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ON GENERALIZED OSTROWSKI TYPE INEQUALITIES FOR FUNCTIONS WHOSE FIRST DERIVATIVES ABSOLUTE VALUE ARE CONVEX

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Abstract. In this paper, we have established some generalized Ostrowski type inequalities for functions whose first derivatives absolute are convex.

1. Introduction

In 1938, Ostrowski established the following interesting integral inequality for differentiable mappings with bounded derivatives [11]:

Theorem 1. *Let $f:[a,b]\to\mathbb{R}$ be a differentiable mapping on (a,b) whoose derivative $f':(a,b)\to\mathbb{R}$ is bounded on (a,b), i.e. $\|f'\|_\infty := \sup_{t\in(a,b)} |f'(t)| < \infty$. Then, we have the inequality*

$$\left| f(x) - \frac{1}{b-a}\int_a^b f(t)dt \right| \leq \left[\frac{1}{4} + \frac{\left(x-\frac{a+b}{2}\right)^2}{(b-a)^2} \right](b-a)\|f'\|_\infty, \tag{1.1}$$

for all $x\in[a,b]$. The constant $\frac{1}{4}$ is the best possible.

This inequality is well known in the literature as the *Ostrowski inequality.*

Definition 1. *The function $f:[a,b]\subset\mathbb{R}\to\mathbb{R}$, is said to be convex if the following inequality holds*

$$f(tx+(1-t)y) \leq tf(x)+(1-t)f(y)$$

for all $x,y\in[a,b]$ and $t\in[0,1]$. We say that f is concave if $(-f)$ is convex.

Let $f: I\subseteq R \to$R be a convex function on the interval I of real numbers and $a,b\in I$ with $a<b$. If f is a convex function then the following double inequality, which is well known in the literature as the Hermite–Hadamard inequality, holds [13]

$$f\left(\frac{a+b}{2}\right) \leq \frac{1}{b-a}\int_a^b f(x)dx \leq \frac{f(a)+f(b)}{2}. \tag{1.2}$$

In [8], Dragomir and Agarwal gave the following important inequality for convex functions:

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Theorem 2. *Let $f : I^{\circ} \subseteq \mathbb{R} \rightarrow \mathbb{R}$ be a differentiable mapping onI°, $a, b \in I^{\circ}$ with $a < b$. If $|f'|$ is convex on $[a, b]$, then the following inequality holds:*

$$\left| \frac{f(a) + f(b)}{2} - \frac{1}{b-a} \int_a^b f(u)du \right| \leq \frac{(b-a)\left|f'(a)\right| + \left|f'(b)\right|}{8}. \tag{1.3}$$

In [12], Ozdemir et al. gave the following Ostrowski type inequalities for functions whose derivatives are convex:

Theorem 3. *Let $I \subset \mathbb{R}$ be an open interval and $f : I \subset \mathbb{R} \rightarrow \mathbb{R}$ be a differentiable function where $a, b \in I$ with $a < b$. If $|f'|^q$ is convex function for $\lambda \in [0, 1]$, $x \in [a, b]$ and $q \in [1, \infty)$, then the following inequality holds:*

$$\begin{aligned}
(1.4) \quad & \left| \frac{1}{b-a} \int_a^b f(u)du \right. \\
& \left. - \frac{(b-x)\left[(1-\lambda) f(x) + \lambda f(b)\right] + (x-a)\left[(1-\lambda) f(x) + \lambda f(a)\right]}{(b-a} \right| \\
\leq \; & (b-a) \left(\frac{2\lambda^2 - 2\lambda + 1}{2} \right)^{\frac{q-1}{q}} \left\{ \left[\left(\frac{2\lambda^3 - 3\lambda + 2}{6} \right) \left(\frac{b-x}{b-a} \right)^{2q+1} \left|f'(a)\right|^q \right. \right. \\
& \left. + \left(\frac{\left[6\lambda^2 - 6\lambda + 3\right] - \left[2\lambda^3 + 3\lambda + 2\right] \left(\frac{b-x}{b-a} \right)}{6} \right) \left(\frac{b-x}{b-a} \right)^{2q} \left|f'b\right|^q \right]^{\frac{1}{q}} \\
& + \left[\left(\frac{\left[6\lambda^2 - 6\lambda + 3\right] - \left[2\lambda^3 + 3\lambda + 2\right] \left(\frac{x-a}{b-a} \right)}{6} \right) \left(\frac{x-a}{b-a} \right)^{2q} \left|f'(a)\right|^q \right. \\
& \left. \left. + \left(\frac{2\lambda^3 - 3\lambda + 2}{6} \right) \left(\frac{x-a}{b-a} \right)^{2q+1} \left|f'(b)\right|^q \right]^{\frac{1}{q}} \right\}.
\end{aligned}$$

For more information and recent advences on Ostrowski type inequalities, please refer to ([1]-[10],[12],[14]-[18]).

The aim of this paper is to establish generalization of the inequality (1.4) and give some special results.

2. Main Results

First, we will give the following calculated integral which are used main results:

$$\int_0^{\frac{b-x}{b-a}} \left| t - \lambda \frac{b-x}{b-a} \right| t dt = \left(\frac{2\lambda^3 - 3\lambda + 2}{6} \right) \left(\frac{b-x}{b-a} \right)^3, \tag{2.1}$$

$$\int_{0}^{\frac{b-x}{b-a}} \left| t-\lambda \frac{b-x}{b-a} \right| (1-t)\, dt = \left(\frac{\left[6\lambda^2-6\lambda+3\right]-\left[2\lambda^3+3\lambda+2\right]\left(\frac{b-x}{b-a}\right)}{6} \right) \left(\frac{b-x}{b-a} \right)^2, \tag{2.2}$$

$$\int_{\frac{b-x}{b-a}}^{1} \left| t-1+\lambda \frac{x-a}{b-a} \right| t dt = \left(\frac{\left[6\lambda^2-6\lambda+3\right]-\left[2\lambda^3+3\lambda+2\right]\left(\frac{x-a}{b-a}\right)}{6} \right) \left(\frac{x-a}{b-a} \right)^2, \tag{2.3}$$

$$\int_{\frac{b-x}{b-a}}^{1} \left| t-1+\lambda \frac{x-a}{b-a} \right| (1-t)\, dt = \left(\frac{2\lambda^3-3\lambda+2}{6} \right) \left(\frac{x-a}{b-a} \right)^{2q+1}, \tag{2.4}$$

$$\int_{0}^{\frac{b-x}{b-a}} \left| t-\lambda \frac{b-x}{b-a} \right| dt = \left(\frac{2\lambda^2-2\lambda+1}{2} \right) \left(\frac{b-x}{b-a} \right)^2, \tag{2.5}$$

$$\int_{\frac{b-x}{b-a}}^{1} \left| t-1+\lambda \frac{x-a}{b-a} \right| dt = \left(\frac{2\lambda^2-2\lambda+1}{2} \right) \left(\frac{x-a}{b-a} \right)^2, \tag{2.6}$$

$$\int_{0}^{\frac{b-x}{b-a}} \left| t-\lambda \frac{b-x}{b-a} \right|^p dt = \left(\frac{\lambda^{p+1}-(1-\lambda)^{p+1}}{p+1} \right) \left(\frac{b-x}{b-a} \right)^{p+1}, \tag{2.7}$$

and

$$\int_{\frac{b-x}{b-a}}^{1} \left| t-1+\lambda \frac{x-a}{b-a} \right|^p dt = \left(\frac{\lambda^{p+1}-(1-\lambda)^{p+1}}{p+1} \right) \left(\frac{x-a}{b-a} \right)^{p+1}. \tag{2.8}$$

We give a important integral identity for differentiable convex functions:

Lemma 1. *Let $f:[a,b]\rightarrow\mathbb{R}$ be a differantiable mapping on (a,b) with $a<b$. If $f'\in L[a,b]$ for $\lambda\in[0,1]$, then, for all $x\in[a,b]$, we have*

$$(b-a)\int_0^1 h(t,\lambda)f'\left[t\left(\mu a+(1-\mu)b\right)+(1-t)\left(\mu b+(1-\mu)a\right)\right]dt \tag{2.9}$$

$$= \frac{(1-\lambda)\,f\left(\mu x+(1-\mu)\left(a+b-x\right)\right)}{(1-2\mu)}$$

$$+\lambda\frac{(b-x)\,f\left(\mu b+(1-\mu)a\right)+(x-a)\,f\left(\mu a+(1-\mu)b\right)}{(b-a)\,(1-2\mu)}$$

$$-\frac{1}{(b-a)\,(1-2\mu)^2}\int_{\mu b+(1-\mu)a}^{\mu a+(1-\mu)b} f(u)du$$

for $\mu\in[0,1]\,/\,\{1/2\}$ where

$$h(t,\lambda)=\begin{cases} t-\lambda\dfrac{b-x}{b-a}, & t\in\left[0,\dfrac{b-x}{b-a}\right] \\[2ex] t-1+\lambda\dfrac{x-a}{b-a}, & t\in\left(\dfrac{b-x}{b-a},1\right]. \end{cases}$$

Proof. Denote

$$\begin{aligned} I &= \int_0^1 h(t,\lambda)f'\left[t\left(\mu a+(1-\mu)b\right)+(1-t)\left(\mu b+(1-\mu)a\right)\right]dt \\ &= \int_0^{\frac{b-x}{b-a}}\left[t-\lambda\frac{b-x}{b-a}\right]f'\left[t\left(\mu a+(1-\mu)b\right)+(1-t)\left(\mu b+(1-\mu)a\right)\right]dt \\ &\quad+\int_{\frac{b-x}{b-a}}^1\left[t-1+\lambda\frac{x-a}{b-a}\right]f'\left[t\left(\mu a+(1-\mu)b\right)+(1-t)\left(\mu b+(1-\mu)a\right)\right]dt \\ &= I_1+I_2. \end{aligned}$$

Integrating by parts,

$$\begin{aligned} I_1 &= \int_0^{\frac{b-x}{b-a}}\left[t-\lambda\frac{b-x}{b-a}\right]f'\left[t\left(\mu a+(1-\mu)b\right)+(1-t)\left(\mu b+(1-\mu)a\right)\right]dt \\ &= \frac{(1-\lambda)\,(b-x)\,f\left(\mu x+(1-\mu)\left(a+b-x\right)\right)}{(b-a)^2\,(1-2\mu)}+\lambda\frac{(b-x)\,f\left(\mu b+(1-\mu)a\right)}{(b-a)^2\,(1-2\mu)} \end{aligned}$$

$$-\frac{1}{(b-a)(1-2\mu)}\int_{0}^{\frac{b-x}{b-a}} f\left[t(\mu a+(1-\mu)b)+(1-t)(\mu b+(1-\mu)a)\right]dt$$

and

$$\begin{aligned}
I_2 &= \int_{\frac{b-x}{b-a}}^{1}\left[t-1+\lambda\frac{x-a}{b-a}\right]f'\left[t(\mu a+(1-\mu)b)+(1-t)(\mu b+(1-\mu)a)\right]dt \\
&= \frac{(1-\lambda)(x-a)f(\mu x+(1-\mu)(a+b-x))}{(b-a)^2(1-2\mu)}+\lambda\frac{(x-a)f(\mu a+(1-\mu)b)}{(b-a)^2(1-2\mu)} \\
&\quad -\frac{1}{(b-a)(1-2\mu)}\int_{\frac{b-x}{b-a}}^{1} f\left[t(\mu a+(1-\mu)b)+(1-t)(\mu b+(1-\mu)a)\right]dt.
\end{aligned}$$

Adding I_1 and I_2, then we have

$$\begin{aligned}
I &= I_1+I_2 \\
&= \frac{(1-\lambda)f(\mu x+(1-\mu)(a+b-x))}{(b-a)(1-2\mu)} \\
&\quad +\lambda\frac{(b-x)f(\mu b+(1-\mu)a)+(x-a)f(\mu a+(1-\mu)b)}{(b-a)^2(1-2\mu)} \\
&\quad -\frac{1}{(b-a)(1-2\mu)}\int_{0}^{1} f\left[t(\mu a+(1-\mu)b)+(1-t)(\mu b+(1-\mu)a)\right]dt.
\end{aligned}$$

If we use the change of the variable $u=t(\mu a+(1-\mu)b)+(1-t)(\mu b+(1-\mu)a)$ with $du=b-a)(1-2\mu)\,dt$, then we have

$$\begin{aligned}
I &= \frac{(1-\lambda)f(\mu x+(1-\mu)(a+b-x))}{(b-a)(1-2\mu)} \\
&\quad +\lambda\frac{(b-x)f(\mu b+(1-\mu)a)+(x-a)f(\mu a+(1-\mu)b)}{(b-a)^2(1-2\mu)} \\
&\quad -\frac{1}{(b-a)^2(1-2\mu)^2}\int_{\mu b+(1-\mu)a}^{\mu a+(1-\mu)b} f(u)du
\end{aligned}$$

which completes the proof. □

Remark 1. *If we choose $\mu=1$ in (2.9), then Lemma 1 reduces the Lemma 1 in* [12].

Theorem 4. *Let $f:[a,b]\to\mathbb{R}$ be a differentiable mapping on (a,b) with $a<b$. If $|f'|^q$, $q\geq 1$, is convex on $[a,b]$ for $\lambda\in[0,1]$ and $x\in[a,b]$, then we have the*

following inequality

$$\begin{aligned}
&\left|\frac{(1-\lambda) f(\mu x+(1-\mu)(a+b-x))}{(1-2\mu)}\right. \\
&+\lambda \frac{(b-x) f(\mu b+(1-\mu)a)+(x-a) f(\mu a+(1-\mu)b)}{(b-a)(1-2\mu)} \\
&\left.-\frac{1}{(b-a)(1-2\mu)^2} \int_{\mu b+(1-\mu)a}^{\mu a+(1-\mu)b} f(u)du\right| \\
\leq\ & (b-a)\left(\frac{2\lambda^2-2\lambda+1}{2}\right)^{1-\frac{1}{q}} \\
&\times\left\{\left[\left(\frac{2\lambda^3-3\lambda+2}{6}\right)\left(\frac{b-x}{b-a}\right)^{2q+1}\left|f'(\mu a+(1-\mu)b)\right|^q\right.\right. \\
&\left.+F(x,\lambda)\left(\frac{b-x}{b-a}\right)^{2q}\left|f'(\mu b+(1-\mu)a)\right|^q\right]^{\frac{1}{q}} \\
&+\left[G(x,\lambda)\left(\frac{x-a}{b-a}\right)^{2q}\left|f'(\mu a+(1-\mu)b)\right|^q\right. \\
&\left.\left.+\left(\frac{2\lambda^3-3\lambda+2}{6}\right)\left(\frac{x-a}{b-a}\right)^{2q+1}\left|f'(\mu b+(1-\mu)a)\right|^q\right]^{\frac{1}{q}}\right\}
\end{aligned} \tag{2.10}$$

where $\mu \in [0,1] / \{1/2\}$. *Here*

$$F(x,\lambda)=\frac{\left[6\lambda^2-6\lambda+3\right]-\left[2\lambda^3+3\lambda+2\right]\left(\frac{b-x}{b-a}\right)}{6}$$

and

$$G(x,\lambda)=\frac{\left[6\lambda^2-6\lambda+3\right]-\left[2\lambda^3+3\lambda+2\right]\left(\frac{x-a}{b-a}\right)}{6}.$$

Proof. Firstly, we suppose that $q=1$. Taking madulus in (2.9), we have

$$\begin{aligned}
&\left|\frac{(1-\lambda) f(\mu x+(1-\mu)(a+b-x))}{(1-2\mu)}\right. \\
&+\lambda \frac{(b-x) f(\mu b+(1-\mu)a)+(x-a) f(\mu a+(1-\mu)b)}{(b-a)(1-2\mu)} \\
&\left.-\frac{1}{(b-a)(1-2\mu)^2} \int_{\mu b+(1-\mu)a}^{\mu a+(1-\mu)b} f(u)du\right|
\end{aligned}$$

$$
\begin{aligned}
&\leq (b-a)\left\{\int_0^{\frac{b-x}{b-a}}\left|t-\lambda\frac{b-x}{b-a}\right|\left|f'\left[t\left(\mu a+(1-\mu)b\right)+(1-t)\left(\mu b+(1-\mu)a\right)\right]\right|dt\right.\\
&\quad\left.+\int_{\frac{b-x}{b-a}}^1\left|t-1+\lambda\frac{x-a}{b-a}\right|\left|f'\left[t\left(\mu a+(1-\mu)b\right)+(1-t)\left(\mu b+(1-\mu)a\right)\right]\right|dt\right\}\\
&= (b-a)\,K_1.
\end{aligned}
$$

Using convexity of $|f'|$, we get

$$
\begin{aligned}
(2.11)\quad &K_1\\
&\leq \int_0^{\frac{b-x}{b-a}}\left|t-\lambda\frac{b-x}{b-a}\right|\left[t\left|f'\left(\mu a+(1-\mu)b\right)\right|+(1-t)\left|f'\left(\mu b+(1-\mu)a\right)\right|\right]dt\\
&\quad+\int_{\frac{b-x}{b-a}}^1\left|t-1+\lambda\frac{x-a}{b-a}\right|\left[t\left|f'\left(\mu a+(1-\mu)b\right)\right|+(1-t)\left|f'\left(\mu b+(1-\mu)a\right)\right|\right]dt\\
&= \left|f'\left(\mu a+(1-\mu)b\right)\right|\int_0^{\frac{b-x}{b-a}}\left|t-\lambda\frac{b-x}{b-a}\right|tdt\\
&\quad+\left|f'\left(\mu b+(1-\mu)a\right)\right|\int_0^{\frac{b-x}{b-a}}\left|t-\lambda\frac{b-x}{b-a}\right|(1-t)\,dt\\
&\quad+\left|f'\left(\mu a+(1-\mu)b\right)\right|\int_{\frac{b-x}{b-a}}^1\left|t-1+\lambda\frac{x-a}{b-a}\right|tdt\\
&\quad+\left|f'\left(\mu b+(1-\mu)a\right)\right|\int_{\frac{b-x}{b-a}}^1\left|t-1+\lambda\frac{x-a}{b-a}\right|(1-t)\,dt.
\end{aligned}
$$

If we use the equalities (2.1)-(2.4) in (2.11), then we complete the proof for the case $q=1$.

Secondly, we suppose that $q>1$. Using Lemma 1 and power mean inequality, we obtain

$$\begin{aligned}
&\left| \frac{(1-\lambda) f(\mu x+(1-\mu)(a+b-x))}{(1-2\mu)} \right. \\
&+\lambda \frac{(b-x) f(\mu b+(1-\mu)a)+(x-a) f(\mu a+(1-\mu)b)}{(b-a)(1-2\mu)} \\
&\left. -\frac{1}{(b-a)(1-2\mu)^2} \int_{\mu b+(1-\mu)a}^{\mu a+(1-\mu)b} f(u)du \right| \\
\leq\ & (b-a)\left\{ \int_0^{\frac{b-x}{b-a}} \left| t-\lambda\frac{b-x}{b-a}\right| \left| f'\left[t(\mu a+(1-\mu)b)+(1-t)(\mu b+(1-\mu)a)\right]\right| dt \right. \\
&\left. +\int_{\frac{b-x}{b-a}}^{1} \left| t-1+\lambda\frac{x-a}{b-a}\right| \left| f'\left[t(\mu a+(1-\mu)b)+(1-t)(\mu b+(1-\mu)a)\right]\right| dt \right\} \\
=\ & (b-a)\left\{ \left(\int_0^{\frac{b-x}{b-a}} \left| t-\lambda\frac{b-x}{b-a}\right| dt \right)^{1-\frac{1}{q}} \right. \\
&\times \left(\int_0^{\frac{b-x}{b-a}} \left| t-\lambda\frac{b-x}{b-a}\right| \left| f'\left[t(\mu a+(1-\mu)b)+(1-t)(\mu b+(1-\mu)a)\right]\right|^q dt \right)^{\frac{1}{q}} \\
&+\left(\int_{\frac{b-x}{b-a}}^{1} \left| t-1+\lambda\frac{x-a}{b-a}\right| dt \right)^{1-\frac{1}{q}} \\
&\left. \left(\int_{\frac{b-x}{b-a}}^{1} \left| t-1+\lambda\frac{x-a}{b-a}\right| \left| f'\left[t(\mu a+(1-\mu)b)+(1-t)(\mu b+(1-\mu)a)\right]\right|^q dt \right)^{\frac{1}{q}} \right\} \\
=\ & (b-a)K_2.
\end{aligned}$$

Using the convexity of $|f'|^q$, we obtain

$$
\begin{aligned}
(2.12)\quad & K_2 \\
\leq & \left(\int_0^{\frac{b-x}{b-a}} \left| t-\lambda \frac{b-x}{b-a} \right| dt \right)^{1-\frac{1}{q}} \\
& \times \left(\int_0^{\frac{b-x}{b-a}} \left| t-\lambda \frac{b-x}{b-a} \right| \left[t\left| f'\left(\mu a+(1-\mu)b\right) \right|^q + (1-t)\left| f'\left(\mu b+(1-\mu)a\right) \right|^q \right] dt \right)^{\frac{1}{q}} \\
& + \left(\int_{\frac{b-x}{b-a}}^1 \left| t-1+\lambda \frac{x-a}{b-a} \right| dt \right)^{1-\frac{1}{q}} \\
& \times \left(\int_{\frac{b-x}{b-a}}^1 \left| t-1+\lambda \frac{x-a}{b-a} \right| \left[t\left| f'\left(\mu a+(1-\mu)b\right) \right|^q + (1-t)\left| f'\left(\mu b+(1-\mu)a\right) \right|^q \right] dt \right)^{\frac{1}{q}} \\
= & \left(\int_0^{\frac{b-x}{b-a}} \left| t-\lambda \frac{b-x}{b-a} \right| dt \right)^{1-\frac{1}{q}} \left(\left| f'\left(\mu a+(1-\mu)b\right) \right|^q \int_0^{\frac{b-x}{b-a}} \left| t-\lambda \frac{b-x}{b-a} \right| tdt \right. \\
& \left. + \left| f'\left(\mu b+(1-\mu)a\right) \right|^q \int_0^{\frac{b-x}{b-a}} \left| t-\lambda \frac{b-x}{b-a} \right| (1-t)\, dt \right)^{\frac{1}{q}} \\
& + \left(\int_{\frac{b-x}{b-a}}^1 \left| t-1+\lambda \frac{x-a}{b-a} \right| dt \right)^{1-\frac{1}{q}} \left(\left| f'\left(\mu a+(1-\mu)b\right) \right|^q \int_{\frac{b-x}{b-a}}^1 \left| t-1+\lambda \frac{x-a}{b-a} \right| tdt \right. \\
& \left. + \left| f'\left(\mu b+(1-\mu)a\right) \right|^q \int_{\frac{b-x}{b-a}}^1 \left| t-1+\lambda \frac{x-a}{b-a} \right| (1-t)\, dt \right)^{\frac{1}{q}}.
\end{aligned}
$$

If we use the equalities (2.1)-(2.6) in (2.12), then we complete the proof completely.

□

Remark 2. *If we choose $\mu = 1$ in Theorem 4, then the inequality (2.10) reduces the inequality (1.4).*

Corollary 1. *Under assumptions of Theorem 4, if we choose $\mu = 0$ in (2.10), then we have the inequality*

$$
\begin{aligned}
(2.13)\quad & \left|(1-\lambda) f(a+b-x) \right. \\
& \left. +\lambda \frac{(b-x) f(a)+(x-a) f(b)}{(b-a)}-\frac{1}{b-a} \int_a^b f(u) du\right| \\
\leq\; & (b-a)\left(\frac{2 \lambda^2-2 \lambda+1}{2}\right)^{1-\frac{1}{q}}\left\{\left[\left(\frac{2 \lambda^3-3 \lambda+2}{6}\right)\left(\frac{b-x}{b-a}\right)^{2 q+1}\left|f'(b)\right|^q\right.\right. \\
& \left.+\left(\frac{\left[6 \lambda^2-6 \lambda+3\right]-\left[2 \lambda^3+3 \lambda+2\right]\left(\frac{b-x}{b-a}\right)}{6}\right)\left(\frac{b-x}{b-a}\right)^{2 q}\left|f'(a)\right|^q\right]^{\frac{1}{q}} \\
& +\left[\left(\frac{\left[6 \lambda^2-6 \lambda+3\right]-\left[2 \lambda^3+3 \lambda+2\right]\left(\frac{x-a}{b-a}\right)}{6}\right)\left(\frac{x-a}{b-a}\right)^{2 q}\left|f'(b)\right|^q\right. \\
& \left.\left.+\left(\frac{2 \lambda^3-3 \lambda+2}{6}\right)\left(\frac{x-a}{b-a}\right)^{2 q+1}\left|f'(a)\right|^q\right]^{\frac{1}{q}}\right\}.
\end{aligned}
$$

Corollary 2. *If choose $\lambda = 0$ in Corollary 1, then we have the inequality*

$$
\begin{aligned}
(2.14)\quad & \left|f(a+b-x)-\frac{1}{b-a} \int_a^b f(u) du\right| \\
\leq\; & \frac{b-a}{2^{1-\frac{1}{q}}}\left\{\left[\frac{1}{3}\left(\frac{b-x}{b-a}\right)^{2 q+1}\left|f'(b)\right|^q\right.\right. \\
& \left.+\left[\frac{1}{2}-\frac{1}{3}\left(\frac{b-x}{b-a}\right)\right]\left(\frac{b-x}{b-a}\right)^{2 q}\left|f'(a)\right|^q\right]^{\frac{1}{q}} \\
& +\left[\left[\frac{1}{2}-\frac{1}{3}\left(\frac{x-a}{b-a}\right)\right]\left(\frac{x-a}{b-a}\right)^{2 q}\left|f'(b)\right|^q\right. \\
& \left.\left.+\frac{1}{3}\left(\frac{x-a}{b-a}\right)^{2 q+1}\left|f'(a)\right|^q\right]^{\frac{1}{q}}\right\}.
\end{aligned}
$$

Remark 3. *If we choose $x = \frac{a+b}{2}$ in Corollary 2, then we have the following midpoint inequality*

$$\left| f\left(\frac{a+b}{2}\right) - \frac{1}{b-a}\int_a^b f(u)du \right| \tag{2.15}$$
$$\leq \frac{b-a}{8}\left\{ \left[\frac{|f'(b)|^q + 2|f'(a)|^q}{3}\right]^{\frac{1}{q}} + \left[\frac{2|f'(b)|^q + |f'(a)|^q}{3}\right]^{\frac{1}{q}} \right\}$$
$$. \leq (b-a)\left(\frac{3^{1-\frac{1}{q}}}{8}\right)\left[|f'(a)| + |f'(b)|\right]$$

which is given by Kirmaci and Ozdemir in [9].

Corollary 3. *If we take $\lambda = 1$ in Corollary 1, the we have the following inequality*

$$\left| \frac{(b-x)f(a) + (x-a)f(b)}{(b-a)} - \frac{1}{b-a}\int_a^b f(u)du \right| \tag{2.16}$$
$$\leq \frac{b-a}{2^{1-\frac{1}{q}}}\left\{ \left[\frac{1}{6}\left(\frac{b-x}{b-a}\right)^{2q+1}|f'(b)|^q + \left[\frac{1}{2} - \frac{b-x}{b-a}\right]\left(\frac{b-x}{b-a}\right)^{2q}|f'(a)|^q\right]^{\frac{1}{q}} \right.$$
$$\left. + \left[\left[\frac{1}{2} - \frac{x-a}{b-a}\right]\left(\frac{x-a}{b-a}\right)^{2q}|f'(b)|^q + \frac{1}{6}\left(\frac{x-a}{b-a}\right)^{2q+1}|f'(a)|^q\right]^{\frac{1}{q}} \right\}.$$

Corollary 4. *If we take $x = \frac{a+b}{2}$ in Corollary 3, the we have the following trapezoid inequality*

$$\left| \frac{f(a) + f(b)}{2} - \frac{1}{b-a}\int_a^b f(u)du \right| \leq \frac{b-a}{8 \times 6^{\frac{1}{q}}}\left[|f'(a)| + |f'(b)|\right].$$

Remark 4. *In Corollary 4, we have an improvement of the constant in Theorem 2, since*

$$\frac{1}{8 \times 6^{\frac{1}{q}}} < \frac{1}{8}$$

for $q \geq 1$.

Theorem 5. *Let $f : [a,b] \to \mathbb{R}$ be a differentiable mapping on (a,b) with $a < b$. If $|f'|^q$, $q > 1$ is convex on $[a,b]$ for $\lambda \in [0,1]$ and $x \in [a,b]$, then we have the*

following inequality

$$
\begin{aligned}
(2.17)\qquad & \left|\frac{(1-\lambda)\, f\left(\mu x+(1-\mu)\,(a+b-x)\right)}{(1-2\mu)}\right. \\
& +\lambda\frac{(b-x)\, f\left(\mu b+(1-\mu)a\right)+(x-a)\, f\left(\mu a+(1-\mu)b\right)}{(b-a)\,(1-2\mu)} \\
& \left.-\frac{1}{(b-a)\,(1-2\mu)^{2}}\int\limits_{\mu b+(1-\mu)a}^{\mu a+(1-\mu)b} f(u)du\right| \\
\leq\ & (b-a)\left(\frac{\lambda^{p+1}+(1-\lambda)^{p+1}}{p+1}\right)^{\frac{1}{p}} \\
& \times\left[\left(\frac{x-a}{b-a}\right)^{\frac{p+1}{p}}\left(\frac{1}{2}\left(\frac{b-x}{b-a}\right)^{2}\left|f'\left(\mu a+(1-\mu)b\right)\right|^{q}\right.\right. \\
& \left.+\left[\frac{1}{2}-\frac{1}{2}\left(\frac{x-a}{b-a}\right)^{2}\right]\left|f'\left(\mu b+(1-\mu)a\right)\right|^{q}\right)^{\frac{1}{q}} \\
& +\left(\frac{b-x}{b-a}\right)^{\frac{p+1}{p}}\left(\left[\frac{1}{2}-\frac{1}{2}\left(\frac{b-x}{b-a}\right)^{2}\right]\left|f'\left(\mu a+(1-\mu)b\right)\right|^{q}\right. \\
& \left.\left.+\frac{1}{2}\left(\frac{x-a}{b-a}\right)\left|f'\left(\mu b+(1-\mu)a\right)\right|^{q}\right)^{\frac{1}{q}}\right]
\end{aligned}
$$

where $\frac{1}{p}+\frac{1}{q}=1$ *and* $\mu\in[0,1]\,/\,\{1/2\}$.

Proof. Taking modulus in Lemma 1 and using Hölder inequality, we have

$$
\begin{aligned}
& \left|\frac{(1-\lambda)\, f\left(\mu x+(1-\mu)\,(a+b-x)\right)}{(1-2\mu)}\right. \\
& +\lambda\frac{(b-x)\, f\left(\mu b+(1-\mu)a\right)+(x-a)\, f\left(\mu a+(1-\mu)b\right)}{(b-a)\,(1-2\mu)} \\
& \left.-\frac{1}{(b-a)\,(1-2\mu)^{2}}\int\limits_{\mu b+(1-\mu)a}^{\mu a+(1-\mu)b} f(u)du\right| \\
\leq\ & (b-a)\left\{\int\limits_{0}^{\frac{b-x}{b-a}}\left|t-\lambda\frac{b-x}{b-a}\right|\left|f'\left[t\left(\mu a+(1-\mu)b\right)+(1-t)\left(\mu b+(1-\mu)a\right)\right]\right|dt\right.
\end{aligned}
$$

$$
\left.+\int_{\frac{b-x}{b-a}}^{1}\left|t-1+\lambda\frac{x-a}{b-a}\right|\left|f'\left[t\left(\mu a+(1-\mu)b\right)+(1-t)\left(\mu b+(1-\mu)a\right)\right]\right|dt\right\}
$$

$$
\begin{aligned}
&\leq (b-a)\left\{\left(\int_{0}^{\frac{b-x}{b-a}}\left|t-\lambda\frac{b-x}{b-a}\right|^{p}dt\right)^{\frac{1}{p}}\right. \\
&\quad\times\left(\int_{0}^{\frac{b-x}{b-a}}\left|f'\left[t\left(\mu a+(1-\mu)b\right)+(1-t)\left(\mu b+(1-\mu)a\right)\right]\right|^{q}dt\right)^{\frac{1}{q}} \\
&\quad+\left(\int_{\frac{b-x}{b-a}}^{1}\left|t-1+\lambda\frac{x-a}{b-a}\right|^{p}dt\right)^{\frac{1}{p}} \\
&\quad\left.\times\left(\int_{\frac{b-x}{b-a}}^{1}\left|f'\left[t\left(\mu a+(1-\mu)b\right)+(1-t)\left(\mu b+(1-\mu)a\right)\right]\right|dt\right)^{\frac{1}{q}}\right\} \\
&= (b-a)K_3.
\end{aligned}
$$

Using the convexity of $|f'|^q$, we obtain

$$
\begin{aligned}
K_3 \leq &\left\{\left(\int_{0}^{\frac{b-x}{b-a}}\left|t-\lambda\frac{b-x}{b-a}\right|^{p}dt\right)^{\frac{1}{p}}\right. \\
&\times\left(\left|f'\left(\mu a+(1-\mu)b\right)\right|^{q}\int_{0}^{\frac{b-x}{b-a}}tdt+\left|f'\left(\mu b+(1-\mu)a\right)\right|^{q}\int_{0}^{\frac{b-x}{b-a}}(1-t)\,dt\right)^{\frac{1}{q}} \\
&+\left(\int_{\frac{b-x}{b-a}}^{1}\left|t-1+\lambda\frac{x-a}{b-a}\right|^{p}dt\right)^{\frac{1}{p}}
\end{aligned}
$$

$$\times\left(\left|f'\left(\mu a+(1-\mu)b\right)\right|^{q}\int_{\frac{b-x}{b-a}}^{1}tdt+\left|f'\left(\mu b+(1-\mu)a\right)\right|^{q}\int_{\frac{b-x}{b-a}}^{1}\left(1-t\right)dt\right)^{\frac{1}{q}}\Bigg\}$$

$$=\Bigg\{\left(\int_{0}^{\frac{b-x}{b-a}}\left|t-\lambda\frac{b-x}{b-a}\right|^{p}dt\right)^{\frac{1}{p}}\left(\frac{1}{2}\left(\frac{b-x}{b-a}\right)^{2}\left|f'\left(\mu a+(1-\mu)b\right)\right|^{q}\right.$$

$$\left.+\left[\frac{1}{2}-\frac{1}{2}\left(\frac{x-a}{b-a}\right)^{2}\right]\left|f'\left(\mu b+(1-\mu)a\right)\right|^{q}\right)^{\frac{1}{q}}$$

$$+\left(\int_{\frac{b-x}{b-a}}^{1}\left|t-1+\lambda\frac{x-a}{b-a}\right|^{p}dt\right)^{\frac{1}{p}}\left(\left[\frac{1}{2}-\frac{1}{2}\left(\frac{b-x}{b-a}\right)^{2}\right]\left|f'\left(\mu a+(1-\mu)b\right)\right|^{q}\right.$$

$$\left.+\frac{1}{2}\left(\frac{x-a}{b-a}\right)^{2}\left|f'\left(\mu b+(1-\mu)a\right)\right|^{q}\right)^{\frac{1}{q}}\Bigg\}.$$

If we use equalities (2.7) and (2.8), then we obtain required result. □

Remark 5. *If we choose $\mu=1$ in (2.17), then the inequality Theorem 5 reduces the Theorem 2 in* [12].

Corollary 5. *Under assumptions of Theorem 5, choosing $\mu=0$, we get the inequality*

$$\begin{aligned}(2.18)\quad &\left|(1-\lambda)f(a+b-x)\right.\\ &\left.+\lambda\frac{(b-x)f(a)+(x-a)f(b)}{(b-a}-\frac{1}{b-a}\int_{a}^{b}f(u)du\right|\\ \leq\ &(b-a)\left(\frac{\lambda^{p+1}+(1-\lambda)^{p+1}}{p+1}\right)^{\frac{1}{p}}\\ &\times\left[\left(\frac{b-x}{b-a}\right)^{\frac{p+1}{p}}\left(\frac{1}{2}\left(\frac{b-x}{b-a}\right)^{2}\left|f'(b)\right|^{q}+\left[\frac{1}{2}-\frac{1}{2}\left(\frac{x-a}{b-a}\right)^{2}\right]\left|f'(a)\right|^{q}\right)^{\frac{1}{q}}\right.\\ &\left.+\left(\frac{b-x}{b-a}\right)^{\frac{p+1}{p}}\left(\left[\frac{1}{2}-\frac{1}{2}\left(\frac{b-x}{b-a}\right)^{2}\right]\left|f'(b)\right|^{q}+\frac{1}{2}\left(\frac{x-a}{b-a}\right)\left|f'(a)\right|^{q}\right)^{\frac{1}{q}}\right].\end{aligned}$$

Corollary 6. *If we take $\lambda = 1$ and $x = \frac{a+b}{2}$ in Corollary 5, then we have the following trapezoid inequality*

$$\begin{aligned}
&\left|\frac{f(a)+f(b)}{2} - \frac{1}{b-a}\int_a^b f(u)du\right| \\
\leq\ & \frac{b-a}{4(p+1)^{\frac{1}{p}}}\left[\left(\frac{3|f'(a)|^q + |f'(b)|^q}{4}\right)^{\frac{1}{q}} + \left(\frac{|f'(a)|^q + 3|f'(b)|^q}{4}\right)^{\frac{1}{q}}\right] \\
\leq\ & \left(\frac{b-a}{4}\right)\left(\frac{4}{p+1}\right)^{\frac{1}{p}}\left[|f'(a)| + |f'(b)|\right].
\end{aligned}$$

Proof. The proof of the first inequality is obvious. For second inequality, let $a_1 = 3|f'(a)|^q$, $a_2 = |f'(a)|^q$, $b_1 = |f'(b)|^q$, $b_2 = 3|f'(b)|^q$. Here $0 < \frac{1}{q} < 1$, for $q > 1$. Using the fact that,

$$\sum_{k=1}^{n}(a_k + b_k)^s \leq \sum_{k=1}^{n} a_k^s + \sum_{k=1}^{n} b_k^s$$

for $(0 < s < 1)$ $a_1, a_2, ..., a_n \geq 0$, $b_1, b_2, ..., b_n \geq 0$, we have

$$\begin{aligned}
&\left(\frac{3|f'(a)|^q + |f'(b)|^q}{4}\right)^{\frac{1}{q}} + \left(\frac{|f'(a)|^q + 3|f'(b)|^q}{4}\right)^{\frac{1}{q}} \\
=\ & \frac{1}{4^{\frac{1}{q}}}\left[\left(3|f'(a)|^q + |f'(b)|^q\right)^{\frac{1}{q}} + \left(|f'(a)|^q + 3|f'(b)|^q\right)^{\frac{1}{q}}\right] \\
\leq\ & \frac{\left(1+3^{\frac{1}{q}}\right)}{4^{\frac{1}{q}}}\left[|f'(a)| + |f'(b)|\right] \\
\leq\ & 4^{1-\frac{1}{q}}\left[|f'(a)| + |f'(b)|\right].
\end{aligned}$$

This completes the proof. □

Remark 6. *If we take $\lambda = 0$ and $x = \frac{a+b}{2}$ in Corollary 5, then we have the following midpoint inequality*

$$\begin{aligned}
&\left|f\left(\frac{a+b}{2}\right) - \frac{1}{b-a}\int_a^b f(u)du\right| \\
\leq\ & \frac{b-a}{4(p+1)^{\frac{1}{p}}}\left[\left(\frac{3|f'(a)|^q + |f'(b)|^q}{4}\right)^{\frac{1}{q}} + \left(\frac{|f'(a)|^q + 3|f'(b)|^q}{4}\right)^{\frac{1}{q}}\right] \\
\leq\ & \left(\frac{b-a}{4}\right)\left(\frac{4}{p+1}\right)^{\frac{1}{p}}\left[|f'(a)| + |f'(b)|\right]
\end{aligned}$$

which is given by Kirmaci in [10].

Theorem 6. *Let $f : [a,b] \to \mathbb{R}$ be a differentiable mapping on (a,b) with $a < b$. If $|f'|^q$, $q > 1$ is convex on $[a,b]$ for $\lambda \in [0,1]$ and $x \in [a,b]$, then we have the*

following inequality

$$\left|\frac{(1-\lambda)f(\mu x+(1-\mu)(a+b-x))}{(1-2\mu)}\right. \tag{2.19}$$
$$+\lambda\frac{(b-x)f(\mu b+(1-\mu)a)+(x-a)f(\mu a+(1-\mu)b)}{(b-a)(1-2\mu)}$$
$$\left.-\frac{1}{(b-a)(1-2\mu)^2}\int_{\mu b+(1-\mu)a}^{\mu a+(1-\mu)b}f(u)du\right|$$
$$\leq \frac{b-a}{2^{\frac{1}{q}}}\left(\frac{\lambda^{p+1}+(1-\lambda)^{p+1}}{p+1}\right)^{\frac{1}{p}}$$
$$\times\left[\left(\frac{b-x}{b-a}\right)^{\frac{p+1}{p}}\left(|f'(\mu x+(1-\mu)(a+b-x))|^q+|f'(\mu b+(1-\mu)a)|^q\right)^{\frac{1}{q}}\right.$$
$$\left.+\left(\frac{x-a}{b-a}\right)^{\frac{p+1}{p}}\left(|f'(\mu a+(1-\mu)b)|^q+|f'(\mu x+(1-\mu)(a+b-x))|^q\right)^{\frac{1}{q}}\right]$$

where $\frac{1}{p}+\frac{1}{q}=1$ *and* $\mu\in[0,1]/\{1/2\}$.

Proof. Taking modulus in Lemma 1 and using Hölder inequality, we have

$$\left|\frac{(1-\lambda)f(\mu x+(1-\mu)(a+b-x))}{(1-2\mu)}\right. \tag{2.20}$$
$$+\lambda\frac{(b-x)f(\mu b+(1-\mu)a)+(x-a)f(\mu a+(1-\mu)b)}{(b-a)(1-2\mu)}$$
$$\left.-\frac{1}{(b-a)(1-2\mu)^2}\int_{\mu b+(1-\mu)a}^{\mu a+(1-\mu)b}f(u)du\right|$$
$$\leq (b-a)\left\{\int_0^{\frac{b-x}{b-a}}\left|t-\lambda\frac{b-x}{b-a}\right|\left|f'\left[t(\mu a+(1-\mu)b)+(1-t)(\mu b+(1-\mu)a)\right]\right|dt\right.$$
$$\left.+\int_{\frac{b-x}{b-a}}^{1}\left|t-1+\lambda\frac{x-a}{b-a}\right|\left|f'\left[t(\mu a+(1-\mu)b)+(1-t)(\mu b+(1-\mu)a)\right]\right|dt\right\}$$
$$\leq (b-a)\left\{\left(\int_0^{\frac{b-x}{b-a}}\left|t-\lambda\frac{b-x}{b-a}\right|^p dt\right)^{\frac{1}{p}}\right.$$

$$\times\left(\int_{0}^{\frac{b-x}{b-a}}\left|f'\left[t\left(\mu a+(1-\mu)b\right)+(1-t)\left(\mu b+(1-\mu)a\right)\right]\right|^{q}dt\right)^{\frac{1}{q}}$$

$$+\left(\int_{\frac{b-x}{b-a}}^{1}\left|t-1+\lambda\frac{x-a}{b-a}\right|^{p}dt\right)^{\frac{1}{p}}$$

$$\left.\times\left(\int_{\frac{b-x}{b-a}}^{1}\left|f'\left[t\left(\mu a+(1-\mu)b\right)+(1-t)\left(\mu b+(1-\mu)a\right)\right]\right|dt\right)^{\frac{1}{q}}\right\}$$

Since convexity of $|f'|^{q}$, using Hermite-Hadamard inequality we have

$$\int_{0}^{\frac{b-x}{b-a}}\left|f'\left[t\left(\mu a+(1-\mu)b\right)+(1-t)\left(\mu b+(1-\mu)a\right)\right]\right|^{q}dt \tag{2.21}$$

$$= \frac{1}{(b-a)(1-2\mu)}\int_{\mu b+(1-\mu)a}^{\mu a+(1-\mu)(a+b-x)}f(u)du$$

$$\leq \frac{\left|f'\left(\mu x+(1-\mu)(a+b-x)\right)\right|^{q}+\left|f'\left(\mu b+(1-\mu)a\right)\right|^{q}}{2}$$

and

$$\int_{\frac{b-x}{b-a}}^{1}\left|f'\left[t\left(\mu a+(1-\mu)b\right)+(1-t)\left(\mu b+(1-\mu)a\right)\right]\right|dt \tag{2.22}$$

$$= \frac{1}{(b-a)(1-2\mu)}\int_{\mu a+(1-\mu)(a+b-x)}^{\mu a+(1-\mu)b}f(u)du$$

$$+\frac{\left|f'\left(\mu a+(1-\mu)b\right)\right|^{q}+\left|f'\left(\mu x+(1-\mu)(a+b-x)\right)\right|^{q}}{2}$$

If we put (2.7)-(2.8) and (2.21)-(2.22) in (2.20), then we complete the proof. □

Corollary 7. *Under assumption of Theorem 6, if we choose $\mu = 1$, then we have the inequality*

$$\left|(1-\lambda) f(x) + \lambda \frac{(b-x) f(b) + (x-a) f(a)}{(b-a)} - \frac{1}{(b-a)} \int_a^b f(u)du \right| \tag{2.23}$$

$$\leq \frac{b-a}{2^{\frac{1}{q}}} \left(\frac{\lambda^{p+1} + (1-\lambda)^{p+1}}{p+1} \right)^{\frac{1}{p}} \left[\left(\frac{b-x}{b-a} \right)^{\frac{p+1}{p}} \left(|f'(x)|^q + |f'(b)|^q \right)^{\frac{1}{q}} \right.$$

$$\left. + \left(\frac{x-a}{b-a} \right)^{\frac{p+1}{p}} \left(|f'(a)|^q + |f'(x)|^q \right)^{\frac{1}{q}} \right].$$

Remark 7. *If we choose $\lambda = 0$ in Corollary 7, then Corollary 7 reduces Theorem 2 in* [3].

Corollary 8. *If we choose $\lambda = 1$ and $x = \frac{a+b}{2}$ in Corollary 7, then we have the following trapezoid inequality*

$$\left| \frac{f(a) + f(b)}{2} - \frac{1}{(b-a)} \int_a^b f(u)du \right|$$

$$\leq \frac{b-a}{4(p+1)^{\frac{1}{p}}} \left[\left(\left| f'\left(\frac{a+b}{2} \right) \right|^q + |f'(b)|^q \right)^{\frac{1}{q}} \right.$$

$$\left. + \left(|f'(a)|^q + \left| f'\left(\frac{a+b}{2} \right) \right|^q \right)^{\frac{1}{q}} \right].$$

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