Received 28/06/16

A NEW INTEGRAL INEQUALITIES FOR $GG-$CONVEX FUNCTIONS

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Abstract. In this paper, we obtained new integral inequalities for the first derivatives of the $GG-$convex functions.

1. Introduction

We will start with the definiton of convexity:

Definition 1. *The function $f : I \subset \mathbb{R} \to \mathbb{R}$ is a convex function on I, if the inequality*

$$f(tx + (1-t)y) \leq tf(x) + (1-t)f(y)$$

holds for all $x, y \in I$ and $t \in [0,1]$. We say that f is concave if $-f$ is convex.

Let $f : I \subset \mathbb{R} \to \mathbb{R}$ be a convex function where $a, b \in I$ with $a < b$. Then the following double inequality hold:

$$f\left(\frac{a+b}{2}\right) \leq \frac{1}{b-a}\int_a^b f(x)dx \leq \frac{f(a)+f(b)}{2}.$$

This inequality is well-known in the literature as Hermite-Hadamard inequality that gives us upper and lower bounds for the mean-value of a convex function. If f is concave function both of the inequalities in above hold in reversed direction.

Anderson *et. al.* mentioned mean function in [2] as following*:*

Definition 2. *A function $M : (0,\infty) \times (0,\infty) \to (0,\infty)$ is called a Mean function if*

(1) $M(x,y) = M(y,x)$,
(2) $M(x,x) = x$,
(3) $x < M(x,y) < y$, whenever $x < y$,
(4) $M(ax,ay) = aM(x,y)$ for all $a > 0$.

Based on the definition of mean function, let us recall special means (See [2])

1. Arithmetic Mean: $M(x,y) = A(x,y) = \frac{x+y}{2}$.
2. Geometric Mean: $M(x,y) = G(x,y) = \sqrt{xy}$.
3. Harmonic Mean: $M(x,y) = H(x,y) = 1/A\left(\frac{1}{x},\frac{1}{y}\right)$.
4. Logarithmic Mean: $M(x,y) = L(x,y) = (x-y)/(\log x - \log y)$ for $x \neq y$ nd $L(x,x) = x$.

1991 *Mathematics Subject Classification.* 26D15, 26A51, 26E60, 41A55.

Key words and phrases. $GG-$convex functions, Hölder inequality, Power-mean integral inequality and Logarithmic Mean.

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RGMIA Res. Rep. Coll. 19 (2016), Art. 111

5. Identric Mean: $M(x,y) = I(x,y) = (1/e)\left(x^x/y^y\right)^{1/(x-y)}$ for $x \neq y$ nd $I(x,x) = x$.

In [2], Anderson *et. al.* also gave a definition that include several different classes of convex functions as the following:

Definition 3. *Let $f : I \to (0,\infty)$ be continuous, where I is subinterval of $(0,\infty)$. Let M and N be any two Mean functions. We say f is MN-convex (concave) if*

$$f(M(x,y)) \leq (\geq) N(f(x), f(y))$$

for all $x, y \in I$.

In [1], Niculescu mentioned the following considerable definition:

Definition 4. *The $GG-$convex functions are those functions $f : I \to J$ (acting on subintervals of $(0,\infty)$) such that*

$$x, y \in I \text{ and } \lambda \in [0,1] \Longrightarrow f\left(x^{1-\lambda}y^{\lambda}\right) \leq f(x)^{1-\lambda} f(y)^{\lambda}. \tag{1.1}$$

Every real analytic function $f(x) = \sum_{n=0}^{\infty} c_n x^n$ with nonnegative coefficients c_n is a $GG-$convex function on $(0,r)$, where r is the radius of convergence of f. The functions such as $\exp, \sinh, \cosh$ are $GG-$convex on $(0,\infty)$; tan, sec, csc, $\frac{1}{x} - \cot x$ are $GG-$convex on $\left(0, \frac{\pi}{2}\right)$; $\frac{1+x}{1-x}$ is $GG-$convex on (0.1). (See [1])

ın [3], authors proved the following lemma and established new inequalities for $GG-$ convex functions.

Lemma 1. *Let $f : I \subseteq IR = (0,\infty) \longrightarrow IR$ be a differentiable function and a,b∈ I° with a‹b . If $f'(x) \in L([a,b])$, then*

$$\frac{b^2 f(b) - a^2 f(a)}{2} - \int_a^b x f(x) du = \frac{\ln b - \ln a}{2} \int_0^1 a^{3(1-t)} b^{3t} f'\left(a^{1-t} b^t\right) dt$$

The main aim of this paper is to prove some new integral inequalities for $GG-$ convex functions.

2. Main Results

We need the following integral identity to get our new results.

Lemma 2. *Let $f : I \subseteq IR = (0,\infty) \longrightarrow IR$ be a differentiable function on I° where a,b∈ I° with a‹b . If $f' \in L[a,b]$, then the following equality holds:*

$$\begin{aligned} & b^2 f(b) - a^2 f(a) - 2\int_a^b u f(u) du \\ = \quad & \frac{\ln b - \ln a}{2} \left[\int_0^1 \left(b^{\frac{3t}{2}} a^{\frac{3(2-t)}{2}} \right) f'\left(b^{\frac{t}{2}} a^{\frac{2-t}{2}} \right) dt + \int_0^1 \left(a^{\frac{3t}{2}} b^{\frac{3(2-t)}{2}} \right) f'\left(a^{\frac{t}{2}} b^{\frac{2-t}{2}} \right) dt \right] \end{aligned}$$

Proof. Let

$$I_1 = \int_0^1 \left(b^{\frac{3t}{2}} a^{\frac{3(2-t)}{2}} \right) f'\left(b^{\frac{t}{2}} a^{\frac{2-t}{2}} \right) dt$$

and

$$I_2=\int_0^1\left(a^{\frac{3t}{2}}b^{\frac{3(2-t)}{2}}\right)f^{'}\left(a^{\frac{t}{2}}b^{\frac{2-t}{2}}\right)dt$$

We notice that

$$\begin{aligned} I_1 &= \int_0^1\left(b^{\frac{3t}{2}}a^{\frac{3(2-t)}{2}}\right)f^{'}\left(b^{\frac{t}{2}}a^{\frac{2-t}{2}}\right)dt \\ &= \frac{2}{\ln b-\ln a}\int_0^1\left(b^ta^{1-t}\right)f^{'}\left(b^{\frac{t}{2}}a^{\frac{2-t}{2}}\right)d\left(b^{\frac{t}{2}}a^{\frac{2-t}{2}}\right). \end{aligned}$$

By the change of the variable $u=b^{\frac{t}{2}}a^{\frac{2-t}{2}}$ and integrating by parts , we have

$$I_1=\frac{2}{\ln b-\ln a}\left[abf\sqrt{ab}-a^2f(a)-2\int_a^{\sqrt{ab}}uf(u)du\right].$$

Conformably, we have

$$I_2=\frac{2}{\ln b-\ln a}\left[b^2f(b)-abf\sqrt{ab}-2\int_{\sqrt{ab}}^b uf(u)du\right]$$

Multiplying I_1 and I_2 by $\frac{\ln b-\ln a}{2}$ and adding the results we get the desired identity. □

Our first result is given in the following Theorem.

Theorem 1. *Let $f:I\subseteq IR_+=(0,\infty)\longrightarrow IR$ be a differentiable function on I° where $a,b\in I^\circ$ with $a<b$,and $f^{'}\in L[a,b]$. If $\left|f^{'}\right|$ is GG− convex on $[a,b]$, then the following inequality holds:*

$$\begin{aligned} &\left|b^2f(b)-a^2f(a)-\int_a^b uf(u)du\right| \\ \leq\ & \frac{\ln b-\ln a}{2}\left(\sqrt{a^3\left|f'(a)\right|}+\sqrt{b^3\left|f'(b)\right|}\right)L\left(\sqrt{a^3\left|f'(a)\right|},\sqrt{b^3\left|f'(b)\right|}\right). \end{aligned}$$

Proof. From Lemma 2, using the property of the modulus and $GG-$ convexity of $\left|f^{'}\right|$ we can write

$$\begin{aligned} &\left|b^2f(b)-a^2f(a)-\int_a^b uf(u)du\right| \\ \leq\ & \frac{\ln b-\ln a}{2}\left[\int_0^1\left(b^{\frac{3t}{2}}a^{\frac{3(2-t)}{2}}\right)\left|f^{'}\left(b^{\frac{t}{2}}a^{\frac{2-t}{2}}\right)\right|dt+\int_0^1\left(a^{\frac{3t}{2}}b^{\frac{3(2-t)}{2}}\right)\left|f^{'}\left(a^{\frac{t}{2}}b^{\frac{2-t}{2}}\right)\right|dt\right] \\ \leq\ & \frac{\ln b-\ln a}{2}\left[\int_0^1\left(b^{\frac{3t}{2}}a^{\frac{3(2-t)}{2}}\right)\left|f^{'}(b)\right|^{\frac{t}{2}}\left|f^{'}(a)\right|^{\frac{2-t}{2}}dt+\int_0^1\left(a^{\frac{3t}{2}}b^{\frac{3(2-t)}{2}}\right)\left|f^{'}(a)\right|^{\frac{t}{2}}\left|f^{'}(b)\right|^{\frac{2-t}{2}}dt\right] \\ =\ & \frac{\ln b-\ln a}{2}\left[a^3\left|f^{'}(a)\right|\int_0^1\left(\frac{\sqrt{b^3\left|f'(b)\right|}}{\sqrt{a^3\left|f'(a)\right|}}\right)^tdt+b^3\left|f^{'}(b)\right|\int_0^1\left(\frac{\sqrt{a^3\left|f'(a)\right|}}{\sqrt{b^3\left|f'(b)\right|}}\right)^tdt\right] \end{aligned}$$

If we calculate the integrals above, we get the desired result. □

Theorem 2. *Let $f : I \subseteq IR_+ = (0, \infty) \longrightarrow IR$ be a differentiable function on I° where $a,b \in I^\circ$ with $a < b$,and $f' \in L[a,b]$. If $\left|f'\right|^q$ is GG− convex on $[a,b]$ for all $x \in [a,b]$, the following inequality*

$$\begin{aligned}&\left|b^2 f(b) - a^2 f(a) - 2\int_a^b u f(u) du\right| \\ \leq\ & \frac{\ln b - \ln a}{2}\left(\sqrt{a^3\left|f'(a)\right|} + \sqrt{b^3\left|f'(b)\right|}\right)\left(L\left(\sqrt{a^{3p}}, \sqrt{b^{3p}}\right)\right)^{\frac{1}{p}}\left(L\left(\sqrt{\left|f'(a)\right|^q}, \sqrt{\left|f'(b)\right|^q}\right)\right)^{\frac{1}{q}}\end{aligned}$$

holds where $q > 1$ and $\frac{1}{p} + \frac{1}{q} = 1$.

Proof. From Lemma 2, using the property of the modulus, $GG-$ convexity of $\left|f'\right|^q$ and Hölder integral inequality, we can write

$$\begin{aligned}&\left|b^2 f(b) - a^2 f(a) - 2\int_a^b u f(u) du\right| \\ =\ & \frac{\ln b - \ln a}{2}\left[\int_0^1 \left(b^{\frac{3t}{2}} a^{\frac{3(2-t)}{2}}\right)\left|f'\left(b^{\frac{t}{2}} a^{\frac{2-t}{2}}\right)\right| dt + \int_0^1 \left(a^{\frac{3t}{2}} b^{\frac{3(2-t)}{2}}\right)\left|f'\left(a^{\frac{t}{2}} b^{\frac{2-t}{2}}\right)\right| dt\right] \\ \leq\ & \frac{\ln b - \ln a}{2}\left\{\left(\int_a^b b^{\frac{3tp}{2}} a^{\frac{3(2-t)}{2}} dt\right)^{\frac{1}{p}}\left(\int_0^1 \left|f'\left(b^{\frac{t}{2}} a^{\frac{2-t}{2}}\right)\right|^q dt\right)^{\frac{1}{q}}\right. \\ & \left. + \left(\int_a^b a^{\frac{3tp}{2}} b^{\frac{3(2-t)}{2}} dt\right)^{\frac{1}{p}}\left(\int_0^1 \left|f'\left(a^{\frac{t}{2}} b^{\frac{2-t}{2}}\right)\right|^q dt\right)^{\frac{1}{q}}\right\} \\ \leq\ & \frac{\ln b - \ln a}{2}\left\{a^3\left|f'(a)\right|\left(\int_0^1 \left(\frac{\sqrt{b^{3p}}}{\sqrt{a^{3p}}}\right)^t dt\right)^{\frac{1}{p}}\left(\int_0^1 \left|f'(b)\right|^{\frac{tq}{2}}\left|f'(a)\right|^{\frac{(2-t)q}{2}} dt\right)^{\frac{1}{q}}\right. \\ & \left. + b^3\left|f'(b)\right|\left(\int_0^1 \left(\frac{\sqrt{a^{3p}}}{\sqrt{b^{3p}}}\right)^t dt\right)^{\frac{1}{p}}\left(\int_0^1 \left|f'(a)\right|^{\frac{tq}{2}}\left|f'(b)\right|^{\frac{(2-t)q}{2}} dt\right)^{\frac{1}{q}}\right\}.\end{aligned}$$

If we calculate the integrals above, we get the desired result. □

Theorem 3. *Under the assumptions of Theorem 2, the following inequality holds:*

$$\begin{aligned}&\left|b^2 f(b) - a^2 f(a) - 2\int_a^b u f(u) du\right| \\ \leq\ & \frac{\ln b - \ln a}{2}\left[\left(\frac{\sqrt{b^{3p}} - 1}{\ln\left(\sqrt{b^{3p}}\right)}\right)^{\frac{1}{p}}\left(\sqrt{a^3\left|f'(a)\right|} \times \left(L\left(\sqrt{\left|f'(b)\right|^q}, \sqrt{\left|f'(a)\right|^q a^{3q}}\right)\right)^{\frac{1}{q}}\right)\right] \\ & + \left[\left(\frac{\sqrt{a^{3p}} - 1}{\ln\left(\sqrt{a^{3p}}\right)}\right)^{\frac{1}{p}}\left(\sqrt{b^3\left|f'(b)\right|} \times \left(L\left(\sqrt{\left|f'(a)\right|^q}, \sqrt{\left|f'(b)\right|^q b^{3q}}\right)\right)^{\frac{1}{q}}\right)\right].\end{aligned}$$

Proof. From Lemma 2, using the property of the modulus, $GG-$ convexity of $\left|f'\right|^q$ and Hölder integral inequality, we can write

$$\begin{aligned}
&\left|b^2f(b)-a^2f(a)-2\int_a^b uf(u)du\right| \\
\leq\ & \frac{\ln b-\ln a}{2}\left\{\left(\int_0^1 b^{\frac{3tp}{2}}dt\right)^{\frac{1}{p}}\left(\int_0^1 a^{\frac{3(2-t)q}{2}}\left|f'\left(b^{\frac{t}{2}}a^{\frac{2-t}{2}}\right)\right|^q dt\right)^{\frac{1}{q}}\right. \\
&\left.+\left(\int_0^1 a^{\frac{3tp}{2}}dt\right)^{\frac{1}{p}}\left(\int_0^1 b^{\frac{3(2-t)q}{2}}\left|f'\left(a^{\frac{t}{2}}b^{\frac{2-t}{2}}\right)\right|^q dt\right)^{\frac{1}{q}}\right\} \\
\leq\ & \frac{\ln b-\ln a}{2}\left\{\left(\int_0^1 b^{\frac{3tp}{2}}dt\right)^{\frac{1}{p}}\left(\int_0^1 a^{\frac{3(2-t)q}{2}}\left|f'(b)\right|^{\frac{tq}{2}}\left|f'(a)\right|^{\frac{(2-t)q}{2}}dt\right)^{\frac{1}{q}}\right. \\
&\left.+\left(\int_0^1 a^{\frac{3tp}{2}}dt\right)^{\frac{1}{p}}\left(\int_0^1 b^{\frac{3(2-t)q}{2}}\left|f'(a)\right|^{\frac{tq}{2}}\left|f'(b)\right|^{\frac{(2-t)q}{2}}\right)^{\frac{1}{q}}\right\}.
\end{aligned}$$

By a simple computation we get the desired result. □

Theorem 4. *Under the assumptions of Theorem 2, the following inequality holds:*

$$\begin{aligned}
&\left|b^2f(b)-a^2f(a)-2\int_a^b f(u)du\right| \\
\leq\ & \frac{\ln b-\ln a}{2}\left(\sqrt{a^3\left|f'(a)\right|}+\sqrt{b^3\left|f'(b)\right|}\right)\left(L\left(\sqrt{b^{3q}\left|f'(b)\right|^q},\sqrt{a^{3q}\left|f'(a)\right|^q}\right)\right)^{\frac{1}{q}}.
\end{aligned}$$

Proof. From Lemma 2, using the property of the modulus, $GG-$ convexity of $\left|f'\right|^q$ and Hölder integral inequality, we can write

$$\begin{aligned}
&\left|b^2f(b)-a^2f(a)-2\int_a^b uf(u)du\right| \\
\leq\ & \frac{\ln b-\ln a}{2}\left\{\left(\int_0^1 dt\right)^{1-\frac{1}{q}}\left(\int_0^1 b^{\frac{3tq}{2}}a^{\frac{3(2-t)q}{2}}\left|f'\left(b^{\frac{t}{2}}a^{\frac{2-t}{2}}\right)\right|^q dt\right)^{\frac{1}{q}}\right. \\
&\left.+\left(\int_0^1 dt\right)^{1-\frac{1}{q}}\left(\int_0^1 a^{\frac{3tq}{2}}b^{\frac{3(2-t)q}{2}}\left|f'\left(a^{\frac{t}{2}}b^{\frac{2-t}{2}}\right)\right|^q dt\right)^{\frac{1}{q}}\right\} \\
\leq\ & \frac{\ln b-\ln a}{2}\left\{a^3\left|f'(a)\right|\left(\int_0^1\left(\frac{\sqrt{b^{3q}\left|f'(b)\right|^q}}{\sqrt{a^{3q}\left|f'(a)\right|^q}}\right)^t dt\right)^{\frac{1}{q}}\right. \\
&\left.+b^3\left|f'(b)\right|\left(\int_0^1\left(\frac{\sqrt{a^{3q}\left|f'(a)\right|^q}}{\sqrt{b^{3q}\left|f'(b)\right|^q}}\right)^t dt\right)^{\frac{1}{q}}\right\}.
\end{aligned}$$

If we calculate the integrals above, we get the desired result. □

Theorem 5. *Under the assumptions of Theorem 2, the following inequality holds:*

$$\begin{aligned}
&\left|b^2 f(b) - a^2 f(a) - 2\int_a^b u f(u)du\right| \\
\leq\ & \frac{\ln b - \ln a}{2}\left\{\left(\frac{2\sqrt{b^3}-2}{3\ln b}\right)^{\frac{1}{p}} a^3 \left|f^{'}(a)\right| L^{\frac{1}{q}}\left(\sqrt{b^3\left|f'(b)\right|^q}, \sqrt{a^{3q}\left|f'(a)\right|^q}\right)\right. \\
&\left. + \left(\frac{2\sqrt{a^3}-2}{3\ln a}\right)^{\frac{1}{p}} b^3 \left|f^{'}(b)\right| L^{\frac{1}{q}}\left(\sqrt{a^3\left|f'(a)\right|^q}, \sqrt{b^{3q}\left|f'(b)\right|^q}\right)\right\}.
\end{aligned}$$

Proof. From Lemma 2, using the property of the modulus, $GG-$ convexity of $\left|f^{'}\right|^q$ and Hölder integral inequality, we can write

$$\begin{aligned}
&\left|b^2 f(b) - a^2 f(a) - 2\int_a^b u f(u)du\right| \\
\leq\ & \frac{\ln b - \ln a}{2}\left\{\left(\int_0^1 b^{\frac{3t}{2}}dt\right)^{\frac{1}{p}}\left(\int_0^1 b^{\frac{3t}{2}} a^{\frac{3(2-t)q}{2}}\left|f^{'}(b)\right|^{\frac{tq}{2}}\left|f^{'}(a)\right|^{\left(1-\frac{t}{2}\right)q}dt\right)^{\frac{1}{q}}\right. \\
&\left.\left(\int_0^1 a^{\frac{3t}{2}}dt\right)^{\frac{1}{p}}\left(\int_0^1 a^{\frac{3t}{2}} b^{\frac{3(2-t)q}{2}}\left|f^{'}(a)\right|^{\frac{tq}{2}}\left|f^{'}(b)\right|^{\left(1-\frac{t}{2}\right)q}dt\right)^{\frac{1}{q}}\right\}.
\end{aligned}$$

If we calculate the integrals above, we get the desired result. □

Theorem 6. *Let $f : I \subseteq IR_+ = (0, \infty) \longrightarrow IR$ be a differentiable function on I° where a,b∈ I° with $a < b$,and $f^{'} \in L[a, b]$. If $\left|f^{'}\right|^q$ is $GG-$ convex on $[a, b]$ for all $x \in [a, b]$, the following inequality*

$$\begin{aligned}
&\left|b^2 f(b) - a^2 f(a) - 2\int_a^b u f(u)du\right| \\
\leq\ & \frac{\ln b - \ln a}{2}\left\{\left(\left(\sqrt{b^3}\right)^{1-\frac{1}{q}} + \left(\sqrt{a^3}\right)^{1-\frac{1}{q}}\right)\left(L(\sqrt{b^3}, \sqrt{a^3})\right)^{1-\frac{1}{q}}\right. \\
&\left.\left(L\left(\sqrt{b^3\left|f'(b)\right|^q}, \sqrt{a^3\left|f'(a)\right|^q}\right)\right)^{\frac{1}{q}}\left(b^{\frac{3}{2q}}\sqrt{\left|f'(b)\right|} + a^{\frac{3}{2q}}\sqrt{\left|f'(a)\right|}\right)\right\}.
\end{aligned}$$

holds for $q \geq 1$.

Proof. From Lemma 2, using the property of the modulus, $GG-$ convexity of $\left|f^{'}\right|^q$ and power-mean integral inequality, we can write

$$
\begin{aligned}
&\left|b^2 f(b)-a^2 f(a)-2\int_a^b u f(u)du\right| \\
\leq\ & \frac{\ln b-\ln a}{2}\left\{\left(\int_0^1 b^{\frac{3t}{2}}a^{\frac{3(2-t)}{2}}dt\right)^{1-\frac{1}{q}}\left(\int_0^1 b^{\frac{3t}{2}}a^{\frac{3(2-t)}{2}}\left|f'\left(b^{\frac{t}{2}}a^{\frac{2-t}{2}}\right)\right|^q dt\right)^{\frac{1}{q}}\right. \\
&\left.+\left(\int_0^1 a^{\frac{3t}{2}}b^{\frac{3(2-t)}{2}}dt\right)^{1-\frac{1}{q}}\left(\int_0^1 a^{\frac{3t}{2}}b^{\frac{3(2-t)}{2}}\left|f'\left(a^{\frac{t}{2}}b^{\frac{2-t}{2}}\right)\right|^q dt\right)^{\frac{1}{q}}\right\} \\
\leq\ & \frac{\ln b-\ln a}{2}\left\{a^{3\left(1-\frac{1}{q}\right)}\left(\int_0^1\left(\sqrt{\frac{b^3}{a^3}}\right)^t dt\right)^{1-\frac{1}{q}} a^{\frac{3}{q}}\left(\int_0^1\left(\sqrt{\frac{b^3}{a^3}}\right)^t\left|f'(b)\right|^{\frac{qt}{2}}\left|f'(a)\right|^{\left(1-\frac{t}{2}\right)q}dt\right)^{\frac{1}{q}}\right. \\
&\left.+b^{3\left(1-\frac{1}{q}\right)}\left(\int_0^1\left(\sqrt{\frac{a^3}{b^3}}\right)^t dt\right)^{1-\frac{1}{q}} b^{\frac{3}{q}}\left(\int_0^1\left(\sqrt{\frac{a^3}{b^3}}\right)^t\left|f'(a)\right|^{\frac{qt}{2}}\left|f'(b)\right|^{\left(1-\frac{t}{2}\right)q}dt\right)^{\frac{1}{q}}\right\}.
\end{aligned}
$$

We get the desired result by a simple calculation. □

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