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A NEW HERMITE-HADAMARD INEQUALITY FOR $h-$CONVEX STOCHASTIC PROCESSES

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Abstract. Firstly, some new definitions which are the special cases of $h-$convex stochastic processes are given. Then, we establish a new refinement of Hermite-Hadamard inequality for $h-$convex stochastic proceses and give some special cases of this result.

1. Introduction

The classical Hermite-Hadamard inequality which was first published in [5] gives us an estimate of the mean value of a convex function $f : I \to \mathbb{R}$,

$$f\left(\frac{a+b}{2}\right) \leq \frac{1}{b-a}\int_a^b f(x)dx \leq \frac{f(a)+f(b)}{2} \tag{1.1}$$

An account the history of this inequality can be found in [3]. Surveys on various generalizations and developments can be found in [2] and [9].

In 1980, Nikodem [10] introduced convex stochastic processes and investigated their regularity properties. In 1992, Skwronski [14] obtained some further results on convex functions.

Let $(\Omega, \mathcal{A}, P)$ be an arbitrary probability space. A function $X : \Omega \to \mathbb{R}$ is called a random variable if it is $\mathcal{A}-$measurable. A function $X : I \times \Omega \to \mathbb{R}$, where $I \subset \mathbb{R}$ is an interval, is called a stochastic process if for every $t \in I$ the function $X(t, .)$ is a random variable.

Recall that the stochastic process $X : I \times \Omega \to \mathbb{R}$ is called

(i) continuous in probability in interval I, if for all $t_0 \in I$ we have

$$P - \lim_{t \to t_0} X(t, .) = X(t_0, .),$$

where $P - \lim$ denotes the limit in probability.

(ii) *mean-square continuous* in the interval I, if for all $t_0 \in I$

$$\lim_{t \to t_0} E\left[(X(t) - X(t_0))^2\right] = 0,$$

where $E[X(t)]$ denotes the expectation value of the random variable $X(t, .)$.

Obviously, *mean-square* continuity implies continuity in probability, but the converse implication is not true.

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Definition 1. *Suppose we are given a sequence $\{\Delta^m\}$ of partitions, $\Delta^m = \{a_{m,0}, ..., a_{m,n_m}\}$. We say that the sequence $\{\Delta^m\}$ is a normal sequence of partitions if the length of the greatest interval in the $n-th$ partition tends to zero, i.e.,*

$$\lim_{m\to\infty} \sup_{1\leq i\leq n_m} |a_{m,i} - a_{m,i-1}| = 0.$$

Now we would like to recall the concept of the mean-square integral. For the definition and basic properties see [15].

Let $X : I \times \Omega \to \mathbb{R}$ be a stochastic process with $E\left[X(t)^2\right] < \infty$ for all $t \in I$. Let $[a, b] \subset I$, $a = t_0 < t_1 < t_2 < ... < t_n = b$ be a partition of $[a, b]$ and $\Theta_k \in [t_{k-1}, t_k]$ for all $k = 1, ..., n$. A random variable $Y : \Omega \to \mathbb{R}$ is called the mean-square integral of the process X on $[a, b]$, if we have

$$\lim_{n\to\infty} E\left[\left(\sum_{k=1}^{n} X(\Theta_k)(t_k - t_{k-1}) - Y\right)^2\right] = 0$$

for all normal sequence of partitions of the interval $[a, b]$ and for all $\Theta_k \in [t_{k-1}, t_k]$, $k = 1, ..., n$. Then, we write

$$Y(\cdot) = \int_a^b X(s, \cdot)\, ds \text{ (a.e.)}.$$

For the existence of the mean-square integral it is enough to assume the mean-square continuity of the stochastic process X.

Throughout the paper we will frequently use the monotonicity of the mean-square integral. If $X(t, \cdot) \leq Y(t, \cdot)$ (a.e.) in some interval $[a, b]$, then

$$\int_a^b X(t, \cdot)\, dt \leq \int_a^b Y(t, \cdot)\, dt \text{ (a.e.)}.$$

Of course, this inequality is the immediate consequence of the definition of the mean-square integral.

Definition 2. *We say that a stochastic processes $X : I \times \Omega \to \mathbb{R}$ is convex, if for all $\lambda \in [0, 1]$ and $u, v \in I$ the inequality*

$$X(\lambda u + (1-\lambda) v, \cdot) \leq \lambda X(u, \cdot) + (1-\lambda) X(v, \cdot) \text{ (a.e.)} \tag{1.2}$$

is satisfied. If the above inequality is assumed only for $\lambda = \frac{1}{2}$, then the process X is Jensen-convex or $\frac{1}{2}-$convex. A stochastic process X is concave if $(-X)$ is convex. Some interesting properties of convex and Jensen-convex processes are presented in [10, 15].

Now, we present some results proved by Kotrys [6] about Hermite-Hadamard inequality for convex stochastic processes.

Lemma 1. *If $X : I \times \Omega \to \mathbb{R}$ is a stochastic process of the form $X(t, \cdot) = A(\cdot)\, t + B(\cdot)$, where $A, B : \Omega \to \mathbb{R}$ are random variables, such that $E\left[A^2\right] < \infty, E\left[B^2\right] < \infty$ and $[a, b] \subset I$, then*

$$\int_a^b X(t, \cdot)\, dt = A(\cdot) \frac{b^2 - a^2}{2} + B(\cdot)(b - a) \text{ (a.e.)}.$$

Proposition 1. *Let $X : I \times \Omega \to \mathbb{R}$ be a convex stochastic process and $t_0 \in intI$. Then there exist a random variable $A : \Omega \to \mathbb{R}$ such that X is supported at t_0 by the process $A(\cdot)(t - t_0) + X(t_0, \cdot)$. That is*

$$X(t, \cdot) \geq A(\cdot)(t - t_0) + X(t_0, \cdot) \ \ \textit{(a.e.)}.$$

for all $t \in I$.

Theorem 1. *Let $X : I \times \Omega \to \mathbb{R}$ be Jensen-convex, mean-square continuous in the interval I stochastic process. Then for any $u, v \in I$ we have*

$$X\left(\frac{u+v}{2}, \cdot\right) \leq \frac{1}{v-u}\int_u^v X(t, \cdot)\,dt \leq \frac{X(u, \cdot) + X(v, \cdot)}{2} \ \ \textit{(a.e.)} \tag{1.3}$$

In [11]*, Sarıkaya et al. proved the following refinement of the inequality (1.3):*

Theorem 2. *If $X : I \times \Omega \to \mathbb{R}$ be Jensen-convex, mean-square continuous in the interval I stochastic process. Then for any $u, v \in I$ and for all $\lambda \in [0, 1]$, we have*

$$X\left(\frac{u+v}{2}, \cdot\right) \leq h(\lambda) \leq \frac{1}{v-u}\int_u^v X(t, \cdot)\,dt \leq H(\lambda) \leq \frac{X(u, \cdot) + X(v, \cdot)}{2}, \tag{1.4}$$

where

$$h(\lambda) := \lambda X\left(\frac{\lambda v + (2-\lambda)u}{2}, \cdot\right) + (1-\lambda) X\left(\frac{(1+\lambda)v + (1-\lambda)u}{2}, \cdot\right)$$

and

$$H(\lambda) := \frac{1}{2}\left(X(\lambda v + (1-\lambda)u, \cdot) + \lambda X(u, \cdot) + (1-\lambda) X(v, \cdot)\right).$$

In [1], Barraez et al. introduced the concept of $h-$convex stochastic process with following definition.

Definition 3. *Let $h : (0, 1) \to \mathbb{R}$ be a non-negative function, $h \neq 0$.we say that a stochastic process $X : I \times \Omega \to \mathbb{R}$ is an $h-$convex stochastic process if, for every $t_1, t_2 \in I$, $\lambda \in (0, 1)$, the following inequality is satisfied*

$$X(\lambda u + (1-\lambda)v, \cdot) \leq h(\lambda)X(u, \cdot) + h(1-\lambda)X(v, \cdot) \ \ \textit{(a.e.)} \tag{1.5}$$

Obviously, if we take $h(\lambda) = \lambda$ and $h(\lambda) = \lambda^s$ in (1.5), then the definition of $h-$convex stochastic process reduces to the definition of classical convex stochastic process [10] and $s-$convex stochastic process in the second sence [12] respectively. Moreover, A stochastic process $X : I \times \Omega \to \mathbb{R}$ is:

1) Godunova-Levin stochastic process if, we take $h(\lambda) = \frac{1}{\lambda}$ in (1.5),

$$X(\lambda u + (1-\lambda)v, \cdot) \leq \frac{X(u, \cdot)}{\lambda} + \frac{X(v, \cdot)}{1-\lambda} \ \ \text{(a.e.)} \tag{1.6}$$

2) $P-$stochastic process if, we take $h(\lambda) = 1$ in (1.5),

$$X(\lambda u + (1-\lambda)v, \cdot) \leq X(u, \cdot) + X(v, \cdot) \ \ \text{(a.e.)} \tag{1.7}$$

Authors proved the following Hermite-Hadamard inequality for $h-$convex stochastic process in [1]:

Theorem 3. *If $X : I\times\Omega\to\mathbb{R}$ Let be $h:(0,1)\to\mathbb{R}$ a non-negative function, $h\neq 0$ and $X : I\times\Omega\to\mathbb{R}$ a non negative, $h-$convex, mean square integrable stochastic process. For every $u,v\in I,(u<v)$, the following inequality is satisfied almost everywhere*

$$\frac{1}{2h\left(\frac{1}{2}\right)}X\left(\frac{u+v}{2},\cdot\right)\leq\frac{1}{v-u}\int_{u}^{v}X\left(t,\cdot\right)dt\leq\left[X\left(u,\cdot\right)+X\left(v,\cdot\right)\right]\int_{0}^{1}h(\lambda)d\lambda.$$

For more information and recent developments on Hermite-Hadamard type inequalities for stochastic process, please refer to ([1], [4], [6]-[8], [11]-[13], [16]).

The aim of this paper is to establish an improvement of Hermite-Hadamard inequality for $h-$convex stochastic process.

2. Main Results

Theorem 4. *If $X : I\times\Omega\to\mathbb{R}$ Let be $h:(0,1)\to\mathbb{R}$ a non-negative function, $h\neq 0$ and $X : I\times\Omega\to\mathbb{R}$ a non negative, $h-$convex, mean square integrable stochastic process. For every $u,v\in I,(u<v)$, we have the following inequality*

$$\begin{aligned}
&\frac{1}{4\left[h\left(\frac{1}{2}\right)\right]^{2}}X\left(\frac{u+v}{2},\cdot\right) \\
\leq\;& \Delta_1\leq\frac{1}{v-u}\int_{u}^{v}X\left(t,\cdot\right)dt \\
\leq\;& \Delta_2\leq\left[X\left(u,\cdot\right)+X\left(v,\cdot\right)\right]\left[\frac{1}{2}+h\left(\frac{1}{2}\right)\right]\int_{0}^{1}h(\lambda)d\lambda
\end{aligned}\tag{2.1}$$

where

$$\Delta_1:=\frac{1}{4h\left(\frac{1}{2}\right)}\left[X\left(\frac{3u+v}{4},\cdot\right)+\left(\frac{u+3v}{4},\cdot\right)\right]$$

and

$$\Delta_2:=\left[\frac{X\left(u,\cdot\right)+X\left(v,\cdot\right)}{2}+X\left(\frac{u+v}{2},\cdot\right)\right]\int_{0}^{1}h(\lambda)d\lambda.$$

Proof. Since $X : I\times\Omega\to\mathbb{R}$ is a $h-$convex stochastic process, we have

$$\begin{aligned}
X\left(\frac{u+\frac{u+v}{2}}{2},\cdot\right)&=X\left(\frac{\lambda u+(1-\lambda)\frac{u+v}{2}+(1-\lambda)u+\lambda\frac{u+v}{2}}{2},\cdot\right) \\
\leq\; h\left(\frac{1}{2}\right)&\left[X\left(\lambda u+(1-\lambda)\frac{u+v}{2},\cdot\right)+X\left((1-\lambda)u+\lambda\frac{u+v}{2},\cdot\right)\right].
\end{aligned}\tag{2.2}$$

Integrating (2.2) from 0 to 1 with respect to λ, we get

$$
\begin{aligned}
(2.3)\quad & X\left(\frac{3u+v}{4},\cdot\right) \\
\leq\ & h\left(\frac{1}{2}\right)\left[\int_0^1 X\left(\lambda u+(1-\lambda)\frac{u+v}{2},\cdot\right)d\lambda+\int_0^1 X\left((1-\lambda)u+\lambda\frac{u+v}{2},\cdot\right)d\lambda\right] \\
\leq\ & h\left(\frac{1}{2}\right)\left[\frac{2}{v-u}\int_u^{\frac{u+v}{2}} X(t,\cdot)\,dt+\frac{2}{v-u}\int_u^{\frac{u+v}{2}} X(t,\cdot)\,dt\right] \\
=\ & \frac{4h\left(\frac{1}{2}\right)}{v-u}\int_u^{\frac{u+v}{2}} X(t,\cdot)\,dt.
\end{aligned}
$$

That is,

$$
\frac{1}{4h\left(\frac{1}{2}\right)}X\left(\frac{3u+v}{4},\cdot\right)\leq\frac{1}{v-u}\int_u^{\frac{u+v}{2}} X(t,\cdot)\,dt. \tag{2.4}
$$

Since X is a $h-$convex stochastic process, we also have

$$
\begin{aligned}
(2.5)\qquad X\left(\frac{\frac{u+v}{2}+v}{2},\cdot\right) &= X\left(\frac{\lambda\frac{u+v}{2}+(1-\lambda)v+(1-\lambda)\frac{u+v}{2}+\lambda v}{2},\cdot\right) \\
&\leq h\left(\frac{1}{2}\right)\left[X\left(\lambda\frac{u+v}{2}+(1-\lambda)v,\cdot\right)+X\left((1-\lambda)\frac{u+v}{2}+\lambda v,\cdot\right)\right].
\end{aligned}
$$

Integrating (2.5) from 0 to 1 with respect to λ, we get

$$
\begin{aligned}
& X\left(\frac{u+3v}{4},\cdot\right) \\
\leq\ & h\left(\frac{1}{2}\right)\left[\int_0^1 X\left(\lambda\frac{u+v}{2}+(1-\lambda)v,\cdot\right)d\lambda+\int_0^1 X\left((1-\lambda)\frac{u+v}{2}+\lambda v,\cdot\right)d\lambda\right] \\
\leq\ & h\left(\frac{1}{2}\right)\left[\frac{2}{v-u}\int_{\frac{u+v}{2}}^{v} X(t,\cdot)\,dt+\frac{2}{v-u}\int_{\frac{u+v}{2}}^{v} X(t,\cdot)\,dt\right] \\
=\ & \frac{4h\left(\frac{1}{2}\right)}{v-u}\int_{\frac{u+v}{2}}^{v} X(t,\cdot)\,dt,
\end{aligned}
$$

i.e.

$$
\frac{1}{4h\left(\frac{1}{2}\right)}X\left(\frac{u+3v}{4},\cdot\right)\leq\frac{1}{v-u}\int_{\frac{u+v}{2}}^{v} X(t,\cdot)\,dt. \tag{2.6}
$$

Summing inequalities (2.4) and (2.6), we obtain

$$\Delta_1 = \frac{1}{4h\left(\frac{1}{2}\right)}\left[X\left(\frac{3u+v}{4},\cdot\right)+\left(\frac{u+3v}{4},\cdot\right)\right] \leq \frac{1}{v-u}\int_u^v X(t,\cdot)\,dt$$

which finishes the proof of second inequality in (2.1).

Applying the Hermite-Hadamard inequality for $h-$convex stochastic process (Theorem 3), we have

$$\begin{aligned}
\frac{1}{v-u}\int_u^v X(t,\cdot)\,dt &= \frac{1}{2}\left[\frac{2}{v-u}\int_u^{\frac{u+v}{2}} X(t,\cdot)\,dt + \frac{2}{v-u}\int_{\frac{u+v}{2}}^v X(t,\cdot)\,dt\right] \\
&\leq \frac{1}{2}\left[\left[X(u,\cdot)+X\left(\frac{u+v}{2},\cdot\right)\right]\int_0^1 h(\lambda)d\lambda\right] \\
&\quad +\frac{1}{2}\left[\left[X\left(\frac{u+v}{2},\cdot\right)+X(v,\cdot)\right]\int_0^1 h(\lambda)d\lambda\right] \\
&= \left[\frac{X(u,\cdot)+X(v,\cdot)}{2}+X\left(\frac{u+v}{2},\cdot\right)\right]\int_0^1 h(\lambda)d\lambda = \Delta_2.
\end{aligned}$$

This completes the proof of third inequality in (2.1).

For the first inequality, using the $h-$convexity of X, we have

$$\begin{aligned}
&\frac{1}{4\left[h\left(\frac{1}{2}\right)\right]^2}X\left(\frac{u+v}{2},\cdot\right) \\
&= \frac{1}{4\left[h\left(\frac{1}{2}\right)\right]^2}X\left(\frac{1}{2}\frac{3u+v}{2}+\frac{1}{2}\frac{u+3v}{4},\cdot\right) \\
&\leq \frac{1}{4\left[h\left(\frac{1}{2}\right)\right]^2}\left[h\left(\frac{1}{2}\right)X\left(\frac{3u+v}{2},\cdot\right)+h\left(\frac{1}{2}\right)X\left(\frac{u+3v}{4},\cdot\right)\right] \\
&= \Delta_1.
\end{aligned}$$

Finally,

$$\begin{aligned}
\Delta_2 &= \left[\frac{X(u,\cdot)+X(v,\cdot)}{2}+X\left(\frac{u+v}{2},\cdot\right)\right]\int_0^1 h(\lambda)d\lambda \\
&\leq \left[\frac{X(u,\cdot)+X(v,\cdot)}{2}+h\left(\frac{1}{2}\right)[X(u,\cdot)+X(v,\cdot)]\right]\int_0^1 h(\lambda)d\lambda \\
&= [X(u,\cdot)+X(v,\cdot)]\left[\frac{1}{2}+h\left(\frac{1}{2}\right)\right]\int_0^1 h(\lambda)d\lambda.
\end{aligned}$$

This completes completely the proof of the Theorem. $\square$

Remark 1. *Under assumption of Theorem 4 with $h(t)=t$, we have*

$$X\left(\frac{u+v}{2},\cdot\right)\leq\Delta_1\leq\frac{1}{v-u}\int\limits_u^v X(t,\cdot)\,dt\leq\Delta_2\leq\frac{X(u,\cdot)+X(v,\cdot)}{2}$$

where

$$\Delta_1:=\frac{1}{2}\left[X\left(\frac{3u+v}{4},\cdot\right)+\left(\frac{u+3v}{4},\cdot\right)\right]$$

and

$$\Delta_2:=\frac{1}{2}\left[\frac{X(u,\cdot)+X(v,\cdot)}{2}+X\left(\frac{u+v}{2},\cdot\right)\right].$$

This inequality is a special case of the Theorem 2 with $\lambda=\frac{1}{2}$.

Corollary 1. *Under assumption of Theorem 4 with $h(t)=t^s$, we have the refinement Hermite-Hadamard inequality for $s-$convex stochastic processes in the second sense*

$$\begin{aligned}2^{2s-2}X\left(\frac{u+v}{2},\cdot\right) &\leq \Delta_1\leq\frac{1}{v-u}\int\limits_u^v X(t,\cdot)\,dt\\ &\leq \Delta_2\leq[X(u,\cdot)+X(v,\cdot)]\left[\frac{1}{2}+\frac{1}{2^s}\right]\frac{1}{s+1}\end{aligned}$$

where

$$\Delta_1=2^{s-2}\left[X\left(\frac{3u+v}{4},\cdot\right)+\left(\frac{u+3v}{4},\cdot\right)\right]$$

and

$$\Delta_2=\left[\frac{X(u,\cdot)+X(v,\cdot)}{2}+X\left(\frac{u+v}{2},\cdot\right)\right]\frac{1}{s+1}.$$

Corollary 2. *Under assumption of Theorem 4 with $h(t)=1$, we have the following Hermite-Hadamard type inequality for $P-$stochastic processes*

$$\frac{1}{4}X\left(\frac{u+v}{2},\cdot\right)\leq\Delta_1\leq\frac{1}{v-u}\int\limits_u^v X(t,\cdot)\,dt\leq\Delta_2\leq\frac{3}{2}[X(u,\cdot)+X(v,\cdot)]$$

where

$$\Delta_1=\frac{1}{4}\left[X\left(\frac{3u+v}{4},\cdot\right)+\left(\frac{u+3v}{4},\cdot\right)\right]$$

and

$$\Delta_2=\left[\frac{X(u,\cdot)+X(v,\cdot)}{2}+X\left(\frac{u+v}{2},\cdot\right)\right].$$

Corollary 3. *Under assumption of Theorem 4 with $h(t)=\frac{1}{t}$, we have the following Hermite-Hadamard type inequality for Godunova-Levin stochastic processes*

$$\frac{1}{16}X\left(\frac{u+v}{2},\cdot\right)\leq\Delta\leq\frac{1}{v-u}\int\limits_u^v X(t,\cdot)\,dt$$

where

$$\Delta=\frac{1}{8}\left[X\left(\frac{3u+v}{4},\cdot\right)+\left(\frac{u+3v}{4},\cdot\right)\right].$$

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