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Selfie Palindromic Magic Squares

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Abstract

This work deals with Selfie palindromic magic squares. By Selfie magic squares, it is understood that the magic squares are upside down, mirror looking and having the same magic sums. Results are obtained for the orders 3×3, 4×4, 5×5, 8×8 and 9×9. In case of 8×8, the results are semi-bimagic and for 9×9 the results are bimagic. For order 7×7, we have upside down palindromic magic square. For orders 6×6, 7×7 and 10×10 still we don't have Selfie magic squares. 3-digit palindromic magic squares are given for the orders 3×3 to 10×10.

1. Definitions

1.1. Palindromic Magic Squares

Magic squares made through palindromic numbers are known as *palindromic magic squares.*

1.2. Upside down Magic Squares

When we make a rotation of 180 degrees in a magic square (180^0) and still remain a magic square, we call it *upside down magic square* or *rotatable magic square*.

Rotatable magic squares are different from reversible magic squares. In reversible magic square we just reverse the order of digits, such as, 13 by 31, 28 by 82, etc.

Out of 10 digits available, we have only 7 rotatable digits. i.e., **0, 1, 2, 5, 6, 8** and **9**. Digits **0, 1** and **8** are always rotatable. After making rotation of 180^0, **6** becomes as **9** and **9** as **6**. The numbers 2 and 5 are not rotatable, but when we write them in digital form, like 2 and 5, they becomes rotatable.

1.3. Selfie Magic Squares

Selfie Magic Squares *are magic squares that remain the same after making the rotation of 180^0 and looking through the mirror. It is not necessary that magic squares always satisfy both the conditions. Selfie magic squares are also known by Universal magic squares.*

More precisely, a *Selfie magic square* should have the following three properties:

(i) Rotatable to 180^0 (upside-down magic square);
(ii) Mirror looking;
(iii) Having the same magic sum (in all the four situations, i.e., magic sum of *original, rotated to 180^0, mirror looking, and rotation of 180^0 of mirror looking magic squares*).

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We observed that the numbers 6 and 9 are not *Selfie numbers*. In case of mirror looking 2 becomes as 5, and 5 as 2. Thus we have only five Selfie numbers, i.e., 0, 1, 2, 5 and 8.

Palindromic letters written in a square is understood as *palindromic grid*.

Here below are examples of *Selfie palindromic magic squares* of different orders. Notations for *magic* and *bimagic sums* are S and Sb respectively. This work is a part of author's [1] work in preparation.

2. 3-Digit Palindromic Magic Squares of Order 3x3

Let us consider the following *palindromic grid* of order 3x3 only with three letters *a, b* and *c*:

bab	*ccc*	*aba*
aca	*bbb*	*cac*
cbc	*aaa*	*bcb*

… (1)

For all $a,b,c \in \{0,1,2,3,4,5,6,7,8,9\}$, the grid (1) represent a *palindromic square*. If $b=(a+c)/2$, then it is always a *palindromic magic square*, otherwise it may or may be a magic square. Sometimes it becomes *semi-magic square*. The sum is given by

$$S_{3\times3}(a,b,c)=(a+b+c)\times111 \text{ or } \frac{S_{3\times3}(a,b,c)}{a+b+c}=111.$$

Here below are some interesting examples:

Example 1. For $a=1, b=2$ and $c=3$ in (1), we have regular 3-digit *palindromic magic square* of order 3×3 with sum $S_{3\times3}(1,2,3)=1110=(1+2+3)\times111=666$:

212	333	121
131	222	313
323	111	232

Example 2. For $a=2, b=5$ and $c=8$ in (1), we have regular 3-digit *palindromic Selfie magic square* of order 3×3 with sum $S_{3\times3}(2,5,8)=(2+5+8)\times111=1665$:

525	888	252
282	555	828
858	222	585

Example 3. For $a=1, b=6$ and $c=9$ in (1), we have 3-digit *palindromic semi-magic square* of order 3×3:

616	999	161
191	666	919
969	111	696

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The semi-magic sum is $S_{3\times3}(1,6,9) = (1+6+9)\times111 = 1776$ (sums of rows and columns) and the diagonals sums are 1796 and 1978 respectively.

3. Palindromic Magic Square of Order 4x4

3.1. 3-Digit Palindromic Magic Squares of Order 4x4

Let us consider the following *palindromic grid* of order 4×4 only with three letters *a, b* and *c*:

bcb	*cdc*	*aaa*	*dbd*
aba	*dad*	*bdb*	*ccc*
ddd	*aca*	*cbc*	*bab*
cac	*bbb*	*dcd*	*ada*

… (2)

For all $a,b,c,d \in \{0,1,2,3,4,5,6,7,8,9\}$, the above grid represent a *palindromic magic square of order* 4×4. Its sum is

$$S_{4\times4}(a,b,c,d) = (a+b+c+d)\times111 \text{ or } \frac{S_{4\times4}(a,b,c,d)}{a+b+c+d} = 111.$$

Here below are some interesting examples:

Example 4. For $a=1, b=2, c=3$ and $d=4$ in (2), we have 3-digit *palindromic magic square* of order 4×4 with sum $S_{4\times4}(1,2,3,4) = (1+2+3+4)\times111 = 1110$:

232	343	111	424
121	414	242	333
444	131	323	212
313	222	434	141

Example 5. For $a=1, b=2, c=5$ and $d=8$ in (2), we have 3-digit *palindromic Selfie magic square* of order 4×4 with sum $S_{4\times4}(1,2,5,8) = (1+2+5+8)\times111 = 1776 = 888+888$:

252	585	111	828
121	818	282	555
888	151	525	212
515	222	858	181

Example 6. For $a=1, b=6, c=8$ and $d=9$ in (2), we have 3-digit *palindromic upside down magic square* of order 4×4 with sum $S_{4\times4}(1,6,8,9) = (1+6+8+9)\times111 = 2664 = 3\times888$:

686	898	111	969
161	919	696	888
999	181	868	616
818	666	989	191

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3.2. 7-Digit Palindromic Pan Diagonal Magic Squares of Order 4x4

Let us consider the following *palindromic grid* of order 4x4 with two letters *a* and *b*:

abbabba	*babbbab*	*aaaaaaa*	*bbababb*
aaabaaa	*bbaaabb*	*abbbbba*	*bababab*
bbbbbbb	*aababaa*	*baabaab*	*abaaaba*
baaaaab	*abababa*	*bbbabbb*	*aabbbaa*

… (3)

For all $a, b \in \{0,1,2,3,4,5,6,7,8,9\}$, above grid represent a *palindromic pan diagonal magic square of order* 4×4 with sum

$$S_{4\times4}(a,b) = 2 \times (aaaaaaa + bbbbbbb) = (a+b) \times 2222222 \text{ or } \frac{S_{4\times4}(a,b)}{a+b} = 2222222.$$

Here below are some examples:

Example 7. For $a = 2$ and $b = 5$ in (3), we have 7-digit *palindromic pan diagonal Selfie magic square* of order 4×4 with sum $S_{4\times4}(2,5) = (2+5) \times 2222222 = 15555554$:

2552552	5255525	2222222	5525255
2225222	5522255	2555552	5252525
5555555	2252522	5225225	2522252
5222225	2525252	5552555	2255522

Example 8. For $a = 1$ and $b = 8$ in (3), we have 7-digit *palindromic pan diagonal Selfie magic square* of order 4×4 with sum $S_{4\times4}(1,8) = (1+8) \times 2222222 = 19999998$:

1881881	8188818	1111111	8818188
1118111	8811188	1888881	8181818
8888888	1181811	8118118	1811181
8111118	1818181	8881888	1188811

Example 9. For $a = 6$ and $b = 9$, we have 7-digit *palindromic pan diagonal upside down magic square* of order 4×4 with sum $S_{4\times4}(6,9) = (6+9) \times 2222222 = 33333330$.

6996996	9699969	6666666	9969699
6669666	9966699	6999996	9696969
9999999	6696966	9669669	6966696
9666669	6969696	9996999	6699966

Above three examples give us following symmetry:

$$\frac{S_{4\times4}(2,5)}{2+5} = \frac{S_{4\times4}(1,8)}{1+8} = \frac{S_{4\times4}(6,9)}{6+9} = 2222222.$$

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4. Palindromic Magic Square of Order 5x5

Let us consider the following *palindromic grid* of order 4×4 five letters *a, b, c, d* and *e*:

aaa	*bbb*	*ccc*	*ddd*	*eee*
dcd	*ede*	*aea*	*bab*	*cbc*
beb	*cac*	*dbd*	*ece*	*ada*
ebe	*aca*	*bdb*	*cec*	*dad*
cdc	*ded*	*eae*	*aba*	*bcb*

… (4)

For all $a, b, c, d, e \in \{0,1,2,3,4,5,6,7,8,9\}$, above grid represent a *pan diagonal palindromic magic square of order* 5×5. Its sum is

$$S_{5\times5}(a,b,c,d,e) = (a+b+c+d+e) \times 111 \text{ or } \frac{S_{4\times4}(a,b,c,d,e)}{a+b+c+d+e} = 111.$$

Here below are some examples:

Example 10. For $a=1, b=2, c=3, d=4$ and $e=5$ in (4), we have 3-digit *palindromic pan diagonal magic square* of order 5×5 with sum $S_{5\times5}(1,2,3,4,5) = (1+2+3+4+5) \times 111 = 1665$:

111	222	333	444	555
434	545	151	212	323
252	313	424	535	141
525	131	242	353	414
343	454	515	121	232

Example 11. For $a=0, b=1, c=2, d=5$ and $e=8$ in (4), we have 3-digit *palindromic pan diagonal Selfie magic square* of order 5×5 with sum $S_{5\times5}(0,1,2,5,8) = (0+1+2+5+8) \times 111 = 1776 = 888 + 888$:

000	111	222	555	888
525	858	080	101	212
181	202	515	828	050
818	020	151	282	505
252	585	808	010	121

Example 12. For $a=0, b=1, c=6, d=8$ and $e=9$ in (4), we have 3-digit *palindromic pan diagonal upside down magic square* of order 5×5 with sum $S_{5\times5}(2,5,6,8,9) = (2+5+6+8+9) \times 111 = 3330$:

222	555	666	888	999
868	989	292	525	656
595	626	858	969	282
959	262	585	696	828
686	898	929	252	565

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5. Palindromic Magic Square of Order 6x6

Let us consider the following *palindromic grid* of order 6×6 only with six letters *a, b, c, d, e* and *f*:

beb	*aba*	*dcd*	*ede*	*fbf*	*cec*
aaa	*ccc*	*fef*	*ded*	*eae*	*bfb*
cac	*fcf*	*ece*	*bbb*	*dfd*	*afa*
dbd	*bdb*	*aca*	*fdf*	*cfc*	*ebe*
efe	*ddd*	*bcb*	*cbc*	*aea*	*faf*
fff	*eee*	*cdc*	*ada*	*bab*	*dad*

… (5)

For all $a,b,c,d,e\in\{0,1,2,3,4,5,6,7,8,9\}$. In this case we don't have many options. See the example below.

Example 13. For $a=1, b=2, c=3, d=4, e=5$ and $f=6$ in (5), we have 3-digit *palindromic magic square* of order 6×6 with sum $S_{6\times6}(1,2,3,4,5,6)=(1+2+3+4+5+6)\times111=2331$:

252	121	434	545	626	353
111	333	656	454	515	262
313	636	535	222	464	161
424	242	131	646	363	525
565	444	232	323	151	616
666	555	343	141	212	414

6. Palindromic Magic Square of Order 7x7

Let us consider the following 3-digit *palindromic grid* of order 7×7 with seven letters *a, b, c, d, e, f* and *g*:

aaa	*bbb*	*ccc*	*ddd*	*eee*	*fff*	*ggg*
fef	*gfg*	*aga*	*bab*	*cbc*	*dcd*	*ede*
dbd	*ece*	*fdf*	*geg*	*afa*	*bgb*	*cac*
bfb	*cgc*	*dad*	*ebe*	*fcf*	*gdg*	*aea*
gcg	*ada*	*beb*	*cfc*	*dgd*	*eae*	*fbf*
ege	*faf*	*gbg*	*aca*	*bdb*	*cec*	*dfd*
cdc	*ded*	*efe*	*fgf*	*gag*	*aba*	*bcb*

… (6)

For all $a,b,c,d,e,f,g\in\{0,1,2,3,4,5,6,7,8,9\}$, above grid represent a 3-digit *palindromic pan diagonal magic square of order* 7×7. Its sum is

$$S_{7\times7}(a,b,c,d,e,f,g)=(a+b+c+d+e+f+g)\times111.$$

Here below are some examples:

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Example 14. For $a=1, b=2, c=3, d=4, e=5, f=6$ and $g=7$ in (6), we have 3-digit *palindromic pan diagonal magic square* of order 7×7 with sum $S_{7\times 7}(1,2,3,4,5,6,7)=(1+2+3+4+5+6+7)\times 111=3107$:

111	222	333	444	555	666	777
656	767	171	212	323	434	545
424	535	646	757	161	272	313
262	373	414	525	636	747	151
737	141	252	363	474	515	626
575	616	727	131	242	353	464
343	454	565	676	717	121	232

Example 15. For $a=1, b=2, c=3, d=5, e=6, f=8$ and $g=9$ in (6), we have 3-digit *palindromic pan diagonal magic square* of order 7×7 with sum $S_{7\times 7}(1,2,3,5,6,8,9)=(1+2+3+5+6+8+9)\times 111=3774$:

111	222	333	555	666	888	999
868	989	191	212	323	535	656
525	636	858	969	181	292	313
282	393	515	626	838	959	161
939	151	262	383	595	616	828
696	818	929	131	252	363	585
353	565	686	898	919	121	232

In this case, still we don't have *Selfie palindromic magic square*, but here below is an example of *upside down palindromic magic square* with 0, 1, 2, 5, 6, 8 and 9:

Example 16. For $a=0, b=1, c=2, d=5, e=6, f=8$ and $g=9$ in (6), we have 3-digit *palindromic pan diagonal upside down magic square* of order 7×7 with sum $S_{7\times 7}(0,1,2,5,6,8,9)=(0+1+2+5+6+8+9)\times 111=3441$:

000	111	222	555	666	888	999
868	989	090	101	212	525	656
515	626	858	969	080	191	202
181	292	505	616	828	959	060
929	050	161	282	595	606	818
696	808	919	020	151	262	585
252	565	686	898	909	010	121

In order to make it upside down, we considered palindromic symmetry in numbers: 000, 010, 020, 050, 060, 080 and 090.

7. Palindromic Magic Square of Order 8x8

This section deals with 3 and 7 and 11-digit palindromes. In each case we have different types of palindromic magic squares.

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7.1. 3-Digit Palindromic Magic Squares of Order 8x8

Let us consider the following 3-digit *palindromic grid* of order 8×8 with eight letters *a, b, c, d, e, f, g* and *h*:

bhb	*faf*	*ede*	*aea*	*dcd*	*hfh*	*ggg*	*cbc*
dbd	*hgh*	*gfg*	*ccc*	*beb*	*fdf*	*eae*	*aha*
aaa	*ehe*	*fef*	*bdb*	*cfc*	*gcg*	*hbh*	*dgd*
cgc	*gbg*	*hch*	*dfd*	*ada*	*eee*	*fhf*	*bab*
efe	*aca*	*bbb*	*fgf*	*gag*	*chc*	*ded*	*hdh*
gdg	*cec*	*dhd*	*hah*	*ege*	*aba*	*bcb*	*fff*
fcf	*bfb*	*aga*	*ebe*	*hhh*	*dad*	*cdc*	*geg*
heh	*ddd*	*cac*	*ghg*	*fbf*	*bgb*	*afa*	*ece*

… (7)

For all $a,b,c,d,e,f,g,h \in \{0,1,2,3,4,5,6,7,8,9\}$, above grid represent a *palindromic magic square of order* 8×8. Its sum is

$$S_{8\times8}(a,b,c,d,e,f,g,h) = (a+b+c+d+e+f+g+h)\times111.$$

Here below are some examples:

Example 17. For $a=1, b=2, c=3, d=4, e=5, f=6, g=7$ and $h=8$ in (7), we have regular 3-digit *palindromic pan diagonal magic square* of order 8×8:

282	616	545	151	434	868	777	323
424	878	767	333	252	646	515	181
111	585	656	242	363	737	828	474
373	727	838	464	141	555	686	212
565	131	222	676	717	383	454	848
747	353	484	818	575	121	232	666
636	262	171	525	888	414	343	757
858	444	313	787	626	272	161	535

Above magic square is *bimagic*. Its sums are

$$S_{8\times8}(1,2,3,4,5,6,7,8) = (1+2+3+4+5+6+7+8)\times111 = 3996$$

and

$$Sb_{8\times8}(1,2,3,4,5,6,7,8) = 2428644.$$

We have total 10 digits, i.e., $\{0,1,2,3,4,5,6,7,8,9\}$. Above example considered first eight numbers in a sequence. If we consider, number not in a sequence, still we have a *pan diagonal bimagic square*. See example below:

Example 18. For $a=1, b=2, c=3, d=4, e=6, f=7, g=8$ and $h=9$ in (7), we have 3-digit *palindromic pan diagonal magic square* of order 8×8:

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292	717	646	161	434	979	888	323
424	989	878	333	262	747	616	191
111	696	767	242	373	838	929	484
383	828	939	474	141	666	797	212
676	131	222	787	818	393	464	949
848	363	494	919	686	121	232	777
737	272	181	626	999	414	343	868
969	444	313	898	727	282	171	636

Above magic square is *bimagic*. Its bimagic sums are

$$S_{8\times 8}(1,2,3,4,6,7,8,9) = (1+2+3+4+6+7+8+9)\times 111 = 4440$$

and

$$Sb_{8\times 8}(1,2,3,4,6,7,8,9) = 3082260 .$$

As we seen before, we don't have 8 rotatable numbers. Maximum we have 7, i.e., 0, 1, 2, 5, 6, 8 and 9. In order to bring *Selfie magic square* of order 8×8, we shall try to reduce this number and increase palindromic digits. This is done in the following two subsections.

7.2. Palindromic Semi Magic Square of Order 8x8

Above sub section is with 3-digit palindromes having eight letters. Let us consider the following 5-digit *palindromic grid* of order 8×8 only with four letters *a, b, c* and *d*:

addda	*ccacc*	*cadac*	*ababa*	*bcccb*	*ddbdd*	*dbcbd*	*babab*
bcbcb	*ddcdd*	*dbbbd*	*bacab*	*adada*	*ccdcc*	*caaac*	*abdba*
aaaaa	*cbdbc*	*cdadc*	*acdca*	*bbbbb*	*dacad*	*dcbcd*	*bdcdb*
bbcbb	*dabad*	*dcccd*	*bdbdb*	*aadaa*	*cbabc*	*cdddc*	*acaca*
cbbbc	*aacaa*	*acbca*	*cdcdc*	*daaad*	*bbdbb*	*bdadb*	*dcdcd*
dadad	*bbabb*	*bdddb*	*dcacd*	*cbcbc*	*aabaa*	*accca*	*cdbdc*
ccccc	*adbda*	*abcba*	*cabac*	*ddddd*	*bcacb*	*badab*	*dbabd*
ddadd	*bcdcb*	*baaab*	*dbdbd*	*ccbcc*	*adcda*	*abbba*	*cacac*

… (8)

Above grid is not always a magic square? In some cases it is a semi-magic. Here below are some examples:

Example 19. For $a=1, b=2, c=3$ and $d=4$ in (8), we have 5-digit *palindromic pan diagonal magic square* of order 5×5 with sum $S_{8\times 8}(1,2,3,4) = (1+2+3+4)\times 22222 = 222220$:

14441	33133	31413	12121	23332	44244	42324	21212
23232	44344	42224	21312	14141	33433	31113	12421
11111	32423	34143	13431	22222	41314	43234	24342
22322	41214	43334	24242	11411	32123	34443	13131
32223	11311	13231	34343	41114	22422	24142	43434
41414	22122	24442	43134	32323	11211	13331	34243
33333	14241	12321	31213	44444	23132	21412	42124
44144	23432	21112	42424	33233	14341	12221	31313

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Each block of order 2×8 is also having the same sum as of magic square. It is semi-bimagic. Bimagic sum of rows and columns is $Sb_{8\times8}(1,2,3,4)=7183217060$, and main diagonals sum is 7183377060.

The following example is a *Selfie palindromic semi-magic square*.

Example 20. For $a=1, b=2, c=5$ and $d=8$ in (8), we have 5-digit *palindromic semi-magic square* of order 8×8:

18881	55155	51815	12121	25552	88288	82528	21212
25252	88588	82228	21512	18181	55855	51115	12821
11111	52825	58185	15851	22222	81518	85258	28582
22522	81218	85558	28282	11811	52125	58885	15151
52225	11511	15251	58585	81118	22822	28182	85858
81818	22122	28882	85158	52525	11211	15551	58285
55555	18281	12521	51215	88888	25152	21812	82128
88188	25852	21112	82828	55255	18581	12221	51515

Semi-magic sum is $S_{5\times5}(1,2,5,8)=(1+2+5+8)\times22222=355552$ (sums of rows and columns). The main diagonals sum is 359592.

Examples 19 and 20 are semi-bimagic, while example 18 is bimagic. Moreover, example 20 is *Selfie palindromic semi-magic square*. In order to bring *Selfie palindromic magic square* of order 8×8, we have reduced the numbers and increased the numbers of digits in palindromes. Here below are examples of 11-digit *Selfie palindromic magic square* of order 8×8 just with two letters.

7.3. 11-Digit Palindromic Magic Squares of Order 8x8

Above subsection is with 5-digit palindromes having four letters. Let us consider the following 11-digit *palindromic grid* of order 8×8 only with two letters *a* and *b*:

aabbbbbbbaa	*babaaaaabab*	*baaabbbaaab*	*aaabaaabaaa*	*abbabababba*	*bbbbababbbb*	*bbabbabbabb*	*abaaabaaaba*
abbaabaabba	*bbbbbabbbbb*	*bbababababb*	*abaababaaba*	*aabbaaabbaa*	*bababbbabab*	*baaaaaaaaab*	*aaabbbbbaaa*
aaaaaaaaaaa	*baabbbbbaab*	*babbaaabbab*	*aababbbabaa*	*abababababa*	*bbaababaabb*	*bbbaabaabbb*	*abbbbabbbba*
ababbabbaba	*bbaaabaaabb*	*bbbababab bb*	*abbbababbba*	*aaaabbbaaaa*	*baabaaabaab*	*babbbbbbbab*	*aabaaaaabaa*
baabababaab	*aaaababaaaa*	*aabaabaabaa*	*babbbabbbab*	*bbaaaaaaabb*	*ababbbbbaba*	*abbbaaabbba*	*bbbabbbabbb*
bbaabbbaabb	*ababaaababa*	*abbbbbbbbba*	*bbbaaaaabbb*	*baabbabbaab*	*aaaaabaaaaa*	*aababababaa*	*babbababbab*
bababababab	*aabbababbaa*	*aaabbabbaaa*	*baaaabaaaab*	*bbbbbbbbbbb*	*abbaaaaabba*	*abaabbbaaba*	*bbabaaababb*
bbbbaaabbbb	*abbabbbabba*	*abaaaaaaaba*	*bbabbbbbabb*	*babaabaabab*	*aabbbabbbaa*	*aaabababaaa*	*baaababaaab*

… (9)

For all $a,b\in\{0,1,2,3,4,5,6,7,8,9\}$, above grid represent a *palindromic square* with sum

$$S_{8\times8}(a,b)=4\times(a+b)\times11111111111 \text{ or } \frac{S_{8\times8}(a,b,c)}{a+b}=44444444444.$$

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Here below are some examples:

Example 21. For $a = 2$ and $b = 5$ in (9), we have 11-digit *Selfie palindromic pan diagonal magic square* of order 8×8:

22555555522	52522222525	52225552225	22252225222	25525252552	55552525555	55255255255	25222522252
25522522552	55555255555	55252525255	25225252252	22552225522	52525552525	52222222225	22255555222
22222222222	52255555225	52552225525	22525552522	25252525252	55225252255	55522522555	25555255552
25255255252	55222522255	55525252555	25555255552	22225552222	52252225225	52555555525	22522222522
52252525225	22225252222	22522522522	52555255525	55222222255	25255555252	25552225552	55525552555
55225552255	25252225252	25555555552	55522222555	52255255225	22222522222	22525252522	52552525525
52525252525	22552525522	22255255222	52222522225	55555555555	25522222552	25225552252	55252225255
55552225555	25525552552	25222222252	55255555255	52522522525	22555255522	22252525222	52225252225

It is a magic square but semi-bimagic. Its sums are

$$S_{8\times8}(2,5) = 4 \times (2+5) \times 11111111111 = 311111111108$$

and

$$Sb_{8\times8}(2,5) = 1391694725183860830527 6 \text{ (rows and columns).}$$

Diagonals sum is 13916947255474608305276

The same we can get with 1 and 8 and still is *Selfie magic square*. If we use 6 and 9, it becomes *upside down palindromic magic square*. See example below

Example 22. For $a = 6$ and $b = 9$ in (9), we have 11-digit *Selfie palindromic pan diagonal magic square* of order 8×8:

66999999966	96966666969	96669996669	66696669666	69969696996	99996969999	99699699699	69666966696
69966966996	99999699999	99696969699	69669696696	66996669966	96969996969	96666666669	66699999666
66666666666	96699999669	96996669969	66969996966	69696969696	99669696699	99966966999	69999699996
69699699696	99666966699	99969696999	69996969996	66669996666	96696669669	96999999969	66966666966
96696969669	66669696666	66966966966	96999699969	99666666699	69699999696	69996669996	99969996999
99669996699	69696669696	69999999996	99966666999	96699699669	66666966666	66969696966	96996969969
96969696969	66996969966	66699699666	96666966669	99999999999	69966666996	69669996696	99696669699
99996669999	69969996996	69666666696	99699999699	96966966969	66999699966	66696969666	96669696669

It is a magic square and semi-bimagic. Its sums are

$$S_{8\times8}(2,5) = 4 \times (6+9) \times 11111111111 = 666666666660$$

and

$$Sb_{8\times8}(2,5) = 57373737374426262626268 \text{ (rows and columns).}$$

Diagonals sum is 57373737378062262626268.

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8. Palindromic Bimagic Squares of Order 9x9

8.1. 3-Digit Palindromic Bimagic Squares of Order 9x9

Let us consider the following 3-digit palindromic grid of order 9×9 with nine letters *a, b, c, d, e, f, g, h* and *k*:

aaa	*bkb*	*cec*	*dhd*	*ede*	*fcf*	*gfg*	*hbh*	*kgk*
dfd	*ebe*	*fgf*	*gag*	*hkh*	*kek*	*aha*	*bdb*	*ccc*
ghg	*hdh*	*kck*	*afa*	*bbb*	*cgc*	*dad*	*eke*	*fef*
ckc	*aea*	*bab*	*fdf*	*dcd*	*ehe*	*kbk*	*ggg*	*hfh*
fbf	*dgd*	*efe*	*kkk*	*geg*	*hah*	*cdc*	*aca*	*bhb*
kdk	*gcg*	*hhh*	*cbc*	*aga*	*bfb*	*fkf*	*ded*	*eae*
beb	*cac*	*aka*	*ece*	*fhf*	*ddd*	*hgh*	*kfk*	*gbg*
ege	*fff*	*dbd*	*heh*	*kak*	*gkg*	*bcb*	*chc*	*ada*
hch	*khk*	*gdg*	*bgb*	*cfc*	*aba*	*eee*	*faf*	*dkd*

… (10)

For all $a,b,c,d,e,f,g,h,k \in \{0,1,2,3,4,5,6,7,8,9\}$, above grid represent a *palindromic magic square of order* 9×9. It sum is

$$S_{9\times 9}(a,b,c,d,e,f,g,h,k) = (a+b+c+d+e+f+g+h+k) \times 111.$$

Here below are some interesting examples:

Example 23. For $a=1, b=2, c=3, d=4, e=5, f=6, g=7$, $h=8$ and $k=9$ in (10), we have 3-digit *palindromic magic square* of order 9×9:

111	292	353	484	545	636	767	828	979
464	525	676	717	898	959	181	242	333
787	848	939	161	222	373	414	595	656
393	151	212	646	434	585	929	777	868
626	474	565	999	757	818	343	131	282
949	737	888	323	171	262	696	454	515
252	313	191	535	686	444	878	969	727
575	666	424	858	919	797	232	383	141
838	989	747	272	363	121	555	616	494

It is a palindromic bimagic square. Its sums are

$$S_{9\times 9}(1,2,3,4,5,6,7,8,9) = (1+2+3+4+5+6+7+8+9) \times 111 = 4995$$

and

$$Sb_{9\times 9}(1,2,3,4,5,6,7,8,9) = 3390285.$$

Above grid is valid for 10 digits, and we have used nine. There are very few possibilities. Let us consider 0 and exclude 6. See the following example.

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Example 24. For $a = 0, b = 1, c = 2, d = 3, e = 4, f = 5, g = 7$, $h = 8$ and $k = 9$ in (10), we have 3-digit *palindromic magic square* of order 9×9:

000	191	242	383	434	525	757	818	979
353	414	575	707	898	949	080	131	222
787	838	929	050	111	272	303	494	545
292	040	101	535	323	484	919	777	858
515	373	454	999	747	808	232	020	181
939	727	888	212	070	151	595	343	404
141	202	090	424	585	333	878	959	717
474	555	313	848	909	797	121	282	030
828	989	737	171	252	010	444	505	393

It is a palindromic bimagic square. Its sums are

$$S_{9\times 9}(0,1,2,3,4,5,7,8,9) = (0+1+2+3+4+5+7+8+9)\times 111 = 4329$$

and

$$Sb_{9\times 9}(0,1,2,3,4,5,7,8,9) = 2906329.$$

8.2. 7-Digit Palindromic Bimagic Squares of Order 9x9

Above subsection is with 3-digit palindromes having nine letters. Let us consider the following 7-digit *palindromic grid* of order 9×9 only with three letters *a, b* and *c*:

aaaaaaa	*abcccba*	*acbbbca*	*bacbcab*	*bbbabbb*	*bcacacb*	*cabcbac*	*cbababc*	*cccaccc*
babcbab	*bbababb*	*bccaccb*	*caaaaac*	*cbcccbc*	*ccbbbcc*	*aacbcaa*	*abbabba*	*acacaca*
cacbcac	*cbbabbc*	*ccacacc*	*aabcbaa*	*abababa*	*accacca*	*baaaaab*	*bbcccbb*	*bcbbbcb*
acccca	*aabbbaa*	*abaaaba*	*bcbabcb*	*baacaab*	*bbcbcbb*	*ccabacc*	*cacacac*	*cbbcbbc*
bcabacb	*bacacab*	*bbbcbbb*	*ccccccc*	*cabbbac*	*cbaaabc*	*acbabca*	*aaacaaa*	*abcbcba*
ccbabcc	*caacaac*	*cbcbcbc*	*acabaca*	*aacacaa*	*abbcbba*	*bcccccb*	*babbbab*	*bbaaabb*
abbbbba	*acaaaca*	*aacccaa*	*bbacabb*	*bccbccb*	*bababab*	*cbcacbc*	*ccbcbcc*	*caabaac*
bbcacbb	*bcbcbcb*	*baabaab*	*cbbbbbc*	*ccaaacc*	*cacccac*	*abacaba*	*accbcca*	*aababaa*
cbacabc	*cccbccc*	*cababac*	*abcacba*	*acbcbca*	*aaabaaa*	*bbbbbbb*	*bcaaacb*	*bacccab*

… (11)

For all $a, b, c \in \{0,1,2,3,4,5,6,7,8,9\}$, above grid represent a *palindromic square* with sum

$$S_{4\times 4}(a,b) = 3\times(a+b+c)\times 1111111 \text{ or } \frac{S_{9\times 9}(a,b,c)}{a+b+c} = 3333333.$$

Here below are some examples:

Example 25. For $a = 2, b = 5$ and $c = 8$ in (11), we have 7-digit *Selfie palindromic magic square* of order 9×9:

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2222222	2588852	2855582	5285825	5552555	5828285	8258528	8525258	8882888
5258525	5525255	5882885	8222228	8588858	8855588	2285822	2552552	2828282
8285828	8552558	8828288	2258522	2525252	2882882	5222225	5588855	5855585
2888882	2255522	2522252	5852585	5228225	5585855	8825288	8282828	8558558
5825285	5282825	5558555	8888888	8255528	8522258	2852582	2228222	2585852
8852588	8228228	8585858	2825282	2282822	2558552	5888885	5255525	5522255
2555552	2822282	2288822	5528255	5885885	5252525	8582858	8858588	8225228
5582855	5858585	5225225	8555558	8822288	8288828	2528252	2885882	2252522
8528258	8885888	8252528	2582852	2858582	2225222	5555555	5822285	5288825

It is *Selfie palindromic bimagic square*. Its sums are

$$S_{9\times 9}(2,5,8) = 49999995{=}(1{+}2{+}3{+}4{+}5{+}6{+}7{+}8{+}9)\times 1111111$$

and

$$Sb_{9\times 9}(2,5,8) = 332323500767679.$$

Example 26. For $a=1, b=6$ and $c=9$, we have 7-digit *upside down palindromic magic square* of order 9×9:

1111111	1699961	1966691	6196916	6661666	6919196	9169619	9616169	9991999
6169616	6616166	6991996	9111119	9699969	9966699	1196911	1661661	1919191
9196919	9661669	9919199	1169611	1616161	1991991	6111116	6699966	6966696
1999991	1166611	1611161	6961696	6119116	6696966	9916199	9191919	9669669
6916196	6191916	6669666	9999999	9166619	9611169	1961691	1119111	1696961
9961699	9119119	9696969	1916191	1191911	1669661	6999996	6166616	6611166
1666661	1911191	1199911	6619166	6996996	6161616	9691969	9969699	9116119
6691966	6969696	6116116	9666669	9911199	9199919	1619161	1996991	1161611
9619169	9996999	9161619	1691961	1969691	1116111	6666666	6911196	6199916

It is *upside down palindromic bimagic square*. Its sums are

$$S_{9\times 9}(1,6,9) = 53333328{=}\left(1{+}2^3{+}4{+}5{+}6{+}7{+}8{+}9\right)\times 1111111$$

and

$$Sb_{9\times 9}(1,6,9) = 415039806496074.$$

Examples 25 and 26 give us following interesting relation

$$\frac{S_{9\times 9}(1,6,9)}{1{+}6{+}9} = \frac{S_{9\times 9}(2,5,8)}{2{+}5{+}8} = 3333333.$$

9. Palindromic Magic Square of Order 10x10

In this case we don't have many possibilities. Here below is 3-digit *palindromic magic square* of order 10x10 with magic sum $S_{10\times 10} = 4995$:

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000	484	151	919	878	232	747	363	595	626
353	111	676	828	292	989	404	535	040	767
616	575	222	494	959	060	383	808	737	141
191	868	545	333	707	424	979	010	656	282
939	202	080	565	444	717	858	696	121	373
848	636	303	787	161	555	090	929	272	414
474	020	797	252	515	343	666	181	909	838
262	949	434	101	686	898	525	777	313	050
727	393	969	646	030	171	212	454	888	505
585	757	818	070	323	606	131	242	464	999

Since we have only 90 palindromes of 3-digits, other 10 numbers are used as 000, 010, 020, …, 090 to complete the magic square. In this case, neither we have *upside down* nor *Selfie palindromic magic squares*.

Final Comments

We are able to bring *Selfie palindromic magic* squares for the orders 3×3, 4×4, 5×5, 8×8 and 9×9. In case of order 7×7, we have just *upside down palindromic magic square*. In some situations the results are either bimagic or semi-bimagic. Still, we don't have *upside down* or *Selfie palindromic magic squares* for the orders 6×6 and 10×10. The construction of 11 *grids* given in (1)-(11) are explained in author's work [1].

References

[1] Taneja, I. J., *Uniformly Distributed, Palindromic, Selfie Magic Squares and Colored Patterns* – In preparation.

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