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A counter example for refined reverse Young inequality with Specht's ratio

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For $a, b \geq 0$, a refined Young inequality with Specht's ratio was shown in [1]:

$$S\left(\left(\frac{a}{b}\right)^{r}\right) a^{1-v} b^{v} \leq (1-v)a + vb \tag{1}$$

where $r \equiv \min\{v, 1-v\}$ with $v \in [0,1]$, and the Specht's ratio is defined by $S(h) \equiv \frac{h^{\frac{1}{h-1}}}{e \log h^{\frac{1}{h-1}}}$ for $h > 0$. The inequality (1) is a refinement of Young inequality in the sense of $S(h) \geq 1$ for $h > 0$.

Instead of the reverse Young inequality given in [2, 3] for $a, b \geq 0$:

$$(1-v)a + vb \leq S\left(\frac{a}{b}\right) a^{1-v} b^{v}, \tag{2}$$

(as a quite natural insight) the following inequality was opened in [4, 5] for $a, b \geq 0$:

$$(1-v)a + vb \leq S\left(\left(\frac{a}{b}\right)^{R}\right) a^{1-v} b^{v} \tag{3}$$

where $R \equiv \max\{v, 1-v\}$ with $v \in [0,1]$.

However, we have counter examples for the inequality (3). Actually, we set $a = 2, b = 1$ and $v = \frac{1}{2}$ for simply, then the inequality (3) becomes

$$\frac{3}{2} \leq S(\sqrt{2})\sqrt{2}.$$

By the numerical computations $S(\sqrt{2})\sqrt{2} \simeq 1.43557$ so that the inequality (3) does not hold in general. (For supplementation, $S(2)\sqrt{2} \simeq 1.50115$.)

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