

SOME NEW LOCAL FRACTIONAL INTEGRAL INEQUALITIES

MEHMET ZEKI SARIKAYA AND HÜSEYİN BUDAK

ABSTRACT. In this study, several new inequalities of local fractional integrals are presented.

1. Introduction

In 1882, Chebyshev [3] proved the following inequality:

Theorem 1. *Let f and g be two integrable functions in $[0, 1]$. If both functions are simultaneously increasing or decreasing for the same values of x in $[0, 1]$, then*

$$(1.1) \quad \int_0^1 f(x)g(x)dx \geq \int_0^1 f(x)dx \int_0^1 g(x)dx$$

If one function is increasing and the other is decreasing for the same values of x in $[0, 1]$, then (1.1) reverses.

Also, author gave the following inequality:

$$(1.2) \quad |T(f, g)| \leq \frac{1}{12}(b - a)^2 \|f'\|_\infty \|g'\|_\infty,$$

where $f, g : [a, b] \rightarrow \mathbb{R}$ are absolutely continuous function, whose first derivatives f' and g' are bounded,

$$(1.3) \quad T(f, g) = \frac{1}{b - a} \int_a^b f(x)g(x)dx - \left(\frac{1}{b - a} \int_a^b f(x)dx \right) \left(\frac{1}{b - a} \int_a^b g(x)dx \right)$$

and $\|\cdot\|_\infty$ denotes the norm in $L_\infty[a, b]$ defined as $\|p\|_\infty = \text{ess sup}_{t \in [a, b]} |p(t)|$. In the last

years, many papers were devoted to the generalization of the inequalities (1.1) and (1.2), we can mention the works [1],[2],[5]-[8],[11]-[13],[19].

The purpose of this paper is to obtain some local fractional integral inequalities similar to inequality (1.1). This paper is divided into the following three sections. In Section 2, we give the definitions of the local fractional derivatives and local fractional integral and introduce several useful notations on fractal space used our main results. In Section 3, the main result is presented.

2000 *Mathematics Subject Classification.* 26D07, 26D10, 26D15, 26A33.
Key words and phrases. Chebyshev inequality, local fractional integrals.

2. PRELIMINARIES

Recall the set R^α of real line numbers and use the Gao-Yang-Kang's idea to describe the definition of the local fractional derivative and local fractional integral, see [20, 21] and so on.

Recently, the theory of Yang's fractional sets [20] was introduced as follows.

For $0 < \alpha \leq 1$, we have the following α -type set of element sets:

Z^α : The α -type set of integer is defined as the set $\{0^\alpha, \pm 1^\alpha, \pm 2^\alpha, \dots, \pm n^\alpha, \dots\}$.

Q^α : The α -type set of the rational numbers is defined as the set $\{m^\alpha = \left(\frac{p}{q}\right)^\alpha : p, q \in Z, q \neq 0\}$.

J^α : The α -type set of the irrational numbers is defined as the set $\{m^\alpha \neq \left(\frac{p}{q}\right)^\alpha : p, q \in Z, q \neq 0\}$.

R^α : The α -type set of the real line numbers is defined as the set $R^\alpha = Q^\alpha \cup J^\alpha$.

If a^α, b^α and c^α belongs the set R^α of real line numbers, then

- (1) $a^\alpha + b^\alpha$ and $a^\alpha b^\alpha$ belongs the set R^α ;
- (2) $a^\alpha + b^\alpha = b^\alpha + a^\alpha = (a + b)^\alpha = (b + a)^\alpha$;
- (3) $a^\alpha + (b^\alpha + c^\alpha) = (a + b)^\alpha + c^\alpha$;
- (4) $a^\alpha b^\alpha = b^\alpha a^\alpha = (ab)^\alpha = (ba)^\alpha$;
- (5) $a^\alpha (b^\alpha c^\alpha) = (a^\alpha b^\alpha) c^\alpha$;
- (6) $a^\alpha (b^\alpha + c^\alpha) = a^\alpha b^\alpha + a^\alpha c^\alpha$;
- (7) $a^\alpha + 0^\alpha = 0^\alpha + a^\alpha = a^\alpha$ and $a^\alpha 1^\alpha = 1^\alpha a^\alpha = a^\alpha$.

The definition of the local fractional derivative and local fractional integral can be given as follows.

Definition 1. [20] *A non-differentiable function $f : R \rightarrow R^\alpha, x \rightarrow f(x)$ is called to be local fractional continuous at x_0 , if for any $\varepsilon > 0$, there exists $\delta > 0$, such that*

$$|f(x) - f(x_0)| < \varepsilon^\alpha$$

holds for $|x - x_0| < \delta$, where $\varepsilon, \delta \in R$. If $f(x)$ is local continuous on the interval (a, b) , we denote $f(x) \in C_\alpha(a, b)$.

Definition 2. [20] *The local fractional derivative of $f(x)$ of order α at $x = x_0$ is defined by*

$$f^{(\alpha)}(x_0) = \left. \frac{d^\alpha f(x)}{dx^\alpha} \right|_{x=x_0} = \lim_{x \rightarrow x_0} \frac{\Delta^\alpha (f(x) - f(x_0))}{(x - x_0)^\alpha},$$

where $\Delta^\alpha (f(x) - f(x_0)) \cong \Gamma(\alpha + 1) (f(x) - f(x_0))$.

If there exists $f^{(k+1)\alpha}(x) = \overbrace{D_x^\alpha \dots D_x^\alpha}^{k+1 \text{ times}} f(x)$ for any $x \in I \subseteq R$, then we denoted $f \in D_{(k+1)\alpha}(I)$, where $k = 0, 1, 2, \dots$

Definition 3. [20] *Let $f(x) \in C_\alpha[a, b]$. Then the local fractional integral is defined by,*

$${}_a I_b^\alpha f(x) = \frac{1}{\Gamma(\alpha + 1)} \int_a^b f(t) (dt)^\alpha = \frac{1}{\Gamma(\alpha + 1)} \lim_{\Delta t \rightarrow 0} \sum_{j=0}^{N-1} f(t_j) (\Delta t_j)^\alpha,$$

with $\Delta t_j = t_{j+1} - t_j$ and $\Delta t = \max\{\Delta t_1, \Delta t_2, \dots, \Delta t_{N-1}\}$, where $[t_j, t_{j+1}]$, $j = 0, \dots, N - 1$ and $a = t_0 < t_1 < \dots < t_{N-1} < t_N = b$ is partition of interval $[a, b]$.

Here, it follows that ${}_a I_b^\alpha f(x) = 0$ if $a = b$ and ${}_a I_b^\alpha f(x) = -{}_b I_a^\alpha f(x)$ if $a < b$. If for any $x \in [a, b]$, there exists ${}_a I_x^\alpha f(x)$, then we denoted by $f(x) \in I_x^\alpha[a, b]$.

Lemma 1. [20]

(1) (Local fractional integration is anti-differentiation) Suppose that $f(x) = g^{(\alpha)}(x) \in C_\alpha[a, b]$, then we have

$${}_a I_b^\alpha f(x) = g(b) - g(a).$$

(2) (Local fractional integration by parts) Suppose that $f(x), g(x) \in D_\alpha[a, b]$ and $f^{(\alpha)}(x), g^{(\alpha)}(x) \in C_\alpha[a, b]$, then we have

$${}_a I_b^\alpha f(x)g^{(\alpha)}(x) = f(x)g(x)|_a^b - {}_a I_b^\alpha f^{(\alpha)}(x)g(x).$$

Lemma 2. [20] We have

$$\begin{aligned} i) \quad \frac{d^\alpha x^{k\alpha}}{dx^\alpha} &= \frac{\Gamma(1+k\alpha)}{\Gamma(1+(k-1)\alpha)} x^{(k-1)\alpha}; \\ ii) \quad \frac{1}{\Gamma(\alpha+1)} \int_a^b x^{k\alpha} (dx)^\alpha &= \frac{\Gamma(1+k\alpha)}{\Gamma(1+(k+1)\alpha)} (b^{(k+1)\alpha} - a^{(k+1)\alpha}), \quad k \in \mathbb{R}. \end{aligned}$$

For more information and recent developments on local fractional theory, please refer to [4],[9],[10],[14]-[18],[20]-[24].

3. Main Results

Theorem 2. Let $f, g : [a, b] \subseteq \mathbb{R} \rightarrow \mathbb{R}^\alpha$ be two synchronous mapping such that $f, g \in I_x^\alpha[a, b]$ and let $h : [a, b] \rightarrow \mathbb{R}^\alpha$ be non-negative such that $h \in I_x^\alpha[a, b]$. Then, we have the following inequality for local fractional integrals

$$\begin{aligned} (3.1) \quad & \frac{(b-a)^\alpha}{\Gamma(1+\alpha)} {}_a I_b^\alpha (fgh)(x) + [{}_a I_b^\alpha (fg)(x)] [{}_a I_b^\alpha h(x)] \\ & \geq [{}_a I_b^\alpha (fh)(x)] [{}_a I_b^\alpha g(x)] + [{}_a I_b^\alpha (f)(x)] [{}_a I_b^\alpha (gh)(x)]. \end{aligned}$$

Proof. Since f and g are synchronous functions on $[a, b]$, for any $x, y \in [a, b]$, we have

$$(3.2) \quad [f(x) - f(y)] [g(x) - g(y)] [h(x) + h(y)] \geq 0$$

and from (3.2) we get,

$$(3.3) \quad \frac{1}{\Gamma^2(1+\alpha)} \int_a^b \int_a^b [f(x) - f(y)] [g(x) - g(y)] [h(x) + h(y)] (dy)^\alpha (dx)^\alpha \geq 0.$$

On the other hand, by using the local fractional integrals, we have

$$\begin{aligned}
(3.4) \quad & \frac{1}{\Gamma^2(1+\alpha)} \int_a^b \int_a^b [f(x) - f(y)] [g(x) - g(y)] [h(x) + h(y)] (dy)^\alpha (dx)^\alpha \\
&= \frac{1}{\Gamma^2(1+\alpha)} \int_a^b \int_a^b [f(x)g(x)h(x) - f(x)g(y)h(x) - f(y)g(x)h(x) \\
&\quad + f(y)g(y)h(x) + f(x)g(x)h(y) \\
&\quad - f(x)g(y)h(y) - f(y)g(x)h(y) + f(y)g(y)h(y)] (dy)^\alpha (dx)^\alpha \\
&= \frac{2^\alpha (b-a)^\alpha}{\Gamma^2(1+\alpha)} \int_a^b f(x)g(x)h(x) (dx)^\alpha \\
&\quad - 2^\alpha \left(\frac{1}{\Gamma(1+\alpha)} \int_a^b f(x)h(x) (dx)^\alpha \right) \left(\frac{1}{\Gamma(1+\alpha)} \int_a^b g(x) (dx)^\alpha \right) \\
&\quad - 2^\alpha \left(\frac{1}{\Gamma(1+\alpha)} \int_a^b f(x) (dx)^\alpha \right) \left(\frac{1}{\Gamma(1+\alpha)} \int_a^b g(x)h(x) (dx)^\alpha \right) \\
&\quad + 2^\alpha \left(\frac{1}{\Gamma(1+\alpha)} \int_a^b f(x)g(x) (dx)^\alpha \right) \left(\frac{1}{\Gamma(1+\alpha)} \int_a^b h(x) (dx)^\alpha \right) \\
&= \frac{2^\alpha (b-a)^\alpha}{\Gamma(1+\alpha)} {}_a I_b^\alpha (fgh) (x) + 2^\alpha [{}_a I_b^\alpha (fg) (x)] [{}_a I_b^\alpha h(x)] \\
&\quad - 2^\alpha [{}_a I_b^\alpha (fh) (x)] [{}_a I_b^\alpha g(x)] - 2^\alpha [{}_a I_b^\alpha f(x)] [{}_a I_b^\alpha (gh) (x)].
\end{aligned}$$

That is, from (3.3) and (3.4), we obtain

$$\begin{aligned}
& \frac{(b-a)^\alpha}{\Gamma(1+\alpha)} {}_a I_b^\alpha (fgh) (x) + [{}_a I_b^\alpha (fg) (x)] [{}_a I_b^\alpha h(x)] \\
& - [{}_a I_b^\alpha (fh) (x)] [{}_a I_b^\alpha (h) (x)] - [{}_a I_b^\alpha (f) (x)] [{}_a I_b^\alpha (gh) (x)] \geq 0
\end{aligned}$$

which completes the proof. \square

Corollary 1. *Under assumption of Theorem 2 with $h(x) \equiv 1^\alpha$, then we have*

$$(3.5) \quad \frac{(b-a)^\alpha}{\Gamma(1+\alpha)} {}_a I_b^\alpha (fg) (x) \geq [{}_a I_b^\alpha f(x)] [{}_a I_b^\alpha g(x)].$$

Theorem 3. *Let $f, g : [a, b] \subseteq \mathbb{R} \rightarrow \mathbb{R}^\alpha$ be a mapping such that $f, g \in C_\alpha [a, b]$. Then, we have the following inequality for local fractional integrals*

$$(3.6) \quad \frac{(b-a)^\alpha}{\Gamma(1+\alpha)} [{}_a I_b^\alpha (f^2) (x) + {}_a I_b^\alpha (g^2) (x)] \geq 2^\alpha [{}_a I_b^\alpha f(x)] [{}_a I_b^\alpha g(x)].$$

Proof. Since

$$[f(x) - g(y)]^2 \geq 0$$

then, we have

$$(3.7) \quad \frac{1}{\Gamma^2(1+\alpha)} \int_a^b \int_a^b [f(x) - g(y)]^2 (dy)^\alpha (dx)^\alpha \geq 0.$$

Also, we get

$$(3.8) \quad \begin{aligned} & \frac{1}{\Gamma^2(1+\alpha)} \int_a^b \int_a^b [f(x) - g(y)]^2 (dy)^\alpha (dx)^\alpha \\ &= \frac{1}{\Gamma^2(1+\alpha)} \int_a^b \int_a^b [f^2(x) - 2^\alpha f(x)g(y) + g^2(y)] (dy)^\alpha (dx)^\alpha \\ &= \frac{(b-a)^\alpha}{\Gamma(1+\alpha)} [{}_aI_b^\alpha (f^2)(x) + {}_aI_b^\alpha (g^2)(x)] - [{}_aI_b^\alpha f(x)] [{}_aI_b^\alpha g(x)]. \end{aligned}$$

If we combine (3.7) and (3.8), then we obtain the required result. \square

Corollary 2. *Under assumption of Theorem 3 with $f(x) \equiv g(x)$ for all $x \in [a, b]$, then we have*

$$(3.9) \quad \frac{(b-a)^\alpha}{\Gamma(1+\alpha)} {}_aI_b^\alpha f^2(x) \geq [{}_aI_b^\alpha f(x)]^2.$$

Theorem 4. *Under assumption of Theorem 3, we have the following inequality*

$$(3.10) \quad [{}_aI_b^\alpha f^2(x)] [{}_aI_b^\alpha g^2(x)] \geq [{}_aI_b^\alpha fg(x)]^2.$$

Proof. Since

$$[f(x)g(y) - f(y)g(x)]^2 \geq 0$$

then, we have

$$(3.11) \quad \frac{1}{\Gamma^2(1+\alpha)} \int_a^b \int_a^b [f(x)g(y) - f(y)g(x)]^2 (dy)^\alpha (dx)^\alpha \geq 0.$$

Moreover, we have

$$\begin{aligned} & \frac{1}{\Gamma^2(1+\alpha)} \int_a^b \int_a^b [f(x)g(y) - f(y)g(x)]^2 (dy)^\alpha (dx)^\alpha \\ &= \frac{1}{\Gamma^2(1+\alpha)} \int_a^b \int_a^b [f^2(x)g^2(y) - 2^\alpha f(x)g(y)f(y)g(x) + f^2(y)g^2(x)] (dy)^\alpha (dx)^\alpha \\ &= 2^\alpha [{}_aI_b^\alpha f^2(x)] [{}_aI_b^\alpha g^2(x)] - 2^\alpha [{}_aI_b^\alpha fg(x)]^2. \end{aligned}$$

This completes the proof. \square

Remark 1. *If we choose $g(x) \equiv 1$ for all $x \in [a, b]$, then the inequality (3.10) reduces (3.9).*

Theorem 5. Let $f : [a, b] \subseteq \mathbb{R} \rightarrow \mathbb{R}^\alpha$ be a mapping such that $f \in D_\alpha(a, b)$ and $f^{(\alpha)} \in C_\alpha[a, b]$. Then, we have the following inequality

$$\left| f(b) - \frac{\Gamma(1+\alpha)}{(b-a)^\alpha} {}_a J_b^\alpha f(x) \right| \leq \frac{\Gamma(1+\alpha)}{\Gamma(1+2\alpha)} \|f^{(\alpha)}\|_\infty (b-a)^\alpha$$

where $\|f^{(\alpha)}\|_\infty$ is defined by

$$\|f^{(\alpha)}\|_\infty = \sup_{t \in [a, b]} |f^{(\alpha)}(t)|.$$

Proof. From the hypothesis, we have the following identity

$$(3.12) \quad f(b) - f(x) = \frac{1}{\Gamma(1+\alpha)} \int_x^b f^{(\alpha)}(t) (dt)^\alpha.$$

Taking the modulus in (3.12), for all $x \in [a, b]$, we have

$$\begin{aligned} |f(b) - f(x)| &\leq \frac{1}{\Gamma(1+\alpha)} \int_x^b |f^{(\alpha)}(t)| (dt)^\alpha \\ &\leq \frac{\|f^{(\alpha)}\|_\infty}{\Gamma(1+\alpha)} (b-x)^\alpha. \end{aligned}$$

Therefore,

$$\begin{aligned} &\left| \frac{1}{(b-a)^\alpha \Gamma(1+\alpha)} \int_a^b f(x) (dx)^\alpha - \frac{f(b)}{\Gamma(1+\alpha)} \right| \\ &= \frac{1}{(b-a)^\alpha \Gamma(1+\alpha)} \int_a^b [f(x) - f(b)] (dx)^\alpha \\ &\leq \frac{1}{(b-a)^\alpha \Gamma(1+\alpha)} \int_a^b |f(b) - f(x)| (dx)^\alpha \\ &\leq \frac{\|f^{(\alpha)}\|_\infty}{(b-a)^\alpha \Gamma^2(1+\alpha)} \int_a^b (b-x)^\alpha (dx)^\alpha \\ &= \frac{\|f^{(\alpha)}\|_\infty}{\Gamma(1+2\alpha)} (b-a)^{2\alpha}. \end{aligned}$$

This completes the proof. \square

Theorem 6. Let $f, g : [a, b] \subseteq \mathbb{R} \rightarrow \mathbb{R}^\alpha$ be two functions such that $f, g \in D_\alpha(a, b)$ and $f^{(\alpha)}, g^{(\alpha)} \in C_\alpha[a, b]$. Then, we have the following inequality

$$\begin{aligned} & |f(b) {}_a I_b^\alpha g(x) + g(b) {}_a I_b^\alpha f(x) - 2^\alpha {}_a I_b^\alpha (fg)(x)| \\ & \leq \frac{1}{\Gamma^2(1+\alpha)} \int_a^b \left[\|f^{(\alpha)}\|_\infty |g(x)| + \|g^{(\alpha)}\|_\infty |f(x)| \right] (b-x)^\alpha (dx)^\alpha. \end{aligned}$$

Proof. From the hypothesis, we have the following identities

$$(3.13) \quad f(b) - f(x) = \frac{1}{\Gamma(1+\alpha)} \int_x^b f^{(\alpha)}(t) (dt)^\alpha$$

and

$$(3.14) \quad g(b) - g(x) = \frac{1}{\Gamma(1+\alpha)} \int_x^b g^{(\alpha)}(t) (dt)^\alpha.$$

Multiplying both sides of (3.13) and (3.14) by $g(x)$ and $f(x)$ respectively and adding the resulting identities, we have

$$\begin{aligned} (3.15) \quad & f(b)g(x) + g(b)f(x) - 2^\alpha f(x)g(x) \\ & = \frac{g(x)}{\Gamma(1+\alpha)} \int_x^b f^{(\alpha)}(t) (dt)^\alpha + \frac{f(x)}{\Gamma(1+\alpha)} \int_x^b g^{(\alpha)}(t) (dt)^\alpha. \end{aligned}$$

Integrating the both sides of equality (3.15) with respect to x over $[a, b]$ and using the properties of modulus, we get

$$\begin{aligned} & |f(b) {}_a I_b^\alpha g(x) + g(b) {}_a I_b^\alpha f(x) - 2^\alpha {}_a I_b^\alpha (fg)(x)| \\ & \leq \frac{1}{\Gamma^2(1+\alpha)} \int_a^b \left[|g(x)| \int_x^b |f^{(\alpha)}(t)| (dt)^\alpha + |f(x)| \int_x^b |g^{(\alpha)}(t)| (dt)^\alpha \right] (dx)^\alpha \\ & \leq \frac{1}{\Gamma^2(1+\alpha)} \int_a^b \left[\|f^{(\alpha)}\|_\infty |g(x)| (b-x)^\alpha + \|g^{(\alpha)}\|_\infty |f(x)| (b-x)^\alpha \right] (dx)^\alpha \end{aligned}$$

which completes the proof. \square

Corollary 3. Under assumption of Theorem 6 with $g(x) \equiv 1^\alpha$, we have

$$\left| f(b) - \frac{\Gamma(1+\alpha)}{(b-a)^\alpha} {}_a I_b^\alpha (f)(x) \right| \leq \frac{\Gamma(1+\alpha)}{\Gamma(1+2\alpha)} \|f^{(\alpha)}\|_\infty (b-a)^\alpha.$$

Theorem 7. Under the assumptions of Theorem 6, we have the inequality

$$\begin{aligned} & \left| {}_a I_b^\alpha (fg)(x) - f(b) {}_a I_b^\alpha g(x) - g(b) {}_a I_b^\alpha f(x) - \frac{f(b)g(b)(b-a)^\alpha}{\Gamma(1+\alpha)} \right| \\ & \leq \frac{\|f^{(\alpha)}\|_\infty \|g^{(\alpha)}\|_\infty \Gamma(1+2\alpha)}{\Gamma^2(1+\alpha) \Gamma(1+3\alpha)} (b-a)^{3\alpha}. \end{aligned}$$

Proof. From (3.13) and (3.14), we observe that

$$\begin{aligned} & f(x)g(x) - f(b)g(x) - g(b)f(x) + f(b)g(b) \\ &= \left[\frac{1}{\Gamma(1+\alpha)} \int_x^b f^{(\alpha)}(t) (dt)^\alpha \right] \left[\frac{1}{\Gamma(1+\alpha)} \int_x^b g^{(\alpha)}(t) (dt)^\alpha \right]. \end{aligned}$$

Integrating the both sides of equality (3.15) with respect to x over $[a, b]$ and using the properties of modulus, we get

$$\begin{aligned} & \left| {}_a I_b^\alpha (fg)(x) - f(b) {}_a I_b^\alpha g(x) - g(b) {}_a I_b^\alpha f(x) - \frac{f(b)g(b)(b-a)^\alpha}{\Gamma(1+\alpha)} \right| \\ & \leq \frac{1}{\Gamma(1+\alpha)} \int_a^b \left[\frac{1}{\Gamma(1+\alpha)} \int_x^b |f^{(\alpha)}(t)| (dt)^\alpha \right] \left[\frac{1}{\Gamma(1+\alpha)} \int_x^b |g^{(\alpha)}(t)| (dt)^\alpha \right] (dx)^\alpha \\ & \leq \frac{\|f^{(\alpha)}\|_\infty \|g^{(\alpha)}\|_\infty}{\Gamma^3(1+\alpha)} \int_a^b (b-x)^{2\alpha} (dx)^\alpha \\ & = \frac{\|f^{(\alpha)}\|_\infty \|g^{(\alpha)}\|_\infty}{\Gamma^2(1+\alpha)} \frac{\Gamma(1+2\alpha)}{\Gamma(1+3\alpha)} (b-a)^{3\alpha}. \end{aligned}$$

The proof is completed. \square

REFERENCES

- [1] S. Belarbi and Z. Dahmani, *On Some new fractional integral inequalities*, J. Inequal. Pure and Appl. Math., 10(3), Art. 86, 2009.
- [2] K. Boukerrioua and A.G. Lakoud, *On generalization of Čebyšev type inequalities*, J. Inequal. Pure and Appl. Math. 8(2), Art 55, 2007.
- [3] P. L. Čebyšev, *Sur less expressions approximatives des integrales definies par les autres prises entre les memes limites*, Proc. Math. Soc. Charkov, 2, 93-98, 1882.
- [4] G-S. Chen, *Generalizations of Hölder's and some related integral inequalities on fractal space*, Journal of Function Spaces and Applications Volume 2013, Article ID 198405.
- [5] A. G. Lakoud and F. Aissaouineu, *Čebyšev type inequalities for double integrals*, Journal Math. Inequal. Volume 5, Number 4 (2011), 453-462.
- [6] S. Hussain and M. W. Alomari, *Weighted Osrowski and Čebyšev type inequalities with applications*, Konuralp Journal of Mathematics, Volume 1 No. 2 pp. 1-16 (2013).
- [7] S. M. Malamud, *Some complements to the Jensen and Chebyshev inequalities and a problem of W. Walter*, Proc. Amer. Math. Soc., 129 (9)(2001), 2671-2678.
- [8] D. S. Mitrinovic, J. E. Pecaric and A. M. Fink, *Inequalities involving functions and their integrals and derivatives*, Kluwer Academic Publishers, Dordrecht, 1991.
- [9] H. Mo, X. Sui and D. Yu, *Generalized convex functions on fractal sets and two related inequalities*, Abstract and Applied Analysis, Volume 2014, Article ID 636751, 7 pages.
- [10] H. Mo, *Generalized Hermite-Hadamard inequalities involving local fractional integral*, arXiv:1410.1062.
- [11] B. G. Pachpatte, *On Čebyšev-Grüss type inequalities via Pecaric's extention of the Montgomery identity*, J. Inequal. Pure and Appl. Math. 7(1), Art 108, 2006.
- [12] J. E. Pecaric, *On the Čebyšev inequality*, Bul. Sti. Tehn. Inst. Politehn "Tralan Vuia" Timișora(Romania), 25(39)(1), 5-9, 1980.
- [13] M. Z. Sarikaya, N. Aktan, H. Yildırım, *Weighted Čebyšev-Grüss type inequalities on time scales*, J. Math. Inequal. 2, 2 (2008), 185-195.
- [14] M. Z. Sarikaya and H Budak, *Generalized Ostrowski type inequalities for local fractional integrals*, RGMIA Research Report Collection, 18(2015), Article 62, 11 pp.

- [15] M. Z. Sarikaya, S.Erden and H. Budak, *Some generalized Ostrowski type inequalities involving local fractional integrals and applications*, RGMIA Research Report Collection, 18(2015), Article 63, 12 pp.
- [16] M. Z. Sarikaya H. Budak, *On generalized Hermite-Hadamard inequality for generalized convex function*, RGMIA Research Report Collection, 18(2015), Article 64, 15 pp.
- [17] M. Z. Sarikaya, S.Erden and H. Budak, *Some integral inequalities for local fractional integrals*, RGMIA Research Report Collection, 18(2015), Article 65, 12 pp.
- [18] M. Z. Sarikaya, H. Budak and S.Erden, *On new inequalities of Simpson's type for generalized convex functions*, RGMIA Research Report Collection, 18(2015), Article 66, 13 pp.
- [19] W. T. Sulaiman, *Some new fractional integral inequalities*, Journal of Mathematical Analysis, 2(2), 2011, 23-28.
- [20] X. J. Yang, *Advanced Local Fractional Calculus and Its Applications*, World Science Publisher, New York, 2012.
- [21] J. Yang, D. Baleanu and X. J. Yang, *Analysis of fractal wave equations by local fractional Fourier series method*, Adv. Math. Phys. , 2013 (2013), Article ID 632309.
- [22] X. J. Yang, *Local fractional integral equations and their applications*, Advances in Computer Science and its Applications (ACSA) 1(4), 2012.
- [23] X. J. Yang, *Generalized local fractional Taylor's formula with local fractional derivative*, Journal of Expert Systems, 1(1) (2012) 26-30.
- [24] X. J. Yang, *Local fractional Fourier analysis*, Advances in Mechanical Engineering and its Applications 1(1), 2012 12-16.

DEPARTMENT OF MATHEMATICS, FACULTY OF SCIENCE AND ARTS, DÜZCE UNIVERSITY, DÜZCE-TURKEY

E-mail address: sarikayamz@gmail.com

E-mail address: hsyn.budak@gmail.com