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Selfie Numbers, Fibonacci Sequence and Selfie Fractions

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SUMMARY

This summary brings author's work on numbers. The study is made in different ways. Specially towards, ***selfie numbers***, ***unified, patterns, symmetrical representations in selfie numbers***, ***Fibonacci sequence and selfie numbers***, ***flexible power Narcissistic and selfie numbers***, ***selfie fractions***, etc. The *selfie numbers* may also be considered as generalized or wild narcissistic numbers, where natural numbers are represented by their own digits with certain operations. The study is also made towards ***equivalent fractions*** and ***palindromic-type*** numbers. This summary is the second part of the work. For the first part of the summary on **crazy representations of natural numbers** in different situations refer to [26].

The whole work is divided in small sections and subsections summarized as:

1 Selfie Numbers

Numbers represented by their own digits by use of certain operations are considered as *"Selfie Number"*. These numbers, we have divided in two categories. These two categories are again divided in two each,

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i.e., one in order of digits appearing in the numbers and their reverse, and the second is in increasing and decreasing order of digits. See below examples in each category:

- **Digit's Order**

$$\begin{aligned} 936 &= (\sqrt{9})!^3 + 6!; \\ 1296 &= \sqrt{(1+2)!^9/6}; \\ 2896 &= 2 \times (8 + (\sqrt{9})!! + 6!); \\ 12969 &= 1 \times 2 \times 9 \times 6! + 9. \end{aligned}$$

- **Reverse Order of Digits**

$$\begin{aligned} 936 &= 6! + (3!)^{\sqrt{9}}; \\ 1296 &= 6^{(\sqrt{9}+2-1)}; \\ 2896 &= (6! + (\sqrt{9})!! + 8) \times 2; \\ 20167 &= 7 + (6 + 1 + 0!)!/2. \end{aligned}$$

- **Increasing Order of Digits**

$$\begin{aligned} 936 &= 3!! + 6^{\sqrt{9})}; \\ 1296 &= (1+2)! \times 6^{\sqrt{9}}; \\ 8397 &= -3 - 7! + 8!/\sqrt{9}; \\ 241965 &= (1 + (2 \times 4)! + 5) \times 6 + 9. \end{aligned}$$

- **Decreasing Order of Digits**

$$\begin{aligned} 936 &= (\sqrt{9})!! + 6^3; \\ 1296 &= ((\sqrt{9})! \times 6)^2 \times 1; \\ 20148 &= (8! - 4)/2 - 10; \\ 435609 &= 9 + (6! - 5!/\sqrt{4})^{(3-0!)}. \end{aligned}$$

Above we have given examples of *selfie numbers* in four different ways. This has been done using the basic operations along with *factorial* and *square-root*. See below more examples:

$$\begin{aligned} 331779 &:= 3 + (31-7)^{\sqrt{7+9}} &&= \sqrt{9} + (7 \times 7 - 1)^3 \times 3. \\ 342995 &:= (3^4 - 2 - 9)^{\sqrt{9}} - 5 &&= -5 + (-9 + 9^2 - \sqrt{4})^3. \\ 759375 &:= (-7 + 59 - 37)^5 &&= (5 + 7 + 3)^{\sqrt{9}-5+7}. \\ 759381 &:= 7 + (5 \times \sqrt{9})^{-3+8} - 1 &&= -1 + (8 \times 3 - 9)^5 + 7. \end{aligned}$$

Below are interesting numbers following sequential order left side. See below:

$$\begin{aligned} 456 &:= 4 \times (5! - 6) &&= (-6 + 5!) \times 4. \\ 3456 &:= 3!! \times 4/5 \times 6 &&= 6!/5 \times 4 \times 3!!. \\ 34567 &:= (3 + 45) \times 6! + 7 &&= 7 + 6! \times (5 + 43). \\ 345678 &:= (3! - \sqrt{4}) \times 5! \times 6! + 78. \end{aligned}$$

For full details, refer to link [1, 2, 3, 6, 7, 8]:

http://rgmia.org/papers/v17/v17a140.pdf
http://rgmia.org/papers/v18/v18a32.pdf
http://rgmia.org/papers/v18/v18a70.pdf
http://rgmia.org/papers/v18/v18a174.pdf
http://rgmia.org/papers/v18/v18a175.pdf.
http://rgmia.org/papers/v19/v19a16.pdf.

1.1 Selfie Numbers with Addition, Subtraction and Factorial

Examples given above uses basic operation along with factorial and square-root. Madachy [40], page 167, 1966, gave few examples with factorial using only the operation of addition. See below.

$$\begin{aligned} 1 &= 1! \\ 2 &= 2! \\ 145 &= 1! + 4! + 5!. \\ 40585 &= 4! + 0! + 5! + 8! + 5! \end{aligned}$$

Question arises, what else we can get using only the operations of addition and subtraction along with factorial? Below are some examples up to 6-digits:

$$145 = 1! + 4! + 5!.$$

$$1463 = -1! + 4! + 6! + 3!!.$$

$$\begin{aligned} 10077 &= -1! - 0! - 0! + 7! + 7!. \\ 40585 &= 4! + 0! + 5! + 8! + 5!. \\ 80518 &= 8! - 0! - 5! - 1! + 8!. \end{aligned}$$

$$\begin{aligned} 317489 &= -3! - 1! - 7! - 4! - 8! + 9!. \\ 352797 &= -3! + 5 - 2! - 7! + 9! - 7!. \end{aligned}$$

$$\begin{aligned} 357592 &= -3! - 5! - 7! - 5! + 9! - 2!. \\ 357941 &= 3! + 5! - 7! + 9! - 4! - 1!. \\ 361469 &= 3! - 6! - 1! + 4! - 6! + 9!. \\ 364292 &= 3!! + 6! - 4! - 2! + 9! - 2!. \\ 397584 &= -3!! + 9! - 7! + 5! + 8! + 4!. \\ 398173 &= 3! + 9! + 8! + 1! - 7! + 3!. \\ 408937 &= -4! + 0! + 8! + 9! + 3!! + 7!. \\ 715799 &= -7! - 1! + 5! - 7! + 9! + 9!. \\ 720599 &= -7! - 2! + 0! - 5! + 9! + 9!. \end{aligned}$$

Still, we can have examples not necessarily having factorial on all the numbers. See below:

$$\begin{aligned} 733 &= 7 + 3!! + 3!. \\ 5177 &= 5! + 17 + 7!. \\ 363239 &= 36 + 323 + 9!. \\ 363269 &= 363 + 26 + 9!. \\ 403199 &= 40319 + 9!. \end{aligned}$$

For more details refer to author's work [28]:

http://rgmia.org/papers/v19/v19a163.pdf

1.2 Unified Representations

From examples above, we observe that there are numbers such as 936, 1296, etc., that can be written in all the four ways. These types of numbers we call *unified selfie numbers*. More clearly,

$$\begin{aligned}\text{Unified Selfie number} &= \text{Order of digits}\\ &= \text{Reverse order of digits}\\ &= \text{Increasing order of digits}\\ &= \text{Decreasing order of digits.}\end{aligned}$$

Below are examples of *unified selfie numbers* written in all the four ways,

- $$\begin{aligned}729 &= (\sqrt{7+2})!! + 9\\ &= 9 + (\sqrt{2+7})!!\\ &= (2+7)^{\sqrt{9}}\\ &= 9^{\sqrt{7+2}}.\end{aligned}$$

- $$\begin{aligned}97632 &= -(\sqrt{9})!! + 7! + 6^{3!} \times 2\\ &= 2 \times 3!^6 + 7! - (\sqrt{9})!!\\ &= 2 \times 3!^6 + 7! - (\sqrt{9})!!\\ &= -(\sqrt{9})!! + 7! + 6^{3!} \times 2.\end{aligned}$$

- $$\begin{aligned}114688 &= (11 \times \sqrt{4} + 6) \times \sqrt{8^8}\\ &= (8 + 8 \times 6) \times \sqrt{4^{11}}\\ &= (11 \times \sqrt{4} + 6) \times \sqrt{8^8}\\ &= (8 + 8 \times 6) \times \sqrt{4^{11}}.\end{aligned}$$

- $$\begin{aligned}139968 &= (13 \times 9 - 9) \times \sqrt{6^8}\\ &= 8 \times 6^{\sqrt{9}} \times 9^{3-1}\\ &= 1 \times \sqrt{36^8}/(\sqrt{9} + 9)\\ &= (-\sqrt{9} + 9)^8/(6 \times (3-1)).\end{aligned}$$

- $$\begin{aligned}326627 &= (3 + 2 + 6^{\sqrt{6^2}}) \times 7\\ &= \sqrt{7^2} \times (6^6 + 2 + 3)\\ &= (\sqrt{2+23} + 6^6) \times 7\\ &= 7 \times (6^6 + \sqrt{3+22}).\end{aligned}$$

- $$\begin{aligned}531439 &= -5 + 3 + (-1+4)^{3+9}\\ &= (9 \times 3)^4 \times 1 + 3 - 5\\ &= 1 - 3 + 3^{\sqrt{4+5}+9}\\ &= 9^{5+4-3} - \sqrt{3+1}.\end{aligned}$$

For more details refer to author's work [4, 6, 7, 8]:

http://rgmia.org/papers/v18/v18a153.pdf
http://rgmia.org/papers/v18/v18a174.pdf
http://rgmia.org/papers/v18/v18a175.pdf
http://rgmia.org/papers/v19/v19a16.pdf.

1.3 Patterns in Selfie Numbers

Numbers extended with same properties multiplying by zero, we consider as *patterns in numbers*. Few examples of this kind are studied long back in 1966 by Madachy [40], page 174–175. See below:

$$\begin{aligned}3^4 \times 425 &= 34425\\ 3^4 \times 4250 &= 344250\\ 3^4 \times 42500 &= 3442500.\end{aligned}$$

$$31^2 \times 325 = 312325$$
$$31^2 \times 3250 = 3123250$$
$$31^2 \times 32500 = 31232500.$$

For simplicity, let us write these numbers as *patterns in selfie numbers*. Subsections below give more examples with factorial and square-root:

- **Digit's Order**

Numbers appearing in this subsection are represented in order of digits.

$$1285 = (1 + 2^8) \times 5$$
$$12850 = (1 + 2^8) \times 50$$
$$128500 = (1 + 2^8) \times 500$$

$$15585 = 1 \times (5^5 - 8) \times 5$$
$$155850 = 1 \times (5^5 - 8) \times 50$$
$$1558500 = 1 \times (5^5 - 8) \times 500$$

$$8192 = 8^{1+\sqrt{9}} \times 2$$
$$81920 = 8^{1+\sqrt{9}} \times 20$$
$$819200 = 8^{1+\sqrt{9}} \times 200$$

$$29435 = \sqrt{29^4} \times 35$$
$$294350 = \sqrt{29^4} \times 350$$
$$2943500 = \sqrt{29^4} \times 3500$$

- **Decreasing Order of Digits**

The numbers appearing in this subsection are represented decreasing order of digits.

$$1827 = 87 \times 21$$
$$18270 = 87 \times 210$$
$$182700 = 87 \times 2100$$

$$19683 = \sqrt{9^8} \times (6 - 3) \times 1$$
$$196830 = \sqrt{9^8} \times (6 - 3) \times 10$$
$$1968300 = \sqrt{9^8} \times (6 - 3) \times 100$$

$$2916 = (9 \times 6)^2 \times 1$$
$$29160 = (9 \times 6)^2 \times 10$$
$$291600 = (9 \times 6)^2 \times 100$$

$$995544 = ((\sqrt{9} + 9)^5 + 54) \times 4$$
$$9955440 = ((\sqrt{9} + 9)^5 + 54) \times 40$$
$$99554400 = ((\sqrt{9} + 9)^5 + 54) \times 400$$

For more details, refer to author's work [6, 7, 8]:

http://rgmia.org/papers/v18/v18a154.pdf
http://rgmia.org/papers/v18/v18a174.pdf
http://rgmia.org/papers/v18/v18a175.pdf.

Above we have written *patterns in selfie numbers*. But there are another kind of patterns in numbers. This study in done in Part 1 [26]. Also refer 7.1.

1.4 Symmetric Consecutive Representations

There are numbers those can be represented in symmetric and consecutive forms of block of 10 or more. In some cases the numbers appears only in one way, i.e., either in order of digits appearing in number or in reverse order of digits. In some cases we have representations is in both ways. See below some examples:

$$14400 := (1+4)!^{\sqrt{4}} + 0 + 0 = 0 + (0!+4)!^{\sqrt{4}} \times 1$$
$$14401 := (1+4)!^{\sqrt{4}} + 0 + 1 = 1 + (0!+4)!^{\sqrt{4}} \times 1$$
$$14402 := (1+4)!^{\sqrt{4}} + 0 + 2 = 2 + (0!+4)!^{\sqrt{4}} \times 1$$
$$14403 := (1+4)!^{\sqrt{4}} + 0 + 3 = 3 + (0!+4)!^{\sqrt{4}} \times 1$$
$$14404 := (1+4)!^{\sqrt{4}} + 0 + 4 = 4 + (0!+4)!^{\sqrt{4}} \times 1$$
$$14405 := (1+4)!^{\sqrt{4}} + 0 + 5 = 5 + (0!+4)!^{\sqrt{4}} \times 1$$
$$14406 := (1+4)!^{\sqrt{4}} + 0 + 6 = 6 + (0!+4)!^{\sqrt{4}} \times 1$$
$$14407 := (1+4)!^{\sqrt{4}} + 0 + 7 = 7 + (0!+4)!^{\sqrt{4}} \times 1$$
$$14408 := (1+4)!^{\sqrt{4}} + 0 + 8 = 8 + (0!+4)!^{\sqrt{4}} \times 1$$
$$14409 := (1+4)!^{\sqrt{4}} + 0 + 9 = 9 + (0!+4)!^{\sqrt{4}} \times 1$$

$$64800 = 6!^{\sqrt{4}}/8 + 0 + 0.$$
$$64801 = 6!^{\sqrt{4}}/8 + 0 + 1.$$
$$64802 = 6!^{\sqrt{4}}/8 + 0 + 2.$$
$$64803 = 6!^{\sqrt{4}}/8 + 0 + 3.$$
$$64804 = 6!^{\sqrt{4}}/8 + 0 + 4.$$
$$64805 = 6!^{\sqrt{4}}/8 + 0 + 5.$$
$$64806 = 6!^{\sqrt{4}}/8 + 0 + 6.$$
$$64807 = 6!^{\sqrt{4}}/8 + 0 + 7.$$
$$64808 = 6!^{\sqrt{4}}/8 + 0 + 8.$$
$$64809 = 6!^{\sqrt{4}}/8 + 0 + 9.$$

In the increasing case, above two sequences can be extended up to 14499 and 64899 respectively, forming sets of 100 consecutive symmetric representations:

$$14410 = (1+4)!^{\sqrt{4}} + 10.$$
$$14411 = (1+4)!^{\sqrt{4}} + 11.$$
......
$$14498 = (1+4)!^{\sqrt{4}} + 98.$$
$$14499 = (1+4)!^{\sqrt{4}} + 99.$$

$$64810 = 6!^{\sqrt{4}}/8 + 10.$$
$$64811 = 6!^{\sqrt{4}}/8 + 11.$$
......
$$64898 = 6!^{\sqrt{4}}/8 + 98.$$
$$64899 = 6!^{\sqrt{4}}/8 + 99.$$

More examples of symmetrical consecutive representations of selfie numbers are follows as:

$$466560 := (4+6) \times 6^5 \times 6 + 0 = 0 + 6^5 \times 6 \times (6+4).$$
$$466561 := (4+6) \times 6^5 \times 6 + 1 = 1 + 6^5 \times 6 \times (6+4).$$
$$466562 := (4+6) \times 6^5 \times 6 + 2 = 2 + 6^5 \times 6 \times (6+4).$$
$$466563 := (4+6) \times 6^5 \times 6 + 3 = 3 + 6^5 \times 6 \times (6+4).$$
$$466564 := (4+6) \times 6^5 \times 6 + 4 = 4 + 6^5 \times 6 \times (6+4).$$
$$466565 := (4+6) \times 6^5 \times 6 + 5 = 5 + 6^5 \times 6 \times (6+4).$$
$$466566 := (4+6) \times 6^5 \times 6 + 6 = 6 + 6^5 \times 6 \times (6+4).$$
$$466567 := (4+6) \times 6^5 \times 6 + 7 = 7 + 6^5 \times 6 \times (6+4).$$
$$466568 := (4+6) \times 6^5 \times 6 + 8 = 8 + 6^5 \times 6 \times (6+4).$$
$$466569 := (4+6) \times 6^5 \times 6 + 9 = 9 + 6^5 \times 6 \times (6+4).$$

$$64840 := 6!^{\sqrt{4}}/8 + 40 = 0 + 4^8 + 4! - 6!.$$
$$64841 := 6!^{\sqrt{4}}/8 + 41 = 1 + 4^8 + 4! - 6!.$$
$$64842 := 6!^{\sqrt{4}}/8 + 42 = 2 + 4^8 + 4! - 6!.$$
$$64843 := 6!^{\sqrt{4}}/8 + 43 = 3 + 4^8 + 4! - 6!.$$
$$64844 := 6!^{\sqrt{4}}/8 + 44 = 4 + 4^8 + 4! - 6!.$$
$$64845 := 6!^{\sqrt{4}}/8 + 45 = 5 + 4^8 + 4! - 6!.$$
$$64846 := 6!^{\sqrt{4}}/8 + 46 = 6 + 4^8 + 4! - 6!.$$
$$64847 := 6!^{\sqrt{4}}/8 + 47 = 7 + 4^8 + 4! - 6!.$$
$$64848 := 6!^{\sqrt{4}}/8 + 48 = 8 + 4^8 + 4! - 6!.$$
$$64849 := 6!^{\sqrt{4}}/8 + 49 = 9 + 4^8 + 4! - 6!.$$

$$15637 := -1+3!+5^6+7 = 7+6+5^{3!}-1.$$
$$15638 := -1+3!+5^6+8 = 8+6+5^{3!}-1.$$
$$15639 := -1+3!+5^6+9 = 9+6+5^{3!}-1.$$

$$30245 = 5+(4+3)!\times(2+0!)!.$$
$$30246 = 6+(4+3)!\times(2+0!)!.$$
$$30247 = 7+(4+3)!\times(2+0!)!.$$
$$30248 = 8+(4+3)!\times(2+0!)!.$$
$$30249 = 9+(4+3)!\times(2+0!)!.$$

$$790 = (\sqrt{9})!!+70.$$
$$791 = (\sqrt{9})!!+71.$$
$$792 = (\sqrt{9})!!+72.$$
$$793 = (\sqrt{9})!!+73.$$
$$794 = (\sqrt{9})!!+74.$$
$$795 = (\sqrt{9})!!+75.$$
$$796 = (\sqrt{9})!!+76.$$

For the numbers 15637 and 30245, we have symmetry only for few numbers. Moreover, these two examples are increasing and decreasing order of digits. For complete details, refer the following links [6, 7, 8]:

http://rgmia.org/papers/v18/v18a174.pdf
http://rgmia.org/papers/v18/v18a175.pdf
http://rgmia.org/papers/v19/v19a16.pdf.

Above examples are *symmetric selfie* representations up to 5-digits. Below are examples of 6-digits *symmetric selfie* representations:

$$518400 := (5+1)!^{8/4}+00 = 00+(4!/8)!!\times(1+5)!.$$
$$518411 := (5+1)!^{8/4}+11 = 11+(4!/8)!!\times(1+5)!.$$
$$518422 := (5+1)!^{8/4}+22 = 22+(4!/8)!!\times(1+5)!.$$
$$518433 := (5+1)!^{8/4}+33 = 33+(4!/8)!!\times(1+5)!.$$
$$518444 := (5+1)!^{8/4}+44 = 44+(4!/8)!!\times(1+5)!.$$
$$518455 := (5+1)!^{8/4}+55 = 55+(4!/8)!!\times(1+5)!.$$
$$518466 := (5+1)!^{8/4}+66 = 66+(4!/8)!!\times(1+5)!.$$
$$518477 := (5+1)!^{8/4}+77 = 77+(4!/8)!!\times(1+5)!.$$
$$518488 := (5+1)!^{8/4}+88 = 88+(4!/8)!!\times(1+5)!.$$
$$518499 := (5+1)!^{8/4}+99 = 99+(4!/8)!!\times(1+5)!.$$

We observe that the above example is *symmetric selfie* representation but is not consecutive. Below is consecutive *symmetric selfie* representation:

$$363390 := 3! + 6! - 3!^3 + 9! + 0 = 0 + 9! + (3 \times 3)!/6! + 3!.$$
$$363391 := 3! + 6! - 3!^3 + 9! + 1 = 1 + 9! + (3 \times 3)!/6! + 3!.$$
$$363392 := 3! + 6! - 3!^3 + 9! + 2 = 2 + 9! + (3 \times 3)!/6! + 3!.$$
$$363393 := 3! + 6! - 3!^3 + 9! + 3 = 3 + 9! + (3 \times 3)!/6! + 3!.$$
$$363394 := 3! + 6! - 3!^3 + 9! + 4 = 4 + 9! + (3 \times 3)!/6! + 3!.$$
$$363395 := 3! + 6! - 3!^3 + 9! + 5 = 5 + 9! + (3 \times 3)!/6! + 3!.$$
$$363396 := 3! + 6! - 3!^3 + 9! + 6 = 6 + 9! + (3 \times 3)!/6! + 3!.$$
$$363397 := 3! + 6! - 3!^3 + 9! + 7 = 7 + 9! + (3 \times 3)!/6! + 3!.$$
$$363398 := 3! + 6! - 3!^3 + 9! + 8 = 8 + 9! + (3 \times 3)!/6! + 3!.$$
$$363399 := 3! + 6! - 3!^3 + 9! + 9 = 9 + 9! + (3 \times 3)!/6! + 3!.$$

Still there are consecutive *symmetric selfie* representations of blocks of 100 with 6-digits. See examples below:

$$158400 := (-(1 + 5)! + 8!) \times 4 + 00.$$
$$158401 := (-(1 + 5)! + 8!) \times 4 + 01.$$
$$158402 := (-(1 + 5)! + 8!) \times 4 + 02.$$
$$158403 := (-(1 + 5)! + 8!) \times 4 + 03.$$
$$\dots \quad \dots \quad \dots$$
$$158451 := (-(1 + 5)! + 8!) \times 4 + 51.$$
$$158452 := (-(1 + 5)! + 8!) \times 4 + 52.$$
$$158453 := (-(1 + 5)! + 8!) \times 4 + 53.$$
$$158454 := (-(1 + 5)! + 8!) \times 4 + 54.$$
$$\dots \quad \dots \quad \dots$$
$$158496 := (-(1 + 5)! + 8!) \times 4 + 96.$$
$$158497 := (-(1 + 5)! + 8!) \times 4 + 97.$$
$$158498 := (-(1 + 5)! + 8!) \times 4 + 98.$$
$$158499 := (-(1 + 5)! + 8!) \times 4 + 99.$$

$$363600 := (3^{6/3})! + 6! + 00.$$
$$363601 := (3^{6/3})! + 6! + 01.$$
$$363602 := (3^{6/3})! + 6! + 02.$$
$$363603 := (3^{6/3})! + 6! + 03.$$
$$\dots \quad \dots \quad \dots$$
$$363651 := (3^{6/3})! + 6! + 51.$$
$$363652 := (3^{6/3})! + 6! + 52.$$
$$363653 := (3^{6/3})! + 6! + 53.$$
$$363654 := (3^{6/3})! + 6! + 54.$$
$$\dots \quad \dots \quad \dots$$
$$363696 := (3^{6/3})! + 6! + 96.$$
$$363697 := (3^{6/3})! + 6! + 97.$$
$$363698 := (3^{6/3})! + 6! + 98.$$
$$363699 := (3^{6/3})! + 6! + 99.$$

For more details refer the following link [29]:

http://rgmia.org/papers/v19/v19a164.pdf

2 Fibonacci Sequence and Selfie Numbers

Fibonacci sequence numbers are well known in literature [32, 33]. This sequence is defined as

$$F(0) = 0, \quad F(1) = 1, \quad F(n + 1) = F(n) + F(n - 1), \quad n \geq 1.$$

In [20], we worked with selfie numbers using the terms of Fibonacci sequences as $F(.)$. See examples below:

$$256 = 2^5 \times F(6).$$
$$46493 = F(4 \times 6) + (-4 + 9)^3.$$
$$882 = 2 \times F(8) \times F(8).$$

$$1631 = F(13) \times (6 + 1).$$
$$54128 = 8 \times (F(2) + F(1 \times 4 \times 5)).$$

The first two examples are in digit's order and last three are in reverse order of digits. In [21], we worked with *composition* of *Fibonacci sequence* values, such as $F(F(.))$, $F(F(F(.)))$. See examples below:

$$235 = 2 + F(F(F(3) + 5)).$$
$$4427 = (F(4) + 4^2) \times F(F(7)).$$

$$63 = 3 \times F(F(6)).$$
$$43956 = (F(F(F(6))) + 5 \times 9 - F(3)) \times 4.$$

The first two examples are in order of digits, and last two examples are in reverse order of digits.

Using the idea of both the papers [20, 21] along with *factorial* are used in [22]. See below more examples,

$$447 = (F(4))!! - F(F((F(4))!)) \times F(7).$$
$$29471 = (F(2) + F(9)) \times F(F(F((F(4))!)))/F(7) + 1.$$

$$433 = F(F(3!))^{F(3)} - F(F(4)!).$$
$$4995 = -5 \times 9 + (9 - F(F(4)))!.$$

For details refer the following works [20, 21, 22]:

http://rgmia.org/papers/v19/v19a142.pdf
http://rgmia.org/papers/v19/v19a143.pdf
http://rgmia.org/papers/v19/v19a156.pdf.

First work is just with $F(.)$, second work is with composition of F, i.e., $F(F(.))$, etc. The third paper combines first and second along with use of *factorial*. Still there are more possibilities of working with *square-root*. See below some examples,

$$954 = F((\sqrt{9})!) \times 5! - F(4)!.$$
$$1439 = 1 + \sqrt{4} \times 3!! - F(\sqrt{9}).$$
$$4394 = F(4 + 3)^{\sqrt{9}} \times \sqrt{4}.$$
$$89735 = (F(F(8) + F(F(\sqrt{9}))) + F(F(7)) + 3) \times 5.$$

$$
\begin{aligned}
1919 &= (F((\sqrt{9})!)!/F(-1+9)) - 1.\\
6498 &= -F(8) \times \sqrt{9} + F(4)^{F(6)}.\\
12784 &= F(F(\sqrt{4})+8) \times (F(7\times 2) - 1).\\
39901 &= -F(10) \times F((\sqrt{9})!) + F(F((\sqrt{9})!)) + F(3!)!.
\end{aligned}
$$

First four examples are in digit's order and in second four are in reverse order of digits. Detailed study on this type of work shall be dealt elsewhere.

More examples with *Fibonacci sequence* values written in symmetric form are given in section below:

2.1 Symmetrical Representations with Fibonacci Sequence Values

Below are some examples of symmetrical representations of *selfie numbers* by use of *Fibonacci sequence* values:

$$
\begin{aligned}
823540 &:= (8-F(2))^{F(3)+5} - F(4) + 0 = 0 - F(4) + (5+F(3))^{-F(2)+8}.\\
823541 &:= (8-F(2))^{F(3)+5} - F(4) + 1 = 1 - F(4) + (5+F(3))^{-F(2)+8}.\\
823542 &:= (8-F(2))^{F(3)+5} - F(4) + 2 = 2 - F(4) + (5+F(3))^{-F(2)+8}.\\
823543 &:= (8-F(2))^{F(3)+5} - F(4) + 3 = 3 - F(4) + (5+F(3))^{-F(2)+8}.\\
823544 &:= (8-F(2))^{F(3)+5} - F(4) + 4 = 4 - F(4) + (5+F(3))^{-F(2)+8}.\\
823545 &:= (8-F(2))^{F(3)+5} - F(4) + 5 = 5 - F(4) + (5+F(3))^{-F(2)+8}.\\
823546 &:= (8-F(2))^{F(3)+5} - F(4) + 6 = 6 - F(4) + (5+F(3))^{-F(2)+8}.\\
823547 &:= (8-F(2))^{F(3)+5} - F(4) + 7 = 7 - F(4) + (5+F(3))^{-F(2)+8}.\\
823548 &:= (8-F(2))^{F(3)+5} - F(4) + 8 = 8 - F(4) + (5+F(3))^{-F(2)+8}.\\
823549 &:= (8-F(2))^{F(3)+5} - F(4) + 9 = 9 - F(4) + (5+F(3))^{-F(2)+8}.
\end{aligned}
$$

$$
\begin{aligned}
54670 &:= 5 \times (F(F(F(4))) + F(F(F(6))) - F(7)) + 0 = 0 + (-F(7) + F(F(F(6))) + F(F(F(4)))) \times 5.\\
54671 &:= 5 \times (F(F(F(4))) + F(F(F(6))) - F(7)) + 1 = 1 + (-F(7) + F(F(F(6))) + F(F(F(4)))) \times 5.\\
54672 &:= 5 \times (F(F(F(4))) + F(F(F(6))) - F(7)) + 2 = 2 + (-F(7) + F(F(F(6))) + F(F(F(4)))) \times 5.\\
54673 &:= 5 \times (F(F(F(4))) + F(F(F(6))) - F(7)) + 3 = 3 + (-F(7) + F(F(F(6))) + F(F(F(4)))) \times 5.\\
54674 &:= 5 \times (F(F(F(4))) + F(F(F(6))) - F(7)) + 4 = 4 + (-F(7) + F(F(F(6))) + F(F(F(4)))) \times 5.\\
54675 &:= 5 \times (F(F(F(4))) + F(F(F(6))) - F(7)) + 5 = 5 + (-F(7) + F(F(F(6))) + F(F(F(4)))) \times 5.\\
54676 &:= 5 \times (F(F(F(4))) + F(F(F(6))) - F(7)) + 6 = 6 + (-F(7) + F(F(F(6))) + F(F(F(4)))) \times 5.\\
54677 &:= 5 \times (F(F(F(4))) + F(F(F(6))) - F(7)) + 7 = 7 + (-F(7) + F(F(F(6))) + F(F(F(4)))) \times 5.\\
54678 &:= 5 \times (F(F(F(4))) + F(F(F(6))) - F(7)) + 8 = 8 + (-F(7) + F(F(F(6))) + F(F(F(4)))) \times 5.\\
54679 &:= 5 \times (F(F(F(4))) + F(F(F(6))) - F(7)) + 9 = 9 + (-F(7) + F(F(F(6))) + F(F(F(4)))) \times 5.
\end{aligned}
$$

$$3840 := (F(3!))!/F(8) \times F(F(4)) + 0 = 0 + F(F(4)) \times 8!/F(F(3!)).$$
$$3841 := (F(3!))!/F(8) \times F(F(4)) + 1 = 1 + F(F(4)) \times 8!/F(F(3!)).$$
$$3842 := (F(3!))!/F(8) \times F(F(4)) + 2 = 2 + F(F(4)) \times 8!/F(F(3!)).$$
$$3843 := (F(3!))!/F(8) \times F(F(4)) + 3 = 3 + F(F(4)) \times 8!/F(F(3!)).$$
$$3844 := (F(3!))!/F(8) \times F(F(4)) + 4 = 4 + F(F(4)) \times 8!/F(F(3!)).$$
$$3845 := (F(3!))!/F(8) \times F(F(4)) + 5 = 5 + F(F(4)) \times 8!/F(F(3!)).$$
$$3846 := (F(3!))!/F(8) \times F(F(4)) + 6 = 6 + F(F(4)) \times 8!/F(F(3!)).$$
$$3847 := (F(3!))!/F(8) \times F(F(4)) + 7 = 7 + F(F(4)) \times 8!/F(F(3!)).$$
$$3848 := (F(3!))!/F(8) \times F(F(4)) + 8 = 8 + F(F(4)) \times 8!/F(F(3!)).$$
$$3849 := (F(3!))!/F(8) \times F(F(4)) + 9 = 9 + F(F(4)) \times 8!/F(F(3!)).$$

Still, we have symmetric representations with numbers $F(2)$, $F(3)$ and $F(4)$.

$$73792 := (7 + F(3)^{F(7)}) \times 9 + F(2) = F(2) + 9 \times (7 + F(3)^{F(7)}).$$
$$73793 := (7 + F(3)^{F(7)}) \times 9 + F(3) = F(3) + 9 \times (7 + F(3)^{F(7)}).$$
$$73794 := (7 + F(3)^{F(7)}) \times 9 + F(4) = F(4) + 9 \times (7 + F(3)^{F(7)}).$$

$$65652 := F(F(F(6))) + 5 \times (F(F(F(6))) - 5) + F(2) = F(2) + (-5 + F(F(F(6)))) \times 5 + F(F(F(6))).$$
$$65653 := F(F(F(6))) + 5 \times (F(F(F(6))) - 5) + F(3) = F(3) + (-5 + F(F(F(6)))) \times 5 + F(F(F(6))).$$
$$65654 := F(F(F(6))) + 5 \times (F(F(F(6))) - 5) + F(4) = F(4) + (-5 + F(F(F(6)))) \times 5 + F(F(F(6))).$$

$$65672 := -F(F(F(6))) - 5 + F(F(F(6))) \times 7 + F(2) = F(2) + 7 \times F(F(F(6))) - 5 - F(F(F(6))).$$
$$65673 := -F(F(F(6))) - 5 + F(F(F(6))) \times 7 + F(3) = F(3) + 7 \times F(F(F(6))) - 5 - F(F(F(6))).$$
$$65674 := -F(F(F(6))) - 5 + F(F(F(6))) \times 7 + F(4) = F(4) + 7 \times F(F(F(6))) - 5 - F(F(F(6))).$$

Above symmetrical examples are just with $F(.)$, $F(F(.))$, etc. Still there are examples with factorial and square-roots. For more details refer to [20, 21, 22]:

***http://rgmia.org/papers/v19/v19a142.pdf*.**
***http://rgmia.org/papers/v19/v19a143.pdf*.**
***http://rgmia.org/papers/v19/v19a156.pdf*.**

2.2 Number Patterns with Fibonacci Sequence Values

Below are representations of numbers those can be extended just multiplying by 10. These numbers we call, number patterns with Fibonacci numbers sequence values:

$$3528 = F(3+5)^2 \times 8$$
$$35280 = F(3+5)^2 \times 80$$
$$352800 = F(3+5)^2 \times 800.$$

$$3635 = (3^6 - F(3)) \times 5$$
$$36350 = (3^6 - F(3)) \times 50$$
$$363500 = (3^6 - F(3)) \times 500.$$

$$1365 = 13 \times F(F(6)) \times 5$$
$$13650 = 13 \times F(F(6)) \times 50$$
$$136500 = 13 \times F(F(6)) \times 500.$$

$$1687 = (F(F(1+6)) + 8) \times 7$$
$$16870 = (F(F(1+6)) + 8) \times 70$$
$$168700 = (F(F(1+6)) + 8) \times 700.$$

3 Narcissistic Numbers

An $n-$digit number that is the sum of the n^{th} powers of its digits is called an $n-$narcissistic numbers. It is also sometimes known as an Armstrong number, perfect digital invariant. Below are examples, of narcissistic numbers with width 3 and 4:

$$153 = 1^3 + 5^3 + 3^3.$$
$$370 = 3^3 + 7^3 + 0^3.$$
$$371 = 3^3 + 7^3 + 1^3.$$
$$407 = 4^3 + 0^3 + 7^3.$$

$$1634 = 1^4 + 6^4 + 3^4 + 4^4.$$
$$4151 = 4^5 + 1^5 + 5^5 + 1^5.$$
$$8208 = 8^4 + 2^4 + 0^4 + 8^4.$$
$$9472 = 9^4 + 4^4 + 7^4 + 2^4.$$

3.1 Flexible Power Narcissistic Numbers

The narcissistic numbers given above are with fixed power and with positive signs. There are many narcissistic numbers with flexibility in power and sign, for example,

$$24 = 2^3 + 4^2.$$
$$43 = 4^2 + 3^3.$$
$$23 = -2^2 + 3^3.$$
$$48 = -4^2 + 8^2.$$

$$2345 = 2^5 + 3^7 + 4^0 + 5^3.$$
$$2352 = 2^3 + 3^7 + 5^3 + 2^5.$$
$$2371 = -2^1 - 3^3 + 7^4 - 1^0.$$
$$2374 = -2^1 - 3^2 + 7^4 - 4^2.$$

$$263 = 2^8 + 6^1 + 3^0.$$
$$267 = 2^1 + 6^3 + 7^2.$$
$$337 = -3^1 - 3^1 + 7^3.$$
$$354 = -3^3 + 5^3 + 4^4.$$

$$10693 = 1^1 + 0^1 + 6^5 + 9^3 + 3^7.$$
$$10694 = 1^1 + 0^1 + 6^2 + 9^4 + 4^6.$$
$$10846 = -1^1 - 0^0 + 8^4 - 4^5 + 6^5.$$
$$10933 = -1^1 - 0^0 + 9^4 + 3^7 + 3^7.$$

The above results are given with positive and negative operations. But the power is always positive and flexible. These types of numbers, we can as *flexible power narcissistic numbers*. For complete detail refer to author's work [10]:

http://rgmia.org/papers/v19/v19a32.pdf.

3.2 Fixed Power Narcissistic Numbers with Divisions

There are very few *narcissistic numbers* with fixed power that can be written in terms of division, for examples,

$$37 = \frac{3^3 + 7^3}{3 + 7}. \qquad 48 = \frac{4^3 + 8^3}{4 + 8}.$$

$$241 = \frac{2^8 + 4^8 + 1^8}{2^4 + 4^4 + 1^1}. \qquad 415 = \frac{4^5 + 1^5 + 5^5}{4 + 1 + 5}.$$

$$2464 = \frac{2^5 + 4^5 + 6^5 + 4^5}{2^0 + 4^0 + 6^0 + 4^0}. \qquad 4714 = \frac{4^5 + 7^5 + 1^5 + 4^5}{4^0 + 7^0 + 1^0 + 4^0}.$$

$$5247 = \frac{5^5 + 2^5 + 4^5 + 7^5}{5^0 + 2^0 + 4^0 + 7^0}. \qquad 8200 = \frac{8^5 + 2^5 + 0^5 + 0^5}{8^0 + 2^0 + 0^0 + 0^0}.$$

$$15501 = \frac{1^9 + 5^9 + 5^9 + 0^9 + 1^9}{1^3 + 5^3 + 5^3 + 0^3 + 1^3}. \qquad 142740 = \frac{1^7 + 4^7 + 2^7 + 7^7 + 4^7 + 0^7}{1^0 + 4^0 + 2^0 + 7^0 + 4^0 + 0^0}.$$

$$231591 = \frac{2^7 + 3^7 + 1^7 + 5^7 + 9^7 + 1^7}{2 + 3 + 1 + 5 + 9 + 1}.$$

The above numbers are with positive coefficients. Allowing negative coefficients, still there are more numbers of similar kind:

$$264 = \frac{2^4 + 6^4 - 4^4}{2 + 6 - 4}. \qquad 407 = \frac{4^6 + 0^6 - 7^6}{4^3 + 0^3 - 7^3}.$$

$$803 = \frac{8^4 + 0^4 - 3^4}{8 + 0 - 3}. \qquad 6181 = \frac{6^7 + 1^7 - 8^7 + 1^7}{6^3 + 1^3 - 8^3 + 1^3}.$$

$$54901 = \frac{5^4 + 4^4 - 9^4 + 0^4 + 1^4}{5 + 4 - 9 + 0 + 1}.$$

3.3 Flexible Power Narcissistic Numbers with Divisions

Following the same idea of previous subsection 3.2 , here also we have written fractions with fixed and flexible power along with positive-negative coefficients, such as,

$$13 = \frac{-1^3 + 3^3}{1^0 + 3^0}. \qquad 63 = \frac{6^3 - 3^3}{6^1 - 3^1}.$$

$$353 = \frac{-3^5 - 5^2 + 3^9}{3^1 + 5^2 + 3^3}. \qquad 355 = \frac{3^{11} - 5^0 - 5^0}{-3^0 - 5^3 + 5^4}.$$

$$1337 = \frac{1^0 + 3^1 + 3^1 + 7^6}{-1^0 + 3^0 + 3^4 + 7^1}. \qquad 1343 = \frac{1^0 + 3^6 - 4^7 + 3^9}{-1^0 - 3^0 + 4^1 + 3^0}.$$

$$10954 = \frac{-1^0 - 0^0 + 9^3 + 5^2 + 4^9}{1^0 + 0^0 + 9^0 + 5^1 + 4^2}. \qquad 10958 = \frac{-1^0 + 0^0 + 9^2 + 5^2 + 8^5}{-1^0 + 0^0 + 9^0 + 5^0 + 8^0}.$$

$$118378 = \frac{1^{12} - 1^{12} + 8^{12} - 3^{12} + 7^{12} - 8^{12}}{1^6 - 1^6 + 8^6 - 3^6 + 7^6 - 8^6}. \qquad 122347 = \frac{1^6 - 2^6 - 2^6 + 3^6 + 4^6 + 7^6}{1^1 + 2^1 - 2^1 + 3^1 + 4^1 - 7^1}.$$

For complete detail refer to author's work [10]:

http://rgmia.org/papers/v19/v19a32.pdf.

3.4 Floor Function and Narcissistic Numbers with Divisions

In this section, we shall bring numbers in such a way that they becomes *narcissistic numbers* with division in terms of *"floor function"*. Below are examples,

$$115 = \left\lfloor \frac{1^5 + 1^5 + 5^5}{1^2 + 1^2 + 5^2}. \right\rfloor.$$

$$21 = \left\lfloor \frac{2^6 + 1^6}{2 + 1}. \right\rfloor.$$

$$23 = \left\lfloor \frac{2^7 + 3^7}{2^4 + 3^4}. \right\rfloor. \qquad 16737 = \left\lfloor \frac{1^{32} + 6^{32} + 7^{32} + 3^{32} + 7^{32}}{1^{27} + 6^{27} + 7^{27} + 3^{27} + 7^{27}}. \right\rfloor.$$

$$102 = \left\lfloor \frac{1^9 + 0^9 + 2^9}{1^2 + 0^2 + 2^2}. \right\rfloor. \qquad 56494 = \left\lfloor \frac{5^{13} + 6^{13} + 4^{13} + 9^{13} + 4^{13}}{5^8 + 6^8 + 4^8 + 9^8 + 4^8}. \right\rfloor.$$

For complete detail refer to author's work [11]:

http://rgmia.org/papers/v19/v19a33.pdf.

4 Flexible Power Selfie Representations

From numbers given in section 3.1 , we observe that there are numbers, 23, 1239, 1364, 1654, 1837, 2137, 2173, 2537, 3125, 3275, 3529 and 4316 uses the same digits as of number along with power too, such as,

$$23 = -2^2 + 3^3.$$
$$1239 = 1^2 + 2^9 - 3^1 + 9^3.$$
$$1364 = 1^6 + 3^1 + 6^4 + 4^3.$$
$$1654 = -1^6 + 6^1 + 5^4 + 4^5.$$
$$1837 = 1^8 - 8^1 + 3^7 - 7^3.$$
$$2137 = -2^1 + 1^3 + 3^7 - 7^2.$$
$$2173 = -2^3 + 1^2 - 7^1 + 3^7.$$
$$2537 = 2^5 - 5^2 + 3^7 + 7^3.$$
$$3125 = -3^2 + 1^1 + 2^3 + 5^5.$$
$$3275 = -3^3 + 2^7 + 7^2 + 5^5.$$
$$3529 = -3^3 + 5^5 + 2^9 - 9^2.$$
$$4316 = 4^6 + 3^1 + 1^4 + 6^3.$$

These representations, we call as *"flexible power selfie numbers"*. Motivated by this idea author wrote *"flexible power selfie numbers"* in different digits, for examples,

$$1 = 1^1.$$
$$23 = -2^2 + 3^3.$$
$$1654 = -1^6 + 6^1 + 5^4 + 4^5.$$
$$3435 = 3^3 + 4^4 + 3^3 + 5^5.$$
$$4355 = 4^5 + 3^4 + 5^3 + 5^5.$$
$$39339 = -3^3 + 9^3 + 3^9 + 3^9 - 9^3.$$
$$46350 = -4^3 + 6^6 - 3^5 + 5^0 + 0^4.$$
$$46360 = 4^0 + 6^6 - 3^4 - 6^3 + 0^6.$$
$$397612 = 3^2 + 9^1 + 7^6 + 6^7 + 1^9 + 2^3.$$
$$423858 = 4^3 + 2^8 + 3^4 + 8^2 + 5^8 + 8^5.$$
$$637395 = 6^5 + 3^3 + 7^3 + 3^9 + 9^6 + 5^7.$$
$$758014 = 7^7 + 5^1 + 8^0 + 0^5 + 1^4 - 4^8.$$
$$778530 = 7^7 + 7^3 + 8^5 - 5^7 + 3^0 + 0^8.$$
$$804637 = 8^0 + 0^4 - 4^8 + 6^6 - 3^3 + 7^7.$$

$$15647982 = 1^5 - 5^9 + 6^2 + 4^4 + 7^7 - 9^1 + 8^8 + 2^6.$$
$$17946238 = 1^6 + 7^8 + 9^4 + 4^2 + 6^9 + 2^3 + 3^1 + 8^7.$$
$$57396108 = -5^6 + 7^9 + 3^5 + 9^3 + 6^7 + 1^1 + 0^0 + 8^8.$$
$$134287690 = 1^2 + 3^8 + 4^7 + 2^4 + 8^9 + 7^3 + 6^6 + 9^0 + 0^1.$$
$$387945261 = 3^3 + 8^2 + 7^6 + 9^9 + 4^7 + 5^8 + 2^4 + 6^1 + 1^5.$$
$$392876054 = 3^0 + 9^9 - 2^2 - 8^5 + 7^8 - 6^7 + 0^3 - 5^4 + 4^6.$$
$$392876540 = -3^0 + 9^9 - 2^4 - 8^5 + 7^8 - 6^7 - 5^3 + 4^6 + 0^2.$$

The work is divided in different categories [12, 13, 14]. For details refer the following works:

http://rgmia.org/papers/v19/v19a49.pdf
http://rgmia.org/papers/v19/v19a50.pdf
http://rgmia.org/papers/v19/v19a51.pdf.

5 Selfie Fractions

A **addable fraction** is a proper fraction where addition signs can be inserted into numerator and denominator, and the resulting fraction is equal to the original. The same is true for operations also, such as with *addition, multiplication, potentiation,* etc. Below are ideas of sum of these kinds:

- **Addable**

$$\frac{96}{352}=\frac{9+6}{3+52},\quad \frac{182}{6734}=\frac{18+2}{6+734},\ \text{etc.}$$

- **Subtractable**

$$\frac{204}{357}=\frac{20-4}{35-7},\quad \frac{726}{1089}=\frac{72-6}{108-9},\ \text{etc.}$$

- **Dottable**

$$\frac{13}{624}=\frac{1\times 3}{6\times 24},\quad \frac{416}{728}=\frac{4\times 16}{7\times 2\times 8},\ \text{etc.}$$

- **Dottable with Potentiation**

$$\frac{95}{342}=\frac{9\times 5}{3^4\times 2},\quad \frac{728}{1456}=\frac{7^2\times 8}{14\times 56},\ \text{etc.}$$

- **Mixed: All Operations**

$$\frac{4980}{5312}=\frac{4-9+80}{5\times(3+1)^2},\quad \frac{3249}{5168}=\frac{(3+2^4)\times 9}{(5-1)\times 68},\ \text{etc.}$$

Observing above examples, the numerator and denominator follows the same order of digits in both sides of each fraction separated by operations. These type of fractions, we call *Selfie fractions*. There are two situations. One when digits appearing in each fraction are distinct, and second, when there are repetitions of digits. Studies on non-repeated digits are summarized in following works [15, 16, 17]:

http://rgmia.org/papers/v19/v19a113.pdf
http://rgmia.org/papers/v19/v19a114.pdf
http://rgmia.org/papers/v19/v19a115.pdf.

5.1 Equivalent Selfie Fractions

Above we have given *selfie fractions* with single value in each case. There are many fractions, that can be written in more than one way, for example,

- **Addable**

$$\frac{1453}{2906}=\frac{1+453}{2+906}=\frac{145+3}{290+6}=\frac{1+45+3}{2+90+6}.$$

- **Subtractable**

$$\frac{932}{1864}=\frac{9-32}{18-64}=\frac{93-2}{186-4}.$$

- **Dottable and Addable**

$$\frac{1680}{59472}=\frac{1\times 6\times 80}{59\times 4\times 72}=\frac{1+6+8+0}{59+472}.$$

- **Dottable, Addable and Subtractable**

$$\frac{302}{8154}=\frac{30\times 2}{81\times 5\times 4}=\frac{3+02}{81+54}=\frac{3-02}{81-54}.$$

- **Symmetric Addable and Subtractable**

$$\frac{645}{1290}=\frac{6-45}{12-90}=\frac{6+45}{12+90}.$$

- **Dottable and Addable together**

$$\frac{284}{639}=\frac{2\times 8+4}{6+39}=\frac{28+4}{6\times(3+9)}.$$

- **Mixed - All Operations**

$$\frac{73842}{90516}=\frac{7-3\times(8-4^2)}{9\times 05-1-6}=\frac{7\times(3+8)+4^2}{90+(5-1)\times 6}=\frac{738+4+2}{905+1+6}.$$

Equivalent expression given in equation (8), let us classify it as *symmetric equivalent fraction*. In this case we just change plus with minus and vice-versa. There are many fractions *double symmetric equivalent fraction* too. In this paper, we shall work with ***equivalent fractions*** given in equations (6)-(10). Below is an example of one of the biggest equivalent selfie fraction with addition and multiplication of 10 digits:

- $$\frac{30927}{61854}=\frac{3+0\times 927}{6^{1854}}=\frac{3\times(0\times 9+2)+7}{6+1^8\times 5\times 4}=\frac{3+0\times 9+2\times 7}{6+1\times 8+5\times 4}=\frac{3+0\times 9+27}{6+1^8\times 54}$$
$$=\frac{(3+09)\times 2+7}{6+(1+8+5)\times 4}=\frac{3+09+27}{6+1\times 8\times(5+4)}=\frac{3\times(09+2)+7}{(6+1+8+5)\times 4}=\frac{3\times 09+2\times 7}{6\times 1\times(8+5)+4}$$
$$=\frac{30\times(9+2\times 7)}{(61+8)\times 5\times 4}=\frac{30+9+2+7}{6+1+85+4}=\frac{3\times(09+2+7)}{6\times(1+8+5+4)}=\frac{30+9\times 2+7}{(6+1)\times 8+54}$$
$$=\frac{30\times(9+27)}{6\times 18\times 5\times 4}=\frac{3^{0\times 9+2}\times 7}{6\times(1^8+5\times 4)}=\frac{3+0\times 9+2+7}{6+1+8+5+4}=\frac{3\times(09\times 2+7)}{61+85+4}$$
$$=\frac{3\times(0\times 9+27)}{6\times(18+5+4)}=\frac{3+09\times(2+7)}{6\times 1\times(8+5\times 4)}=\frac{(30+9)\times 2+7}{6+(1+8\times 5)\times 4}=\frac{(3+09+2)\times 7}{(6+1)\times(8+5\times 4)}$$
$$=\frac{3+092+7}{(6+(1+8)\times 5)\times 4}=\frac{3\times(09+27)}{(6+18)\times(5+4)}=\frac{30+9^2+7}{(6\times(1+8)+5)\times 4}=\frac{3+09+2^7}{(6+1\times 8)\times 5\times 4}$$
$$=\frac{(3+09\times 2)\times 7}{6\times((1+8)\times 5+4)}=\frac{30+9\times 2\times 7}{6\times 1\times(8+5)\times 4}=\frac{(3+09)\times 2\times 7}{6\times(1+8+5)\times 4}=\frac{30\times(9+2+7)}{6\times(1+8)\times 5\times 4}$$
$$=\frac{3\times 0927}{618\times(5+4)}=\frac{3\times 09\times(2+7)}{6\times(1+8)\times(5+4)}=\frac{3+09\times 27}{6+(1+8)\times 54}=\frac{3\times(0\times 9+2+7)}{6\times 1^8\times(5+4)}$$
$$=\frac{(30+92)\times 7}{61\times(8+5\times 4)}=\frac{3\times 09\times 2\times 7}{(6+1\times 8)\times 54}=\frac{3+0927}{6+1854}=\frac{3\times 0\times 9+2\times 7}{6+1^{85}\times 4}$$
$$=\frac{3\times 09^{2+7}}{6\times(1+8)^{5+4}}=\frac{(30+9\times 2)\times 7}{618+54}.$$

The work on *equivalent selfie fractions* is divided in different papers given as follows [18, 19]:

http://rgmia.org/papers/v19/v19a116.pdf

http://rgmia.org/papers/v19/v19a117.pdf .

6 Equivalent Fractions

This section deals with *equivalent fractions* without use of any operations, just with different digits. Below are some examples to understand the idea of *equivalent fractions* for different digits:

- $\frac{3}{24}=\frac{4}{32}$, • $\frac{4}{356}=\frac{6}{534}$, • $\frac{27}{3456}=\frac{42}{5376}$,

- $\frac{315}{4620}=\frac{342}{5016}$, • $\frac{123}{58764}=\frac{164}{78352}$,

- $\frac{357}{26418}=\frac{618}{45732}=\frac{714}{52836}=\frac{732}{54168}=\frac{738}{54612}$, etc.

Some studies in this direction can be seen in [39, 44, 45]. Work of these authors is concentrated on equivalent fractions for the digits 1 to 9, calling *pandigital fractions*. While, we worked for all digits, i.e., for 3 to 10 digits. For example, in case of 3-digits, we tried to found equivalent fractions 3 by 3, i.e., [0, 1, 2], [1, 2, 3], [2, 3, 4], [3, 4, 5], [4, 5, 6], [5, 6, 7], [6, 7, 8] and [7, 8, 9]. In this case, we found only one result, i.e., $\frac{3}{24}=\frac{4}{32}$. The same procedure is done for 4 by 4, 5 by 5, etc. Below are highest possible equivalent fractions for 6 digits onwards. Before we have only few examples just with 2 expressions.

- **6-Digits Higher Equivalent Fractions: 3 Expressions**

In this case we have five possibilities, i.e., 0 to 5, 1 to 6, 2 to 7, 3 to 8, and 4 to 9. The number we got is for 2 to 7 and 4 to 9. Both with 3 expressions:

- $\frac{267}{534}=\frac{273}{546}=\frac{327}{654}$.

- $\frac{497}{568}=\frac{749}{856}=\frac{854}{976}$.

- **7-Digits Higher Equivalent Fractions: 5 Expressions**

In this case we have four possibilities, i.e., 0 to 6, 1 to 7, 2 to 8, and 3 to 9. The only higher number we got is in case of 3 to 9. See below:

- $\frac{697}{3485}=\frac{769}{3845}=\frac{937}{4685}=\frac{967}{4835}=\frac{973}{4865}$.

- **8-Digits Higher Equivalent Fractions: 12 Expressions**

In this case we have three possibilities, i.e., 0 to 7, 1 to 8, and 2 to 9. The only higher number we got is in case of 1 to 8. See below:

- $\frac{1728}{3456}=\frac{1764}{3528}=\frac{1782}{3564}=\frac{1827}{3654}=\frac{2178}{4356}=\frac{2358}{4716}=\frac{2718}{5436}=\frac{2817}{5634}=\frac{3564}{7128}=\frac{3582}{7164}=\frac{4176}{8352}=\frac{4356}{8712}$.

- **9-Digits Higher Equivalent Fractions: 46 Expressions**

In this case we have two possibilities, i.e., 0 to 8 and 1 to 9. The only higher number we got is in case of 1 to 9. See below:

- $$\begin{aligned}\frac{3187}{25496} &= \frac{4589}{36712} = \frac{4591}{36728} = \frac{4689}{37512} = \frac{4691}{37528} = \frac{4769}{38152} = \frac{5237}{41896} = \frac{5371}{42968} = \frac{5789}{46312} = \frac{5791}{46328} = \frac{5839}{46712} \\ &= \frac{5892}{47136} = \frac{5916}{47328} = \frac{5921}{47368} = \frac{6479}{51832} = \frac{6741}{53928} = \frac{6789}{54312} = \frac{6791}{54328} = \frac{6839}{54712} = \frac{7123}{56984} = \frac{7312}{58496} \\ &= \frac{7364}{58912} = \frac{7416}{59328} = \frac{7421}{59368} = \frac{7894}{63152} = \frac{7941}{63528} = \frac{8174}{65392} = \frac{8179}{65432} = \frac{8394}{67152} = \frac{8419}{67352} = \frac{8439}{67512} \\ &= \frac{8932}{71456} = \frac{8942}{71536} = \frac{8953}{71624} = \frac{8954}{71632} = \frac{9156}{73248} = \frac{9158}{73264} = \frac{9182}{73456} = \frac{9316}{74528} = \frac{9321}{74568} = \frac{9352}{74816} \\ &= \frac{9416}{75328} = \frac{9421}{75368} = \frac{9523}{76184} = \frac{9531}{76248} = \frac{9541}{76328}.\end{aligned}$$

- **10-Digits Higher Equivalent Fractions: 7 Expressions**

In this case we have only one possibility, i.e., 0 to 9. See below the higher equivalent fraction with 78 expressions:

- $$\begin{aligned}\frac{16748}{20935} &= \frac{18476}{23095} = \frac{18940}{23675} = \frac{18964}{23705} = \frac{19076}{23845} = \frac{19708}{24635} = \frac{24716}{30895} = \frac{24876}{31095} = \frac{25496}{31870} = \frac{28940}{36175} = \frac{29180}{36475} = \frac{29684}{37105} \\ &= \frac{32716}{40895} = \frac{32876}{41095} = \frac{36712}{45890} = \frac{36728}{45910} = \frac{37512}{46890} = \frac{37528}{46910} = \frac{38092}{47615} = \frac{38152}{47690} = \frac{39012}{48765} = \frac{41896}{52370} = \frac{42968}{53710} \\ &= \frac{46312}{57890} = \frac{46328}{57910} = \frac{46712}{58390} = \frac{47136}{58920} = \frac{47328}{59160} = \frac{47368}{59210} = \frac{48732}{60915} = \frac{49380}{61725} = \frac{49708}{62135} = \frac{51832}{64790} = \frac{53928}{67410} \\ &= \frac{54312}{67890} = \frac{54328}{67910} = \frac{54712}{68390} = \frac{56984}{71230} = \frac{58496}{73120} = \frac{58912}{73640} = \frac{59328}{74160} = \frac{59368}{74210} = \frac{63124}{78905} = \frac{63140}{78925} = \frac{63152}{78940} \\ &= \frac{63284}{79105} = \frac{63528}{79410} = \frac{64732}{80915} = \frac{65392}{81740} = \frac{65432}{81790} = \frac{67124}{83905} = \frac{67140}{83925} = \frac{67152}{83940} = \frac{67352}{84190} = \frac{67512}{84390} = \frac{71236}{89045} \\ &= \frac{71364}{89205} = \frac{71456}{89320} = \frac{71460}{89325} = \frac{71536}{89420} = \frac{71624}{89530} = \frac{71632}{89540} = \frac{72836}{91045} = \frac{73248}{91560} = \frac{73264}{91580} = \frac{73284}{91605} = \frac{73456}{91820} \\ &= \frac{73460}{91825} = \frac{73684}{92105} = \frac{74108}{92635} = \frac{74528}{93160} = \frac{74568}{93210} = \frac{74816}{93520} = \frac{75328}{94160} = \frac{75368}{94210} = \frac{76184}{95230} = \frac{76248}{95310} = \frac{76328}{95410}.\end{aligned}$$

For complete details refer to author's work [23, 24, 25]:

http://rgmia.org/papers/v19/v19a148.pdf
http://rgmia.org/papers/v19/v19a149.pdf
http://rgmia.org/papers/v19/v19a150.pdf.

The first work [23] is from 3 to 8 digits. The second work [24] is for 9 digits and the third work [25] is for 10 digits. All the three papers are for different digits. The repetition of digits give much more equivalent fractions. This shall be dealt elsewhere.

7 Palindromic-Type Numbers

A palindromic number is the number that remains the same when its digits are reversed, for example, 121, 3333, 161161, etc. The *palindromic-type* numbers are not palindromes but appears like palindromes. They are separated by operations of additions and/or multiplications, for example, 1343×3421, 1225×5221, $16+61$, $1825+5281$, etc. When we remove the sign of operation, they becomes palindromes. These types of numbers we call as *palindromic-type* numbers. Some studies in this direction can be seen in [**?**]. Let us separate these numbers in two categories:

- **Type 1:**

$$234 \times 111 + 111 \times 432 = 25974 + 47952 := 73926.$$
$$10011 \times 11 + 11 \times 11001 = 110121 + 121011 := 231132.$$

- **Type 2:**

$$1596 \times 6951 = 2793 \times 3972 := 11093796.$$
$$616248 \times 842616 = 645408 \times 804546 := 519260424768.$$

The difference is that the first type is with *addition* and *multiplication*, while second type is just with *multiplication*. This work is concentrated only on **Type 1** kind of numbers, i.e., *palindromic-type* numbers with *addition* and *multiplication*. Below are examples of *palindromic-type* numbers, where each digit appears once and multiplicative factors are of same width.

$$12 \times 21 + 21 \times 12 = 252 + 252 := 504.$$
$$12 \times 12 + 21 \times 21 = 144 + 441 := 585.$$

$$102 \times 210 + 201 \times 012 = 21420 + 02412 := 23832.$$
$$201 \times 102 + 201 \times 102 = 20502 + 20502 := 41004.$$
$$102 \times 012 + 210 \times 201 = 01224 + 42210 := 43434.$$
$$102 \times 102 + 201 \times 201 = 10404 + 40401 := 50805 :$$

$$2031 \times 1032 + 2301 \times 1302 = 2095992 + 2995902 := 5091894.$$
$$2103 \times 1203 + 3021 \times 3012 = 2529909 + 9099252 := 11629161.$$
$$2013 \times 1023 + 3201 \times 3102 = 2059299 + 9929502 := 11988801.$$

Below are numbers, where squaring each multiplicative factor in each palindromic-type number lead us to final sum as a palindrome:

$$12^2 + 21^2 = 144 + 441 := 585.$$
$$111^2 + 111^2 = 12321 + 12321 := 24642.$$
$$1022^2 + 2201^2 = 1044484 + 4844401 := 5888885.$$
$$10031^2 + 13001^2 = 100620961 + 169026001 := 269646962.$$
$$100301^2 + 103001^2 = 10060290601 + 10609206001 := 20669496602.$$

For complete details refer to author's work [26]:

http://rgmia.org/papers/v19/v19a159.pdf

7.1 Patterns in Palindromic-Type Numbers

Below are some examples of patterns in palindromic-type numbers:

$$\begin{aligned}
101 \times 44 + 44 \times 101 &= 4444 + 4444 := 8888\\
202 \times 22 + 22 \times 202 &= 4444 + 4444 := 8888\\
404 \times 11 + 11 \times 404 &= 4444 + 4444 := 8888.
\end{aligned}$$

$$\begin{aligned}
1001 \times 444 + 444 \times 1001 &= 444444 + 444444 := 888888\\
2002 \times 222 + 222 \times 2002 &= 444444 + 444444 := 888888\\
4004 \times 111 + 111 \times 4004 &= 444444 + 444444 := 888888.
\end{aligned}$$

$$\begin{aligned}
10001 \times 4444 + 4444 \times 10001 &= 44444444 + 44444444 := 88888888\\
20002 \times 2222 + 2222 \times 20002 &= 44444444 + 44444444 := 88888888\\
40004 \times 1111 + 1111 \times 40004 &= 44444444 + 44444444 := 88888888.
\end{aligned}$$

$$\begin{aligned}
1001 \times 44 + 44 \times 1001 &= 44044 + 44044 := 88088\\
2002 \times 22 + 22 \times 2002 &= 44044 + 44044 := 88088\\
4004 \times 11 + 11 \times 4004 &= 44044 + 44044 := 88088.
\end{aligned}$$

$$\begin{aligned}
10001 \times 404 + 404 \times 10001 &= 4040404 + 4040404 := 8080808\\
20002 \times 202 + 202 \times 20002 &= 4040404 + 4040404 := 8080808\\
40004 \times 101 + 101 \times 40004 &= 4040404 + 4040404 := 8080808.
\end{aligned}$$

$$\begin{aligned}
10001 \times 4004 + 4004 \times 10001 &= 40044004 + 40044004 := 80088008\\
20002 \times 2002 + 2002 \times 20002 &= 40044004 + 40044004 := 80088008\\
40004 \times 1001 + 1001 \times 40004 &= 40044004 + 40044004 := 80088008.
\end{aligned}$$

For complete details refer to author's work [26]:

http://rgmia.org/papers/v19/v19a159.pdf

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