Crazy Representations of Natural Numbers, Selfie Numbers, Fibonacci Sequence, and Selfie Fractions

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S U M M A R Y

This summary brings author's work on numbers. The study is made in different ways. Specially, towards, *Crazy Representations of Natural Numbers*, *Running Expressions*, *Selfie Numbers* etc. Natural numbers are represented in different situations, such as, writing in terms of 1 to 9 or reverse, flexible power of same digits as bases, single digit, single letter, etc. Expressions appearing with equalities having 1 to 9 or 9 to 1 or 9 to 0, calling *running expressions* are also presented. In continuation, there is work is on *selfie numbers*, *unified, patterns, symmetrical representations in selfie numbers*, *Fibonacci sequence and selfie numbers*, *flexible power Narcissistic and selfie numbers*, *selfie fractions*, etc. The *selfie numbers* may also be considered as generalized or wild narcissistic numbers, where natural numbers are represented by their own digits with certain operations. The study is also made towards *equivalent fractions* and *palindromic-type* numbers. This summary is separated by sections and subsections given as follows:

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For extra reading, we suggest readers the references: [48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 62, 63, 64, 65, 66, 67, 68]

1 Crazy Representations of Natural Numbers

In 2014, author [1] wrote natural numbers in increasing and decreasing orders of 1 to 9 and 9 to 1. See examples below:

$$100 = 1 + 2 + 3 + 4 + 5 + 6 + 7 + 8 \times 9 = 9 \times 8 + 7 + 6 + 5 + 4 + 3 + 2 + 1.$$
$$101 = 1 + 2 + 34 + 5 + 6 \times 7 + 8 + 9 = 9 \times 8 + 7 + 6 + 5 + 4 + 3 \times 2 + 1.$$
$$102 = 12 + 3 \times 4 \times 5 + 6 + 7 + 8 + 9 = 9 + 8 + 7 + 6 + 5 + 4^3 + 2 + 1.$$
$$103 = 1 \times 2 \times 34 + 5 + 6 + 7 + 8 + 9 = 9 + 8 + 7 \times 6 + 5 \times 4 + 3 + 21.$$
$$104 = 1 + 23 + 4 + 5 + 6 + 7 \times 8 + 9 = 9 + 8 + 7 + 65 + 4 \times 3 + 2 + 1.$$
$$105 = 1 + 2 \times 3 \times 4 + 56 + 7 + 8 + 9 = 9 + 8 \times 7 + 6 \times 5 + 4 + 3 + 2 + 1.$$
$$106 = 12 + 3 + 4 \times 5 + 6 + 7 \times 8 + 9 = 9 + 8 \times 7 + 6 \times 5 + 4 + 3 \times 2 + 1.$$
$$107 = 1 \times 23 + 4 + 56 + 7 + 8 + 9 = 9 + 8 + 76 + 5 + 4 + 3 + 2 \times 1.$$
$$108 = 1 + 2 + 3 + 4 + 5 + 6 + 78 + 9 = 9 + 8 + 76 + 5 + 4 + 3 + 2 + 1.$$

Below are more examples,

$$999 = 12 \times 3 \times (4 + 5) + (67 + 8) \times 9 = 9 + 8 + 7 + 654 + 321.$$
$$2535 = 1 + 2345 + (6 + 7 + 8) \times 9 = 9 + 87 \times (6 + 5 \times 4 + 3) + 2 + 1.$$
$$2607 = 123 \times 4 \times 5 + 6 + (7 + 8) \times 9 = 987 + 6 \times 54 \times (3 + 2) \times 1.$$
$$10958 = 12 \times 3 + \sqrt{4} + 5! \times (67 + 8 \times \sqrt{9}) = (9 + 8 \times 7 \times 65 + 4) \times 3 - 2 + 1.$$
$$11807 = 1 \times 234 \times (5 + 6 \times 7) + 89 = -9 + 8 + 7 \times (6 + 5) \times (4 \times 3)^2 \times 1.$$

We observe that the number 10958 is the only number among 0 to 11111, where use *square-root* and *factorial*. All other numbers are just with basic operations. For full work, refer to link below [1] (Jan., 2014):

http://arxiv.org/abs/1302.1479.

For comments on this work see [46, 47, 60, 61].

2 Flexible Power Representations

Let us consider two numbers, 1 and 2. Using the idea of power and the operations of *addition* and *subtraction*, we can write following 3 numbers in terms of 1 and 2, as $1 = -1^2 + 2^1$, $3 = 1^2 + 2^1$ and $5 = 1^1 + 2^2$. In this situation, we observe that *bases* and *exponents* are of same digits. Permutations of exponent values helps in bringing different numbers. In case of repeated values, for example, $3 = 1^2 + 2^1 = -1^1 + 2^2$, only possibilities is considered. There is only one number having single digit, i.e., $1 = 1^1$. For simplicity, let us represent the above procedure as $(1, 2)^{(1,2)}$, resulting in three possible values. The above procedure is with two digits. Instead having two digits, we can work with two letters, such as, $(a, b)^{(a,b)}$, where $a \neq b$ and $a, b \in \{1, 2, 3, 4, 5, 6, 8, 9\}$. This process can be extended for more number of letters. See below:

$$(a, b)^{(a,b)};$$
$$(a, b, c)^{(a,b,c)};$$
$$(a, b, c, d)^{(a,b,c,d)};$$
$$(a, b, c, d, e)^{(a,b,c,d,e)}$$
$$(a, b, c, d, e, f)^{(a,b,c,d,e,f)};$$
$$(a, b, c, d, e, f, g)^{(a,b,c,d,e,f,g)};$$
$$(a, b, c, d, e, f, g, h)^{(a,b,c,d,e,f,g,h)};$$
$$(a, b, c, d, e, f, g, h, i)^{(a,b,c,d,e,f,g,h,i)}.$$

where $a, b, c, d, e, f, g, h, i \in \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$, all distinct.

Based on above procedure, natural numbers are written in two different procedures.

2.1 Unequal String Lengths

$100 := 2^6 + 6^2.$

$101 := 1^1 + 2^6 + 6^2.$

$102 := -2^5 + 3^2 + 5^3.$

$103 := 1^1 - 2^5 + 3^2 + 5^3.$

$104 := -1^1 + 2^3 + 3^4 + 4^2.$

$105 := 2^3 + 3^4 + 4^2.$

$106 := 2^7 + 3^3 - 7^2.$

$107 := -1^2 + 2^7 - 3^3 + 7^1.$

$108 := 1^7 + 2^6 + 6^2 + 7^1.$

$109 := 1^2 + 2^7 - 3^3 + 7^1.$

$110 := 1^9 + 2^6 + 6^2 + 9^1.$

$111 := -1^3 + 2^7 - 3^2 - 7^1.$

$112 := 3^5 - 4^4 + 5^3.$

$113 := -1^5 - 2^1 - 3^2 + 5^3.$

$114 := -2^2 + 3^5 - 5^3.$

$115 := 1^5 - 2^1 - 3^2 + 5^3.$

$116 := 2^2 + 3^5 - 4^4 + 5^3.$

$117 := -1^1 + 3^5 - 5^3.$

$118 := 3^5 - 5^3.$

$119 := 1^1 + 3^5 - 5^3.$

See more examples,

$$666 := -2^5 + 3^2 + 4^3 + 5^4.$$
$$786 := -1^4 + 3^6 + 4^3 - 6^1.$$
$$9711 := 1^3 + 2^4 + 3^8 + 4^2 + 5^5 - 8^1.$$
$$9777 := 1^9 + 2^1 + 4^7 - 7^2 - 9^4.$$
$$11110 := 1^1 + 2^2 + 3^9 - 5^6 + 6^5 - 9^3.$$
$$11111 := -1^1 + 2^7 + 3^8 - 4^2 + 7^3 + 8^4.$$

By unequal string length, we mean that each number is not represented by same digits. It uses minimum necessary number of expressions. The digits used in the powers are the same as of bases but with different permutations. For complete representations of numbers from 0 to 11111 refer to link [17]:

http://rgmia.org/papers/v19/v19a31.pdf.

2.2   Equal String Lengths

Instead working with unequal strings as of previous section, here we worked with equal string using the digits 0 to 9, i.e., using all the 10 digits, {0,1,2,3,4,5,6,7,8,9}. The results obtained are symmetric, i.e., writing in 0 to 9 or 9 to 0, the resulting number is same. See some examples below,

$201 := 0^3 + 1^9 + 2^4 + 3^7 - 4^8 + 5^1 + 6^6 + 7^5 + 8^2 + 9^0.$

$202 := 0^0 + 1^9 + 2^6 + 3^8 - 4^7 + 5^5 + 6^3 + 7^2 + 8^1 + 9^4.$

$203 := 0^3 - 1^9 + 2^4 + 3^7 - 4^8 + 5^0 + 6^6 + 7^5 + 8^2 + 9^1.$

$204 := 0^8 + 1^9 + 2^5 + 3^7 - 4^6 + 5^1 + 6^4 + 7^2 + 8^0 + 9^3.$

$205 := 0^3 + 1^9 + 2^4 + 3^7 - 4^8 + 5^0 + 6^6 + 7^5 + 8^2 + 9^1.$

$206 := 0^7 - 1^9 - 2^5 - 3^8 + 4^6 + 5^1 + 6^3 + 7^4 + 8^0 + 9^2.$

$207 := 0^8 + 1^9 + 2^5 + 3^7 - 4^6 + 5^0 + 6^4 + 7^2 + 8^1 + 9^3.$

$208 := 0^7 + 1^9 - 2^5 - 3^8 + 4^6 + 5^1 + 6^3 + 7^4 + 8^0 + 9^2.$

$209 := 0^7 - 1^9 - 2^5 - 3^8 + 4^6 + 5^0 + 6^3 + 7^4 + 8^1 + 9^2.$

$210 := 0^5 - 1^7 - 2^8 - 3^9 + 4^1 + 5^6 + 6^0 + 7^3 + 8^4 + 9^2.$

$211 := 0^7 + 1^9 - 2^5 - 3^8 + 4^6 + 5^0 + 6^3 + 7^4 + 8^1 + 9^2.$

$212 := 0^5 + 1^7 - 2^8 - 3^9 + 4^1 + 5^6 + 6^0 + 7^3 + 8^4 + 9^2.$

$213 := 0^5 + 1^8 - 2^7 - 3^9 + 4^1 + 5^6 + 6^3 + 7^0 + 8^4 + 9^2.$

$214 := 0^5 + 1^7 - 2^8 - 3^9 + 4^0 + 5^6 + 6^1 + 7^3 + 8^4 + 9^2.$

$215 := 0^5 + 1^9 + 2^8 + 3^7 - 4^6 + 5^0 + 6^4 + 7^2 + 8^3 + 9^1.$

$216 := 0^1 - 1^7 + 2^8 - 3^9 + 4^5 + 5^6 + 6^0 + 7^4 + 8^3 + 9^2.$

$217 := 0^7 - 1^9 + 2^5 - 3^8 + 4^6 + 5^2 + 6^3 + 7^4 + 8^1 + 9^0.$

$218 := 0^1 + 1^7 + 2^8 - 3^9 + 4^5 + 5^6 + 6^0 + 7^4 + 8^3 + 9^2.$

$219 := 0^7 + 1^9 + 2^5 - 3^8 + 4^6 + 5^2 + 6^3 + 7^4 + 8^1 + 9^0.$

$220 := 0^7 + 1^9 + 2^5 - 3^8 + 4^6 + 5^2 + 6^3 + 7^4 + 8^0 + 9^1.$

Below are more examples,

$$11080 := 0^8 + 1^9 + 2^7 + 3^6 + 4^2 + 5^5 + 6^0 + 7^1 + 8^3 + 9^4.$$
$$11081 := 0^8 - 1^9 + 2^6 + 3^7 + 4^4 + 5^1 + 6^5 + 7^0 + 8^2 + 9^3.$$
$$11082 := 0^8 + 1^9 + 2^6 + 3^7 + 4^1 + 5^4 + 6^5 + 7^3 + 8^0 + 9^2.$$
$$11083 := 0^8 + 1^9 + 2^6 + 3^7 + 4^4 + 5^1 + 6^5 + 7^0 + 8^2 + 9^3.$$
$$11084 := 0^7 + 1^9 + 2^8 + 3^6 + 4^1 + 5^5 + 6^0 + 7^3 + 8^2 + 9^4.$$
$$11085 := 0^8 + 1^9 + 2^6 + 3^7 + 4^4 + 5^0 + 6^5 + 7^1 + 8^2 + 9^3.$$
$$11086 := 0^7 + 1^9 + 2^8 + 3^6 + 4^0 + 5^5 + 6^1 + 7^3 + 8^2 + 9^4.$$
$$11087 := 0^6 + 1^9 - 2^8 + 3^7 + 4^2 + 5^4 + 6^5 + 7^0 + 8^1 + 9^3.$$

For complete representations of numbers from 0 to 11111 refer to link [34]:

http://rgmia.org/papers/v19/v19a131.pdf.

Analysing the procedures given in sections 1 and 2 , we observe that in the section 1 , all the 9 digits from 1 to 9 or 9 to 1 are used to bring natural numbers, where each digit appears only once. In this case, the operations used are, *addition, subtraction, multiplication, division, potentiation, factorial* and *square-root*. The section 2 works with representations of natural numbers written in a way that we use each digit twice, where *bases* and *exponents* are of same digits with different permutations. Subsection 2.1 choose the digits from 1 to 9, according to necessity, while subsection 2.2 works with all the 10 digits, i.e., 0 to 9, along with the operations of *addition* and *subtraction*.

3   Pyramidical–Type Representations

This section deals with pyramidical–type of representations of natural numbers in two different ways. One is based on the procedure used in section 1 and second is based on procedure given in section 2 . This happens because, we can write same number in different ways.

3.1   Crazy Representations

Following the procedure of section 2 , we can write the natural numbers in pyramidical forms, for examples,

- $33 = 32 + 1 \times 0!$
  $= 4 \times 3 + 21 \times 0!$
  $= 5 + 4 + 3 \times (-2 + 10)$
  $= 6 + 5 + 43 - 21 \times 0!$
  $= 76 - 5 + 4 - 32 - 10$
  $= 8 + 7 + 65 - 4 \times 3! \times 2 + 1 \times 0!$
  $= -9 - 8 + 7 - 6 + 54 - 3 - 2 \times 1 \times 0!.$

- $48 = 3! \times (-2 + 10)$
  $= 4! + 3 + 21 \times 0!$
  $= -54 \times 3 + 210$
  $= 6 + 5 + 4! + 3!/2 + 10$
  $= 7 \times (6 + 5) + 4 - 32 - 1 \times 0!$
  $= 8 + 7 + 6 - 5 + 4 \times 3 + 2 \times 10$
  $= -9 + 8 \times 76 - 543 + 2 - 10.$

- $434 = 432 + 1 + 0!$
  $= 54 \times (3! + 2) + 1 + 0!$
  $= 6 \times 54 + (3 + 2)! - 10$
  $= -7 + 654 - 3 - 210$
  $= 8 \times (7 + 6) \times 5 - 4! - 3 \times 21 + 0!$
  $= (9 + 8 + 7 + 6 + 5 - 4) \times (-3! + 2 \times 10).$

- $729 = 3^{(2+1)! \times 0!}$
  $= (4! + 3)^2 - 1 + 0!$
  $= (5 + 4)^3 \times (2 - 1) \times 0!$
  $= 6! + 5 + 4 + 321 \times 0$
  $= 765 - 4 \times 3! - 2 - 10$
  $= (8 + 76 + 5 - 4^3 + 2)^{(1+0!)}$
  $= (9 + 8 + 7 + 6 - 54 - 3)^2 \times 1 \times 0!.$

- $895 = -5 + (4! + 3!)^2 \times 1 \times 0!$
  $= 6 \times (5 + 4 \times 3!^2 \times 1) + 0!$
  $= 7 \times (65 + 4^3) + 2 - 10$
  $= -8 - 7 + 65 \times (4 \times 3 + 2) + 1 \times 0$
  $= 9 + 876 + 5 - 4 + 3^2 \times 1 \times 0!.$

- $947 = -5! + 43 + 2^{10}$
  $= 6! + 5 + 4 \times 3 + 210$
  $= 7 + ((6 + 5) \times 4 + 3) \times 2 \times 10$
  $= (8 + 7) \times 65 - 4! - 3! + 2 + 1 \times 0$
  $= -9 + 8 - 76 + 5! + 43 \times 21 + 0!.$

For complete representations of natural numbers from 0 to 1000 refer to link [32]:

http://rgmia.org/papers/v19/v19a130.pdf.

3.2 Flexible Power

Following the procedure of section 2 , we can write the natural numbers in pyramidical forms, for examples,

- $1 = 0^0$
  $= 0^1 + 1^0$
  $= 0^2 - 1^0 + 2^1$
  $= 0^0 + 1^3 - 2^2 + 3^1$
  $= 0^3 + 1^4 - 2^2 + 3^1 + 4^0$
  $= 0^2 + 1^4 - 2^5 + 3^3 + 4^1 + 5^0$
  $= 0^5 - 1^0 - 2^6 - 3^4 + 4^2 + 5^3 + 6^1$
  $= 0^2 + 1^7 + 2^5 - 3^6 + 4^3 + 5^4 + 6^0 + 7^1$
  $= 0^3 + 1^7 + 2^5 - 3^8 + 4^6 + 5^2 + 6^1 + 7^4 + 8^0$
  $= 0^5 - 1^8 + 2^9 + 3^7 - 4^6 + 5^4 + 6^2 + 7^0 + 8^1 + 9^3$.

- $22 = 0^1 - 1^0 - 2^2 + 3^3$
  $= 0^2 + 1^3 + 2^4 + 3^0 + 4^1$
  $= 0^4 - 1^5 + 2^3 + 3^2 + 4^0 + 5^1$
  $= 0^2 + 1^6 + 2^5 - 3^4 + 4^3 + 5^1 + 60$
  $= 0^5 + 1^7 - 2^6 - 3^4 + 4^1 + 5^3 + 6^2 + 7^0$
  $= 0^1 + 1^4 + 2^8 + 3^5 - 4^7 + 5^6 + 6^3 + 7^0 + 8^2$
  $= 0^6 - 1^9 + 2^8 - 3^7 + 4^5 + 5^4 + 6^3 + 7^1 + 8^0 + 9^2$.

- $666 = 0^1 - 1^3 + 2^5 + 3^2 + 4^0 + 5^4$
  $= 0^0 + 1^5 - 2^6 + 3^1 + 4^3 + 5^4 + 6^2$
  $= 0^5 + 1^7 - 2^6 + 3^1 + 4^3 + 5^4 + 6^2 + 7^0$
  $= 0^2 - 1^7 - 2^6 - 3^8 + 4^3 + 5^5 + 6^1 + 7^0 + 8^4$
  $= 0^7 + 1^9 - 2^5 - 3^8 + 4^6 + 5^2 + 6^1 + 7^4 + 8^0 + 9^3$.

- $1089 = 0^1 + 1^0 + 2^3 + 3^4 + 4^5 - 5^2$
  $= 0^4 - 1^6 + 2^1 + 3^3 + 4^5 + 5^0 + 6^2$
  $= 0^2 + 1^6 - 2^7 + 3^5 + 4^1 + 5^4 + 6^0 + 7^3$
  $= 0^0 - 1^7 + 2^4 - 3^8 + 4^6 + 5^5 + 6^1 + 7^3 + 8^2$
  $= 0^6 - 1^9 + 2^7 - 3^8 + 4^1 + 5^5 + 6^3 + 7^0 + 8^4 + 9^2$.

- $1179 = 0^1 + 1^0 + 2^5 + 3^6 + 4^4 + 5^3 + 6^2$
  $= 0^2 + 1^6 + 2^4 - 3^7 + 4^0 + 5^5 + 6^3 + 7^1$
  $= 0^6 + 1^7 - 2^8 + 3^5 + 4^1 + 5^4 + 6^0 + 7^2 + 8^3$
  $= 0^6 + 1^9 - 2^8 - 3^7 + 4^5 + 5^3 + 6^1 + 7^4 + 8^2 + 9^0$.

We observe that the digits appearing in bases and exponents are same in each number. For complete representations of natural numbers from 0 to 1500 refer to link [25]:

http://rgmia.org/papers/v19/v19a31.pdf.

4 Double Sequential Representations

This section deals with representations of natural numbers written in a sequential way of 3 to 8 digits ending in 0, such as {2,1,0}, {3,2,1,0},..., {9,8,7,6,5,4,32,1,0}. These representations are done combining both the processes given in sections 1 and 2 . It is interesting to observe that the processes given in subsection 1 uses operations such as, *addition, subtraction, multiplication, division, potentiation, square-root* and *factorial* with each digit appearing once. In case of process given in section 2 only addition and subtractions are used but each digit appears twice, once in base and another as potentiation. Below are some examples,

- $1 = 2^1 - 1^0 + 0^2$
  $= 2 - 1 \times 0!$.

- $2 = 2^0 + 1^2 + 0^1$
  $= 2 \times 1 \times 0!$.

- $11 = 3^2 + 2^0 + 1^3 + 0^1$
  $= 3 - 2 + 10$.

- $25 = 3^3 - 2^2 + 1^1 + 0^0$
  $= 3 + 21 + 0!$.

- $20 = 4^2 + 3^0 + 2^1 + 1^4 + 0^3$
    $= 4 + 3 \times 2 + 10.$

- $21 = 4^2 + 3^1 + 2^0 + 1^4 + 0^3$
    $= (4 - 3) \times 21 \times 0!.$

- $116 = 5^2 + 4^0 + 3^4 + 2^3 + 1^5 + 0^1$
    $= 54 + 3 \times 21 - 0!.$

- $120 = 5^2 + 4^1 + 3^4 + 2^3 + 1^5 + 0^0$
    $= (5 + 4321 \times 0)!.$

- $1406 = 6^4 + 5^1 + 4^3 + 3^2 + 2^5 - 1^6 + 0^0$
    $= 6 + 5! + 4 \times 32 \times 10.$

- $1411 = 6^3 + 5^2 + 4^5 + 3^4 + 2^6 + 1^0 + 0^1$
    $= 6! - 5 - 4! + (3 \times 2 \times 1)! \times 0!.$

- $78 = -7^4 + 6^3 - 5^1 + 4^2 + 3^7 + 2^6 + 1^0 + 0^5$
    $= -76 - 54 - 3 + 210.$

- $227 = -7^5 + 6^1 + 5^4 + 4^7 + 3^2 + 2^3 + 1^6 + 0^0$
    $= (765 - 4!)/3 - 2 \times 10.$

- $1048 = -7^3 + 6^4 + 5^2 + 4^1 + 3^0 + 2^6 + 1^7 + 0^5$
    $= 7 - 6 + 5 \times 4 + 3 + 2^{10}.$

- $2016 = 7^3 + 6^4 + 5^0 + 4^1 + 3^5 + 2^7 + 1^6 + 0^2$
    $= (7 + 65) \times (\sqrt{4} \times 3^2 + 10).$

- $661 = 8^2 - 7^3 + 6^1 + 5^4 + 4^0 + 3^5 + 2^6 + 1^8 + 0^7$
    $= 8 + 7 + 654 - 3^2 + 1 \times 0!.$

- $1087 = -8^2 + 7^1 + 6^0 + 5^3 + 4^4 + 3^6 + 2^5 + 1^8 + 0^7$
    $= -87 - 6 + (5 \times 4 \times 3! - 2) \times 10.$

- $192 = 9^2 - 8^4 + 7^1 + 6^3 + 5^5 + 4^0 + 3^6 + 2^7 + 1^9 + 0^8$
    $= 98 + 76 - 5 + 4 - 3 + 21 + 0!.$

- $1417 = -9^0 + 8^3 - 7^4 + 6^1 + 5^2 + 4^5 + 3^7 + 2^6 + 1^9 + 0^8$
    $= 9 \times (8 - 7) + (6 + 5) \times 4 \times 32 \times 1 \times 0!.$

For complete representations of numbers refer to links [20, 24, 31, 32]:

http://rgmia.org/papers/v19/v19a48.pdf.
http://rgmia.org/papers/v19/v19a57.pdf.
http://rgmia.org/papers/v19/v19a128.pdf.
http://rgmia.org/papers/v19/v19a129.pdf.

In the first work, the results are up 7 digits, i.e., from $\{2,1,0\}$ to $\{6,5,4,3,2,1,0\}$. The second paper give the results for 8 digits, i.e., for $\{7,6,5,4,3,2,1,0\}$. The third paper give the results for 8 digits, i.e., for $\{8,7,6,5,4,3,2,1,0\}$, and finally the forth paper given the results for digits $\{9,8,7,6,5,4,3,2,1,0\}$. Since all numbers ending in 0, obviously, all these results are in decreasing order.

5   Triple Representations of Numbers

This section deals with the representations of natural numbers in three different ways. In each case the same digits are used. The first way is based on the representations given in section 2 . The second and third are based on section 1 in increasing and decreasing order of digits. It is not possible to write all the numbers in three ways. Only those numbers are written in the work, when representations are possible in all the three ways. For an idea see some examples below:

- $3 = 1^2 + 2^1$
  $= 1 + 2$
  $= 2 + 1.$

- $1 = -2^3 + 3^2$
  $= -2 + 3$
  $= 3 - 2.$

- $32 = 1^1 + 2^2 + 3^3$
  $= 32 \times 1.$

- $13 = -2^2 + 3^4 - 4^3$
  $= 4 + 3^2.$

- $138 = -3^5 + 4^4 + 5^3$
  $= -3! + 4! + 5!$
  $= 5! + 4! - 3!.$

- $10 = -1^4 - 2^3 + 3^1 + 4^2$
  $= \sqrt{12 \times 3} + 4$
  $= 4 + 3 + 2 + 1.$

- $60 = 2^5 - 3^4 - 4^2 + 5^3$
  $= (2^3 + 4) \times 5$
  $= 54 + 3 \times 2.$

- $4278 = -3^3 - 4^6 + 5^4 + 6^5$
  $= 3! \times (-\sqrt{4} - 5 + 6!)$
  $= (6! - 5 - \sqrt{4}) \times 3!.$

- $31 = 1^1 - 2^3 - 3^5 + 4^4 + 5^2$
  $= 1 \times 2 + 34 - 5$
  $= (5 - 4) \times 32 - 1.$

- $2009 = 3^7 - 4^3 - 5^6 - 6^4 + 7^5$
  $= -3!! + 4! \times (5! - 6) - 7$
  $= -7 + (-6 + 5!) \times 4! - 3!!.$

- $1840 = -4^5 - 5^8 + 6^7 + 7^6 - 8^4$
  $= 4! \times (5 + 6) \times 7 - 8$
  $= -8 + 7 \times (6 + 5) \times 4!.$

- $121 = -1^5 - 2^4 + 3^2 + 4^1 + 5^3$
  $= 1^{234} + 5!$
  $= 5 \times 4 \times 3 \times 2 + 1.$

- $194 = -2^6 + 3^5 + 4^4 - 5^2 - 6^3$
  $= 2 \times 34 + 5! + 6$
  $= -6 + 5 \times (4 + 3!^2).$

- $425 = 1^6 + 2^4 + 3^5 + 4^1 + 5^3 + 6^2$
  $= -1 + 23 \times 4! - 5! - 6$
  $= 6! + 5 \times (4 - 3 \times 21).$

- $905 = 2^6 + 3^2 - 4^7 + 5^4 - 6^3 + 7^5$
  $= 2 + (3 \times 45 - 6) \times 7$
  $= 7!/6 + 5 + 4! + 3!^2.$

For complete detail refer refer to links [35]:

http://rgmia.org/papers/v19/v19a134.pdf.

The above work give the results only up to width 6. We can extend the results for higher width too. See examples below:

- $1008 = 1^6 - 2^7 + 3^5 + 4^1 + 5^4 + 6^3 + 7^2$
  $= (12 - 3) \times (45 + 67)$
  $= (-7 + 6 + 5) \times 4 \times 3 \times 21.$

- $109 = 2^8 - 3^3 - 4^7 - 5^2 + 6^6 + 7^4 - 8^5$
  $= 234 + 5 \times 67 + 8$
  $= 87 + (6 - 5 + 43)/2.$

- $944 = 3^9 + 4^8 - 5^7 + 6^6 - 7^5 + 8^4 - 9^3$
    $= (-3 + 4!) \times 5 + 6! + 7 \times (8 + 9)$
    $= 987 \times (6 - 5) + 43.$

- $288 = -2^3 - 3^8 - 4^9 + 5^4 - 6^7 + 7^5 + 8^2 + 9^6$
    $= 2 \times (34 - 5 \times (67 - 89))$
    $= (-9 + 87 - 6 \times 5) \times 4 \times 3/2.$

- $512 = 1^7 + 2^5 - 3^8 + 4^6 + 5^2 + 6^1 + 7^4 + 8^3$
    $= 1 - 23 + 456 + 78$
    $= 8 \times ((7 - 6)^{54} + 3 \times 21).$

- $1172 = -2^9 - 3^2 + 4^8 - 5^7 - 6^6 + 7^4 - 8^3 + 9^5$
    $= (2 \times 3)^4 + 5 + 6 - (7 + 8) \times 9$
    $= (9 + 8) \times 76 - 5 \times 4 \times 3 \times 2.$

The work on above type of examples is under preparation and shall be dealt later on.

6 Single Digit Representations

In section 1, all the nine digits are used to write natural numbers. Here the work is done writing numbers for each digit separately. See examples below:

$$717 = (1 + 1)^{11} - 11^{(1+1+1)}$$
$$= 22^2 + 222 + 22/2$$
$$= 3^{(3+3)} - 3 - 3 \times 3$$
$$= 4 \times (4 \times 44 + 4) - 4 + 4/4$$
$$= (55 \times (55 + 5 + 5) + 5 + 5)/5$$
$$= (6 \times 6/(6 + 6))^6 - 6 - 6$$
$$= 777 - 7 \times 7 - 77/7$$
$$= 8 \times 88 + (88 + 8 + 8)/8$$
$$= 9 \times 9 \times 9 - (99 + 9)/9.$$

$$995 = (11 - 1)^{(1+1+1)} - (11 - 1)/(1 + 1)$$
$$= 22 + 2 \times (22^2 + 2) + 2/2$$
$$= 3 \times 333 - 3 - 3/3$$
$$= 4 \times (4^4 - 4 - 4) + 4 - 4/4$$
$$= 5 \times (5 + 5) \times (5 \times 5 - 5) - 5$$
$$= 666 + 6 \times 66 - 66 - 6/6$$
$$= (7 + 7) \times (77 - 7) + 7 + 7 + 7/7$$
$$= 888 + 88 + 8 + 88/8$$
$$= 999 - (9 + 9 + 9 + 9)/9.$$

$$666 = (1 + 1) \times (1 + 1 + 1) \times 111$$
$$= (2 + 2/2) \times 222$$
$$= 3 \times ((3 + 3)^3 + 3 + 3)$$
$$= 444 \times (4 + (4 + 4)/4)/4$$
$$= 555 + 555/5$$
$$= 666$$
$$= 777 - 777/7$$
$$= 888 \times (8 - (8 + 8)/8)/8$$
$$= 9 \times (9 \times 9 - 9) + 9 + 9.$$

$$1000 = (11 - 1)^{(1+1+1)}$$
$$= 2 \times (22^2 + 2^{(2+2)})$$
$$= (3 \times 3 + 3/3)^3$$
$$= 4 \times (4^4 - 4) - 4 - 4$$
$$= 5 \times (5 + 5) \times (5 \times 5 - 5)$$
$$= ((66 - 6)/6)^{(6 \times 6/(6+6))}$$
$$= (7 + 7 + 7 - 7/7) \times (7 \times 7 + 7/7)$$
$$= 888 + 88 + 8 + 8 + 8$$
$$= 999 + 9/9.$$

Values are calculated up to 1.000.000, but the work is written only from 0 to 1000. For details, refer to link [2]:

http://arxiv.org/abs/1502.03501.

7 Single Letter Representations

We observe that the numbers written in previous section 6 are not in a symmetric way. But there are numbers, that can be written in a symmetric way, see examples below:

$$\bullet\, 5 = \frac{11-1}{1+1} = \frac{22-2}{2+2} = \frac{33-3}{3+3} = \frac{44-4}{4+4} = \frac{55-5}{5+5} = \frac{66-6}{6+6} = \frac{77-7}{7+7} = \frac{88-8}{8+8} = \frac{99-9}{9+9}.$$

$$\bullet\, 6 = \frac{11+1}{1+1} = \frac{22+2}{2+2} = \frac{33+3}{3+3} = \frac{44+4}{4+4} = \frac{55+5}{5+5} = \frac{66+6}{6+6} = \frac{77+7}{7+7} = \frac{88+8}{8+8} = \frac{99+9}{9+9}.$$

$$\bullet\, 55 = \frac{111-1}{1+1} = \frac{222-2}{2+2} = \frac{333-3}{3+3} = \frac{444-4}{4+4} = \frac{555-5}{5+5} = \frac{666-6}{6+6} = \frac{777-7}{7+7} = \frac{888-8}{8+8} = \frac{999-9}{9+9}.$$

$$\bullet\, 56 = \frac{111+1}{1+1} = \frac{222+2}{2+2} = \frac{333+3}{3+3} = \frac{444+4}{4+4} = \frac{555+5}{5+5} = \frac{666+6}{6+6} = \frac{777+7}{7+7} = \frac{888+8}{8+8} = \frac{999+9}{9+9}.$$

Motivated by this idea, instead working for each digit separately, we can work with a *single letter "a"*. See examples below:

$$5 := (aa - a)/(a + a).$$
$$6 := (aa + a)/(a + a).$$
$$55 := (aaa - a)/(a + a).$$
$$56 := (aaa + a)/(a + a).$$
$$561 := (aaaa + aa)/(a + a).$$
$$666 := aaa \times (aa + a)/((a + a) \times a).$$
$$925 := (aaaaa - aa)/(aa + a).$$

$$1089 := (aaaa - aa - aa)/a.$$
$$1991 := (aaaaaa/aaa \times (a + a) - aa)/a.$$
$$2020 := (aaaaa - a)/aa \times (a + a)/a.$$
$$2035 := (aaaa - a)/(a + a + a) \times aa/(a + a).$$
$$4477 := (aaa/(a + a + a) \times aa \times aa)/(a \times a).$$
$$4999 := (aaaaa - aaaa - a - a)/(a + a).$$
$$5000 := (aaaaa - aaaa)/(a + a).$$

where $a \in \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$, and $aa = 10^2 \times a + a$, $aaa = 10^3 \times a + 10^2 \times a + a$, etc.

For full work, refer to links below [4, 8]:

http://rgmia.org/papers/v18/v18a40.pdf

http://rgmia.org/papers/v18/v18a73.pdf

The first link is up to 3000 numbers, while second link extend it to 5000 numbers.

7.1 Single Letter Power Representations

Above there are numbers written in terms of single letter "a". Using same idea, below are examples numbers with of exponential values written in terms of letter "a":

Power 2.

$$4 := 2^2 = ((a + a)/a)^{(a+a)/a}.$$
$$9 := 3^2 = ((a + a + a)/a)^{(a+a)/a}.$$
$$16 := 4^2 = ((a + a + a + a)/a)^{(a+a)/a}.$$
$$25 := 5^2 = ((aa - a)/(a + a))^{(a+a)/a}.$$
$$36 := 6^2 = ((aa + a)/(a + a))^{(a+a)/a}.$$
$$49 := 7^2 = ((aa - a - a - a - a)/a)^{(a+a)/a}.$$

Power 3.

$$8 := 2^3 = ((a + a)/a)^{(a+a+a)/a}.$$
$$27 := 3^3 = ((a + a + a)/a)^{(a+a+a)/a}.$$
$$64 := 4^3 = ((a + a + a + a)/a)^{(a+a+a)/a}.$$
$$125 := 5^3 = ((aa - a)/(a + a))^{(a+a+a)/a}.$$
$$216 := 6^3 = ((aa + a)/(a + a))^{(a+a+a)/a}.$$
$$343 := 7^3 = ((aa - a - a - a - a)/a)^{(a+a+a)/a}.$$
$$512 := 8^3 = ((aa - a - a - a)/a)^{(a+a+a)/a}.$$

Power of 2.

$$4 := 2^2 = ((a + a)/a)^{(a+a)/a}.$$
$$8 := 2^3 = ((a + a)/a)^{(a+a+a)/a}.$$
$$16 := 2^4 = ((a + a)/a)^{(a+a+a+a)/a}.$$
$$32 := 2^5 = ((a + a)/a)^{(aa-a)/(a+a)}.$$
$$64 := 2^6 = ((a + a)/a)^{(aa+a)/(a+a)}.$$
$$128 := 2^7 = ((a + a)/a)^{(aa-a-a-a-a)/a}.$$

Power of 3.

$$9 := 3^2 = ((a + a + a)/a)^{(a+a)/a}.$$
$$27 := 3^3 = ((a + a + a)/a)^{(a+a+a)/a}.$$
$$81 := 3^4 = ((a + a + a)/a)^{(a+a+a+a)/a}.$$
$$243 := 3^5 = ((a + a + a)/a)^{(aa-a)/(a+a)}.$$
$$729 := 3^6 = ((a + a + a)/a)^{(aa+a)/(a+a)}.$$
$$2187 := 3^7 = ((a + a + a)/a)^{(aa-a-a-a-a)/a}.$$
$$6561 := 3^8 = ((a + a + a)/a)^{(aa-a-a-a)/a}.$$

For full work, refer to link below [8]:

http://rgmia.org/papers/v18/v18a73.pdf

7.2 Palindromic and Number Patterns

The idea of single letter representations of numbers given above can be applied to palindromic and number patterns. The study is also extended to prime patterns, doubly symmetric patterns, etc. See some examples below:

$$11 := (a \times aa)/(a \times a).$$
$$121 := (aa \times aa)/(a \times a).$$
$$12321 := (aaa \times aaa)/(a \times a).$$
$$1234321 := (aaaa \times aaaa)/(a \times a).$$
$$123454321 := (aaaaa \times aaaaa)/(a \times a).$$
$$12345654321 := (aaaaaa \times aaaaaa)/(a \times a).$$
$$1234567654321 := (aaaaaaa \times aaaaaaa)/(a \times a).$$
$$123456787654321 := (aaaaaaaa \times aaaaaaaa)/(a \times a).$$
$$12345678987654321 := (aaaaaaaaa \times aaaaaaaaa)/(a \times a).$$

$$1156 = 34^2 \qquad := ((aa + aa + aa + a)/a)^{(a+a)/a}.$$
$$111556 = 334^2 \qquad := ((aaa + aaa + aaa + a)/a)^{(a+a)/a}.$$
$$11115556 = 3334^2 \qquad := ((aaaa + aaaa + aaaa + a)/a)^{(a+a)/a}.$$
$$1111155556 = 33334^2 \qquad := ((aaaaa + aaaaa + aaaaa + a)/a)^{(a+a)/a}.$$
$$111111555556 = 333334^2 \qquad := ((aaaaaa + aaaaaa + aaaaaa + a)/a)^{(a+a)/a}.$$
$$11111115555556 = 3333334^2 \quad := ((aaaaaaa + aaaaaaa + aaaaaaa + a)/a)^{(a+a)/a}.$$
$$1111111155555556 = 33333334^2 := ((aaaaaaaa + aaaaaaaa + aaaaaaaa + a)/a)^{(a+a)/a}.$$

$$99 = 98 + 1 \qquad := (aaa - aa - a)/(a \times a).$$
$$999 = 987 + 12 \qquad := (aaaa - aaa - a)/(a \times a).$$
$$9999 = 9876 + 123 \qquad := (aaaaa - aaaa - a)/(a \times a).$$
$$99999 = 98765 + 1234 \qquad := (aaaaaa - aaaaa - a)/(a \times a).$$
$$999999 = 987654 + 12345 \qquad := (aaaaaaa - aaaaaa - a)/(a \times a).$$
$$9999999 = 9876543 + 123456 \qquad := (aaaaaaaa - aaaaaaa - a)/(a \times a).$$
$$99999999 = 98765432 + 1234567 \qquad := (aaaaaaaaa - aaaaaaaa - a)/(a \times a).$$
$$999999999 = 987654321 + 12345678 \quad := (aaaaaaaaaa - aaaaaaaaa - a)/(a \times a).$$
$$9999999999 = 9876543210 + 123456789 := (aaaaaaaaaaa - aaaaaaaaaa - a)/(a \times a).$$

$$33 = 12 + 21 \qquad := (a + a + a) \times aa/(a \times a)$$
$$444 = 123 + 321 \qquad := (a + a + a + a) \times aaa/(a \times a)$$
$$5555 = 1234 + 4321 \qquad := (a + a + a + a + a) \times aaaa/(a \times a)$$
$$66666 = 12345 + 54321 \qquad := (a + a + a + a + a + a) \times aaaaa/(a \times a)$$
$$777777 = 123456 + 654321 \qquad := (aa - a - a - a - a) \times aaaaaa/(a \times a)$$
$$8888888 = 1234567 + 7654321 \qquad := (aa - a - a - a) \times aaaaaaa/(a \times a)$$
$$99999999 = 12345678 + 87654321 := (aa - a - a) \times aaaaaaaa/(a \times a)$$

$$111111111 = 12345679 \times 9 \times 1 := aaaaaaaaa \times a/(a \times a)$$
$$222222222 = 12345679 \times 9 \times 2 := aaaaaaaaa \times (a + a)/(a \times a)$$
$$333333333 = 12345679 \times 9 \times 3 := aaaaaaaaa \times (a + a + a)/(a \times a)$$
$$444444444 = 12345679 \times 9 \times 4 := aaaaaaaaa \times (a + a + a + a)/(a \times a)$$
$$555555555 = 12345679 \times 9 \times 5 := aaaaaaaaa \times (a + a + a + a + a)/(a \times a)$$
$$666666666 = 12345679 \times 9 \times 6 := aaaaaaaaa \times (a + a + a + a + a + a)/(a \times a)$$
$$777777777 = 12345679 \times 9 \times 7 := aaaaaaaaa \times (aa - a - a - a - a)/(a \times a)$$
$$888888888 = 12345679 \times 9 \times 8 := aaaaaaaaa \times (aa - a - a - a)/(a \times a)$$
$$999999999 = 12345679 \times 9 \times 9 := aaaaaaaaa \times (aa - a - a)/(a \times a).$$

The number 12345679 also appears as a division of 1/81, i.e.

$$\frac{1}{81} = 0.012345679\ 012345679\ 012345679\ 012345679\ .... = 0.\overline{012345679}$$

For full work, refer to links below [9, 10]:

http://rgmia.org/papers/v18/v18a77.pdf.

http://rgmia.org/papers/v18/v18a99.pdf.

8 Running Expressions

It is well know that one can write, $12 = 3 \times 4$, $56 = 7 \times 8$. Here 9 remains alone. The aim of this work is to see how we can write expressions using 9 digits in a sequence in increasing and/or decreasing way. In the decreasing case, the possibility of 9 to 0 is also considered. The expressions are separated either by single or by double equality signs. See example below:

$$12 = 3 + 4 + (5 \times 6 + 7 + 8)/9$$
$$123 = 4 + 5 + 6 \times 7 + 8 \times 9$$
$$1234 = -5 + 6! \ + 7 + 8^{\sqrt{9}}$$

$$12 + 3 \times 4 + 5 \times (6 + 7) = 89$$
$$1 + 23 + 45 + 6! = 789$$

$$98 = (7 + 6) \times 5 + 4 \times 3 + 21$$
$$987 = 6! \ + 5! \ + (4 + 3) \times 21$$

$$98 - 7 \times (6 + 5) \times (4 - 3) = 21$$
$$\sqrt{9} \times 87 + 6 + 54 = 321$$
$$9 - 8 + 7! \ - 6 \times 5! = 4321$$

$$9 - 8 + 7 - 6 + 5 + 4 - 3 + 2 = 10$$
$$9 \times (8 + 7) + 6 + 5 + 4^3 = 210$$
$$(9 - 87 + 6!) \times 5! \ /4! = 3210.$$

Above examples give running expressions in increasing or decreasing orders of 1 to 9 or 9 to 1 or 9 to 0 separated by single equality sign. But there are numbers, that can be separated by more equality signs, for example,

$$16 := 12/3 \times 4 \qquad = 5 + 6 + (7 + 8)/\sqrt{9}$$
$$:= (9 + 87)/6 \qquad = 5 + 4 + 3 \times 2 + 1.$$

$$18 := 12 + 3! \qquad = \sqrt{4 + 5} \times 6 \qquad = 7 + 8 + \sqrt{9}$$
$$:= \sqrt{9} + 8 + 7 \qquad = \sqrt{6 \times 54} \qquad = -3 + 21 \ = \ 3! + 2 + 10.$$

$$24 := 1 + 23 \qquad = 4 + 5!/6 \qquad = 7 + 8 + 9$$
$$:= 9 + 8 + 7 \qquad = (6 - 5) \times 4! \qquad = 3 + 21$$

$$120 := 1 \times (2 + 3)! \qquad = 4 + 5!/6 + 7 + 89$$
$$:= 98 + 7 + 6 + 5 + 4 = (3 + 2)! \times 1$$
$$:= \sqrt{9} + 87 + 6 \times 5 \quad = \sqrt{4} \times 3 \times 2 \times 10.$$

For full work, refer to the link below [3]:

http://rgmia.org/papers/v18/v18a27.pdf.

9 Selfie Numbers

Numbers represented by their own digits by use of certain operations are considered as *"Selfie Number"*. These numbers, we have divided in two categories. These two categories are again divided in two each, i.e., one in order of digits appearing in the numbers and their reverse, and the second is in increasing and decreasing order of digits. See below examples in each category:

- **Digit's Order**

$$936 = (\sqrt{9})!^3 + 6!;$$
$$1296 = \sqrt{(1+2)!^9/6};$$
$$2896 = 2 \times (8 + (\sqrt{9})!! + 6!);$$
$$12969 = 1 \times 2 \times 9 \times 6! + 9.$$

- **Reverse Order of Digits**

$$936 = 6! + (3!)^{\sqrt{9}};$$
$$1296 = 6^{(\sqrt{9}+2-1)};$$
$$2896 = (6! + (\sqrt{9})!! + 8) \times 2;$$
$$20167 = 7 + (6 + 1 + 0!)!/2.$$

- **Increasing Order of Digits**

$$936 = 3!! + 6^{\sqrt{9}};$$
$$1296 = (1+2)! \times 6^{\sqrt{9}};$$
$$8397 = -3 - 7! + 8!/\sqrt{9};$$
$$241965 = (1 + (2 \times 4)! + 5) \times 6 + 9.$$

- **Decreasing Order of Digits**

$$936 = (\sqrt{9})!! + 6^3;$$
$$1296 = ((\sqrt{9})! \times 6)^2 \times 1;$$
$$20148 = (8! - 4)/2 - 10;$$
$$435609 = 9 + (6! - 5!/\sqrt{4})^{(3-0!)}.$$

Above we have given examples of *selfie numbers* in four different ways. This has been done using the basic operations along with *factorial* and *square-root*. See below more examples:

$$331779 := 3 + (31 - 7)^{\sqrt{7+9}} \qquad = \sqrt{9} + (7 \times 7 - 1)^3 \times 3.$$
$$342995 := (3^4 - 2 - 9)^{\sqrt{9}} - 5 \quad = -5 + (-9 + 9^2 - \sqrt{4})^3.$$
$$759375 := (-7 + 59 - 37)^5 \qquad = (5 + 7 + 3)^{\sqrt{9}-5+7}.$$
$$759381 := 7 + (5 \times \sqrt{9})^{-3+8} - 1 = -1 + (8 \times 3 - 9)^5 + 7.$$

Below are interesting numbers following sequential order left side. See below:

$$456 := 4 \times (5! - 6) \qquad = (-6 + 5!) \times 4.$$
$$3456 := 3!! \times 4/5 \times 6 \qquad = 6!/5 \times 4 \times 3!!.$$
$$34567 := (3 + 45) \times 6! + 7 \qquad = 7 + 6! \times (5 + 43).$$
$$345678 := (3! - \sqrt{4}) \times 5! \times 6! + 78.$$

For full details, refer to link [5, 6, 7, 13, 14, 15]:

http://rgmia.org/papers/v17/v17a140.pdf
http://rgmia.org/papers/v18/v18a32.pdf
http://rgmia.org/papers/v18/v18a70.pdf
http://rgmia.org/papers/v18/v18a174.pdf
http://rgmia.org/papers/v18/v18a175.pdf .
http://rgmia.org/papers/v19/v19a16.pdf .

9.1   Selfie Numbers with Addition, Subtraction and Factorial

Examples given above uses basic operation along with factorial and square–root. Madachy [59], page 167, 1966, gave few examples with factorial using only the operation of addition. See below.

$$1 = 1!$$
$$2 = 2!$$
$$145 = 1! + 4! + 5!.$$
$$40585 = 4! + 0! + 5! + 8! + 5!$$

Question arises, what else we can get using only the operations of addition and subtraction along with factorial? Below are some examples up to 6–digits:

$$145 = 1! + 4! + 5!.$$

$$1463 = -1! + 4! + 6! + 3!!.$$

$$10077 = -1! - 0! - 0! + 7! + 7!.$$
$$40585 = 4! + 0! + 5! + 8! + 5!.$$
$$80518 = 8! - 0! - 5! - 1! + 8!.$$

$$317489 = -3! - 1! - 7! - 4! - 8! + 9!.$$
$$352797 = -3! + 5 - 2! - 7! + 9! - 7!.$$

$$357592 = -3! - 5! - 7! - 5! + 9! - 2!.$$
$$357941 = 3! + 5! - 7! + 9! - 4! - 1!.$$
$$361469 = 3! - 6! - 1! + 4! - 6! + 9!.$$
$$364292 = 3!! + 6! - 4! - 2! + 9! - 2!.$$
$$397584 = -3!! + 9! - 7! + 5! + 8! + 4!.$$
$$398173 = 3! + 9! + 8! + 1! - 7! + 3!.$$
$$408937 = -4! + 0! + 8! + 9! + 3!! + 7!.$$
$$715799 = -7! - 1! + 5! - 7! + 9! + 9!.$$
$$720599 = -7! - 2! + 0! - 5! + 9! + 9!.$$

Still, we can have examples not necessarily having factorial on all the numbers. See below:

$$733 = 7 + 3!! + 3!.$$
$$5177 = 5! + 17 + 7!.$$
$$363239 = 36 + 323 + 9!.$$
$$363269 = 363 + 26 + 9!.$$
$$403199 = 40319 + 9!.$$

For more details refer to author's work [44]:

http://rgmia.org/papers/v19/v19a163.pdf

9.2 Unified Representations

From examples above, we observe that there are numbers such as 936, 1296, etc., that can be written in all the four ways. These types of numbers we call *unified selfie numbers*. More clearly,

$$\text{Unified Selfie number} = \text{Order of digits}$$
$$= \text{Reverse order of digits}$$
$$= \text{Increasing order of digits}$$
$$= \text{Decreasing order of digits}.$$

Below are examples of *unified selfie numbers* written in all the four ways,

- $729 = (\sqrt{7+2})!! + 9$
  $= 9 + (\sqrt{2+7})!!$
  $= (2+7)^{\sqrt{9}}$
  $= 9^{\sqrt{7+2}}.$

- $97632 = -(\sqrt{9})!! + 7! + 6^{3!} \times 2$
  $= 2 \times 3!^6 + 7! - (\sqrt{9})!!$
  $= 2 \times 3!^6 + 7! - (\sqrt{9})!!$
  $= -(\sqrt{9})!! + 7! + 6^{3!} \times 2.$

- $114688 = (11 \times \sqrt{4} + 6) \times \sqrt{8^8}$
  $= (8 + 8 \times 6) \times \sqrt{4^{11}}$
  $= (11 \times \sqrt{4} + 6) \times \sqrt{8^8}$
  $= (8 + 8 \times 6) \times \sqrt{4^{11}}.$

- $139968 = (13 \times 9 - 9) \times \sqrt{6^8}$
  $= 8 \times 6^{\sqrt{9}} \times 9^{3-1}$
  $= 1 \times \sqrt{36^8}/(\sqrt{9} + 9)$
  $= (-\sqrt{9} + 9)^8/(6 \times (3 - 1)).$

- $326627 = (3 + 2 + 6^{\sqrt{6^2}}) \times 7$
  $= \sqrt{7^2} \times (6^6 + 2 + 3)$
  $= (\sqrt{2 + 23} + 6^6) \times 7$
  $= 7 \times (6^6 + \sqrt{3 + 22}).$

- $531439 = -5 + 3 + (-1 + 4)^{3+9}$
  $= (9 \times 3)^4 \times 1 + 3 - 5$
  $= 1 - 3 + 3^{\sqrt{4+5}+9}$
  $= 9^{5+4-3} - \sqrt{3 + 1}.$

For more details refer to author's work [11, 13, 14, 15]:

http://rgmia.org/papers/v18/v18a153.pdf
http://rgmia.org/papers/v18/v18a174.pdf
http://rgmia.org/papers/v18/v18a175.pdf
http://rgmia.org/papers/v19/v19a16.pdf .

9.3   Patterns in Selfie Numbers

Numbers extended with same properties multiplying by zero, we consider as *patterns in numbers*. Few examples of this kind are studied long back in 1966 by Madachy [59], page 174–175. See below:

$$3^4 \times 425 = 34425$$
$$3^4 \times 4250 = 344250$$
$$3^4 \times 42500 = 3442500.$$

$$31^2 \times 325 = 312325$$
$$31^2 \times 3250 = 3123250$$
$$31^2 \times 32500 = 31232500.$$

For simplicity, let us write these numbers as *patterns in selfie numbers*. Subsections below give more examples with factorial and square-root:

- **Digit's Order**

Numbers appearing in this subsection are represented in order of digits.

$$1285 = (1 + 2^8) \times 5$$
$$12850 = (1 + 2^8) \times 50$$
$$128500 = (1 + 2^8) \times 500$$

$$15585 = 1 \times (5^5 - 8) \times 5$$
$$155850 = 1 \times (5^5 - 8) \times 50$$
$$1558500 = 1 \times (5^5 - 8) \times 500$$

$$8192 = 8^{1+\sqrt{9}} \times 2$$
$$81920 = 8^{1+\sqrt{9}} \times 20$$
$$819200 = 8^{1+\sqrt{9}} \times 200$$

$$29435 = \sqrt{29^4} \times 35$$
$$294350 = \sqrt{29^4} \times 350$$
$$2943500 = \sqrt{29^4} \times 3500$$

- **Decreasing Order of Digits**

The numbers appearing in this subsection are represented decreasing order of digits.

$$1827 = 87 \times 21$$
$$18270 = 87 \times 210$$
$$182700 = 87 \times 2100$$

$$19683 = \sqrt{9^8} \times (6 - 3) \times 1$$
$$196830 = \sqrt{9^8} \times (6 - 3) \times 10$$
$$1968300 = \sqrt{9^8} \times (6 - 3) \times 100$$

$$2916 = (9 \times 6)^2 \times 1$$
$$29160 = (9 \times 6)^2 \times 10$$
$$291600 = (9 \times 6)^2 \times 100$$

$$995544 = ((\sqrt{9} + 9)^5 + 54) \times 4$$
$$9955440 = ((\sqrt{9} + 9)^5 + 54) \times 40$$
$$99554400 = ((\sqrt{9} + 9)^5 + 54) \times 400$$

For more details, refer to author's work [13, 14, 15]:

http://rgmia.org/papers/v18/v18a154.pdf
http://rgmia.org/papers/v18/v18a174.pdf
http://rgmia.org/papers/v18/v18a175.pdf .

Above we have written *patterns in selfie numbers*. But there are another kind of patterns in numbers. This study in done in [42]. Also refer 15.1 for different kind of *palindromic-type pattern in numbers*.

9.4   Symmetric Consecutive Representations

There are numbers those can be represented in symmetric and consecutive forms of block of 10 or more. In some cases the numbers appears only in one way, i.e., either in order of digits appearing in number or in reverse order of digits. In some cases we have representations is in both ways. See below some examples:

$$14400 := (1+4)!^{\sqrt{4}} + 0 + 0 = 0 + (0!+4)!^{\sqrt{4}} \times 1 \qquad 64800 = 6!^{\sqrt{4}}/8 + 0 + 0.$$
$$14401 := (1+4)!^{\sqrt{4}} + 0 + 1 = 1 + (0!+4)!^{\sqrt{4}} \times 1 \qquad 64801 = 6!^{\sqrt{4}}/8 + 0 + 1.$$
$$14402 := (1+4)!^{\sqrt{4}} + 0 + 2 = 2 + (0!+4)!^{\sqrt{4}} \times 1 \qquad 64802 = 6!^{\sqrt{4}}/8 + 0 + 2.$$
$$14403 := (1+4)!^{\sqrt{4}} + 0 + 3 = 3 + (0!+4)!^{\sqrt{4}} \times 1 \qquad 64803 = 6!^{\sqrt{4}}/8 + 0 + 3.$$
$$14404 := (1+4)!^{\sqrt{4}} + 0 + 4 = 4 + (0!+4)!^{\sqrt{4}} \times 1 \qquad 64804 = 6!^{\sqrt{4}}/8 + 0 + 4.$$
$$14405 := (1+4)!^{\sqrt{4}} + 0 + 5 = 5 + (0!+4)!^{\sqrt{4}} \times 1 \qquad 64805 = 6!^{\sqrt{4}}/8 + 0 + 5.$$
$$14406 := (1+4)!^{\sqrt{4}} + 0 + 6 = 6 + (0!+4)!^{\sqrt{4}} \times 1 \qquad 64806 = 6!^{\sqrt{4}}/8 + 0 + 6.$$
$$14407 := (1+4)!^{\sqrt{4}} + 0 + 7 = 7 + (0!+4)!^{\sqrt{4}} \times 1 \qquad 64807 = 6!^{\sqrt{4}}/8 + 0 + 7.$$
$$14408 := (1+4)!^{\sqrt{4}} + 0 + 8 = 8 + (0!+4)!^{\sqrt{4}} \times 1 \qquad 64808 = 6!^{\sqrt{4}}/8 + 0 + 8.$$
$$14409 := (1+4)!^{\sqrt{4}} + 0 + 9 = 9 + (0!+4)!^{\sqrt{4}} \times 1 \qquad 64809 = 6!^{\sqrt{4}}/8 + 0 + 9.$$

In the increasing case, above two sequences can be extended up to 14499 and 64899 respectively, forming sets of 100 consecutive symmetric representations:

$$14410 = (1+4)!^{\sqrt{4}} + 10. \qquad 64810 = 6!^{\sqrt{4}}/8 + 10.$$
$$14411 = (1+4)!^{\sqrt{4}} + 11. \qquad 64811 = 6!^{\sqrt{4}}/8 + 11.$$
$$14412 = (1+4)!^{\sqrt{4}} + 12. \qquad 64812 = 6!^{\sqrt{4}}/8 + 12.$$
$$14413 = (1+4)!^{\sqrt{4}} + 13. \qquad 64813 = 6!^{\sqrt{4}}/8 + 13.$$
$$\ldots\ldots \qquad \ldots\ldots$$
$$14496 = (1+4)!^{\sqrt{4}} + 96. \qquad 64896 = 6!^{\sqrt{4}}/8 + 96.$$
$$14497 = (1+4)!^{\sqrt{4}} + 97. \qquad 64897 = 6!^{\sqrt{4}}/8 + 97.$$
$$14498 = (1+4)!^{\sqrt{4}} + 98. \qquad 64898 = 6!^{\sqrt{4}}/8 + 98.$$
$$14499 = (1+4)!^{\sqrt{4}} + 99. \qquad 64899 = 6!^{\sqrt{4}}/8 + 99.$$

More examples of symmetrical consecutive representations of selfie numbers are follows as:

$$466560 := (4+6) \times 6^5 \times 6 + 0 = 0 + 6^5 \times 6 \times (6+4).$$
$$466561 := (4+6) \times 6^5 \times 6 + 1 = 1 + 6^5 \times 6 \times (6+4).$$
$$466562 := (4+6) \times 6^5 \times 6 + 2 = 2 + 6^5 \times 6 \times (6+4).$$
$$466563 := (4+6) \times 6^5 \times 6 + 3 = 3 + 6^5 \times 6 \times (6+4).$$
$$466564 := (4+6) \times 6^5 \times 6 + 4 = 4 + 6^5 \times 6 \times (6+4).$$
$$466565 := (4+6) \times 6^5 \times 6 + 5 = 5 + 6^5 \times 6 \times (6+4).$$
$$466566 := (4+6) \times 6^5 \times 6 + 6 = 6 + 6^5 \times 6 \times (6+4).$$
$$466567 := (4+6) \times 6^5 \times 6 + 7 = 7 + 6^5 \times 6 \times (6+4).$$
$$466568 := (4+6) \times 6^5 \times 6 + 8 = 8 + 6^5 \times 6 \times (6+4).$$
$$466569 := (4+6) \times 6^5 \times 6 + 9 = 9 + 6^5 \times 6 \times (6+4).$$

$$64840 := 6!^{\sqrt{4}}/8 + 40 = 0 + 4^8 + 4! - 6!.$$
$$64841 := 6!^{\sqrt{4}}/8 + 41 = 1 + 4^8 + 4! - 6!.$$
$$64842 := 6!^{\sqrt{4}}/8 + 42 = 2 + 4^8 + 4! - 6!.$$
$$64843 := 6!^{\sqrt{4}}/8 + 43 = 3 + 4^8 + 4! - 6!.$$
$$64844 := 6!^{\sqrt{4}}/8 + 44 = 4 + 4^8 + 4! - 6!.$$
$$64845 := 6!^{\sqrt{4}}/8 + 45 = 5 + 4^8 + 4! - 6!.$$
$$64846 := 6!^{\sqrt{4}}/8 + 46 = 6 + 4^8 + 4! - 6!.$$
$$64847 := 6!^{\sqrt{4}}/8 + 47 = 7 + 4^8 + 4! - 6!.$$
$$64848 := 6!^{\sqrt{4}}/8 + 48 = 8 + 4^8 + 4! - 6!.$$
$$64849 := 6!^{\sqrt{4}}/8 + 49 = 9 + 4^8 + 4! - 6!.$$

$$15637 := -1 + 3! + 5^6 + 7 = 7 + 6 + 5^{3!} - 1.$$
$$15638 := -1 + 3! + 5^6 + 8 = 8 + 6 + 5^{3!} - 1.$$
$$15639 := -1 + 3! + 5^6 + 9 = 9 + 6 + 5^{3!} - 1.$$

$$790 = (\sqrt{9})!! + 70.$$
$$791 = (\sqrt{9})!! + 71.$$
$$792 = (\sqrt{9})!! + 72.$$
$$793 = (\sqrt{9})!! + 73.$$
$$794 = (\sqrt{9})!! + 74.$$
$$795 = (\sqrt{9})!! + 75.$$
$$796 = (\sqrt{9})!! + 76.$$

$$30245 = 5 + (4+3)! \times (2+0!)!.$$
$$30246 = 6 + (4+3)! \times (2+0!)!.$$
$$30247 = 7 + (4+3)! \times (2+0!)!.$$
$$30248 = 8 + (4+3)! \times (2+0!)!.$$
$$30249 = 9 + (4+3)! \times (2+0!)!.$$

For the numbers 15637 and 30245, we have symmetry only for few numbers. Moreover, these two examples are increasing and decreasing order of digits. For complete details, refer the following links [13, 14, 15]:

http://rgmia.org/papers/v18/v18a174.pdf
http://rgmia.org/papers/v18/v18a175.pdf
http://rgmia.org/papers/v19/v19a16.pdf .

Above examples are *symmetric selfie* representations up to 5-digits. Below are examples of 6-digits *symmetric selfie* representations:

$$518400 := (5+1)!^{8/4} + 00 = 00 + (4!/8)!! \times (1+5)!.$$
$$518411 := (5+1)!^{8/4} + 11 = 11 + (4!/8)!! \times (1+5)!.$$
$$518422 := (5+1)!^{8/4} + 22 = 22 + (4!/8)!! \times (1+5)!.$$
$$518433 := (5+1)!^{8/4} + 33 = 33 + (4!/8)!! \times (1+5)!.$$
$$518444 := (5+1)!^{8/4} + 44 = 44 + (4!/8)!! \times (1+5)!.$$

$$518455 := (5 + 1)!^{8/4} + 55 = 55 + (4!/8)!! \times (1 + 5)!.$$
$$518466 := (5 + 1)!^{8/4} + 66 = 66 + (4!/8)!! \times (1 + 5)!.$$
$$518477 := (5 + 1)!^{8/4} + 77 = 77 + (4!/8)!! \times (1 + 5)!.$$
$$518488 := (5 + 1)!^{8/4} + 88 = 88 + (4!/8)!! \times (1 + 5)!.$$
$$518499 := (5 + 1)!^{8/4} + 99 = 99 + (4!/8)!! \times (1 + 5)!.$$

We observe that the above example is *symmetric selfie* representation but is not consecutive. Below is consecutive *symmetric selfie* representation:

$$363390 := 3! + 6! - 3!^3 + 9! + 0 = 0 + 9! + (3 \times 3)!/6! + 3!.$$
$$363391 := 3! + 6! - 3!^3 + 9! + 1 = 1 + 9! + (3 \times 3)!/6! + 3!.$$
$$363392 := 3! + 6! - 3!^3 + 9! + 2 = 2 + 9! + (3 \times 3)!/6! + 3!.$$
$$363393 := 3! + 6! - 3!^3 + 9! + 3 = 3 + 9! + (3 \times 3)!/6! + 3!.$$
$$363394 := 3! + 6! - 3!^3 + 9! + 4 = 4 + 9! + (3 \times 3)!/6! + 3!.$$
$$363395 := 3! + 6! - 3!^3 + 9! + 5 = 5 + 9! + (3 \times 3)!/6! + 3!.$$
$$363396 := 3! + 6! - 3!^3 + 9! + 6 = 6 + 9! + (3 \times 3)!/6! + 3!.$$
$$363397 := 3! + 6! - 3!^3 + 9! + 7 = 7 + 9! + (3 \times 3)!/6! + 3!.$$
$$363398 := 3! + 6! - 3!^3 + 9! + 8 = 8 + 9! + (3 \times 3)!/6! + 3!.$$
$$363399 := 3! + 6! - 3!^3 + 9! + 9 = 9 + 9! + (3 \times 3)!/6! + 3!.$$

Still there are consecutive *symmetric selfie* representations of blocks of 100 with 6–digits. See examples below:

$$158400 := (-(1 + 5)! + 8!) \times 4 + 00.$$
$$158401 := (-(1 + 5)! + 8!) \times 4 + 01.$$
$$158402 := (-(1 + 5)! + 8!) \times 4 + 02.$$
$$158403 := (-(1 + 5)! + 8!) \times 4 + 03.$$
$$\ldots \qquad \ldots \qquad \ldots$$
$$158451 := (-(1 + 5)! + 8!) \times 4 + 51.$$
$$158452 := (-(1 + 5)! + 8!) \times 4 + 52.$$
$$158453 := (-(1 + 5)! + 8!) \times 4 + 53.$$
$$158454 := (-(1 + 5)! + 8!) \times 4 + 54.$$
$$\ldots \qquad \ldots \qquad \ldots$$
$$158496 := (-(1 + 5)! + 8!) \times 4 + 96.$$
$$158497 := (-(1 + 5)! + 8!) \times 4 + 97.$$
$$158498 := (-(1 + 5)! + 8!) \times 4 + 98.$$
$$158499 := (-(1 + 5)! + 8!) \times 4 + 99.$$

$$363600 := (3^{6/3})! + 6! + 00.$$
$$363601 := (3^{6/3})! + 6! + 01.$$
$$363602 := (3^{6/3})! + 6! + 02.$$
$$363603 := (3^{6/3})! + 6! + 03.$$
$$\ldots \qquad \ldots \qquad \ldots$$
$$363651 := (3^{6/3})! + 6! + 51.$$
$$363652 := (3^{6/3})! + 6! + 52.$$
$$363653 := (3^{6/3})! + 6! + 53.$$
$$363654 := (3^{6/3})! + 6! + 54.$$
$$\ldots \qquad \ldots \qquad \ldots$$
$$363696 := (3^{6/3})! + 6! + 96.$$
$$363697 := (3^{6/3})! + 6! + 97.$$
$$363698 := (3^{6/3})! + 6! + 98.$$
$$363699 := (3^{6/3})! + 6! + 99.$$

For more details refer the following link [45]:

http://rgmia.org/papers/v19/v19a164.pdf

10 Fibonacci Sequence and Selfie Numbers

Fibonacci sequence numbers are well known in literature [51, 52]. This sequence is defined as

$$F(0) = 0, \quad F(1) = 1, \quad F(n+1) = F(n) + F(n-1), \quad n \geq 1.$$

In [36], we worked with selfie numbers using the terms of Fibonacci sequences as $F(.)$. See examples below:

$$256 = 2^5 \times F(6).$$
$$46493 = F(4 \times 6) + (-4+9)^3.$$
$$882 = 2 \times F(8) \times F(8).$$

$$1631 = F(13) \times (6+1).$$
$$54128 = 8 \times (F(2) + F(1 \times 4 \times 5)).$$

The first two examples are in digit's order and last three are in reverse order of digits. In [37], we worked with *composition* of *Fibonacci sequence* values, such as $F(F(.))$, $F(F(F(.)))$. See examples below:

$$235 = 2 + F(F(F(3) + 5)).$$
$$4427 = (F(4) + 4^2) \times F(F(7)).$$

$$63 = 3 \times F(F(6)).$$
$$43956 = (F(F(F(6))) + 5 \times 9 - F(3)) \times 4.$$

The first two examples are in order of digits, and last two examples are in reverse order of digits.

Using the idea of both the papers [36, 37] along with *factorial* are used in [38]. See below more examples,

$$447 = (F(4))!! - F(F((F(4))!)) \times F(7).$$
$$29471 = (F(2) + F(9)) \times F(F(F((F(4))!)))/F(7) + 1.$$

$$433 = F(F(3!))^{F(3)} - F(F(4)!).$$
$$4995 = -5 \times 9 + (9 - F(F(4)))!.$$

For details refer the following works [36, 37, 38]:

http://rgmia.org/papers/v19/v19a142.pdf
http://rgmia.org/papers/v19/v19a143.pdf
http://rgmia.org/papers/v19/v19a156.pdf .

First work is just with $F(.)$, second work is with composition of F, i.e., $F(F(.))$, etc. The third paper combines first and second along with use of *factorial*. Still there are more possibilities of working with *square-root*. See below some examples,

$$954 = F((\sqrt{9})!) \times 5! - F(4)!.$$
$$1439 = 1 + \sqrt{4} \times 3!! - F(\sqrt{9}).$$
$$4394 = F(4 + 3)^{\sqrt{9}} \times \sqrt{4}.$$
$$89735 = (F(F(8) + F(F(\sqrt{9}))) + F(F(7)) + 3) \times 5.$$

$$1919 = (F((\sqrt{9})!)!/F(-1 + 9)) - 1.$$
$$6498 = -F(8) \times \sqrt{9} + F(4)^{F(6)}.$$
$$12784 = F(F(\sqrt{4}) + 8) \times (F(7 \times 2) - 1).$$
$$39901 = -F(10) \times F((\sqrt{9})!) + F(F((\sqrt{9})!)) + F(3!)!.$$

First four examples are in digit's order and in second four are in reverse order of digits. Detailed study on this type of work shall be dealt elsewhere.

More examples with *Fibonacci sequence* values written in symmetric form are given in section below:

10.1   Symmetrical Representations with Fibonacci Sequence Values

Below are some examples of symmetrical representations of *selfie numbers* by use of *Fibonacci sequence* values:

$$823540 := (8 - F(2))^{F(3)+5} - F(4) + 0 = 0 - F(4) + (5 + F(3))^{-F(2)+8}.$$
$$823541 := (8 - F(2))^{F(3)+5} - F(4) + 1 = 1 - F(4) + (5 + F(3))^{-F(2)+8}.$$
$$823542 := (8 - F(2))^{F(3)+5} - F(4) + 2 = 2 - F(4) + (5 + F(3))^{-F(2)+8}.$$
$$823543 := (8 - F(2))^{F(3)+5} - F(4) + 3 = 3 - F(4) + (5 + F(3))^{-F(2)+8}.$$
$$823544 := (8 - F(2))^{F(3)+5} - F(4) + 4 = 4 - F(4) + (5 + F(3))^{-F(2)+8}.$$
$$823545 := (8 - F(2))^{F(3)+5} - F(4) + 5 = 5 - F(4) + (5 + F(3))^{-F(2)+8}.$$
$$823546 := (8 - F(2))^{F(3)+5} - F(4) + 6 = 6 - F(4) + (5 + F(3))^{-F(2)+8}.$$
$$823547 := (8 - F(2))^{F(3)+5} - F(4) + 7 = 7 - F(4) + (5 + F(3))^{-F(2)+8}.$$
$$823548 := (8 - F(2))^{F(3)+5} - F(4) + 8 = 8 - F(4) + (5 + F(3))^{-F(2)+8}.$$
$$823549 := (8 - F(2))^{F(3)+5} - F(4) + 9 = 9 - F(4) + (5 + F(3))^{-F(2)+8}.$$

$54670 := 5 \times (F(F(F(4))) + F(F(F(6))) - F(7)) + 0 = 0 + (-F(7) + F(F(F(6))) + F(F(F(4)))) \times 5.$

$54671 := 5 \times (F(F(F(4))) + F(F(F(6))) - F(7)) + 1 = 1 + (-F(7) + F(F(F(6))) + F(F(F(4)))) \times 5.$

$54672 := 5 \times (F(F(F(4))) + F(F(F(6))) - F(7)) + 2 = 2 + (-F(7) + F(F(F(6))) + F(F(F(4)))) \times 5.$

$54673 := 5 \times (F(F(F(4))) + F(F(F(6))) - F(7)) + 3 = 3 + (-F(7) + F(F(F(6))) + F(F(F(4)))) \times 5.$

$54674 := 5 \times (F(F(F(4))) + F(F(F(6))) - F(7)) + 4 = 4 + (-F(7) + F(F(F(6))) + F(F(F(4)))) \times 5.$

$54675 := 5 \times (F(F(F(4))) + F(F(F(6))) - F(7)) + 5 = 5 + (-F(7) + F(F(F(6))) + F(F(F(4)))) \times 5.$

$54676 := 5 \times (F(F(F(4))) + F(F(F(6))) - F(7)) + 6 = 6 + (-F(7) + F(F(F(6))) + F(F(F(4)))) \times 5.$

$54677 := 5 \times (F(F(F(4))) + F(F(F(6))) - F(7)) + 7 = 7 + (-F(7) + F(F(F(6))) + F(F(F(4)))) \times 5.$

$54678 := 5 \times (F(F(F(4))) + F(F(F(6))) - F(7)) + 8 = 8 + (-F(7) + F(F(F(6))) + F(F(F(4)))) \times 5.$

$54679 := 5 \times (F(F(F(4))) + F(F(F(6))) - F(7)) + 9 = 9 + (-F(7) + F(F(F(6))) + F(F(F(4)))) \times 5.$

$3840 := (F(3!))!/F(8) \times F(F(4)) + 0 = 0 + F(F(4)) \times 8!/F(F(3!)).$

$3841 := (F(3!))!/F(8) \times F(F(4)) + 1 = 1 + F(F(4)) \times 8!/F(F(3!)).$

$3842 := (F(3!))!/F(8) \times F(F(4)) + 2 = 2 + F(F(4)) \times 8!/F(F(3!)).$

$3843 := (F(3!))!/F(8) \times F(F(4)) + 3 = 3 + F(F(4)) \times 8!/F(F(3!)).$

$3844 := (F(3!))!/F(8) \times F(F(4)) + 4 = 4 + F(F(4)) \times 8!/F(F(3!)).$

$3845 := (F(3!))!/F(8) \times F(F(4)) + 5 = 5 + F(F(4)) \times 8!/F(F(3!)).$

$3846 := (F(3!))!/F(8) \times F(F(4)) + 6 = 6 + F(F(4)) \times 8!/F(F(3!)).$

$3847 := (F(3!))!/F(8) \times F(F(4)) + 7 = 7 + F(F(4)) \times 8!/F(F(3!)).$

$3848 := (F(3!))!/F(8) \times F(F(4)) + 8 = 8 + F(F(4)) \times 8!/F(F(3!)).$

$3849 := (F(3!))!/F(8) \times F(F(4)) + 9 = 9 + F(F(4)) \times 8!/F(F(3!)).$

Still, we have symmetric representations with numbers $F(2)$, $F(3)$ and $F(4)$.

$73792 := (7 + F(3)^{F(7)}) \times 9 + F(2) = F(2) + 9 \times (7 + F(3)^{F(7)}).$

$73793 := (7 + F(3)^{F(7)}) \times 9 + F(3) = F(3) + 9 \times (7 + F(3)^{F(7)}).$

$73794 := (7 + F(3)^{F(7)}) \times 9 + F(4) = F(4) + 9 \times (7 + F(3)^{F(7)}).$

$65652 := F(F(F(6))) + 5 \times (F(F(F(6))) - 5) + F(2) = F(2) + (-5 + F(F(F(6)))) \times 5 + F(F(F(6))).$

$65653 := F(F(F(6))) + 5 \times (F(F(F(6))) - 5) + F(3) = F(3) + (-5 + F(F(F(6)))) \times 5 + F(F(F(6))).$

$65654 := F(F(F(6))) + 5 \times (F(F(F(6))) - 5) + F(4) = F(4) + (-5 + F(F(F(6)))) \times 5 + F(F(F(6))).$

$65672 := -F(F(F(6))) - 5 + F(F(F(6))) \times 7 + F(2) = F(2) + 7 \times F(F(F(6))) - 5 - F(F(F(6))).$

$65673 := -F(F(F(6))) - 5 + F(F(F(6))) \times 7 + F(3) = F(3) + 7 \times F(F(F(6))) - 5 - F(F(F(6))).$

$65674 := -F(F(F(6))) - 5 + F(F(F(6))) \times 7 + F(4) = F(4) + 7 \times F(F(F(6))) - 5 - F(F(F(6))).$

Above symmetrical examples are just with $F(.)$, $F(F(.))$, etc. Still there are examples with factorial and square-roots. For more details refer to [36, 37, 38]:

http://rgmia.org/papers/v19/v19a142.pdf .

http://rgmia.org/papers/v19/v19a143.pdf .

http://rgmia.org/papers/v19/v19a156.pdf .

10.2 Number Patterns with Fibonacci Sequence Values

Below are representations of numbers those can be extended just multiplying by 10. These numbers we call, number patterns with Fibonacci numbers sequence values:

$$3528 = F(3+5)^2 \times 8$$
$$35280 = F(3+5)^2 \times 80$$
$$352800 = F(3+5)^2 \times 800.$$

$$1365 = 13 \times F(F(6)) \times 5$$
$$13650 = 13 \times F(F(6)) \times 50$$
$$136500 = 13 \times F(F(6)) \times 500.$$

$$3635 = (3^6 - F(3)) \times 5$$
$$36350 = (3^6 - F(3)) \times 50$$
$$363500 = (3^6 - F(3)) \times 500.$$

$$1687 = (F(F(1+6)) + 8) \times 7$$
$$16870 = (F(F(1+6)) + 8) \times 70$$
$$168700 = (F(F(1+6)) + 8) \times 700.$$

11 Narcissistic Numbers

An $n-$digit number that is the sum of the nth powers of its digits is called an $n-$narcissistic numbers. It is also sometimes known as an Armstrong number, perfect digital invariant. Below are examples, of narcissistic numbers with width 3 and 4:

$$153 = 1^3 + 5^3 + 3^3.$$
$$370 = 3^3 + 7^3 + 0^3.$$
$$371 = 3^3 + 7^3 + 1^3.$$
$$407 = 4^3 + 0^3 + 7^3.$$

$$1634 = 1^4 + 6^4 + 3^4 + 4^4.$$
$$4151 = 4^5 + 1^5 + 5^5 + 1^5.$$
$$8208 = 8^4 + 2^4 + 0^4 + 8^4.$$
$$9472 = 9^4 + 4^4 + 7^4 + 2^4.$$

11.1 Flexible Power Narcissistic Numbers

The narcissistic numbers given above are with fixed power and with positive signs. There are many narcissistic numbers with flexibility in power and sign, for example,

$$24 = 2^3 + 4^2.$$
$$43 = 4^2 + 3^3.$$
$$23 = -2^2 + 3^3.$$
$$48 = -4^2 + 8^2.$$

$$2345 = 2^5 + 3^7 + 4^0 + 5^3.$$
$$2352 = 2^3 + 3^7 + 5^3 + 2^5.$$
$$2371 = -2^1 - 3^3 + 7^4 - 1^0.$$
$$2374 = -2^1 - 3^2 + 7^4 - 4^2.$$

$$263 = 2^8 + 6^1 + 3^0.$$
$$267 = 2^1 + 6^3 + 7^2.$$
$$337 = -3^1 - 3^1 + 7^3.$$
$$354 = -3^3 + 5^3 + 4^4.$$

$$10693 = 1^1 + 0^1 + 6^5 + 9^3 + 3^7.$$
$$10694 = 1^1 + 0^1 + 6^2 + 9^4 + 4^6.$$
$$10846 = -1^1 - 0^0 + 8^4 - 4^5 + 6^5.$$
$$10933 = -1^1 - 0^0 + 9^4 + 3^7 + 3^7.$$

The above results are given with positive and negative operations. But the power is always positive and flexible. These types of numbers, we can as *flexible power narcissistic numbers.* For complete detail refer to author's work [18]:

http://rgmia.org/papers/v19/v19a32.pdf.

11.2   Fixed Power Narcissistic Numbers with Divisions

There are very few *narcissistic numbers* with fixed power that can be written in terms of division, for examples,

$$37 = \frac{3^3 + 7^3}{3 + 7}. \qquad\qquad 48 = \frac{4^3 + 8^3}{4 + 8}.$$

$$241 = \frac{2^8 + 4^8 + 1^8}{2^4 + 4^4 + 1^1}. \qquad\qquad 415 = \frac{4^5 + 1^5 + 5^5}{4 + 1 + 5}.$$

$$2464 = \frac{2^5 + 4^5 + 6^5 + 4^5}{2^0 + 4^0 + 6^0 + 4^0}. \qquad\qquad 4714 = \frac{4^5 + 7^5 + 1^5 + 4^5}{4^0 + 7^0 + 1^0 + 4^0}.$$

$$5247 = \frac{5^5 + 2^5 + 4^5 + 7^5}{5^0 + 2^0 + 4^0 + 7^0}. \qquad\qquad 8200 = \frac{8^5 + 2^5 + 0^5 + 0^5}{8^0 + 2^0 + 0^0 + 0^0}.$$

$$15501 = \frac{1^9 + 5^9 + 5^9 + 0^9 + 1^9}{1^3 + 5^3 + 5^3 + 0^3 + 1^3}. \qquad 142740 = \frac{1^7 + 4^7 + 2^7 + 7^7 + 4^7 + 0^7}{1^0 + 4^0 + 2^0 + 7^0 + 4^0 + 0^0}.$$

$$231591 = \frac{2^7 + 3^7 + 1^7 + 5^7 + 9^7 + 1^7}{2 + 3 + 1 + 5 + 9 + 1}.$$

The above numbers are with positive coefficients. Allowing negative coefficients, still there are more numbers of similar kind:

$$264 = \frac{2^4 + 6^4 - 4^4}{2 + 6 - 4}. \qquad\qquad 407 = \frac{4^6 + 0^6 - 7^6}{4^3 + 0^3 - 7^3}.$$

$$803 = \frac{8^4 + 0^4 - 3^4}{8 + 0 - 3}. \qquad\qquad 6181 = \frac{6^7 + 1^7 - 8^7 + 1^7}{6^3 + 1^3 - 8^3 + 1^3}.$$

$$54901 = \frac{5^4 + 4^4 - 9^4 + 0^4 + 1^4}{5 + 4 - 9 + 0 + 1}.$$

11.3   Flexible Power Narcissistic Numbers with Divisions

Following the same idea of previous subsection 11.2 , here also we have written fractions with fixed and flexible power along with positive–negative coefficients, such as,

$$13 = \frac{-1^3 + 3^3}{1^0 + 3^0}.$$

$$63 = \frac{6^3 - 3^3}{6^1 - 3^1}.$$

$$353 = \frac{-3^5 - 5^2 + 3^9}{3^1 + 5^2 + 3^3}.$$

$$355 = \frac{3^{11} - 5^0 - 5^0}{-3^0 - 5^3 + 5^4}.$$

$$1337 = \frac{1^0 + 3^1 + 3^1 + 7^6}{-1^0 + 3^0 + 3^4 + 7^1}.$$

$$1343 = \frac{1^0 + 3^6 - 4^7 + 3^9}{-1^0 - 3^0 + 4^1 + 3^0}.$$

$$10954 = \frac{-1^0 - 0^0 + 9^3 + 5^2 + 4^9}{1^0 + 0^0 + 9^0 + 5^1 + 4^2}.$$

$$10958 = \frac{-1^0 + 0^0 + 9^2 + 5^2 + 8^5}{-1^0 + 0^0 + 9^0 + 5^0 + 8^0}.$$

$$118378 = \frac{1^{12} - 1^{12} + 8^{12} - 3^{12} + 7^{12} - 8^{12}}{1^6 - 1^6 + 8^6 - 3^6 + 7^6 - 8^6}.$$

$$122347 = \frac{1^6 - 2^6 - 2^6 + 3^6 + 4^6 + 7^6}{1^1 + 2^1 - 2^1 + 3^1 + 4^1 - 7^1}.$$

For complete detail refer to author's work [18]:

http://rgmia.org/papers/v19/v19a32.pdf.

11.4   Floor Function and Narcissistic Numbers with Divisions

In this section, we shall bring numbers in such a way that they becomes *narcissistic numbers* with division in terms of *"floor function"*. Below are examples,

$$115 = \left\lfloor \frac{1^5 + 1^5 + 5^5}{1^2 + 1^2 + 5^2}. \right\rfloor.$$

$$21 = \left\lfloor \frac{2^6 + 1^6}{2 + 1}. \right\rfloor.$$

$$23 = \left\lfloor \frac{2^7 + 3^7}{2^4 + 3^4}. \right\rfloor.$$

$$16737 = \left\lfloor \frac{1^{32} + 6^{32} + 7^{32} + 3^{32} + 7^{32}}{1^{27} + 6^{27} + 7^{27} + 3^{27} + 7^{27}}. \right\rfloor.$$

$$102 = \left\lfloor \frac{1^9 + 0^9 + 2^9}{1^2 + 0^2 + 2^2}. \right\rfloor.$$

$$56494 = \left\lfloor \frac{5^{13} + 6^{13} + 4^{13} + 9^{13} + 4^{13}}{5^8 + 6^8 + 4^8 + 9^8 + 4^8}. \right\rfloor.$$

For complete detail refer to author's work [19]:

http://rgmia.org/papers/v19/v19a33.pdf.

12   Flexible Power Selfie Representations

From numbers given in section 3.1 , we observe that there are numbers, 23, 1239, 1364, 1654, 1837, 2137, 2173, 2537, 3125, 3275, 3529 and 4316 uses the same digits as of number along with power too, such as,

$$23 = -2^2 + 3^3.$$
$$1239 = 1^2 + 2^9 - 3^1 + 9^3.$$
$$1364 = 1^6 + 3^1 + 6^4 + 4^3.$$
$$1654 = -1^6 + 6^1 + 5^4 + 4^5.$$

$$1837 = 1^8 - 8^1 + 3^7 - 7^3.$$
$$2137 = -2^1 + 1^3 + 3^7 - 7^2.$$
$$2173 = -2^3 + 1^2 - 7^1 + 3^7.$$
$$2537 = 2^5 - 5^2 + 3^7 + 7^3.$$

$$3125 = -3^2 + 1^1 + 2^3 + 5^5.$$
$$3275 = -3^3 + 2^7 + 7^2 + 5^5.$$
$$3529 = -3^3 + 5^5 + 2^9 - 9^2.$$
$$4316 = 4^6 + 3^1 + 1^4 + 6^3.$$

These representations, we call as *"flexible power selfie numbers"*. Motivated by this idea author wrote *"flexible power selfie numbers"* in different digits, for examples,

$$1 = 1^1.$$
$$23 = -2^2 + 3^3.$$
$$1654 = -1^6 + 6^1 + 5^4 + 4^5.$$
$$3435 = 3^3 + 4^4 + 3^3 + 5^5.$$
$$4355 = 4^5 + 3^4 + 5^3 + 5^5.$$
$$39339 = -3^3 + 9^3 + 3^9 + 3^9 - 9^3.$$
$$46350 = -4^3 + 6^6 - 3^5 + 5^0 + 0^4.$$

$$46360 = 4^0 + 6^6 - 3^4 - 6^3 + 0^6.$$
$$397612 = 3^2 + 9^1 + 7^6 + 6^7 + 1^9 + 2^3.$$
$$423858 = 4^3 + 2^8 + 3^4 + 8^2 + 5^8 + 8^5.$$
$$637395 = 6^5 + 3^3 + 7^3 + 3^9 + 9^6 + 5^7.$$
$$758014 = 7^7 + 5^1 + 8^0 + 0^5 + 1^4 - 4^8.$$
$$778530 = 7^7 + 7^3 + 8^5 - 5^7 + 3^0 + 0^8.$$
$$804637 = 8^0 + 0^4 - 4^8 + 6^6 - 3^3 + 7^7.$$

$$15647982 = 1^5 - 5^9 + 6^2 + 4^4 + 7^7 - 9^1 + 8^8 + 2^6.$$
$$17946238 = 1^6 + 7^8 + 9^4 + 4^2 + 6^9 + 2^3 + 3^1 + 8^7.$$
$$57396108 = -5^6 + 7^9 + 3^5 + 9^3 + 6^7 + 1^1 + 0^0 + 8^8.$$
$$134287690 = 1^2 + 3^8 + 4^7 + 2^4 + 8^9 + 7^3 + 6^6 + 9^0 + 0^1.$$
$$387945261 = 3^3 + 8^2 + 7^6 + 9^9 + 4^7 + 5^8 + 2^4 + 6^1 + 1^5.$$
$$392876054 = 3^0 + 9^9 - 2^2 - 8^5 + 7^8 - 6^7 + 0^3 - 5^4 + 4^6.$$
$$392876540 = -3^0 + 9^9 - 2^4 - 8^5 + 7^8 - 6^7 - 5^3 + 4^6 + 0^2.$$

The work is divided in different categories [21, 22, 23]. For details refer the following works:

http://rgmia.org/papers/v19/v19a49.pdf
http://rgmia.org/papers/v19/v19a50.pdf
http://rgmia.org/papers/v19/v19a51.pdf .

13 Selfie Fractions

A **addable fraction** is a proper fraction where addition signs can be inserted into numerator and denominator, and the resulting fraction is equal to the original. The same is true for operations also, such as with *addition, multiplication, potentiation,* etc. Below are ideas of sum of these kinds:

- **Addable**

$$\frac{96}{352} = \frac{9+6}{3+52}, \quad \frac{182}{6734} = \frac{18+2}{6+734}, \text{ etc.}$$

- **Subtractable**

$$\frac{204}{357} = \frac{20-4}{35-7}, \quad \frac{726}{1089} = \frac{72-6}{108-9}, \text{ etc.}$$

- **Dottable**

$$\frac{13}{624} = \frac{1 \times 3}{6 \times 24}, \quad \frac{416}{728} = \frac{4 \times 16}{7 \times 2 \times 8}, \text{ etc.}$$

- **Dottable with Potentiation**

$$\frac{95}{342} = \frac{9 \times 5}{3^4 \times 2}, \quad \frac{728}{1456} = \frac{7^2 \times 8}{14 \times 56}, \text{ etc.}$$

- **Mixed: All Operations**

$$\frac{4980}{5312} = \frac{4-9+80}{5 \times (3+1)^2}, \quad \frac{3249}{5168} = \frac{(3+2^4) \times 9}{(5-1) \times 68}, \text{ etc.}$$

Observing above examples, the numerator and denominator follows the same order of digits in both sides of each fraction separated by operations. These type of fractions, we call *Selfie fractions*. There are two situations. One when digits appearing in each fraction are distinct, and second, when there are repetitions of digits. Studies on non–repeated digits are summarized in following works [26, 27, 28]:

http://rgmia.org/papers/v19/v19a113.pdf

http://rgmia.org/papers/v19/v19a114.pdf

http://rgmia.org/papers/v19/v19a115.pdf .

13.1   Equivalent Selfie Fractions

Above we have given *selfie fractions* with single value in each case. There are many fractions, that can be written in more than one way, for example,

- **Addable**

$$\frac{1453}{2906} = \frac{1+453}{2+906} = \frac{145+3}{290+6} = \frac{1+45+3}{2+90+6}.$$

- **Subtractable**

$$\frac{932}{1864} = \frac{9-32}{18-64} = \frac{93-2}{186-4}.$$

- **Dottable and Addable**

$$\frac{1680}{59472} = \frac{1 \times 6 \times 80}{59 \times 4 \times 72} = \frac{1+6+8+0}{59+472}.$$

- **Dottable, Addable and Subtractable**

$$\frac{302}{8154} = \frac{30 \times 2}{81 \times 5 \times 4} = \frac{3+02}{81+54} = \frac{3-02}{81-54}.$$

- **Symmetric Addable and Subtractable**

$$\frac{645}{1290} = \frac{6 - 45}{12 - 90} = \frac{6 + 45}{12 + 90}.$$

- **Dottable and Addable together**

$$\frac{284}{639} = \frac{2 \times 8 + 4}{6 + 39} = \frac{28 + 4}{6 \times (3 + 9)}.$$

- **Mixed – All Operations**

$$\frac{73842}{90516} = \frac{7 - 3 \times (8 - 4^2)}{9 \times 05 - 1 - 6} = \frac{7 \times (3 + 8) + 4^2}{90 + (5 - 1) \times 6} = \frac{738 + 4 + 2}{905 + 1 + 6}.$$

Equivalent expression given in equation (8), let us classify it as *symmetric equivalent fraction*. In this case we just change plus with minus and vice-versa. There are many fractions *double symmetric equivalent fraction* too. In this paper, we shall work with **equivalent fractions** given in equations (6)–(10). Below is an example of one of the biggest equivalent selfie fraction with addition and multiplication of 10 digits:

- $\dfrac{30927}{61854} = \dfrac{3 + 0 \times 927}{6^{1^{854}}}$

$$= \frac{(3 + 09) \times 2 + 7}{6 + (1 + 8 + 5) \times 4}$$

$$= \frac{30 \times (9 + 2 \times 7)}{(61 + 8) \times 5 \times 4}$$

$$= \frac{30 \times (9 + 27)}{6 \times 18 \times 5 \times 4}$$

$$= \frac{3 \times (0 \times 9 + 27)}{6 \times (18 + 5 + 4)}$$

$$= \frac{3 + 092 + 7}{(6 + (1 + 8) \times 5) \times 4}$$

$$= \frac{(3 + 09 \times 2) \times 7}{6 \times ((1 + 8) \times 5 + 4)}$$

$$= \frac{3 \times 0927}{618 \times (5 + 4)}$$

$$= \frac{(30 + 92) \times 7}{61 \times (8 + 5 \times 4)}$$

$$= \frac{3 \times 09^{2+7}}{6 \times (1 + 8)^{5+4}}$$

$$= \frac{3 \times (0 \times 9 + 2) + 7}{6 + 1^8 \times 5 \times 4}$$

$$= \frac{3 + 09 + 27}{6 + 1 \times 8 \times (5 + 4)}$$

$$= \frac{30 + 9 + 2 + 7}{6 + 1 + 85 + 4}$$

$$= \frac{3^{0 \times 9 + 2} \times 7}{6 \times (1^8 + 5 \times 4)}$$

$$= \frac{3 + 09 \times (2 + 7)}{6 \times 1 \times (8 + 5 \times 4)}$$

$$= \frac{3 \times (09 + 27)}{(6 + 18) \times (5 + 4)}$$

$$= \frac{30 + 9 \times 2 \times 7}{6 \times 1 \times (8 + 5) \times 4}$$

$$= \frac{3 \times 09 \times (2 + 7)}{6 \times (1 + 8) \times (5 + 4)}$$

$$= \frac{3 \times 09 \times 2 \times 7}{(6 + 1 \times 8) \times 54}$$

$$= \frac{(30 + 9 \times 2) \times 7}{618 + 54}.$$

$$= \frac{3 + 0 \times 9 + 2 \times 7}{6 + 1 \times 8 + 5 \times 4}$$

$$= \frac{3 \times (09 + 2) + 7}{(6 + 1 + 8 + 5) \times 4}$$

$$= \frac{3 \times (09 + 2 + 7)}{6 \times (1 + 8 + 5 + 4)}$$

$$= \frac{3 + 0 \times 9 + 2 + 7}{6 + 1 + 8 + 5 + 4}$$

$$= \frac{(30 + 9) \times 2 + 7}{6 + (1 + 8 \times 5) \times 4}$$

$$= \frac{30 + 9^2 + 7}{(6 \times (1 + 8) + 5) \times 4}$$

$$= \frac{(3 + 09) \times 2 \times 7}{6 \times (1 + 8 + 5) \times 4}$$

$$= \frac{3 + 09 \times 27}{6 + (1 + 8) \times 54}$$

$$= \frac{3 + 0927}{6 + 1854}$$

$$= \frac{3 + 0 \times 9 + 27}{6 + 1^8 \times 54}$$

$$= \frac{3 \times 09 + 2 \times 7}{6 \times 1 \times (8 + 5) + 4}$$

$$= \frac{30 + 9 \times 2 + 7}{(6 + 1) \times 8 + 54}$$

$$= \frac{3 \times (09 \times 2 + 7)}{61 + 85 + 4}$$

$$= \frac{(3 + 09 + 2) \times 7}{(6 + 1) \times (8 + 5 \times 4)}$$

$$= \frac{3 + 09 + 2^7}{(6 + 1 \times 8) \times 5 \times 4}$$

$$= \frac{30 \times (9 + 2 + 7)}{6 \times (1 + 8) \times 5 \times 4}$$

$$= \frac{3 \times (0 \times 9 + 2 + 7)}{6 \times 1^8 \times (5 + 4)}$$

$$= \frac{3 \times 0 \times 9 + 2 \times 7}{6 + 1^{85} \times 4}$$

The work on *equivalent selfie fractions* is divided in different papers given as follows [29, 30]:

http://rgmia.org/papers/v19/v19a116.pdf
http://rgmia.org/papers/v19/v19a117.pdf .

14 Equivalent Fractions

This section deals with *equivalent fractions* without use of any operations, just with different digits. Below are some examples to understand the idea of *equivalent fractions* for different digits:

$$\bullet \frac{3}{24} = \frac{4}{32}, \qquad \bullet \frac{4}{356} = \frac{6}{534}, \qquad \bullet \frac{27}{3456} = \frac{42}{5376},$$

$$\bullet \frac{315}{4620} = \frac{342}{5016}, \qquad \bullet \frac{123}{58764} = \frac{164}{78352},$$

$$\bullet \frac{357}{26418} = \frac{618}{45732} = \frac{714}{52836} = \frac{732}{54168} = \frac{738}{54612}, \text{ etc.}$$

Some studies in this direction can be seen in [58, 67, 68]. Work of these authors is concentrated on equivalent fractions for the digits 1 to 9, calling *pandigital fractions*. While, we worked for all digits, i.e., for 3 to 10 digits. For example, in case of 3-digits, we tried to found equivalent fractions 3 by 3, i.e., [0, 1, 2], [1, 2, 3], [2, 3, 4], [3, 4, 5], [4, 5, 6], [5, 6, 7], [6, 7, 8] and [7, 8, 9]. In this case, we found only one result, i.e., $\frac{3}{24} = \frac{4}{32}$. The same procedure is done for 4 by 4, 5 by 5, etc. Below are highest possible equivalent fractions for 6 digits onwards. Before we have only few examples just with 2 expressions.

- **6-Digits Higher Equivalent Fractions: 3 Expressions**

In this case we have five possibilities, i.e., 0 to 5, 1 to 6, 2 to 7, 3 to 8, and 4 to 9. The number we got is for 2 to 7 and 4 to 9. Both with 3 expressions:

$$\bullet \frac{267}{534} = \frac{273}{546} = \frac{327}{654}.$$

$$\bullet \frac{497}{568} = \frac{749}{856} = \frac{854}{976}.$$

- **7-Digits Higher Equivalent Fractions: 5 Expressions**

In this case we have four possibilities, i.e., 0 to 6, 1 to 7, 2 to 8, and 3 to 9. The only higher number we got is in case of 3 to 9. See below:

$$\bullet \frac{697}{3485} = \frac{769}{3845} = \frac{937}{4685} = \frac{967}{4835} = \frac{973}{4865}.$$

- **8-Digits Higher Equivalent Fractions: 12 Expressions**

In this case we have three possibilities, i.e., 0 to 7, 1 to 8, and 2 to 9. The only higher number we got is in case of 1 to 8. See below:

$$\bullet \frac{1728}{3456} = \frac{1764}{3528} = \frac{1782}{3564} = \frac{1827}{3654} = \frac{2178}{4356} = \frac{2358}{4716} = \frac{2718}{5436} = \frac{2817}{5634} = \frac{3564}{7128} = \frac{3582}{7164} = \frac{4176}{8352} = \frac{4356}{8712}.$$

- **9–Digits Higher Equivalent Fractions: 46 Expressions**

In this case we have two possibilities, i.e., 0 to 8 and 1 to 9. The only higher number we got is in case of 1 to 9. See below:

$$
\begin{aligned}
\bullet\ \frac{3187}{25496} &= \frac{4589}{36712} = \frac{4591}{36728} = \frac{4689}{37512} = \frac{4691}{37528} = \frac{4769}{38152} = \frac{5237}{41896} = \frac{5371}{42968} = \frac{5789}{46312} = \frac{5791}{46328} = \frac{5839}{46712} \\
&= \frac{5892}{47136} = \frac{5916}{47328} = \frac{5921}{47368} = \frac{6479}{51832} = \frac{6741}{53928} = \frac{6789}{54312} = \frac{6791}{54328} = \frac{6839}{54712} = \frac{7123}{56984} = \frac{7312}{58496} \\
&= \frac{7364}{58912} = \frac{7416}{59328} = \frac{7421}{59368} = \frac{7894}{63152} = \frac{7941}{63528} = \frac{8174}{65392} = \frac{8179}{65432} = \frac{8394}{67152} = \frac{8419}{67352} = \frac{8439}{67512} \\
&= \frac{8932}{71456} = \frac{8942}{71536} = \frac{8953}{71624} = \frac{8954}{71632} = \frac{9156}{73248} = \frac{9158}{73264} = \frac{9182}{73456} = \frac{9316}{74528} = \frac{9321}{74568} = \frac{9352}{74816} \\
&= \frac{9416}{75328} = \frac{9421}{75368} = \frac{9523}{76184} = \frac{9531}{76248} = \frac{9541}{76328}.
\end{aligned}
$$

- **10–Digits Higher Equivalent Fractions: 7 Expressions**

In this case we have only one possibility, i.e., 0 to 9. See below the higher equivalent fraction with 78 expressions:

$$
\begin{aligned}
\bullet\ \frac{16748}{20935} &= \frac{18476}{23095} = \frac{18940}{23675} = \frac{18964}{23705} = \frac{19076}{23845} = \frac{19708}{24635} = \frac{24716}{30895} = \frac{24876}{31095} = \frac{25496}{31870} = \frac{28940}{36175} = \frac{29180}{36475} = \frac{29684}{37105} \\
&= \frac{32716}{40895} = \frac{32876}{41095} = \frac{36712}{45890} = \frac{36728}{45910} = \frac{37512}{46890} = \frac{37528}{46910} = \frac{38092}{47615} = \frac{38152}{47690} = \frac{39012}{48765} = \frac{41896}{52370} = \frac{42968}{53710} \\
&= \frac{46312}{57890} = \frac{46328}{57910} = \frac{46712}{58390} = \frac{47136}{58920} = \frac{47328}{59160} = \frac{47368}{59210} = \frac{48732}{60915} = \frac{49380}{61725} = \frac{49708}{62135} = \frac{51832}{64790} = \frac{53928}{67410} \\
&= \frac{54312}{67890} = \frac{54328}{67910} = \frac{54712}{68390} = \frac{56984}{71230} = \frac{58496}{73120} = \frac{58912}{73640} = \frac{59328}{74160} = \frac{59368}{74210} = \frac{63124}{78905} = \frac{63140}{78925} = \frac{63152}{78940} \\
&= \frac{63284}{79105} = \frac{63528}{79410} = \frac{64732}{80915} = \frac{65392}{81740} = \frac{65432}{81790} = \frac{67124}{83905} = \frac{67140}{83925} = \frac{67152}{83940} = \frac{67352}{84190} = \frac{67512}{84390} = \frac{71236}{89045} \\
&= \frac{71364}{89205} = \frac{71456}{89320} = \frac{71460}{89325} = \frac{71536}{89420} = \frac{71624}{89530} = \frac{71632}{89540} = \frac{72836}{91045} = \frac{73248}{91560} = \frac{73264}{91580} = \frac{73284}{91605} = \frac{73456}{91820} \\
&= \frac{73460}{91825} = \frac{73684}{92105} = \frac{74108}{92635} = \frac{74528}{93160} = \frac{74568}{93210} = \frac{74816}{93520} = \frac{75328}{94160} = \frac{75368}{94210} = \frac{76184}{95230} = \frac{76248}{95310} = \frac{76328}{95410}.
\end{aligned}
$$

For complete details refer to author's work [39, 40, 41]:

http://rgmia.org/papers/v19/v19a148.pdf

http://rgmia.org/papers/v19/v19a149.pdf

http://rgmia.org/papers/v19/v19a150.pdf.

The first work [39] is from 3 to 8 digits. The second work [40] is for 9 digits and the third work [41] is for 10 digits. All the three papers are for different digits. The repetition of digits give much more equivalent fractions. This shall be dealt elsewhere.

15 Palindromic–Type Numbers

A palindromic number is the number that remains the same when its digits are reversed, for example, 121, 3333, 161161, etc. The *palindromic–type* numbers are not palindromes but appears like palindromes. They

are separated by operations of additions and/or multiplications, for example, 1343×3421, 1225×5221, $16 + 61$, $1825 + 5281$, etc. When we remove the sign of operation, they becomes palindromes. These types of numbers we call as *palindromic–type* numbers. Some studies in this direction can be seen in [**?**]. Let us separate these numbers in two categories:

- **Type 1:**

$$234 \times 111 + 111 \times 432 = 25974 + 47952 := 73926.$$
$$10011 \times 11 + 11 \times 11001 = 110121 + 121011 := 231132.$$

- **Type 2:**

$$1596 \times 6951 = 2793 \times 3972 := 11093796.$$
$$616248 \times 842616 = 645408 \times 804546 := 519260424768.$$

The difference is that the first type is with *addition* and *multiplication*, while second type is just with *multiplication*. This work is concentrated only on **Type 1** kind of numbers, i.e., *palindromic–type* numbers with *addition* and *multiplication*. Below are examples of *palindromic–type* numbers, where each digit appears once and multiplicative factors are of same width.

$$12 \times 21 + 21 \times 12 = 252 + 252 := 504.$$
$$12 \times 12 + 21 \times 21 = 144 + 441 := 585.$$

$$102 \times 210 + 201 \times 012 = 21420 + 02412 := 23832.$$
$$201 \times 102 + 201 \times 102 = 20502 + 20502 := 41004.$$
$$102 \times 012 + 210 \times 201 = 01224 + 42210 := 43434.$$
$$102 \times 102 + 201 \times 201 = 10404 + 40401 := 50805 :$$

$$2031 \times 1032 + 2301 \times 1302 = 2095992 + 2995902 := 5091894.$$
$$2103 \times 1203 + 3021 \times 3012 = 2529909 + 9099252 := 11629161.$$
$$2013 \times 1023 + 3201 \times 3102 = 2059299 + 9929502 := 11988801.$$

Below are numbers, where squaring each multiplicative factor in each palindromic–type number lead us to final sum as a palindrome:

$$
\begin{array}{lll}
12^2 + 21^2 & = 144 + 441 & := 585. \\
111^2 + 111^2 & = 12321 + 12321 & := 24642. \\
1022^2 + 2201^2 & = 1044484 + 4844401 & := 5888885. \\
10031^2 + 13001^2 & = 100620961 + 169026001 & := 269646962. \\
100301^2 + 103001^2 & = 10060290601 + 10609206001 & := 20669496602.
\end{array}
$$

For complete details refer to author's work [42]:

http://rgmia.org/papers/v19/v19a159.pdf

15.1   Patterns in Palindromic–Type Numbers

Below are some examples of patterns in palindromic–type numbers:

$$101 \times 44 + 44 \times 101 = 4444 + 4444 := 8888$$
$$202 \times 22 + 22 \times 202 = 4444 + 4444 := 8888$$
$$404 \times 11 + 11 \times 404 = 4444 + 4444 := 8888.$$

$$1001 \times 444 + 444 \times 1001 = 444444 + 444444 := 888888$$
$$2002 \times 222 + 222 \times 2002 = 444444 + 444444 := 888888$$
$$4004 \times 111 + 111 \times 4004 = 444444 + 444444 := 888888.$$

$$10001 \times 4444 + 4444 \times 10001 = 44444444 + 44444444 := 88888888$$
$$20002 \times 2222 + 2222 \times 20002 = 44444444 + 44444444 := 88888888$$
$$40004 \times 1111 + 1111 \times 40004 = 44444444 + 44444444 := 88888888.$$

$$1001 \times 44 + 44 \times 1001 = 44044 + 44044 := 88088$$
$$2002 \times 22 + 22 \times 2002 = 44044 + 44044 := 88088$$
$$4004 \times 11 + 11 \times 4004 = 44044 + 44044 := 88088.$$

$$10001 \times 404 + 404 \times 10001 = 4040404 + 4040404 := 8080808$$
$$20002 \times 202 + 202 \times 20002 = 4040404 + 4040404 := 8080808$$
$$40004 \times 101 + 101 \times 40004 = 4040404 + 4040404 := 8080808.$$

$$10001 \times 4004 + 4004 \times 10001 = 40044004 + 40044004 := 80088008$$
$$20002 \times 2002 + 2002 \times 20002 = 40044004 + 40044004 := 80088008$$
$$40004 \times 1001 + 1001 \times 40004 = 40044004 + 40044004 := 80088008.$$

For complete details refer to author's work [42]:

http://rgmia.org/papers/v19/v19a159.pdf

Acknowledgement

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