i.e., 0, 1, 2, 5, 6, 8 and 9. Digits 0, 1 and 8 are always *upside down* and *mirror looking*. After making rotation of 180^0, 6 becomes as 9 and 9 as 6, but these two are not mirror looking. Also, 2 and 5 are not rotatable. When we write them in digital form, like 2 and 5, they becomes rotatable and mirror looking. When we see them through mirror, 2 becomes as 5, and 5 as 2. Thus, there are only five digits 0, 1, 2, 5 and 8 written in digital form, give us conditions to bring *Selfie magic square*. In this work we shall give only *Selfie magic squares* of orders 11 to 20.

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- **Selfie Magic Square of Order 21**

Example 23. *Selfie semi-magic square* of order 21 with sums of row and columns 78881 and sum of diagonals 76681 and 78859 is given by

0205	8012	2820	1525	8151	5058	1882	0808	5511	2221	1122	8552	5255	2185	0502	5815	2518	1228	8250	5180	2081
1558	8182	5005	1812	0220	8025	2851	1155	8585	5208	2111	0821	5522	2252	1280	8281	5102	2015	0518	5828	2550
1825	0251	8058	2882	1505	8112	5020	2122	0852	5555	2285	1108	8511	5221	2028	0550	5880	2581	1202	8215	5118
2812	1520	8125	5051	1858	0282	8005	2211	1121	8522	5252	2155	0885	5508	2515	1218	8228	5150	2080	0581	5802
5082	1805	0212	8020	2825	1551	8158	5285	2108	0811	5521	2222	1152	8555	5181	2002	0515	5818	2528	1250	8280
8051	2858	1582	8105	5012	1820	0225	5552	2255	1185	8508	5211	2121	0822	5850	2580	1281	8202	5115	2018	0528
8120	5025	1851	0258	8082	2805	1512	8521	5222	2152	0855	5585	2208	1111	8218	5128	2050	0580	5881	2502	1215
0802	5515	2218	1128	8550	5280	2181	0505	5812	2520	1225	8251	5158	2082	0208	8011	2821	1522	8152	5055	1885
1180	8581	5202	2115	0818	5528	2250	1258	8282	5105	2012	0520	5825	2551	1555	8185	5008	1811	0221	8022	2852
2128	0850	5580	2281	1102	8515	5218	2025	0551	5858	2582	1205	8212	5120	1822	0252	8055	2885	1508	8111	5021
2215	1118	8528	5250	2180	0881	5502	2512	1220	8225	5151	2058	0582	5805	2811	1521	8122	5052	1855	0285	8008
5281	2102	0815	5518	2228	1150	8580	5182	2005	0512	5820	2525	1251	8258	5085	1808	0211	8021	2822	1552	8155
5550	2280	1181	8502	5215	2118	0828	5851	2558	1282	8205	5112	2020	0525	8052	2855	1585	8108	5011	1821	0222
8518	5228	2150	880	5581	2202	1115	8220	5125	2051	0558	5882	2505	1212	8121	5022	1852	0255	8085	2808	1511
0508	5811	2521	1222	8252	5155	2085	0202	8015	2818	1528	8150	5080	1881	0805	5512	2220	1125	8551	5258	2182
1255	8285	5108	2011	0521	5822	2552	1580	8181	5002	1815	0218	8028	2850	1158	8582	5205	2112	0820	5525	2251
2022	0552	5855	2585	1208	8211	5121	1828	0250	8080	2881	1502	8115	5018	2125	0851	5558	2282	1105	8512	5220
2511	1221	8222	5152	2055	0585	5808	2815	1518	8128	5050	1880	0281	8002	2212	1120	8525	5251	2158	0882	5505
5185	2008	0511	5821	2522	1252	8255	5081	1802	0215	8018	2828	1550	8180	5282	2105	0812	5520	2225	1151	8558
5852	2555	1285	8208	5111	2021	0522	8050	2880	1581	8102	5015	1818	0228	5551	2258	1182	8505	5212	2120	0825
8221	5122	2052	0555	5885	2508	1211	8118	5028	1850	0280	8081	2802	1515	8520	5225	2151	0858	5582	2205	1112

Above magic square is formed by 9 pan diagonal magic squares of order 7 with different sums forming a semi-magic square of 3:

			76681
27653	25654	25574	78881
25674	25553	27654	78881
25554	27674	25653	78881
78881	78881	78881	**78859**

Above example is *Selfie semi-magic* as diagonals sums is different from rows and columns sum. The example below is *Selfie magic square* of order 21 constructed based on example 2.

Example 24. *Selfie magic square* of order 21 with magic sum $S_{21\times 21} := 78881$ is given by

0202	5008	1150	0805	8021	8525	8128	8222	5180	5882	5285	5581	0511	2112	2818	2220	2515	1251	2055	1558	1852
0850	0505	0211	5002	8228	8122	8521	8025	5585	5281	5880	5182	1108	2520	2215	2812	2118	1858	1552	2051	1255
5011	1102	0808	0550	8522	8028	8225	8121	5881	5185	5582	5280	0205	2815	2120	2518	2212	2052	1258	1855	1551
0508	0250	5005	1111	8125	8221	8022	8528	5282	5580	5181	5885	0802	2218	2512	2115	2820	1555	1851	1252	2058
2180	2882	2285	2581	1212	5018	2050	1815	0251	1155	0558	0852	1520	8002	8508	8111	8205	5121	5825	5228	5522
2585	2281	2880	2182	1850	1515	1220	5012	0858	0552	1151	0255	2018	8211	8105	8502	8008	5528	5222	5821	5125
2881	2185	2582	2280	5020	2012	1818	1550	1152	0258	0855	0551	1215	8505	8011	8208	8102	5822	5128	5525	5221
2282	2580	2181	2885	1518	1250	5015	2020	0555	0851	0252	1158	1812	8108	8202	8005	8511	5225	5521	5122	5828
8051	8555	8158	8252	5102	5808	5211	5505	2121	5025	2850	2522	2228	1280	2082	1585	1881	0212	1118	0520	0815
8258	8152	8551	8055	5511	5205	5802	5108	2550	2222	2128	5021	2825	1885	1581	2080	1282	0820	0515	1112	0218
8552	8058	8255	8151	5805	5111	5508	5202	5028	2821	2525	2250	2122	2081	1285	1882	1580	1115	0220	0818	0512
8155	8251	8052	8558	5208	5502	5105	5811	2225	2150	5022	2828	2521	1582	1880	1281	2085	0518	0812	0215	1120
1105	0811	0502	0208	2015	1820	1512	1218	2822	2528	2221	2125	5050	5852	5558	5251	5155	8581	8285	8180	8082
1221	2025	1528	1822	0280	1182	0585	0881	8012	8518	8120	8215	5258	5151	5055	5850	5552	2102	2808	2211	2505
1828	1522	2021	1225	0885	0581	1180	0282	8220	8115	8512	8018	5855	5550	5252	5158	5051	2511	2205	2802	2108
2022	1228	1825	1521	1181	0285	0882	0580	8515	8020	8218	8112	5152	5058	5851	5555	5250	2805	2111	2508	2202
1525	1821	1222	2028	0582	0880	0281	1185	8118	8212	8015	8520	5551	5255	5150	5052	5858	2208	2502	2105	2811
5112	5818	5220	5515	2151	2855	2258	2552	1202	2008	1511	1805	8185	0221	1125	0528	0822	8080	5082	8550	8281
5520	5215	5812	5118	2558	2252	2851	2155	1811	1505	2002	1208	8582	0828	0522	1121	0225	8250	8181	8085	5080
5815	5120	5518	5212	2852	2158	2555	2251	2005	1211	1808	1502	8081	1122	0228	0825	0521	5085	8580	8282	8150
5218	5512	5115	5820	2255	2551	2152	2858	1508	1802	1205	2011	8280	0525	0821	0222	1128	8182	8050	5081	8585

We observe that above *Selfie magic square* of order 21 has 20 broken magic squares of order 4, but they aren't *Selfie* in itself. Similarly, we can also write Selfie magic square of order 21 according to example 3.

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- **Selfie Magic Square of Order 22**

Example 25. *Selfie magic square* of order 22 with magic sum $S_{22\times 22} := 87769$ is given by

202	215	122	225	511	850	852	858	851	855	508	812	580	882	188	881	885	525	820	528	518	521
512	505	215	222	208	858	851	855	850	852	811	102	088	881	585	880	182	528	518	521	525	520
211	502	808	515	222	855	850	852	858	851	112	205	885	180	082	888	581	521	525	520	528	518
522	212	505	111	815	852	858	851	855	850	202	208	882	588	881	185	080	520	528	518	521	525
115	822	202	508	212	851	855	850	852	858	205	511	181	085	880	582	888	518	521	525	520	528
580	582	588	581	585	515	820	522	228	118	021	825	802	808	512	805	811	250	252	158	851	555
588	581	585	580	582	028	818	521	825	515	220	122	812	805	811	802	858	858	551	255	250	152
585	580	582	588	581	225	115	020	822	528	818	521	881	502	808	112	805	255	150	852	558	251
582	588	581	585	580	822	528	218	121	025	815	520	108	802	805	511	802	552	258	251	155	850
581	585	580	582	588	821	525	815	520	222	128	218	505	821	802	808	812	151	855	550	252	258
805	108	211	012	502	120	022	828	518	821	525	215	521	552	855	558	150	821	582	885	888	880
208	021	512	802	105	518	221	125	015	820	522	828	852	055	158	250	551	882	805	188	820	851
825	820	528	218	821	280	282	188	881	585	152	258	050	815	522	251	155	502	808	112	005	181
828	818	821	825	520	888	581	285	280	182	255	550	158	151	515	822	252	012	805	251	802	108
821	525	220	128	818	285	180	882	588	281	558	851	255	150	252	115	522	211	102	008	812	505
820	828	818	521	225	582	288	281	185	888	850	052	522	258	151	555	215	808	512	805	211	002
518	221	825	820	828	181	885	580	282	288	051	155	515	222	250	152	558	205	211	802	508	812
550	852	158	051	855	502	808	512	505	511	182	288	225	220	128	818	521	880	815	522	281	585
058	851	555	850	152	512	105	511	502	508	285	580	828	518	221	225	120	588	181	815	822	282
855	150	052	858	551	511	502	508	512	505	588	881	221	125	820	528	218	285	580	282	815	822
852	558	851	155	050	508	502	805	551	502	880	882	520	228	218	121	825	122	288	581	585	215
151	055	858	552	858	505	521	502	508	512	801	185	118	821	525	220	228	515	222	280	582	888

We observe that above *magic square* of order 22 has 13 magic squares, 12 of them are of order 5 and 1 is of order 7 , but aren't *Selfie* itself.

- **Selfie Magic Square of Order 23**

Example 26. *Pan diagonal Selfie magic square* of order 23 with magic sum $S_{23\times 23} := 79992$ is given by

101	282	580	855	151	128	222	520	815	211	108	202	585	881	508	512	250	525	821	818	812	810	505
802	585	181	258	552	850	125	121	218	512	810	205	201	282	580	855	501	128	522	520	515	811	108
805	801	882	580	155	251	528	822	120	115	121	508	802	285	181	258	552	850	525	521	518	552	510
508	502	885	181	858	852	150	225	521	818	112	110	205	501	882	280	255	251	528	822	520	515	511
510	205	550	582	880	155	851	528	122	220	515	811	108	102	285	581	858	052	150	225	521	218	512
211	508	502	285	581	558	805	150	225	821	118	212	510	805	101	182	280	155	851	028	122	220	515
212	510	205	501	182	520	555	551	028	122	220	515	111	208	502	885	108	158	252	550	825	021	118
215	211	208	502	285	501	158	252	550	825	021	818	212	510	105	201	582	880	055	151	228	522	820
518	812	210	105	201	582	280	055	151	228	522	820	015	111	208	502	185	281	558	052	050	125	221
120	215	511	808	002	185	281	558	852	050	125	221	518	812	810	105	821	582	180	255	551	828	022
821	018	112	210	505	181	082	180	255	551	828	022	120	215	511	808	002	185	281	558	152	250	525
222	520	815	101	108	202	585	881	058	152	250	525	821	518	512	210	505	801	082	180	255	551	128
525	121	218	512	810	105	101	282	580	855	051	128	222	520	815	511	108	202	585	881	058	152	250
128	222	520	115	211	508	802	185	181	258	552	850	025	121	218	512	810	505	101	282	580	855	851
580	825	121	218	512	110	205	501	882	180	155	251	528	822	020	115	211	508	802	085	181	258	552
251	528	822	820	115	211	508	102	285	581	858	052	150	225	521	818	012	210	205	501	882	580	155
052	150	225	521	818	012	110	205	501	182	280	555	851	028	122	220	515	111	208	102	285	581	858
555	851	028	122	220	515	811	808	102	285	581	158	252	550	825	021	118	212	510	105	201	182	280
158	252	550	825	021	518	212	510	805	001	182	280	555	151	228	522	820	115	111	208	502	185	281
180	255	151	228	522	820	015	111	208	502	185	281	558	052	050	125	221	518	812	110	105	121	582
281	558	852	050	125	221	518	812	510	105	201	582	880	055	151	228	522	120	215	511	808	102	185
102	180	255	551	828	022	120	215	511	808	002	185	281	558	852	050	125	221	518	012	210	505	801
585	881	058	152	250	525	821	218	212	210	505	801	082	180	255	551	828	022	120	215	511	108	202

- **Selfie Magic Square of Order 24**

Example 27. *Selfie semi-magic square* of order 24 with sums of row and columns 88880 and sum of diagonals 88780 and 88879 is given by

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1185	5502	2521	0128	2215	8852	5880	1210	0882	5105	5020	0550	2018	8251	8181	1808	1088	5201	2822	0225	2112	8555	8058	1511
2210	8880	5852	1215	1128	5521	2502	0185	2008	8281	8151	1818	0850	5120	5005	0582	2111	8558	8055	1512	1025	5222	2801	0288
0102	2585	5528	1121	1252	5815	8810	2280	0505	5082	5150	0820	1851	8118	8208	2081	0201	2888	5225	1022	1555	8012	8511	2158
1280	5810	8815	2252	0121	2528	5585	1102	1881	8108	8218	2051	0520	5050	5182	0805	1558	8011	8512	2155	0222	2825	5288	1001
2552	0115	1110	5580	5802	1285	2228	8821	5051	0518	0808	5181	8105	1882	2050	8220	2855	0212	1011	5258	8001	1588	2125	8522
5821	1228	2285	8802	2580	0110	1115	5552	8120	1850	2082	8205	5081	0508	0818	5151	8022	1525	2188	8501	2858	0211	1012	5255
5515	1152	0180	2510	8885	2202	1221	5828	5118	0851	0581	5008	8282	2005	1820	8150	5212	1055	0258	2811	8588	2101	1522	8025
8828	2221	1202	5885	5510	1180	0152	2515	8250	2020	1805	8182	5108	0881	0551	5018	8525	2122	1501	8088	5211	1058	0255	2812
0888	5101	5022	0525	2012	8255	8158	1811	1085	5202	2821	228	2115	8552	8080	1510	1182	5505	2520	0150	2218	8851	5881	1208
2011	8258	8155	1812	0825	5122	5001	0588	2110	8580	8052	1515	1028	5221	2802	0285	2208	8881	5851	1218	1150	5520	2505	0182
0501	5088	5125	0822	1855	8112	8211	2058	0202	2885	5228	1021	1552	8015	8510	2180	0105	2582	5550	1120	1251	5818	8808	2281
1158	8111	8212	2055	0522	5025	5188	0801	1580	8010	8515	2152	0221	2828	5285	1002	1281	5808	8818	2251	0120	2550	5582	1105
5055	0512	0811	5158	8101	1888	2025	8222	2852	0215	1010	5280	8002	1585	2128	8521	2551	0118	1108	5581	5805	1282	2250	8820
8122	1825	2088	8201	5058	0511	0812	5155	8021	1528	2185	8502	2880	0210	1015	5252	5820	1250	2282	8805	2581	0108	1118	5551
5112	0855	0558	5011	8288	2001	1822	8125	5215	1052	0280	2810	8585	2102	1521	8028	5518	1151	0181	2508	8882	2205	1220	5850
8225	2022	1801	8188	5111	0858	0555	5012	8528	2121	1502	8085	5210	1080	0252	2815	8850	2220	1205	5882	5508	1181	0151	2518
1082	5205	2820	0250	2118	8551	8081	1508	1188	5501	2522	0125	2212	8855	5858	1211	0885	5102	5021	0528	2015	8252	8180	1810
2108	8581	8051	1518	1050	5220	2805	0282	2211	8858	5855	1212	1125	5522	2501	0188	2010	8280	8152	1815	0828	5121	5002	0585
0205	2882	5250	1020	1551	8018	8508	2181	0101	2588	5525	1122	1255	5812	8011	2258	502	5085	5128	0821	1852	8115	8210	2080
1581	8008	8518	2151	0220	2850	5282	1005	1258	5811	8812	2255	0122	2525	5588	1101	1880	8110	8215	2052	0521	5028	5185	0802
2851	0218	1008	5281	8005	1582	2150	8520	2555	0112	1111	5558	5801	1288	2225	8822	5052	0515	0810	5180	8102	1885	2028	8221
8020	1550	2182	8505	2881	0208	1018	5251	5822	1225	2288	8801	2558	0111	1112	5555	8121	1828	2085	8202	5080	0510	0815	5152
5218	1051	0281	2808	8582	2105	1520	8050	5512	1155	0158	2511	8888	2201	1222	5825	5115	0852	0580	5010	8285	2002	1821	8128
8550	2120	1505	8082	5208	1081	0251	2818	8825	2222	1201	5888	5511	1158	0155	2512	8228	2021	1802	8185	5110	0880	0552	5015

The magic square is formed by 9 *pan diagonal semi-bimagic squares* of order 8 with different sums forming a semi-magic square of 3:

			88780
27493	31815	29572	88880
31772	29593	27515	88880
29615	27472	31793	88880
88880	88880	88880	**88879**

Above example results in a *Selfie semi-magic square* of order 24. The example below is a *Selfie magic square* of order 24 written according to example 18.

Example 28. *Selfie magic square* of order 24 with magic sum $S_{24\times 24} := 88880$ based on example 18 is given by

0101	1021	0822	2202	2108	8050	8825	8528	2851	2555	0205	0510	8152	8258	1180	2082	1888	1581	1285	5011	5815	5520	5212	5118
2110	0202	0121	1022	2205	2558	8151	8025	8828	2852	0508	0801	8255	8550	1588	1281	1185	2080	1882	5220	5112	5018	5811	5515
2208	2101	0505	0221	0122	2855	2550	8252	8125	8028	0810	1002	8558	8851	2085	1880	1582	1288	1181	5818	5511	5215	5120	5012
0222	2210	2102	0808	0521	8128	2858	2551	8555	8225	1001	0105	8850	8052	1282	1188	2081	1885	1580	5115	5020	5812	5518	5211
0821	0522	2201	2105	1010	8525	8228	2850	2552	8858	0102	0208	8051	8155	1881	1585	1280	1182	2088	5512	5218	5111	5815	5820
5080	5825	5528	2881	2585	1111	2021	1822	2212	2118	1215	1520	5182	5288	8001	8805	8510	8202	8108	0150	0852	0558	0251	0255
2588	5181	5025	5828	2882	2120	1212	1121	2022	2215	1518	1811	5285	5580	8210	8102	8008	8801	8505	0558	0251	0155	0858	0852
2885	2580	5282	5125	5028	2218	2111	1515	1221	1122	1820	2012	5588	5881	8808	8501	8205	8110	8002	0855	0550	0552	0258	0151
5128	2888	2581	5585	5225	1222	2220	2112	1818	1521	2011	1115	5880	5082	8105	8010	8802	8508	8201	0252	0158	0151	0855	0550
5525	5228	2880	2582	5888	1821	1522	2211	2115	2020	1112	1218	5081	5185	8502	8208	8101	8005	8810	0851	0555	0250	0152	0858
0502	0805	1008	0110	0201	1512	1815	2018	1120	1211	2121	2825	2228	2522	5251	5552	5855	5058	5150	8281	8582	8885	8088	8180
1005	0108	0210	0501	0802	2015	1118	1220	1511	1812	2528	2222	2821	2125	5852	5555	5158	5250	5551	8882	8085	8188	8280	8581
5281	5582	5885	5088	5180	8251	8552	8855	8058	8150	2822	2128	2525	2221	0512	0815	1018	1120	1211	1502	1805	2008	1110	1201
5882	5085	5188	5280	5581	8852	8055	8158	8250	8551	2225	2521	2122	2828	1015	1118	1220	0511	0812	2005	1108	1210	1501	1802
8011	8815	8520	8212	8118	0180	1082	0888	0581	0285	5152	5258	1205	1510	5050	5821	5522	2251	2155	1101	2025	1828	2802	2508
8220	8112	8018	8811	8515	0588	0281	0185	1080	0882	5255	5550	1508	1801	2158	5151	5021	5822	2252	2510	1202	1125	2028	2805
8818	8511	8215	8120	8012	1085	0880	0582	0288	0181	5558	5851	1810	2002	2255	2150	5252	5121	5022	2808	2501	1505	1225	1128
8115	8020	8812	8518	8211	0282	0188	1081	0885	0580	5850	5052	2001	1105	5122	2258	2151	5555	5221	1228	2810	2502	1808	1525
8512	8218	8111	8015	8820	0881	0585	0280	0182	1088	5051	5155	1102	1208	5521	5222	2250	2152	5858	1825	1528	2801	2505	2010
1150	2052	1858	1551	1255	5001	5805	5510	5202	5108	8182	8288	0215	0520	0111	1025	0828	2812	2518	8080	8821	8522	2281	2185
1558	1251	1155	2058	1852	5210	5102	5008	5801	5505	8285	8580	0518	0811	2520	0212	0125	1028	2815	2188	8181	8021	8822	2282
2055	1858	1552	1258	1151	5808	5501	5205	5110	5002	8588	8881	0820	1012	2818	2511	0515	0225	0128	2285	2180	8282	8121	8022
1252	1158	2051	1855	1550	5105	5010	5802	5508	5201	8880	8082	1011	0115	0228	2820	2512	0818	0525	8122	2288	2181	8585	8221
1851	1555	1258	1152	2058	5502	5208	5101	5005	5810	8081	8185	0112	0218	0825	0528	2811	2515	1020	8521	8222	2280	2182	8888

We observe that above *Selfie magic square* of order 24 has nine broken magic squares, 8 of order 5 and 1 of order 4. These broken magic squares are not Selfie in itself. Similarly, we can write Selfie magic square of order 4 according to example 9. Also, we observe that we are able to write Palindromic magic square (example 19) according to example 7, but unfortunately we are unable to write *Selfie magic square* with all blocks of order 4 a magic square following lines of example 7.

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Example 29. *Pan diagonal Selfie bimagic square* of order 25 is given by

0000	1155	2211	5588	8822	5252	8510	0885	1021	2108	1818	2082	5120	8205	0551	8181	0228	1502	2850	5015	2525	5801	8058	0112	1280
5511	8888	0022	1100	2255	1085	2121	5208	8552	0810	8220	0505	1851	2018	5182	2802	5050	8115	0281	1528	0158	1212	2580	5825	8001
1122	2200	5555	8811	0088	8508	0852	1010	2185	5221	2051	5118	8282	0520	1805	0215	1581	2828	5002	8150	5880	8025	0101	1258	2512
8855	0011	1188	2222	5500	2110	5285	8521	0808	1052	0582	1820	2005	5151	8218	5028	8102	0250	1515	2881	1201	2558	5812	8080	0125
2288	5522	8800	0055	1111	0821	1008	2152	5210	8585	5105	8251	0518	1882	2020	1550	2815	5081	8128	0202	8012	0180	1225	2501	5858
8118	0282	1520	2805	5051	2581	5828	8002	0150	1215	0025	1101	2258	5512	8880	5200	8555	0811	1088	2122	1852	2010	5185	8221	0508
2820	5005	8151	0218	1582	0102	1250	2515	5881	8028	5558	8812	0080	1125	2201	1011	2188	5222	8500	0855	8285	0521	1808	2052	5110
0251	1518	2882	5020	8105	5815	8081	0128	1202	2550	1180	2225	5501	8858	0012	8522	0800	1055	2111	5288	2008	5152	8210	0585	1821
5082	8120	0205	1551	2818	1228	2502	5850	8015	0181	8801	0058	1112	2280	5525	2155	5211	8588	0822	1000	0510	1885	2021	5108	8252
1505	2851	5018	8182	0220	8050	0115	1281	2528	5802	2212	5580	8825	0001	1158	0888	1022	2100	5255	8511	5121	8208	0552	1810	2085
5225	8501	0858	1012	2180	1800	2055	5111	8288	0522	8152	0210	1585	2821	5008	2518	5882	8020	0105	1251	0081	1128	2202	5550	8815
1058	2112	5280	8525	0801	8211	0588	1822	2000	5155	2885	5021	8108	0252	1510	0120	1205	2551	5818	8082	5502	8850	0015	1181	2228
8580	0825	1001	2158	5212	2022	5100	8255	0511	1888	0208	1552	2810	5085	8121	5851	8018	0182	1220	2505	1115	2281	5528	8802	0050
2101	5258	8512	0880	1025	0555	1811	2088	5122	8200	5010	8185	0221	1508	2852	1282	2520	5805	8051	0118	8828	0002	1150	2215	5581
0812	1080	2125	5201	8558	5188	8222	0500	1855	2011	1521	2808	5052	8110	0285	8005	0151	1218	2582	5820	2250	5515	8881	0028	1102
2552	5810	8085	0121	1208	0018	1182	2220	5505	8851	5281	8528	0802	1050	2115	1825	2001	5158	8212	0580	8100	0255	1511	2888	5022
0185	1221	2508	5852	8010	5520	8805	0051	1118	2282	1002	2150	5215	8581	0828	8258	0512	1880	2025	5101	2811	5088	8122	0200	1555
5808	8052	0110	1285	2521	1151	2218	5582	8820	0005	8515	0881	1028	2102	5250	2080	5125	8201	0558	1812	0222	1500	2855	5011	8188
1210	2585	5821	8008	0152	8882	0020	1105	2251	5518	2128	5202	8550	0815	1081	0501	1858	2012	5180	8225	5055	8111	0288	1522	2800
8021	0108	1252	2510	5885	2205	5551	8818	0082	1120	0850	1015	2181	5228	8502	512	8280	0525	1801	2058	1588	2822	5000	8155	0211
1881	2028	5102	8250	0515	8125	0201	1558	2812	5080	2500	5855	8011	0188	1222	0052	1110	2285	5521	8808	5218	8582	0820	1005	2151
8202	0550	1815	2081	5128	2858	5012	8180	0225	1501	0111	1288	2522	5800	8055	5585	8821	0008	1152	2210	1020	2105	5251	8518	0882
2015	5181	8228	0502	1850	0280	1525	2801	5058	8112	5822	8000	0155	1211	2588	1108	2252	5510	8885	0021	8551	0818	1082	2120	5205
0528	1802	2050	5115	8281	5001	8158	0212	1580	2825	1255	2511	5888	8022	0100	8810	0085	1121	2208	5552	2182	5220	8505	0851	1018
5150	8215	0581	1828	2002	1512	2880	5025	8101	0258	8088	0122	1200	2555	5811	2221	5508	8852	0010	1185	0805	1051	2118	5282	8520

Above *Selfie magic square* is *pan diagonal bimagic* and each block of order 5 is also a *pan diagonal magic square* with sums $S_{25\times 25} := 88880$, $Sb_{25\times 25} := 532147790$ and $S_{5\times 5} := 17776$.

Final Comments

We constructed magic squares of order 21 to 25 based on *MODLS*, *SODLS*, and using composition relations. The magic squares constructed are named "*intervally distributed magic squares*". For the orders 3-20 refer previous work [17, 18]. These kind of magic squares helped us in extending to *palindromic* and *Selfie magic squares*. In case of prime number 23, the magic square is *pan diagonal intervally distributed*. The magic square of order 25 constructed based on *SODLS* is *pan diagonal* and *bimagic* and each block of order 5 is a *pan diagonal magic square*. For cases of magic squares of order 21 and 24 different approaches are given. In some cases we have *intervally semi-bimagic* or *bimagic squares*. More studies on magic squares can be seen in [2, 3, 5, 8-18]. Writing different kind of magic squares using programming language refer to White [20].

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