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INEQUALITIES VIA $GG-$CONVEXITY

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Abstract. In this paper, we established some integral inequalities for functions whose derivatives of absolute values are $GG-$convex.

1. Introduction

We will start with the definiton of convexity:

Definition 1. *The function $f : I \subset \mathbb{R} \to \mathbb{R}$ is a convex function on I, if the inequality*

$$f(tx + (1-t)y) \leq tf(x) + (1-t)f(y)$$

holds for all $x, y \in I$ and $t \in [0,1]$. We say that f is concave if $-f$ is convex.

Let $f : I \subset \mathbb{R} \to \mathbb{R}$ be a convex function where $a, b \in I$ with $a < b$. Then the following double inequality hold:

$$f\left(\frac{a+b}{2}\right) \leq \frac{1}{b-a}\int_a^b f(x)dx \leq \frac{f(a)+f(b)}{2}.$$

This inequality is well-known in the literature as Hermite-Hadamard inequality that gives us upper and lower bounds for the mean-value of a convex function. If f is concave function both of the inequalities in above hold in reversed direction.

Anderson *et. al.* mentioned mean function in [5] as following:

Definition 2. *A function $M : (0,\infty) \times (0,\infty) \to (0,\infty)$ is called a Mean function if*

(1) $M(x,y) = M(y,x)$,
(2) $M(x,x) = x$,
(3) $x < M(x,y) < y$, whenever $x < y$,
(4) $M(ax, ay) = aM(x,y)$ for all $a > 0$.

Based on the definition of mean function, let us recall special means (See [5])

1. Arithmetic Mean: $M(x,y) = A(x,y) = \frac{x+y}{2}$.
2. Geometric Mean: $M(x,y) = G(x,y) = \sqrt{xy}$.
3. Harmonic Mean: $M(x,y) = H(x,y) = 1/A\left(\frac{1}{x}, \frac{1}{y}\right)$.
4. Logarithmic Mean: $M(x,y) = L(x,y) = (x-y)/(\log x - \log y)$ for $x \neq y$ nd $L(x,x) = x$.
5. Identric Mean: $M(x,y) = I(x,y) = (1/e)(x^x/y^y)^{1/(x-y)}$ for $x \neq y$ nd $I(x,x) = x$.

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In [5], Anderson *et. al.* also gave a definition that include several different classes of convex functions as the following:

Definition 3. *Let $f : I \to (0,\infty)$ be continuous, where I is subinterval of $(0,\infty)$. Let M and N be any two Mean functions. We say f is MN-convex (concave) if*

$$f(M(x,y)) \leq (\geq) N(f(x), f(y))$$

for all $x, y \in I$.

In [4], Niculescu mentioned the following considerable definition:

Definition 4. *The $GG-$convex functions are those functions $f : I \to J$ (acting on subintervals of $(0,\infty)$) such that*

$$x, y \in I \text{ and } \lambda \in [0,1] \Longrightarrow f\left(x^{1-\lambda} y^{\lambda}\right) \leq f(x)^{1-\lambda} f(y)^{\lambda}. \tag{1.1}$$

Every real analytic function $f(x) = \sum_{n=0}^{\infty} c_n x^n$ with nonnegative coefficients c_n is a $GG-$convex function on $(0,r)$, where r is the radius of convergence of f. The functions such as $\exp, \sinh, \cosh$ are $GG-$convex on $(0,\infty)$; tan, sec, csc, $\frac{1}{x} - \cot x$ are $GG-$convex on $\left(0, \frac{\pi}{2}\right)$; $\frac{1+x}{1-x}$ is $GG-$convex on (0.1). (See [4])

Anderson et al. gave the following Theorem and an immediate consequence of this Theorem in [5].

Theorem 1. *Let $I = (0,b)$, $0 < b < \infty$ and let $f : I \to (0,\infty)$ be continuous. f is $GG-$convex(concave) on I if and only if $\log f(be^{-t})$ is convex (concave) on $(0,\infty)$.*

Corollary 1. *Let $I = (0,b)$, $0 < b < \infty$ and let $f : I \to (0,\infty)$ be differentiable. f is $GG-$convex(concave) on I if and only if $xf'(x)/f(x)$ is increasing (decreasing).*

For some results about $GG-$convex functions one may see the references [1]-[9].

The main aim of this paper is to prove some new integral inequalities for $GG-$convex functions by using a new integral identity.

2. Main Results

We need the following integral identity to get our new results.

Lemma 1. *Let $f : I \subseteq IR = (0,\infty) \longrightarrow IR$ be a differentiable function on I° where a,b∈ I° with a‹b . If $f^{'} \in L[a,b]$, then the following equality holds:*

$$\begin{aligned} & bf(b) - af(a) - \int_a^b f(u)du \\ = & \frac{\ln b - \ln a}{2}\left[\int_0^1 \left(b^t a^{2-t}\right) f^{'}\left(b^{\frac{t}{2}} a^{\frac{2-t}{2}}\right) dt + \int_0^1 \left(a^t b^{2-t}\right) f^{'}\left(a^{\frac{t}{2}} b^{\frac{2-t}{2}}\right) dt\right] \end{aligned}$$

Proof. Let

$$I_1 = \int_0^1 \left(b^t a^{2-t}\right) f^{'}\left(b^{\frac{t}{2}} a^{\frac{2-t}{2}}\right) dt$$

and

$$I_2 = \int_0^1 \left(a^t b^{2-t}\right) f^{'}\left(a^{\frac{t}{2}} b^{\frac{2-t}{2}}\right) dt$$

We notice that

$$\begin{aligned} I_1 &= \int_0^1 \left(b^t a^{2-t}\right) f'\left(b^{\frac{t}{2}} a^{\frac{2-t}{2}}\right) dt \\ &= \frac{2}{\ln b - \ln a} \int_0^1 \left(b^{\frac{t}{2}} a^{\frac{2-t}{2}}\right) f'\left(b^{\frac{t}{2}} a^{\frac{2-t}{2}}\right) d\left(b^{\frac{t}{2}} a^{\frac{2-t}{2}}\right). \end{aligned}$$

By the change of the variable $u = b^{\frac{t}{2}} a^{\frac{2-t}{2}}$ and integrating by parts , we have

$$I_1 = \frac{2}{\ln b - \ln a}\left[\sqrt{ab} f\sqrt{ab} - af(a) - \int_a^{\sqrt{ab}} f(u)du\right].$$

Conformably, we have

$$I_2 = \frac{2}{\ln b - \ln a}\left[bf(b) - \sqrt{ab} f\sqrt{ab} - \int_{\sqrt{ab}}^b f(u)du\right]$$

Multiplying I_1 and I_2 by $\frac{\ln b - \ln a}{2}$ and adding the results we get the desired identity. $\square$

Our first result is given in the following Theorem.

Theorem 2. *Let $f : I \subseteq IR_+ = (0,\infty) \longrightarrow IR$ be a differentiable function on I° where $a, b \in I^\circ$ with $a < b$,and $f' \in L[a,b]$. If $\left|f'\right|$ is $GG-$ convex on $[a,b]$, then the following inequality holds:*

$$\begin{aligned} &\left|bf(b) - af(a) - \int_a^b f(u)du\right| \\ \leq\ & \frac{\ln b - \ln a}{2}\left(a\sqrt{|f'(a)|} + b\sqrt{|f'(b)|}\right) L\left(a\sqrt{|f'(a)|}, b\sqrt{|f'(b)|}\right) \end{aligned}$$

Proof. From Lemma 1, using the property of the modulus and $GG-$ convexity of $\left|f'\right|$ we can write

$$\begin{aligned} &\left|bf(b) - af(a) - \int_a^b f(u)du\right| \\ \leq\ & \frac{\ln b - \ln a}{2}\left[\int_0^1 \left(b^t a^{2-t}\right)\left|f'\left(b^{\frac{t}{2}} a^{\frac{2-t}{2}}\right)\right| dt + \int_0^1 \left(a^t b^{2-t}\right)\left|f'\left(a^{\frac{t}{2}} b^{\frac{2-t}{2}}\right)\right| dt\right] \\ \leq\ & \frac{\ln b - \ln a}{2}\left[\int_0^1 \left(b^t a^{2-t}\right)\left|f'(b)\right|^{\frac{t}{2}}\left|f'(a)\right|^{\frac{2-t}{2}} dt + \int_0^1 \left(a^t b^{2-t}\right)\left|f'(a)\right|^{\frac{t}{2}}\left|f'(b)\right|^{\frac{2-t}{2}} dt\right] \\ =\ & \frac{\ln b - \ln a}{2}\left[a^2\left|f'(a)\right|\int_0^1 \left(\frac{b\sqrt{|f'(b)|}}{a\sqrt{|f'(a)|}}\right)^t dt + b^2\left|f'(b)\right|\int_0^1 \left(\frac{a\sqrt{|f'(a)|}}{b\sqrt{|f'(b)|}}\right)^t dt\right] \end{aligned}$$

If we calculate the integrals above, we get the desired result. $\square$

Theorem 3. *Let $f : I \subseteq IR_+ = (0,\infty) \longrightarrow IR$ be a differentiable function on I° where $a,b\in I^\circ$ with $a<b$,and $f^{'} \in L[a,b]$. If $\left|f^{'}\right|^q$ is $GG-$ convex on $[a,b]$ for all $x\in[a,b]$, the following inequality*

$$\begin{aligned}
&\left|bf(b)-af(a)-\int_a^b f(u)du\right| \\
\leq\ & \frac{\ln b-\ln a}{2}\left(a\sqrt{|f'(a)|}+b\sqrt{|f'(b)|}\right)\left(L\left(a^p,b^p\right)\right)^{\frac{1}{p}}\left(L\left(\sqrt{|f'(a)|^q},\sqrt{|f'(b)|^q}\right)\right)^{\frac{1}{q}}
\end{aligned}$$

holds where $q>1$ and $\frac{1}{p}+\frac{1}{q}=1$.

Proof. From Lemma 1, using the property of the modulus, $GG-$ convexity of $\left|f^{'}\right|^q$ and Hölder integral inequality, we can write

$$\begin{aligned}
&\left|bf(b)-af(a)-\int_a^b f(u)du\right| \\
=\ & \frac{\ln b-\ln a}{2}\left[\int_0^1\left(b^ta^{2-t}\right)\left|f^{'}\left(b^{\frac{t}{2}}a^{\frac{2-t}{2}}\right)\right|dt+\int_0^1\left(a^tb^{2-t}\right)\left|f^{'}\left(a^{\frac{t}{2}}b^{\frac{2-t}{2}}\right)\right|dt\right] \\
\leq\ & \frac{\ln b-\ln a}{2}\left\{\left(\int_a^b b^{tp}a^{(2-t)p}dt\right)^{\frac{1}{p}}\left(\int_0^1\left|f^{'}\left(b^{\frac{t}{2}}a^{\frac{2-t}{2}}\right)\right|^q dt\right)^{\frac{1}{q}}\right. \\
&\left.+\left(\int_a^b a^{tp}b^{(2-t)p}dt\right)^{\frac{1}{p}}\left(\int_0^1\left|f^{'}\left(a^{\frac{t}{2}}b^{\frac{2-t}{2}}\right)\right|^q dt\right)^{\frac{1}{q}}\right\} \\
\leq\ & \frac{\ln b-\ln a}{2}\left\{a^2\left(\int_0^1\left(\frac{b^p}{a^p}\right)^t dt\right)^{\frac{1}{p}}\left(\int_0^1\left|f^{'}(b)\right|^{\frac{tq}{2}}\left|f^{'}(a)\right|^{\frac{(2-t)q}{2}}dt\right)^{\frac{1}{q}}\right. \\
&\left.+b^2\left(\int_0^1\left(\frac{a^p}{b^p}\right)^t dt\right)^{\frac{1}{p}}\left(\int_0^1\left|f^{'}(a)\right|^{\frac{tq}{2}}\left|f^{'}(b)\right|^{\frac{(2-t)q}{2}}dt\right)^{\frac{1}{q}}\right\}.
\end{aligned}$$

If we calculate the integrals above, we get the desired result. □

Theorem 4. *Under the assumptions of Theorem 3, the following inequality holds:*

$$\begin{aligned}
&\left|bf(b)-af(a)-\int_a^b f(u)du\right| \\
\leq\ & \frac{\ln b-\ln a}{2}\left(L\left(\sqrt{|f'(a)|^q},\sqrt{|f'(b)|^q}\right)\right)^{\frac{1}{q}} \\
&\times\left[\left(\frac{b^{p+1}-pb-b}{p+1}\right)^{\frac{1}{p}}a^2\sqrt{|f'(a)|}+\left(\frac{a^{p+1}-pa-a}{p+1}\right)^{\frac{1}{p}}b^2\sqrt{|f'(b)|}\right].
\end{aligned}$$

Proof. From Lemma 1, using the property of the modulus, $GG-$ convexity of $\left|f^{'}\right|^q$ and Hölder integral inequality, we can write

$$\begin{aligned}
&\left|bf(b)-af(a)-\int_a^b f(u)du\right| \\
\leq\ & \frac{\ln b-\ln a}{2}\left\{\left(\int_0^1 b^{tp}dt\right)^{\frac{1}{p}}\left(\int_0^1 a^{(2-t)q}\left|f'\left(b^{\frac{t}{2}}a^{\frac{2-t}{2}}\right)\right|^q dt\right)^{\frac{1}{q}}\right. \\
&\left.+\left(\int_0^1 a^{tp}dt\right)^{\frac{1}{p}}\left(\int_0^1 b^{(2-t)q}\left|f'\left(a^{\frac{t}{2}}b^{\frac{2-t}{2}}\right)\right|^q dt\right)^{\frac{1}{q}}\right\} \\
\leq\ & \frac{\ln b-\ln a}{2}\left\{\left(\int_0^1 b^{tp}dt\right)^{\frac{1}{p}}\left(\int_0^1 a^{(2-t)q}\left|f'(b)\right|^{\frac{tq}{2}}\left|f'(a)\right|^{\frac{(2-t)q}{2}}dt\right)^{\frac{1}{q}}\right. \\
&\left.+\left(\int_0^1 a^{tp}dt\right)^{\frac{1}{p}}\left(\int_0^1 b^{(2-t)q}\left|f'(a)\right|^{\frac{tq}{2}}\left|f'(b)\right|^{\frac{(2-t)q}{2}}\right)^{\frac{1}{q}}\right\}.
\end{aligned}$$

By a simple computation we get the desired result. $\square$

Theorem 5. *Under the assumptions of Theorem 3, the following inequality holds:*

$$\begin{aligned}
&\left|bf(b)-af(a)-\int_a^b f(u)du\right| \\
\leq\ & \frac{\ln b-\ln a}{2}\left(a\sqrt{|f'(a)|}+b\sqrt{|f'(b)|}\right)\left(L\left(a^q\sqrt{|f'(a)|^q},b^q\sqrt{|f'(b)|^q}\right)\right)^{\frac{1}{q}}.
\end{aligned}$$

Proof. From Lemma 1, using the property of the modulus, $GG-$ convexity of $\left|f'\right|^q$ and Hölder integral inequality, we can write

$$\begin{aligned}
&\left|bf(b)-af(a)-\int_a^b f(u)du\right| \\
\leq\ & \frac{\ln b-\ln a}{2}\left\{\left(\int_0^1 dt\right)^{1-\frac{1}{q}}\left(\int_0^1 b^{tq}a^{(2-t)q}\left|f'\left(b^{\frac{t}{2}}a^{\frac{2-t}{2}}\right)\right|^q dt\right)^{\frac{1}{q}}\right. \\
&\left.+\left(\int_0^1 dt\right)^{1-\frac{1}{q}}\left(\int_0^1 a^{tq}b^{(2-t)q}\left|f'\left(a^{\frac{t}{2}}b^{\frac{2-t}{2}}\right)\right|^q dt\right)^{\frac{1}{q}}\right\} \\
\leq\ & \frac{\ln b-\ln a}{2}\left\{a^2\left|f'(a)\right|\left(\int_0^1\left(\frac{b^q\left|f'(b)\right|^{\frac{q}{2}}}{a^q\left|f'(a)\right|^{\frac{q}{2}}}\right)^t dt\right)^{\frac{1}{q}}\right. \\
&\left.+b^2\left|f'(b)\right|\left(\int_0^1\left(\frac{a^q\left|f'(a)\right|^{\frac{q}{2}}}{b^q\left|f'(b)\right|^{\frac{q}{2}}}\right)^t dt\right)^{\frac{1}{q}}\right\}.
\end{aligned}$$

If we calculate the integrals above, we get the desired result. □

Theorem 6. *Under the assumptions of Theorem 3, the following inequality holds:*

$$\begin{aligned}
&\left| bf(b) - af(a) - \int_a^b f(u)du \right| \\
\leq\ & \frac{\ln b - \ln a}{2} \left\{ \left(\frac{b-1}{\ln b} \right)^{\frac{1}{p}} a\sqrt{|f'(a)|} L^{\frac{1}{q}} \left(a^q \sqrt{|f'(a)|^q}, b\sqrt{|f'(b)|^q} \right) \right. \\
&\left. + \left(\frac{a-1}{\ln a} \right)^{\frac{1}{p}} b\sqrt{|f'(b)|} L^{\frac{1}{q}} \left(a\sqrt{|f'(a)|^q}, b^q \sqrt{|f'(b)|^q} \right) \right\}.
\end{aligned}$$

Proof. From Lemma 1, using the property of the modulus, $GG-$ convexity of $\left|f'\right|^q$ and Hölder integral inequality, we can write

$$\begin{aligned}
&\left| bf(b) - af(a) - \int_a^b f(u)du \right| \\
\leq\ & \frac{\ln b - \ln a}{2} \left\{ \left(\int_0^1 b^t dt \right)^{\frac{1}{p}} \left(\int_0^1 b^t a^{(2-t)q} \left|f'(b)\right|^{\frac{tq}{2}} \left|f'(a)\right|^{\left(1-\frac{t}{2}\right)q} dt \right)^{\frac{1}{q}} \right. \\
&\left. \left(\int_0^1 a^t dt \right)^{\frac{1}{p}} \left(\int_0^1 a^t b^{(2-t)q} \left|f'(a)\right|^{\frac{tq}{2}} \left|f'(b)\right|^{\left(1-\frac{t}{2}\right)q} dt \right)^{\frac{1}{q}} \right\}.
\end{aligned}$$

If we calculate the integrals above, we get the desired result. □

Theorem 7. *Under the assumptions of Theorem 3, the following inequality holds:*

$$\begin{aligned}
&\left| bf(b) - af(a) - \int_a^b f(u)du \right| \\
\leq\ & \frac{\ln b - \ln a}{2} \left(a^{\frac{p}{q}} \sqrt{|f'(a)|} + b^{\frac{p}{q}} \sqrt{|f'(b)|} \right) \\
&\times \left(L\left(a^{\frac{q-p}{q-1}}, b^{\frac{q-p}{q-1}} \right) \right)^{1-\frac{1}{q}} \left(L\left(a^p \sqrt{|f'(a)|^q} + b^p \sqrt{|f'(b)|^q} \right) \right)^{\frac{1}{q}}.
\end{aligned}$$

Proof. From Lemma 1, using the property of the modulus, $GG-$ convexity of $\left|f'\right|^q$ and Hölder integral inequality, we can write

$$\begin{aligned} & \left|bf(b)-af(a)-\int_a^b f(u)du\right| \\ \leq \quad & \frac{\ln b-\ln a}{2}\left\{\left(\int_0^1\left(b^t a^{2-t}\right)^{\frac{q-p}{q-1}}dt\right)^{1-\frac{1}{q}}\left(\int_0^1\left(b^t a^{2-t}\right)^p\left|f^{'}(a)\right|^{\left(\frac{2-t}{2}\right)q}\left|f^{'}(b)\right|^{\frac{tq}{2}}dt\right)^{\frac{1}{q}}\right. \\ & \left.+\left(\int_0^1\left(a^t b^{2-t}\right)^{\frac{q-p}{q-1}}dt\right)^{1-\frac{1}{q}}\left(\int_0^1\left(a^t b^{2-t}\right)^p\left|f^{'}(b)\right|^{\left(\frac{2-t}{2}\right)q}\left|f^{'}(a)\right|^{\frac{tq}{2}}\right)^{\frac{1}{q}}\right\} \end{aligned}$$

If we calculate the integrals above, we get the desired result. □

Theorem 8. *Let $f: I\subseteq IR_+=(0,\infty)\longrightarrow IR$ be a differentiable function on I° where $a,b\in I^\circ$ with $a<b$,and $f^{'}\in L[a,b]$. If $\left|f^{'}\right|^q$ is $GG-$ convex on $[a,b]$ for all $x\in[a,b]$, the following inequality*

$$\begin{aligned} & \left|bf(b)-af(a)-\int_a^b f(u)du\right| \\ \leq \quad & \frac{\ln b-\ln a}{2}\left(a\sqrt{f^{'}(a)}+b\sqrt{f^{'}(b)}\right)(L(a,b))^{1-\frac{1}{q}}\left(L\left(a\sqrt{\left|f^{'}(a)\right|^q},b\sqrt{\left|f^{'}(b)\right|^q}\right)\right)^{\frac{1}{q}} \end{aligned}$$

holds for $q\geq 1$.

Proof. From Lemma 1, using the property of the modulus, $GG-$ convexity of $\left|f^{'}\right|^q$ and power-mean integral inequality, we can write

$$\begin{aligned} & \left|bf(b)-af(a)-\int_a^b f(u)du\right| \\ \leq \quad & \frac{\ln b-\ln a}{2}\left\{\left(\int_0^1 b^t a^{2-t}dt\right)^{1-\frac{1}{q}}\left(\int_0^1 b^t a^{2-t}\left|f^{'}\left(b^{\frac{t}{2}}a^{\frac{2-t}{2}}\right)\right|^q dt\right)^{\frac{1}{q}}\right. \\ & \left.+\left(\int_0^1 a^t b^{2-t}dt\right)^{1-\frac{1}{q}}\left(\int_0^1 a^t b^{2-t}\left|f^{'}\left(a^{\frac{t}{2}}b^{\frac{2-t}{2}}\right)\right|^q dt\right)^{\frac{1}{q}}\right\} \\ \leq \quad & \frac{\ln b-\ln a}{2}\left\{a^{2\left(1-\frac{1}{q}\right)}\left(\int_0^1\left(\frac{b}{a}\right)^t dt\right)^{1-\frac{1}{q}}a^{\frac{2}{q}}\left(\int_0^1\left(\frac{b}{a}\right)^t\left|f^{'}(b)\right|^{\frac{qt}{2}}\left|f^{'}(a)\right|^{\left(1-\frac{t}{2}\right)q}dt\right)^{\frac{1}{q}}\right. \\ & \left.+b^{2\left(1-\frac{1}{q}\right)}\left(\int_0^1\left(\frac{a}{b}\right)^t dt\right)^{1-\frac{1}{q}}b^{\frac{2}{q}}\left(\int_0^1\left(\frac{a}{b}\right)^t\left|f^{'}(a)\right|^{\frac{qt}{2}}\left|f^{'}(b)\right|^{\left(1-\frac{t}{2}\right)q}dt\right)^{\frac{1}{q}}\right\}. \end{aligned}$$

We get the desired result by a simple calculation. □

Remark 1. *In Theorem 8, if we choose $q=1$, Theorem 8 reduces to Theorem 2.*

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