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ON GENERALIZATION OF WEIGHTED OSTROWSKI TYPE INEQUALITIES FOR FUNCTIONS OF BOUNDED VARIATION

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ABSTRACT. In this paper, some generalization of weighted Ostrowski type integral inequalities for mappings of bounded variation are obtained and some interesting inequalities as special cases are given.

1. INTRODUCTION

In 1938, Ostrowski established the following interesting integral inequality for differentiable mappings with bounded derivatives [16]:

Theorem 1. *Let $f:[a,b]\to\mathbb{R}$ be a differentiable mapping on (a,b) whoose derivative $f':(a,b)\to\mathbb{R}$ is bounded on (a,b), i.e. $\|f'\|_\infty:=\sup_{t\in(a,b)}|f'(t)|<\infty$. Then, we have the inequality*

$$\left|f(x)-\frac{1}{b-a}\int_a^b f(t)dt\right|\leq\left[\frac{1}{4}+\frac{\left(x-\frac{a+b}{2}\right)^2}{(b-a)^2}\right](b-a)\|f'\|_\infty,\tag{1.1}$$

for all $x\in[a,b]$. The constant $\frac{1}{4}$ is the best possible.

This inequality is well known in the literature as the *Ostrowski inequality.*

Definition 1. *Let $P:a=x_0<x_1<...<x_n=b$ be any partition of $[a,b]$ and let $\Delta f(x_i)=f(x_{i+1})-f(x_i)$ Then $f(x)$ is said to be of bounded variation if the sum*

$$\sum_{i=1}^{n}|\Delta f(x_i)|$$

is bounded for all such partitions.

Let f be of bounded variation on $[a,b]$, and $\sum(P)$ denotes the sum $\sum_{i=1}^{n}|\Delta f(x_i)|$ corresponding to the partition P of $[a,b]$. The number

$$\bigvee_a^b(f):=\sup\left\{\sum(P):P\in P([a,b])\right\},$$

is called the total variation of f on $[a,b]$. Here $P([a,b])$ denotes the family of partitions of $[a,b]$.

A similar result (1.1) is obtained by Dragomir in [9] for functions of bounded variation as follow:

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Theorem 2. *Let $f:[a,b]\to\mathbb{R}$ be a mapping of bounded variation on $[a,b]$. Then*

$$\left|\int_a^b f(t)dt-(b-a)f(x)\right|\leq\left[\frac{1}{2}(b-a)+\left|x-\frac{a+b}{2}\right|\right]\bigvee_a^b(f) \tag{1.2}$$

holds for all $x\in[a,b]$. The constant $\frac{1}{2}$ is the best possible.

Moreover, Dragomir obtained following Ostrowski type inequality for functions of bounded variation [7]:

Theorem 3. *Let $I_n: a=x_0<x_1<...<x_n=b$ be a division of the interval $[a,b]$ and $\alpha_i\,(i=0,1,...,n+1)$ be $n+2$ points so that $\alpha_0=a$, $\alpha_i\in[x_{i-1},x_i]$ $(i=1,...,n)$, $\alpha_{n+1}=b$. If $f:[a,b]\to\mathbb{R}$ is of bounded variation on $[a,b]$, then we have the inequalities:*

$$\begin{aligned}
&\left|\int_a^b f(x)dx-\sum_{i=0}^{n}(\alpha_{i+1}-\alpha_i)f(x_i)\right| \\
\leq\;&\left[\frac{1}{2}\upsilon(h)+\max\left|\alpha_{i+1}-\frac{x_i+x_{i+1}}{2}\right|,i=0,1,...,n-1\right]\bigvee_a^b(f) \\
\leq\;&\upsilon(h)\bigvee_a^b(f)
\end{aligned} \tag{1.3}$$

where $\upsilon(h):=\max\{h_i|\ i=0,...,n-1\}$, $h_i:=x_{i+1}-x_i\,(i=0,1,...,k-1)$ and $\bigvee_a^b(f)$ is the total variation of f on the interval $[a,b]$.

In [15], Liu proved the following weighted Ostrowski type inequality for functions of bounded variation:

Theorem 4. *Let $f:[a,b]\to\mathbb{R}$ be a mapping of bounded variation, $g:[a,b]\to(0,\infty)$ continious and positive mapping on (a,b). Then for any $x\in[a,b]$ and $\alpha\in[0,1]$ we have*

$$\begin{aligned}
&\Bigg|(1-\alpha)\left(\int_a^b g(u)du\right)f(x) \\
&+\alpha\left[\left(\int_a^x g(u)du\right)f(a)+\left(\int_x^b g(u)du\right)f(b)\right]-\int_a^b f(t)g(t)dt\Bigg| \\
\leq\;&\left[\frac{1}{2}+\left|\frac{1}{2}-\alpha\right|\right]\left[\frac{1}{2}\int_a^b g(u)du+\left|\int_a^x g(u)du-\frac{1}{2}\int_a^b g(u)du\right|\right]\bigvee_a^b(f)
\end{aligned} \tag{1.4}$$

where $\bigvee_a^b(f)$ is the total variation of f on the interval $[a,b]$. The constant $\left[\frac{1}{2}+\left|\frac{1}{2}-\alpha\right|\right]$ is the best possible.

In [3], Budak and Sarıkaya gave the following weighted Ostrowski's type inequalities for mapping of bounded variation.

Theorem 5. *Let I_n, $\alpha_i\,(i=0,1,...,n+1)$, $\upsilon(h)$, and $h_i\,(i=0,1,...,k-1)$ be as the Theorem 3. If $f:[a,b]\to\mathbb{R}$ is of bounded variation on $[a,b]$ and $w:[a,b]\to(0,\infty)$ be continious and positive mapping on (a,b), then we have the inequalities:*

$$\left|\sum_{i=0}^{n}\left(\int_{\alpha_i}^{\alpha_{i+1}} w(u)du\right) f(x_i)-\int_a^b f(t)w(t)dt\right| \tag{1.5}$$
$$\leq \left[\frac{1}{2}\upsilon(L)+\max_{i\in\{0,1,\dots,n-1\}}\frac{1}{2}\left|\int_{x_i}^{\alpha_{i+1}} w(u)du-\int_{\alpha_{i+1}}^{x_{i+1}} w(u)du\right|\right]\bigvee_a^b(f)$$
$$\leq \upsilon(L)\bigvee_a^b(f)$$

where $\upsilon(L):=\max\{L_i|\ i=0,...,n-1\}$, $L_i=\int_{x_i}^{x_{i+1}} w(u)du\ (i=0,1,...,n-1)$ and $\bigvee_a^b(f)$ is the total variation of f on the interval $[a,b]$.

A weighted generalization of trapezoid inequality for mappings of bounded variation was considered by Tseng et. al. [17]. Recently, in [2, 3, 20] the authors proved a generalization of weighted Ostrowski type inequality for mappings of bounded variation. The purpose of this paper is to obtain some weighted integral inequalities, which not only provide a weighted form of Theorem 5, but also give some other interesting inequalities as special cases.

For recent new results regarding Ostrowski's type inequalities see [1]-[15],[17]-[21].

2. Main Results

Firstly, we will give the following notations which are used in main Theorem:

Let $I_n: a=x_0<x_1<...<x_n=b$ be a partition of the interval $[a,b]$, $\alpha_i\,(i=0,1,...,n+1)$ be $n+2$ points so that $\alpha_0=a$, $\alpha_i\in[x_{i-1},x_i]\ (i=1,...,n)$, $\alpha_{n+1}=b$. Let $w:[a,b]\to(0,\infty)$ be continious and positive mapping on (a,b), and

$$\upsilon(h):=\max\{h_i|\ i=0,...,n-1\},\quad h_i:=x_{i+1}-x_i\ \ (i=0,1,...,n-1),$$

$$\upsilon(L):=\max\{L_i|\ i=0,...,n-1\},\ L_i=\int_{x_i}^{x_{i+1}} w(u)du\ \ (i=0,1,...,n-1).$$

Lemma 1. *If* $f:[a,b]\to\mathbb{R}$ *is of bounded variation on* $[a,b]$, *then we have the equality*

$$\sum_{i=0}^{n-1}\left[\int_{x_i}^{x_{i+1}}K_i(t)df(t)\right] = (1-\mu)\sum_{i=0}^{n-1}\left(\int_{x_i}^{x_{i+1}}w(u)du\right)f(\alpha_{i+1}) +\mu\sum_{i=0}^{n}\left(\int_{\alpha_i}^{\alpha_{i+1}}w(u)du\right)f(x_i)-\int_a^b f(t)w(t)dt \tag{2.1}$$

where

$$K_i(t)=\begin{cases}(1-\mu)\int_{x_i}^{t}w(u)du+\mu\int_{\alpha_{i+1}}^{t}w(u)du, & t\in[x_i,\alpha_{i+1})\\ (1-\mu)\int_{x_{i+1}}^{t}w(u)du+\mu\int_{\alpha_{i+1}}^{t}w(u)du, & t\in[\alpha_{i+1},x_{i+1}]\end{cases}$$

for $i\in\{0,1,2,...,n-1\}$ *and* $\mu\in[0,1]$.

Proof. Integrating by parts, we have

$$\begin{aligned}
&\int_{x_i}^{x_{i+1}}K_i(t)df(t) \qquad (2.2)\\
&= \int_{x_i}^{\alpha_{i+1}}\left((1-\mu)\int_{x_i}^{t}w(u)du+\mu\int_{\alpha_{i+1}}^{t}w(u)du\right)df(t)\\
&\quad \int_{\alpha_{i+1}}^{x_{i+1}}\left((1-\mu)\int_{x_{i+1}}^{t}w(u)du+\mu\int_{\alpha_{i+1}}^{t}w(u)du\right)df(t)\\
&= (1-\mu)\left(\int_{x_i}^{\alpha_{i+1}}w(u)du\right)f(\alpha_{i+1})+\mu\left(\int_{x_i}^{\alpha_{i+1}}w(u)du\right)f(x_i)-\int_{x_i}^{\alpha_{i+1}}f(t)w(t)dt\\
&\quad +\mu\left(\int_{\alpha_{i+1}}^{x_{i+1}}w(u)du\right)f(x_{i+1})+(1-\mu)\left(\int_{\alpha_{i+1}}^{x_{i+1}}w(u)du\right)f(\alpha_{i+1})-\int_{\alpha_{i+1}}^{x_{i+1}}f(t)w(t)dt\\
&= (1-\mu)\left(\int_{x_i}^{x_{i+1}}w(u)du\right)f(\alpha_{i+1})+\mu\left[f(x_i)\int_{x_i}^{\alpha_{i+1}}w(u)du+f(x_{i+1})\int_{\alpha_{i+1}}^{x_{i+1}}w(u)du\right]\\
&\quad -\int_{x_i}^{x_{i+1}}f(t)w(t)dt.
\end{aligned}$$

Summing (2.2) over i from 0 to $n-1$, we have

$$\sum_{i=0}^{n-1}\left[\int_{x_i}^{x_{i+1}} K_i(t)df(t)\right] \tag{2.3}$$

$$= (1-\mu)\sum_{i=0}^{n-1}\left(\int_{x_i}^{x_{i+1}} w(u)du\right) f(\alpha_{i+1}) +$$

$$\mu\left[\sum_{i=0}^{n-1}\left(\int_{x_i}^{\alpha_{i+1}} w(u)du\right) f(x_i) + \sum_{i=0}^{n-1}\left(\int_{\alpha_{i+1}}^{x_{i+1}} w(u)du\right) f(x_{i+1})\right]$$

$$-\int_a^b f(t)w(t)dt.$$

Using the equalities

$$\sum_{i=0}^{n-1}\left(\int_{x_i}^{\alpha_{i+1}} w(u)du\right) f(x_i) = \left(\int_a^{\alpha_1} w(u)du\right) f(a) + \sum_{i=1}^{n-1}\left(\int_{x_i}^{\alpha_{i+1}} w(u)du\right) f(x_i)$$

and

$$\sum_{i=0}^{n-1}\left(\int_{\alpha_{i+1}}^{x_{i+1}} w(u)du\right) f(x_{i+1}) = \sum_{i=1}^{n}\left(\int_{\alpha_i}^{x_i} w(u)du\right) f(x_i)$$

$$= \sum_{i=1}^{n-1}\left(\int_{\alpha_i}^{x_i} w(u)du\right) f(x_i) + \left(\int_{\alpha_n}^{b} w(u)du\right) f(b)$$

in (2.3), we have

$$\sum_{i=0}^{n-1}\left[\int_{x_i}^{x_{i+1}} K_i(t)df(t)\right]$$

$$= (1-\mu)\sum_{i=0}^{n-1}\left(\int_{x_i}^{x_{i+1}} w(u)du\right) f(\alpha_{i+1})$$

$$+\mu\left[\left(\int_a^{\alpha_1} w(u)du\right) f(a) + \sum_{i=1}^{n-1}\left(\int_{\alpha_i}^{\alpha_{i+1}} w(u)du\right) f(x_i) + \left(\int_{\alpha_n}^{b} w(u)du\right) f(b)\right]$$

$$-\int_a^b f(t)w(t)dt.$$

With $\alpha_0 = a$ and $\alpha_{n+1} = b$, the last equality completes the proof. □

Theorem 6. *If $f:[a,b]\to\mathbb{R}$ is of bounded variation on $[a,b]$, then we have the inequality*

$$\begin{aligned}
&\left|(1-\mu)\sum_{i=0}^{n-1}\left(\int_{x_i}^{x_{i+1}}w(u)du\right)f(\alpha_{i+1})\right. \\
&\left.+\mu\sum_{i=0}^{n}\left(\int_{\alpha_i}^{\alpha_{i+1}}w(u)du\right)f(x_i)-\int_a^b f(t)w(t)dt\right| \\
\leq\;&\left[\frac{1}{2}+\left|\frac{1}{2}-\mu\right|\right] \\
&\times\left[\frac{1}{2}\upsilon(L)+\max_{i\in\{0,1,\dots,n-1\}}\left|\int_{x_i}^{\alpha_{i+1}}w(u)du-\frac{1}{2}\int_{x_i}^{x_{i+1}}w(u)du\right|\right]\bigvee_a^b(f)
\end{aligned}\tag{2.4}$$

where $\mu\in[0,1]$ and $\bigvee_a^b(f)$ denotes the total variation of f on $[a,b]$.

Proof. Taking the modulus in Lemma 1, then ve have

$$\begin{aligned}
&\left|(1-\mu)\sum_{i=0}^{n-1}\left(\int_{x_i}^{x_{i+1}}w(u)du\right)f(\alpha_{i+1})\right. \\
&\left.+\mu\sum_{i=0}^{n}\left(\int_{\alpha_i}^{\alpha_{i+1}}w(u)du\right)f(x_i)-\int_a^b f(t)w(t)dt\right| \\
=\;&\left|\sum_{i=0}^{n-1}\left[\int_{x_i}^{x_{i+1}}K_i(t)df(t)\right]\right| \\
\leq\;&\sum_{i=0}^{n-1}\left|\int_{x_i}^{x_{i+1}}K_i(t)df(t)\right|.
\end{aligned}\tag{2.5}$$

It is well known that if $g,h:[c,d]\to\mathbb{R}$ are such that g is continuous on $[c,d]$ and h is of bounded variation on $[c,d]$, then $\int_c^d g(t)dh(t)$ exist and

$$\left|\int_c^d g(t)dh(t)\right|\leq\sup_{t\in[c,d]}|g(t)|\bigvee_a^b(h).\tag{2.6}$$

Using (2.6), we get

$$\left|\int_{x_i}^{x_{i+1}}K_i(t)df(t)\right|\leq\sup_{t\in[x_i,x_{i+1}]}|K_i(t)|\bigvee_{x_i}^{x_{i+1}}(f)\tag{2.7}$$

for each $i\in\{0,1,2,...,n-1\}$.

Denote

$$p_i(t) = (1-\mu)\int_{x_i}^{t} w(u)du + \mu \int_{\alpha_{i+1}}^{t} w(u)du,\ t \in [x_i, \alpha_{i+1})$$

and

$$q_i(t) = (1-\mu)\int_{x_{i+1}}^{t} w(u)du + \mu \int_{\alpha_{i+1}}^{t} w(u)du,\ t \in [\alpha_{i+1}, x_{i+1}]$$

for $i \in \{0, 1, 2, ..., n-1\}$.

It is obvious that $p_i(t)$ is increasing on $[x_i, \alpha_{i+1})$ and $q_i(t)$ is increasing on $[\alpha_{i+1}, x_{i+1}]$, since $p_i'(t) = q_i'(t) = w(t) > 0$ for $i \in \{0, 1, 2, ..., n-1\}$.

Therefore,

$$\begin{aligned}
(2.8)\quad \sup_{t\in[x_i,x_{i+1}]} |K_i(t)| &= \max\left\{ \sup_{t\in[x_i,\alpha_{i+1})} |K_i(t)|, \sup_{t\in[\alpha_{i+1},x_{i+1}]} |K_i(t)| \right\} \\
&= \max\left\{ \max\left\{ (1-\mu)\int_{x_i}^{\alpha_{i+1}} w(u)du, \mu \int_{x_i}^{\alpha_{i+1}} w(u)du \right\}, \right. \\
&\qquad \left. \left\{ (1-\mu)\int_{\alpha_{i+1}}^{x_{i+1}} w(u)du, \mu \int_{\alpha_{i+1}}^{x_{i+1}} w(u)du \right\} \right\} \\
&= \left[\frac{1}{2} + \left|\frac{1}{2} - \mu\right|\right] \max\left\{ \int_{x_i}^{\alpha_{i+1}} w(u)du, \int_{\alpha_{i+1}}^{x_{i+1}} w(u)du \right\}
\end{aligned}$$

Using properties of maximum and putting (2.7) and (2.8) in (2.5), we obtain

$$\begin{aligned}
&\left| (1-\mu)\sum_{i=0}^{n-1} \left(\int_{x_i}^{x_{i+1}} w(u)du \right) f(\alpha_{i+1}) \right. \\
&\left. + \sum_{i=0}^{n} \left(\int_{\alpha_i}^{\alpha_{i+1}} w(u)du \right) f(x_i) - \int_a^b f(t)w(t)dt \right| \\
\leq\ & \left[\frac{1}{2} + \left|\frac{1}{2} - \mu\right|\right] \\
&\times \sum_{i=0}^{n-1} \left[\frac{1}{2}\int_{x_i}^{x_{i+1}} w(u)du + \left| \int_{x_i}^{\alpha_{i+1}} w(u)du - \frac{1}{2}\int_{x_i}^{x_{i+1}} w(u)du \right| \right] \bigvee_{x_i}^{x_{i+1}} (f)
\end{aligned}$$

$$\begin{aligned}
&\leq \left[\frac{1}{2}+\left|\frac{1}{2}-\mu\right|\right] \\
&\quad\times \max_{i\in\{0,1,\dots,n-1\}}\left[\frac{1}{2}\int_{x_i}^{x_{i+1}} w(u)du+\left|\int_{x_i}^{\alpha_{i+1}} w(u)du-\frac{1}{2}\int_{x_i}^{x_{i+1}} w(u)du\right|\right]\sum_{i=0}^{n-1}\bigvee_{x_i}^{x_{i+1}}(f) \\
&\leq \left[\frac{1}{2}+\left|\frac{1}{2}-\mu\right|\right] \\
&\quad\times\left[\frac{1}{2}\upsilon(L)+\max_{i\in\{0,1,\dots,n-1\}}\left|\int_{x_i}^{\alpha_{i+1}} w(u)du-\frac{1}{2}\int_{x_i}^{x_{i+1}} w(u)du\right|\right]\bigvee_a^b(f)
\end{aligned}$$

which is the completes the proof. □

Remark 1. *If we choose $\mu = 1$ in Theorem 6, then the inequality (2.4) reduces the inequality (1.5).*

Remark 2. *If we take $a < \alpha_1 < b$ and $\alpha_1 = x$ in Theorem 6, then the inequality (2.4) reduces the inequality (1.4).*

Corollary 1. *Under assumption of Theorem 6 with $w(u) \equiv 1$, we have*

$$\begin{aligned}
&\left|(1-\mu)\sum_{i=0}^{n-1} f(\alpha_{i+1})h_i+\mu\sum_{i=0}^{n}(\alpha_{i+1}-\alpha_i)f(x_i)-\int_a^b f(t)dt\right| \qquad (2.9)\\
&\leq \left[\frac{1}{2}+\left|\frac{1}{2}-\mu\right|\right] \\
&\quad\times\left[\frac{1}{2}\upsilon(h)+\max_{i\in\{0,1,\dots,n-1\}}\left|\alpha_{i+1}-\frac{x_i+x_{i+1}}{2}\right|\right]\bigvee_a^b(f) \\
&\leq \upsilon(h)\bigvee_a^b(f)
\end{aligned}$$

where $\mu \in [0,1]$.

Remark 3. *If we choose $\mu = 1$ in Corollary 1, then the the inequalities (2.9) reduces the inequalities (1.3).*

Corollary 2. *If we choose $\mu = 0$ in Corollary 1, then we the inequalities*

$$\begin{aligned}
&\left|\sum_{i=0}^{n-1} f(\alpha_{i+1})h_i-\int_a^b f(t)w(t)dt\right| \\
&\leq \left[\frac{1}{2}\upsilon(h)+\max_{i\in\{0,1,\dots,n-1\}}\left|\alpha_{i+1}-\frac{x_i+x_{i+1}}{2}\right|\right]\bigvee_a^b(f) \\
&\leq \upsilon(h)\bigvee_a^b(f).
\end{aligned}$$

Remark 4. *If we take $a < \alpha_1 < b$ and $\alpha_1 = x$ in Corollary 2, then we get the inequality (1.2)*

Corollary 3. *Under assumption of Theorem 6 with $\mu = 0$, then we have the inequality*

$$\begin{aligned} &\left|\sum_{i=0}^{n-1}\left(\int_{x_i}^{x_{i+1}} w(u)du\right) f(\alpha_{i+1}) - \int_a^b f(t)w(t)dt\right| \\ \leq\ &\left[\frac{1}{2}\upsilon(L) + \max_{i\in\{0,1,\ldots,n-1\}}\left|\int_{x_i}^{\alpha_{i+1}} w(u)du - \frac{1}{2}\int_{x_i}^{x_{i+1}} w(u)du\right|\right]\bigvee_a^b(f) \end{aligned}$$

where $\mu \in [0,1]$.

Remark 5. *If we take $a < \alpha_1 < b$ and $\alpha_1 = x$ in Corollary 3, then we get the "weighted Ostrowski inequality"*

$$\begin{aligned} &\left|\left(\int_a^b w(u)du\right) f(x) - \int_a^b f(t)w(t)dt\right| \\ \leq\ &\left[\frac{1}{2}\int_a^b w(u)du + \left|\int_a^x w(u)du - \frac{1}{2}\int_a^b w(u)du\right|\right]\bigvee_a^b(f). \end{aligned}$$

Corollary 4. *Under the assumption of Theorem 6. Suppose that $f \in C^1[a,b]$, then we have*

$$\begin{aligned} &\left|(1-\mu)\sum_{i=0}^{n-1}\left(\int_{x_i}^{x_{i+1}} w(u)du\right) f(\alpha_{i+1})\right. \\ &\left. +\mu\sum_{i=0}^{n}\left(\int_{\alpha_i}^{\alpha_{i+1}} w(u)du\right) f(x_i) - \int_a^b f(t)w(t)dt\right| \\ \leq\ &\left[\frac{1}{2} + \left|\frac{1}{2} - \mu\right|\right] \\ &\times\left[\frac{1}{2}\upsilon(L) + \max_{i\in\{0,1,\ldots,n-1\}}\left|\int_{x_i}^{\alpha_{i+1}} w(u)du - \frac{1}{2}\int_{x_i}^{x_{i+1}} w(u)du\right|\right]\|f'\|_1. \end{aligned}$$

Here as subsequently $\|.\|_1$ is the L_1-norm

$$\|f'\|_1 := \int_a^b f'(t)dt.$$

Corollary 5. *Under the assumption of Theorem 6. Let* $f:[a,b]\to\mathbb{R}$ *be a Lipschitzian with the constant* $L>0$. *Then the inequality*

$$\left|(1-\mu)\sum_{i=0}^{n-1}\left(\int_{x_i}^{x_{i+1}}w(u)du\right)f(\alpha_{i+1})\right.$$
$$\left.+\mu\sum_{i=0}^{n}\left(\int_{\alpha_i}^{\alpha_{i+1}}w(u)du\right)f(x_i)-\int_a^b f(t)w(t)dt\right|$$
$$\leq \left[\frac{1}{2}+\left|\frac{1}{2}-\mu\right|\right]$$
$$\times\left[\frac{1}{2}\upsilon(L)+\max_{i\in\{0,1,\ldots,n-1\}}\left|\int_{x_i}^{\alpha_{i+1}}w(u)du-\frac{1}{2}\int_{x_i}^{x_{i+1}}w(u)du\right|\right](b-a)L$$

holds.

Corollary 6. *Under the assumption of Theorem 6. Let* $f:[a,b]\to\mathbb{R}$ *be a monotone mapping on* $[a,b]$. *Then we have*

$$\left|(1-\mu)\sum_{i=0}^{n-1}\left(\int_{x_i}^{x_{i+1}}w(u)du\right)f(\alpha_{i+1})\right.$$
$$\left.+\mu\sum_{i=0}^{n}\left(\int_{\alpha_i}^{\alpha_{i+1}}w(u)du\right)f(x_i)-\int_a^b f(t)w(t)dt\right|$$
$$\leq \left[\frac{1}{2}+\left|\frac{1}{2}-\mu\right|\right]$$
$$\times\left[\frac{1}{2}\upsilon(L)+\max_{i\in\{0,1,\ldots,n-1\}}\left|\int_{x_i}^{\alpha_{i+1}}w(u)du-\frac{1}{2}\int_{x_i}^{x_{i+1}}w(u)du\right|\right]|f(b)-f(a)|.$$

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