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ON SOME INTEGRAL INEQUALITIES FOR CONFORMABLE FRACTIONAL INTEGRALS

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Abstract. In this article, we obtain integral inequalities for Conformal fractional integrals and Chebyshev functional by using synchronous functions.

1. Introduction

Let us consider the functional in [1]

$$T(f,g) := \frac{1}{b-a}\int_b^a f(x)g(x)dx - \left(\frac{1}{b-a}\int_a^b f(x)dx\right)\left(\frac{1}{b-a}\int_a^b g(x)dx\right) \tag{1.1}$$

where f and g are two synchronous and integrable functions on $[a,b]$.

In the last few decades, much significant development of integral inequalities had been established. Integral inequalities have been frequently employed in the theory of applied sciences, differential equations, and functional analysis. In the last two decades, they have been the focus of attention in ([1], [2]-[4]).

In this paper, we obtain some integral inequalities for conformable fractional integrals.

2. Definitions and properties of conformable fractional derivative and integral

The following definitions and theorems with respect to conformable fractional derivative and integral were referred in [5]-[10].

Definition 1. *(**Conformable fractional derivative**) Given a function $f:[0,\infty)\to\mathbb{R}$. Then the "conformable fractional derivative" of f of order α is defined by*

$$D_\alpha(f)(t) = \lim_{\varepsilon\to 0}\frac{f\left(t+\varepsilon t^{1-\alpha}\right)-f(t)}{\varepsilon} \tag{2.1}$$

for all $t>0$, $\alpha\in(0,1)$. If f is $\alpha-$differentiable in some $(0,a)$, $\alpha>0$, $\lim\limits_{t\to 0^+} f^{(\alpha)}(t)$ exist, then define

$$f^{(\alpha)}(0) = \lim_{t\to 0^+} f^{(\alpha)}(t). \tag{2.2}$$

We can write $f^{(\alpha)}(t)$ for $D_\alpha(f)(t)$ to denote the conformable fractional derivatives of f of order α. In addition, if the conformable fractional derivative of f of order α exists, then we simply say f is $\alpha-$differentiable.

Theorem 1. *Let $\alpha\in(0,1]$ and f,g be $\alpha-$differentiable at a point $t>0$. Then*

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i. $D_\alpha (af+bg) = aD_\alpha (f) + bD_\alpha (g)$, for all $a, b \in \mathbb{R}$,

ii. $D_\alpha (\lambda) = 0$, for all constant functions $f(t) = \lambda$,

iii. $D_\alpha (fg) = fD_\alpha (g) + gD_\alpha (f)$,

iv. $D_\alpha \left(\frac{f}{g}\right) = \frac{fD_\alpha (g) - gD_\alpha (f)}{g^2}$.

If f is differentiable, then $D_\alpha (f)(t) = t^{1-\alpha} \frac{df}{dt}(t)$.
Also:
1. $D_\alpha (1) = 0$

2. $D_\alpha (e^{ax}) = ax^{1-\alpha} e^{ax}, \ a \in \mathbb{R}$

3. $D_\alpha (\sin(ax)) = ax^{1-\alpha} \cos(ax), \ a \in \mathbb{R}$

4. $D_\alpha (\cos(ax)) = -ax^{1-\alpha} \sin(ax), \ a \in \mathbb{R}$

5. $D_\alpha \left(\frac{1}{\alpha} t^\alpha\right) = 1$

6. $D_\alpha \left(\sin(\frac{t^\alpha}{\alpha})\right) = \cos(\frac{t^\alpha}{\alpha})$

7. $D_\alpha \left(\cos(\frac{t^\alpha}{\alpha})\right) = -\sin(\frac{t^\alpha}{\alpha})$

8. $D_\alpha \left(e^{(\frac{t^\alpha}{\alpha})}\right) = e^{(\frac{t^\alpha}{\alpha})}$.

Theorem 2 (Mean value theorem for conformable fractional differentiable functions)**.** *Let $\alpha \in (0,1]$ and $f : [a,b] \to \mathbb{R}$ be a continuous on $[a,b]$ and an α-fractional differentiable mapping on (a,b) with $0 \le a < b$. Then, there exists $c \in (a,b)$, such that*

$$D_\alpha (f)(c) = \frac{f(b) - f(a)}{\frac{b^\alpha}{\alpha} - \frac{a^\alpha}{\alpha}}.$$

Definition 2 (Conformable fractional integral)**.** *Let $\alpha \in (0,1]$ and $0 \le a < b$. A function $f : [a,b] \to \mathbb{R}$ is α-fractional integrable on $[a,b]$ if the integral*

$$\int_a^b f(x)\, d_\alpha x := \int_a^b f(x)\, x^{\alpha-1} dx \tag{2.3}$$

exists and is finite. All α-fractional integrable on $[a,b]$ is indicated by $L^1_\alpha([a,b])$

Remark 1.

$$I^a_\alpha (f)(t) = I^a_1 \left(t^{\alpha-1} f\right) = \int_a^t \frac{f(x)}{x^{1-\alpha}} dx,$$

where the integral is the usual Riemann improper integral, and $\alpha \in (0,1]$.

Theorem 3. *Let $f : (a,b) \to \mathbb{R}$ be differentiable and $0 < \alpha \le 1$. Then, for all $t > a$ we have*

$$I^a_\alpha D^a_\alpha f(t) = f(t) - f(a). \tag{2.4}$$

Theorem 4. ***(Integration by parts)*** *Let $f, g : [a, b] \to \mathbb{R}$ be two functions such that fg is differentiable. Then*

$$\int_a^b f(x)\, D_\alpha^a (g)(x)\, d_\alpha x = fg|_a^b - \int_a^b g(x)\, D_\alpha^a (f)(x)\, d_\alpha x. \tag{2.5}$$

Theorem 5. *Assume that $f : [a, \infty) \to \mathbb{R}$ such that $f^{(n)}(t)$ is continuous and $\alpha \in (n, n+1]$. Then, for all $t > a$ we have*

$$D_\alpha^a f(t)\, I_\alpha^a = f(t).$$

Theorem 6. *Let $\alpha \in (0, 1]$ and $f : [a, b] \to \mathbb{R}$ be a continuous on $[a, b]$ with $0 \leq a < b$. Then,*

$$\left| I_\alpha^a (f)(x) \right| \leq I_\alpha^a |f|(x).$$

In this paper, we establish integral inequalities for Conformable fractional integrals and Chebyshev functional by using synchronous functions.

3. Main Results

Theorem 7. *Let f and g be two synchronous functions on $[0, \infty)$. Then for $t > a$, $\alpha > 0$;*

$$I_\alpha^a (fg)(t) \geq \frac{\alpha}{t^\alpha - a^\alpha} I_\alpha^a f(t) I_\alpha^a g(t). \tag{3.1}$$

Proof. For f and g synchronous functions, we have

$$\left(f(\tau) - f(\rho) \right) \left(g(\tau) - g(\rho) \right) \geq 0. \tag{3.2}$$

From (3.2), it can be written as

$$f(\tau)g(\tau) + f(\rho)g(\rho) \geq f(\tau)g(\rho) + f(\rho)g(\tau). \tag{3.3}$$

If we multiply both the sides of (3.3) with $\tau^{\alpha-1}$, $\tau \in (a, t)$, we get

$$\tau^{\alpha-1} f(\tau)g(\tau) + \tau^{\alpha-1} f(\rho)g(\rho) \geq \tau^{\alpha-1} f(\tau)g(\rho) + \tau^{\alpha-1} f(\rho)g(\tau). \tag{3.4}$$

Integrating (3.4) with respect to τ over (a, t), we obtain:

$$\begin{aligned} &\textstyle\int_a^t \tau^{\alpha-1} f(\tau)g(\tau)d\tau + \int_a^t \tau^{\alpha-1} f(\rho)g(\rho)d\tau \\ &\geq \textstyle\int_a^t \tau^{\alpha-1} f(\tau)g(\rho)d\tau + \int_a^t \tau^{\alpha-1} f(\rho)g(\tau)d\tau. \end{aligned} \tag{3.5}$$

Consequently,

$$\begin{aligned} &\textstyle\int_a^t f(\tau)g(\tau)d_\alpha\tau + \int_a^t f(\rho)g(\rho)d_\alpha\tau \\ &\geq \textstyle\int_a^t f(\tau)g(\rho)d_\alpha\tau + \int_a^t f(\rho)g(\tau)d_\alpha\tau. \end{aligned} \tag{3.6}$$

So we have

$$I_\alpha^a (fg)(t) + \frac{t^\alpha - a^\alpha}{\alpha} f(\rho)g(\rho) \geq g(\rho) I_\alpha^a (f)(t) + f(\rho) I_\alpha^a (g)(t). \tag{3.7}$$

Now multiplying two sides of (3.7) by $\rho^{\alpha-1}$, $\rho \in (a, t)$, we obtain:

$$\rho^{\alpha-1} I_\alpha^a (fg)(t) + \rho^{\alpha-1} \frac{t^\alpha - a^\alpha}{\alpha} f(\rho)g(\rho) \geq \rho^{\alpha-1} g(\rho) I_\alpha^a (f)(t) + \rho^{\alpha-1} f(\rho) I_\alpha^a (g)(t). \tag{3.8}$$

By integrating (3.8) over (a,t), we get:

$$I_\alpha^a(fg)(t)\int_a^t\rho^{\alpha-1}d\rho+\frac{t^\alpha-a^\alpha}{\alpha}\int_a^t\rho^{\alpha-1}f(\rho)g(\rho)d\rho \tag{3.9}$$
$$\geq I_\alpha^a f(t)\int_a^t\rho^{\alpha-1}g(\rho)d\rho+I_\alpha^a g\,(t)\int_a^t\rho^{\alpha-1}f(\rho)d\rho.$$

The inequality can be written as the following at the same time,

$$I_\alpha^a(fg)(t)\geq\left(\frac{t^\alpha-a^\alpha}{\alpha}\right)^{-1}I_\alpha^a f(t)I_\alpha^a g(t). \tag{3.10}$$

This completes the proof. □

Theorem 8. *Let f and g be two synchronous functions on $[a,b]$. Then for $t>a$, $\alpha>0$, and $\beta>0$,*

$$\frac{t^\beta-a^\beta}{\beta}I_\alpha^a\left(fg\right)(t)+\frac{t^\alpha-a^\alpha}{\alpha}I_\alpha^\beta\left(fg\right)(t)\geq I_\alpha^a f(t)I_\alpha^\beta g(t)+I_\alpha^\beta f(t)I_\alpha^a g(t). \tag{3.11}$$

Proof. If we multiply two sides of (3.7) by $\rho^{\beta-1}$, we obtain:

$$\rho^{\beta-1}I_\alpha^a(fg)(t)+\rho^{\beta-1}\frac{t^\alpha-a^\alpha}{\alpha}f(\rho)g(\rho)\geq\rho^{\beta-1}g(\rho)I_\alpha^a(f)(t)+\rho^{\beta-1}f(\rho)I_\alpha^a(g)(t). \tag{3.12}$$

Integrating (3.12) over (a,t), we get:

$$I_\alpha^a(fg)(t)\int\limits_a^t\rho^{\beta-1}d\rho+\frac{t^\alpha-a^\alpha}{\alpha}\int\limits_a^t\rho^{\beta-1}f(\rho)g(\rho)d\rho \tag{3.13}$$
$$\geq I_\alpha^a f(t)\int\limits_a^t\rho^{\beta-1}g(\rho)dt+I_\alpha^a g(t)\int\limits_a^t\rho^{\beta-1}f(\rho)dt.$$

Consequently,

$$\frac{t^\beta-a^\beta}{\beta}I_\alpha^a\left(fg\right)(t)+\frac{t^\alpha-a^\alpha}{\alpha}I_\alpha^\beta\left(fg\right)(t)\geq I_\alpha^a f(t)I_\alpha^\beta g(t)+I_\alpha^\beta f(t)I_\alpha^a g(t). \tag{3.14}$$

This completes the proof. □

Remark 2. *Applying Theorem 8 for $\alpha=\beta$, we obtain Theorem 7.*

Theorem 9. *Let $(f_i)_{i=1,\dots n}$ be n positive increasing functions on $[0,\infty)$. Then for all $t>a$, $\alpha>0$,*

$$I_\alpha^a(\prod_{i=1}^n f_i)(t)\geq\left(\frac{t^\alpha-a^\alpha}{\alpha}\right)^{1-n}\left(\prod_{i=1}^n I_\alpha^a f_i\right)(t) \tag{3.15}$$

Proof. We will prove this theorem by induction.It is clear that for $n=1$ and all $t>0$, $\alpha>0$, we have $I_\alpha^a(f_1)(t)\geq I_\alpha^a f_1(t)$. And for $n=2$, we obtain (3.1),

$$I_\alpha^a(f_1f_2)(t)\geq\left(\frac{t^\alpha-a^\alpha}{\alpha}\right)^{-1}\left(I_\alpha^a f_1\right)(t)\left(I_\alpha^a f_2\right)(t). \tag{3.16}$$

Now assume that(induction hypothesis)

$$I_\alpha^a(\prod_{i=1}^{n-1} f_i)(t)\geq\left(\frac{t^\alpha-a^\alpha}{\alpha}\right)^{2-n}\left(\prod_{i=1}^{n-1} I_\alpha^a f_i\right)(t). \tag{3.17}$$

If $(f_i)_{i=1,\dots n}$ are positive increasing functions, then $\left(\prod_{i=1}^{n-1} f_i\right)(t)$ is an increasing function. So we can use Theorem 7 for functions $\prod_{i=1}^{n-1} f_i = g$, and $f_n = f$, therefore we obtain

$$I_\alpha^a(\prod_{i=1}^{n} f_i)(t) = I_\alpha^a(fg)(t) \geq \left(\frac{t^\alpha - a^\alpha}{\alpha}\right)^{-1} \left(\prod_{i=1}^{n-1} I_\alpha^a f_i\right)(t)\left(I_\alpha^a f_n\right)(t). \tag{3.18}$$

By (3.17)

$$I_\alpha^a(\prod_{i=1}^{n} f_i)(t) \geq \left(\frac{t^\alpha - a^\alpha}{\alpha}\right)^{-1} \left(\frac{t^\alpha - a^\alpha}{\alpha}\right)^{2-n} \left(\prod_{i=1}^{n-1} I_\alpha^a f_i\right)(t)\left(I_\alpha^a f_n\right)(t). \tag{3.19}$$

This completes the proof. □

Theorem 10. *If f is an increasing and g is a differentiable functions and there exist a real number $m := \inf_{t\geq 0} g'(t)$ on $[0, +\infty)$. Then for all $t \in [a, b]$ and $\alpha > 0$,*

$$\begin{aligned} I_\alpha^a(fg)(t) &\geq \left(\frac{t^\alpha - a^\alpha}{\alpha}\right)^{-1} I_\alpha^a f(t) I_\alpha^a g(t) \\ &-m\frac{t^{\alpha+1} - a^{\alpha+1}}{\alpha+1} I_\alpha^a f(t) + m I_\alpha^a\left(tf(t)\right). \end{aligned} \tag{3.20}$$

Proof. Consider the given function $H(t) = g(t) - mt$. It is clear that H is an increasing function and differentiable on $[0, +\infty)$. Then using Theorem 7 we obtain

$$\begin{aligned} &I_\alpha^a(Hf)(t) = I_\alpha^a(\left(g(t) - mt\right) f(t)) \\ &\geq \left(\frac{t^\alpha - a^\alpha}{\alpha}\right)^{-1} I_\alpha^a f(t)\left[I_\alpha^a g(t) - m I_\alpha^a\left(t\right)\right] \\ &\geq \left(\frac{t^\alpha - a^\alpha}{\alpha}\right)^{-1} I_\alpha^a f(t) I_\alpha^a g(t) - m\frac{t^{\alpha+1} - a^{\alpha+1}}{\alpha+1} I_\alpha^a f(t). \end{aligned} \tag{3.21}$$

Also,

$$\begin{aligned} &I_\alpha^a(Hf)(t) \\ &= I_\alpha^a(\left(g(t) - mt\right) f(t)) \\ &= I_\alpha^a(fg)(t) - m I_\alpha^a\left(tf(t)\right) \end{aligned} \tag{3.22}$$

From equations (3.21) and (3.22), we get:

$$\begin{aligned} I_\alpha^a(fg)(t) &\geq \left(\frac{t^\alpha - a^\alpha}{\alpha}\right)^{-1} I_\alpha^a f(t) I_\alpha^a g(t) \\ &-m\frac{t^{\alpha+1} - a^{\alpha+1}}{\alpha+1} I_\alpha^a f(t) + m I_\alpha^a\left(tf(t)\right). \end{aligned} \tag{3.23}$$

This completes the proof. □

Corollary 1. *If f is an increasing and g is a differentiable functions on $[0,+\infty)$. Then for all $t \in [a,b]$ and $\alpha > 0$,*

I. If there exist real numbers $m_1 := \inf_{t \geq 0} f'(x)$, and $m_2 := \inf_{t \geq 0} g'(t)$. Then we have:

$$I_\alpha^a(fg)(t) - m_1 I_\alpha^a\left(tg\left(t\right)\right) - m_2 I_\alpha^a\left(tf\left(t\right)\right) + m_1 m \left(\frac{t^{\alpha+2} - a^{\alpha+2}}{\alpha+2}\right)$$
$$\geq \left(\frac{t^\alpha - a^\alpha}{\alpha}\right)^{-1} \left[I_\alpha^a f(t) I_\alpha^a g(t) - m_1 \frac{t^{\alpha+1} - a^{\alpha+1}}{\alpha+1} I_\alpha^a g(t) \right. \tag{3.24}$$
$$\left. - m_2 \frac{t^{\alpha+1} - a^{\alpha+1}}{\alpha+1} I_\alpha^a f(t) + m_1 m_2 \left(\frac{t^{\alpha+1} - a^{\alpha+1}}{\alpha+1}\right)^2 \right].$$

II. If there exist real numbers $M_1 := \sup_{t \geq 0} f'(x)$, and $M_2 := \sup_{t \geq 0} g'(t)$. Then we have:

$$I_\alpha^a(fg)(t) - M_1 I_\alpha^a\left(tg(t)\right) - M_2 I_\alpha^a\left(tf(t)\right) + M_1 M_2 \left(\frac{t^{\alpha+2} - a^{\alpha+2}}{\alpha+2}\right)$$
$$\geq \left(\frac{t^\alpha - a^\alpha}{\alpha}\right)^{-1} \left[I_\alpha^a f(t) I_\alpha^a g(t) - M_1 \frac{t^{\alpha+1} - a^{\alpha+1}}{\alpha+1} I_\alpha^a g(t) \right. \tag{3.25}$$
$$\left. - M_2 \frac{t^{\alpha+1} - a^{\alpha+1}}{\alpha+1} I_\alpha^a f(t) + M_1 M_2 \left(\frac{t^{\alpha+1} - a^{\alpha+1}}{\alpha+1}\right)^2 \right].$$

Proof. Consider the given function $F(t) = f(t) - m_1 t$ and $G(t) = g(t) - m_2 t$. It is clear that F and G are increasing functions and differentiable on $[0,+\infty)$. Then using Theorem 7 we obtain

$$\begin{aligned}
I_\alpha^a(FG)(t) &= I_\alpha^a\left(f(t) - m_1 t\right)\left(g(t) - m_2 t\right) \\
&\geq \left(\frac{t^\alpha - a^\alpha}{\alpha}\right)^{-1} I_\alpha^a\left(f(t) - m_1 t\right) I_\alpha^a\left(g(t) - m_2 t\right) \\
&\geq \left(\frac{t^\alpha - a^\alpha}{\alpha}\right)^{-1} \left[I_\alpha^a f(t) I_\alpha^a g(t) - m_1 \frac{t^\alpha - a^\alpha}{\alpha} I_\alpha^a g(t) \right. \\
&\quad \left. - m_2 \frac{t^\alpha - a^\alpha}{\alpha} I_\alpha^a f(t) + m_1 m_2 \left(\frac{t^\alpha - a^\alpha}{\alpha}\right)^2 \right]
\end{aligned}$$

Therefore

$$I_{\alpha}^{a}(fg)(t)-m_1I_{\alpha}^{a}\left(tg(t)\right)-m_2I_{\alpha}^{a}(tf\left(t\right))+m_1m_2\left(\frac{t^{\alpha+2}-a^{\alpha+2}}{\alpha+2}\right)$$

$$\geq\left(\frac{t^{\alpha}-a^{\alpha}}{\alpha}\right)^{-1}\left[I_{\alpha}^{a}f(t)I_{\alpha}^{a}g(t)-m_1\frac{t^{\alpha+1}-a^{\alpha+1}}{\alpha+1}I_{\alpha}^{a}g(t)\right.$$

$$\left.-m_2\frac{t^{\alpha+1}-a^{\alpha+1}}{\alpha+1}I_{\alpha}^{a}f(t)+m_1m_2\left(\frac{t^{\alpha+1}-a^{\alpha+1}}{\alpha+1}\right)^2\right].$$

This completes the proof of (I).

Consider the given function $F(t)=f(t)-M_1t$, $G(t)=g(t)-M_2t$. It is clear that F and G are increasing functions and differentiable on $[0,+\infty)$. Then using Theorem 7 we obtain

$$\begin{aligned}
I_{\alpha}^{a}(FG)(t) &= I_{\alpha}^{a}\left(f(t)-M_1t\right)\left(g(t)-M_2t\right)\\
&\geq\left(\frac{t^{\alpha}-a^{\alpha}}{\alpha}\right)^{-1}I_{\alpha}^{a}\left(f(t)-M_1t\right)I_{\alpha}^{a}\left(g(t)-M_2t\right)\\
&\geq\left(\frac{t^{\alpha}-a^{\alpha}}{\alpha}\right)^{-1}\left[I_{\alpha}^{a}f(t)I_{\alpha}^{a}g(t)-M\frac{t^{\alpha+1}-a^{\alpha+1}}{\alpha+1}I_{\alpha}^{a}g(t)\right.\\
&\left.-m_2\frac{t^{\alpha+1}-a^{\alpha+1}}{\alpha+1}I_{\alpha}^{a}f(t)+M_1M_2\left(\frac{t^{\alpha+1}-a^{\alpha+1}}{\alpha+1}\right)^2\right]
\end{aligned}$$

Therefore

$$I_{\alpha}^{a}(fg)(t)-M_1I_{\alpha}^{a}\left(tg(t)\right)-M_2I_{\alpha}^{a}(tf\left(t\right))+M_1M_2\left(\frac{t^{\alpha+2}-a^{\alpha+2}}{\alpha+2}\right)$$

$$\geq\left(\frac{t^{\alpha}-a^{\alpha}}{\alpha}\right)^{-1}\left[I_{\alpha}^{a}f(t)I_{\alpha}^{a}g(t)-\frac{t^{\alpha+1}-a^{\alpha+1}}{\alpha+1}M_1I_{\alpha}^{a}g(t)\right.$$

$$\left.-M_2\frac{t^{\alpha+1}-a^{\alpha+1}}{\alpha+1}I_{\alpha}^{a}f(t)+M_1M_2\left(\frac{t^{\alpha+1}-a^{\alpha+1}}{\alpha+1}\right)^2\right].$$

This completes the proof of (II). □

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