Palindromic Prime Embedded Trees

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Abstract

*The idea of embedding **palindromic prime (palprime)** numbers in the form of tree is very famous in the literature, where previous palprime is in the middle of next, and so on. In this case there is no limit where it ends, because always we find next palprime containing previous one. In this work, we brought embedded palprimes trees starting with 3 digits.*

Contents

1 Introduction

Embedded palindromic prime numbers are very much famous in the literature [1, 2]. It is generally known by **pyramid palprimes**. Since, the previous palprime is in the middle of next one, we call it embedded palprimes. The work here is concentrated on all the palprimes of digits 3 and 5. We can always find a next palprime containing previous, we limit our study to length 6 for the 3 digits and length 5 for digits 5. By length here we understand that total number of primes in each pattern. In each case we can always find next.

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There are only 4 primes of single digit, i.e., 2, 3, 5 and 7. There is only one palprime of two digits, i.e., 11. There are total 15 palprimes of digits 3 given by

101 131 151 181 191 313 353
373 383 727 757 787 797 919 929.

101	101	131	131	151	191
31013	91019	11311	71317	31513	71917
313	353	373	383	727	757
93139	33533	93739	13831	37273	37573

757	787	797	797	929
97579	97879	17971	77977	39293

Thus, we work with 15 palprimes of digit 3 and 76 of 5 digits. There are much more possibilities of palprimes that we have given below. The aim is not to given more results, but cover all the possible palprimes of digits 3 and 5. We considered all the possible digits, i.e., 0 to 9. Next our aim is to work with restricted digits, such as, 1,3; 1, 6, 9; 1, 2, 5; etc. In some cases complimentary embedded palprimes are also given. By complimentary, we under stand that if we change one digit with other still it remains palprime pyramid. A general study of embedded palprimes of 3 and 5 digits can be seen in author's recent work [16].

2 Comparison Study

2.1 Embedded Palprime Patterns

Let us consider the following embedded palprime pattern up to length 10:

▶ 101
31013
3310133
933101339
1593310133951
13159331013395131
171315933101339513171
1617131593310133951317161
96161713159331013395131716169
363961617131593310133951317161693363

... (1)

More study on this embedded palprimes can be seen in [1, 2, 3, 4].

2.2 Fixed Digits Prime Patterns

Let us consider following pattern of prime numbers with fixed digit's repetition:

▶ **562 9933**
562 192 **9933**
562 192 192 **9933**
562 192 192 192 **9933**
562 192 192 192 192 **9933**
562 192 192 192 192 192 **9933**
562 192 192 192 192 192 192 **9933**
562 192 192 192 192 192 192 192 **9933**
562 192 192 192 192 192 192 192 192 **9933**
562 192 192 192 192 192 192 192 192 192 **9933**.

... (2)

The example (1) is of prime pattern with fixed digits repetition. The example (2) is embedded palprimes pattern, where previous palprimes is in the next. In case of example (1), we have digit 218 repeating horizontally as well as vertically. In total we have 10 prime numbers, while repetition is only in the nine prime numbers. In this case the next number is not a prime numbers, see below:

$$5621921921921921921921921921929933 = 911 \times 14264443 \times 4326250025512145227415633321.$$

This means the process is limited. While in case of example (2), we can easily have further palprimes containing previous ones, see below:

32363961617131593310133951317161693632 3
7723236396161713159331013395131716169363 23277

... ...

This means that we can always find next palprime embedding previous one, while the in case of prime pattern (1) , the process is limited. For more study on prime patterns refer author's work [8, 9, 10, 11, 12].

2.3 Symmetric Embedded

Consider the following examples,

619
111619111
666111619111999
6666611161911199999
11666666661116191119999999911
1116611166666666111619111999999911199111
66111661116666666611161911199999999111991 1199.

619
161619191
11616191911
61111616191911119
6666116161919119999
11666611616191911999911
611111166661161619191199991111119.

... (3)

The above group of prime numbers are having the property of embedding primes, but are not palindromic. Moreover, they are **symmetric in 6 and 9**, but not changeable, i.e., if we replace 6 by 9 and 9 by 6, they are no more prime numbers.

2.4 Complimentary Embedded Palprimes

Below are examples of complimentary embedded palprimes.

16661	19991
1191166611911	1161199911611
1111111191166611911111111	1111111161199911611111111

... (4)

131	191
71317	71917
77771317777	77771917777
11111777771317777711111	11111777771917777711111
111111111111177777131777771111111111111	111111111111177777191777771111111111111

... (5)

131	191
71317	71917
77771317777	77771917777
11777777771317777777711	11777777771917777777711
11111111111111777777771317777777711111111111111	11111111111111777777771917777777711111111111111

... (6)

In examples given in (4), the digits 6 and 9 are changeable. In examples given in (5) and (6), the digits 3 and 7 are changeable. Due this we call these types of examples as **complimentary or paired embedded palprimes**. Detailed study of this kind of results shall be dealt elsewhere.

2.5 Magic-Square-Type Palprimes

Let us consider the following two examples of **magic-square-type palprimes** of order 9×9

```
1 9 3 1 9 1 3 9 1     3 7 3 1 7 1 3 7 3     9 9 1 7 3 7 1 9 9
9 0 1 6 0 6 1 0 9     7 6 1 9 6 9 1 6 7     9 1 1 8 6 8 1 1 9
3 1 8 1 8 1 8 1 3     3 1 9 9 0 9 9 1 3     1 1 8 6 8 6 8 1 1
1 6 1 5 3 5 1 6 1     1 9 9 5 1 5 9 9 1     7 8 6 8 4 8 6 8 7
9 0 8 3 6 3 8 0 9     7 6 0 1 1 1 0 6 7     3 6 8 4 1 4 8 6 3
1 6 1 5 3 5 1 6 1     1 9 9 5 1 5 9 9 1     7 8 6 8 4 8 6 8 7
3 1 8 1 8 1 8 1 3     3 1 9 9 0 9 9 1 3     1 1 8 6 8 6 8 1 1
9 0 1 6 0 6 1 0 9     7 6 1 9 6 9 1 6 7     9 1 1 8 6 8 1 1 9
1 9 3 1 9 1 3 9 1     3 7 3 1 7 1 3 7 3     9 9 1 7 3 7 1 9 9
```

... (6)

The above three examples are of **symmetric-magic-square-type** palprimes, i.e., all the numbers are palprimes in rows, columns and in principal diagonals. Moreover they have the property of row-wise embedding. See below:

908363809

161535161908363809161535161

318181813161535161908363809161535161318181813

901606109318181813161535161908363809161535161318181813901606109

193191391901606109318181813161535161908363809161535161318181813901606109193191391

760111067
199515991760111067199515991
319909913199515991760111067199515991319909913
761969167319909913199515991760111067199515991319909913761969167
373171373761969167319909913199515991760111067199515991319909913761969167373171373

368414863
786848687368414863786848687
118686811786848687368414863786848687118686811
911868119118686811786848687368414863786848687118686811911868119
991737199911868119118686811786848687368414863786848687118686811911868119991737199

... (7)

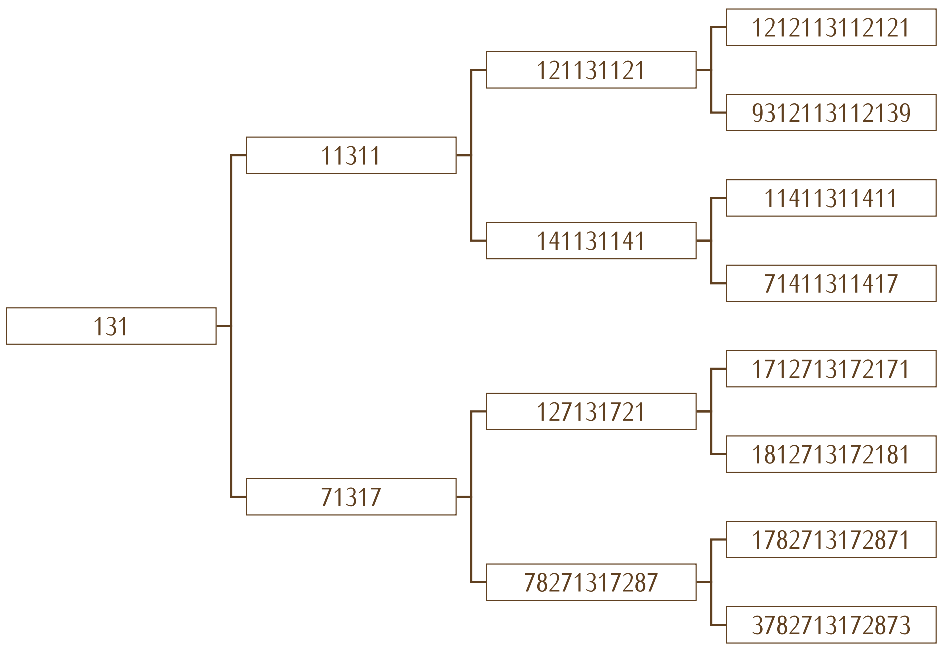
All the three examples appearing in (7) have the property of embedding and also are palprimes. For detailed study refer author's work [13, 14, 15].

3 Palprime Embedded Trees

Let us consider a 3 digits palprime 131. It is embedded in two palprimes 11311 and 71317. Again, 11311 is embedded in 121131121 and 141131141. Further 121131121 is embedded in 1212113112121 and 9312113112139. And 141131141 embedded in 11411311411 and 71411311417. Now the second one i.e., 71317 is embedded in 127131721 and 78271317287. Again 127131721 is embedded in 1712713172171 and 1812713172181. And 78271317287 is embedded in 1782713172871 and 3782713172873, and so on. In other words:

$$131 \rightarrow 11311 \rightarrow 121131121 \rightarrow 1212113112121 \rightarrow \ldots$$
$$131 \rightarrow 11311 \rightarrow 141131141 \rightarrow 11411311411 \rightarrow \ldots$$
$$\ldots \qquad \ldots \qquad \ldots$$

See below the above structures in terms of tree:



... (8)

Alternatively, we can write the above structure in terms of 11311 and 71317 as embedded palprimes:

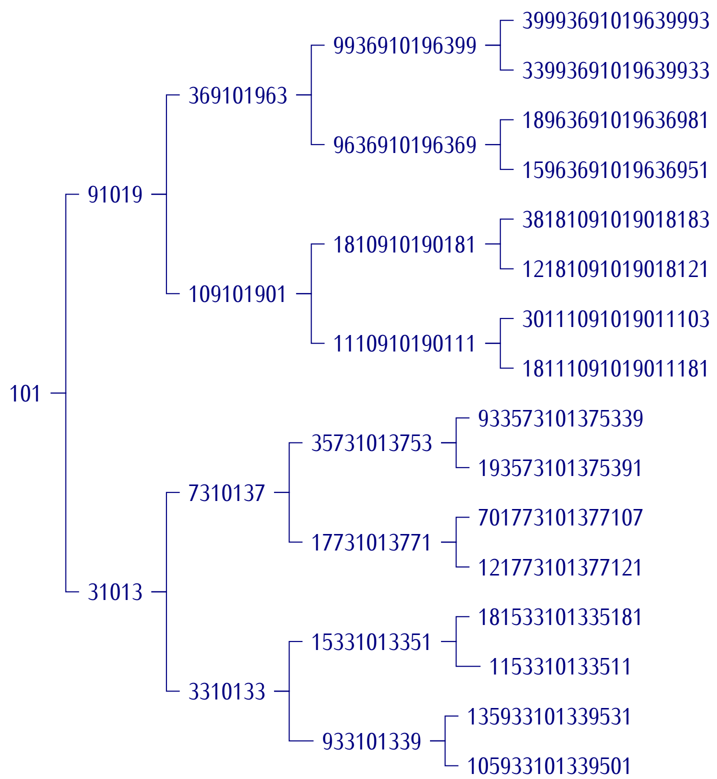
131	131	131	131
11311	11311	11311	11311
121131121	121131121	141131141	141131141
1212113112121	9312113112139	11411311411	71411311417
... ...	... ...	... ...	... ...
131	131	131	131
71317	71317	71317	71317
127131721	127131721	78271317287	78271317287
1712713172171	1812713172181	1782713172871	3782713172873
... ...	... ...	... ...	... ...

Remark 3.1. *By no way, we can say that the above two-by-two example is unique. We may have other palprimes giving more splitting results. Also, this is not the end. This process continues as long as we go on finding more palprimes.*

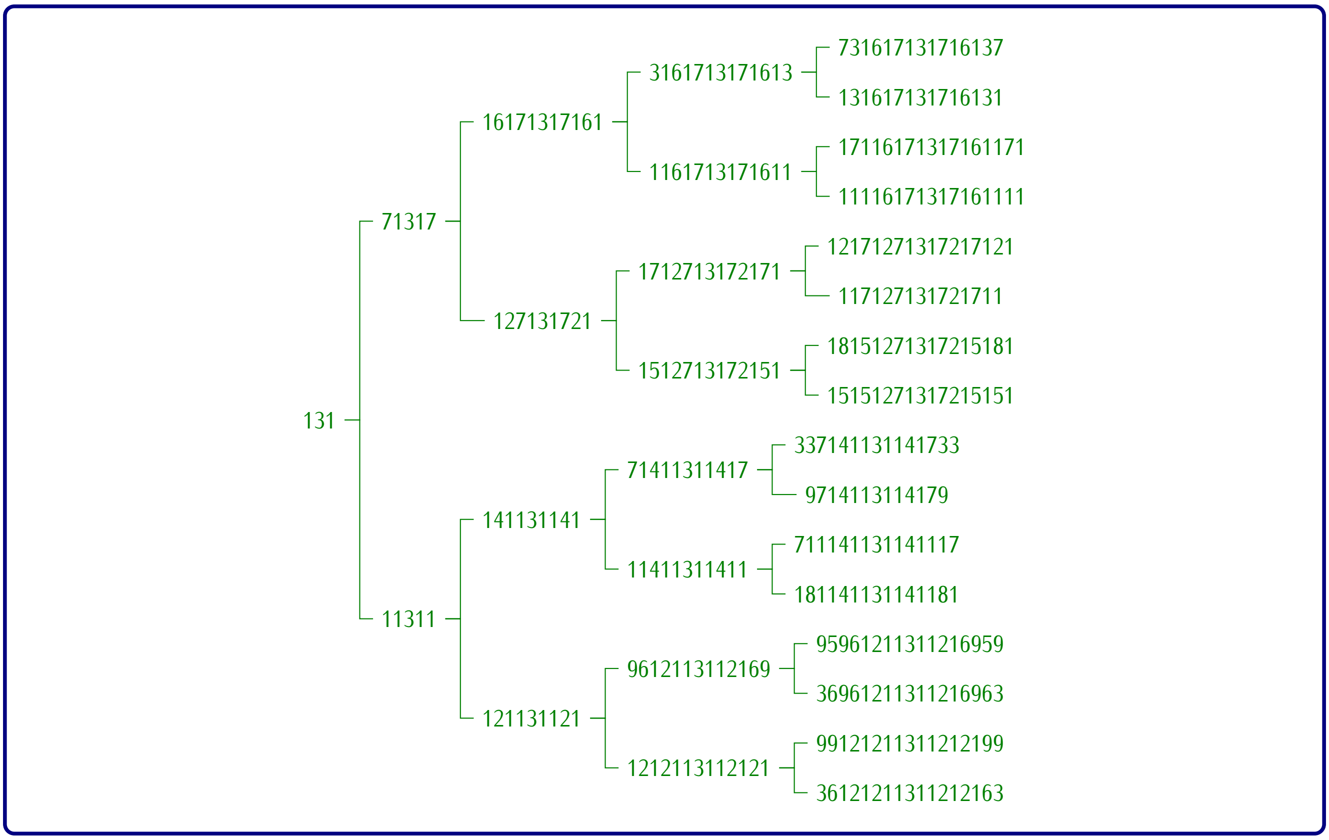
3.1 Palindromic Prime Embedded Trees

We have only 11 palprimes of with 3 digits. Below are palprime embedded trees starting with 3-digits. The example (8) given above goes up to 4th order and ending in 8 palprimes. The results given below goes up to 5th order and ends in 16 palprimes. We have splitted in two-by-two system, but there are much more possibilities.

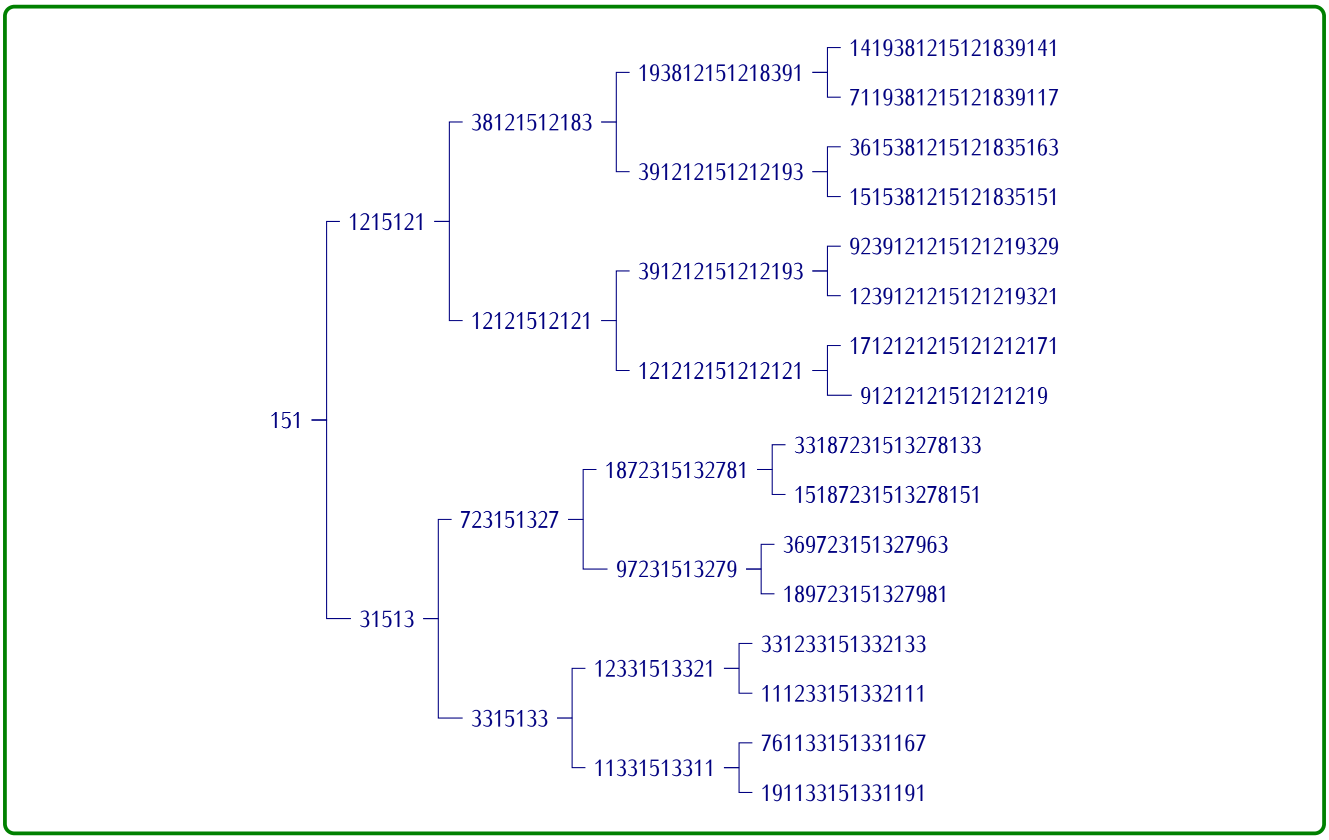
3.2 Embedded Tree with 101



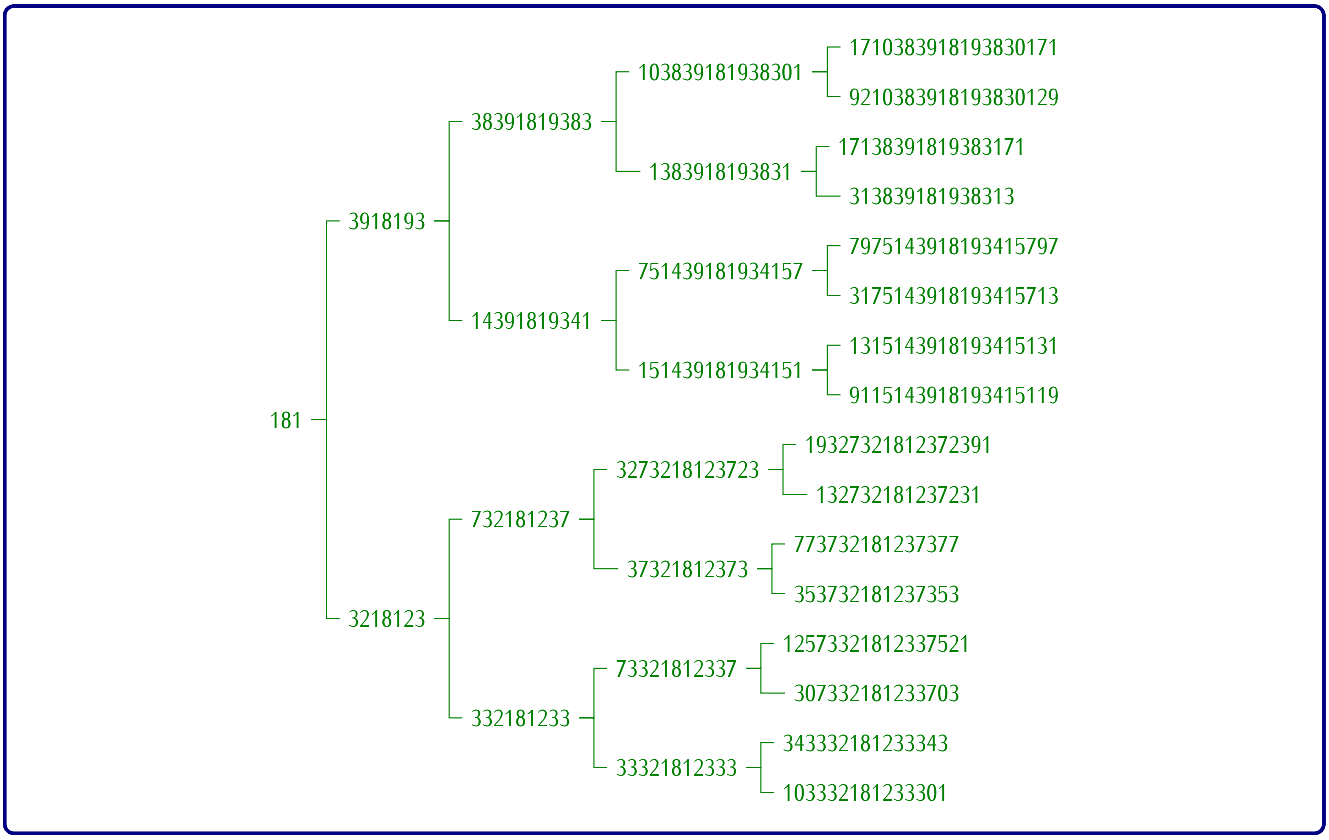
3.2.1 Embedded Tree with 131



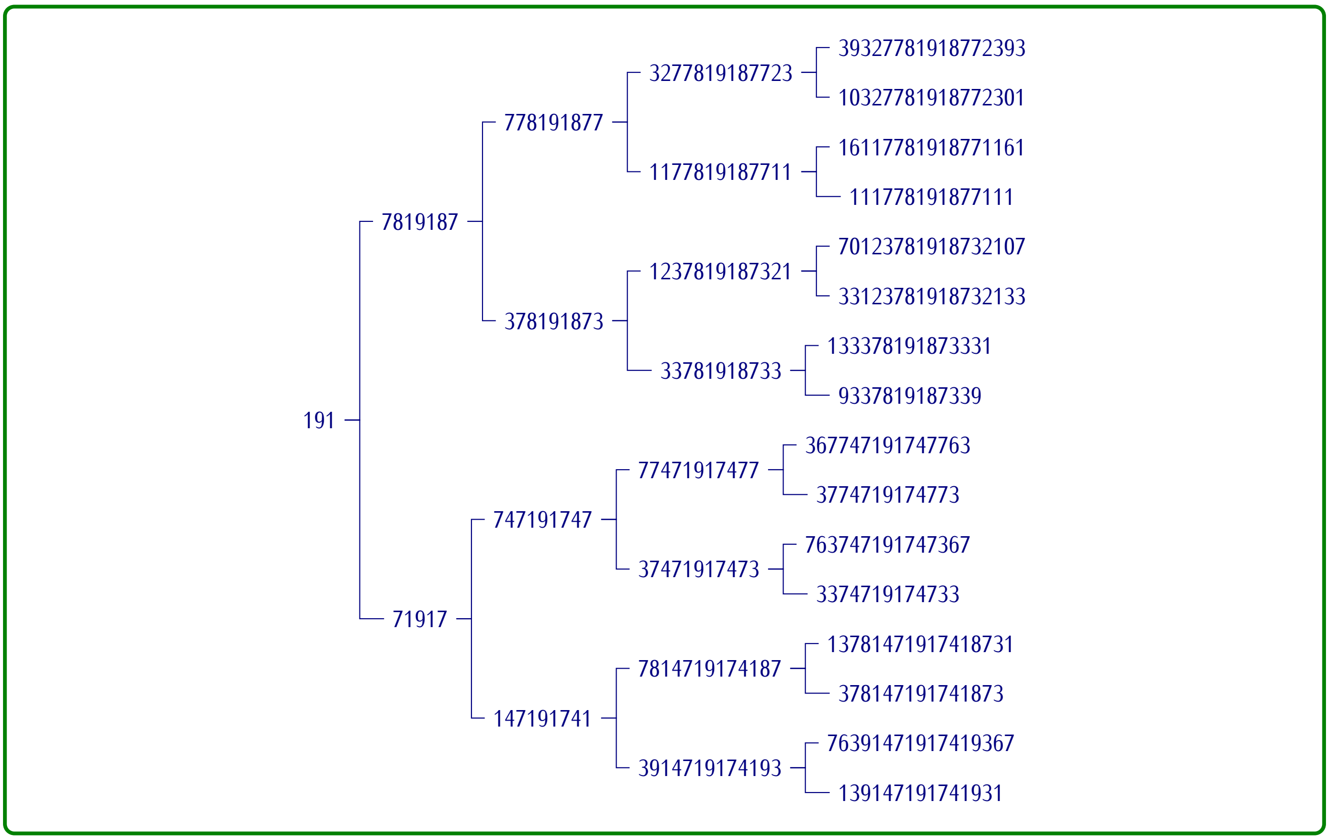
3.2.2 Embedded Tree with 151



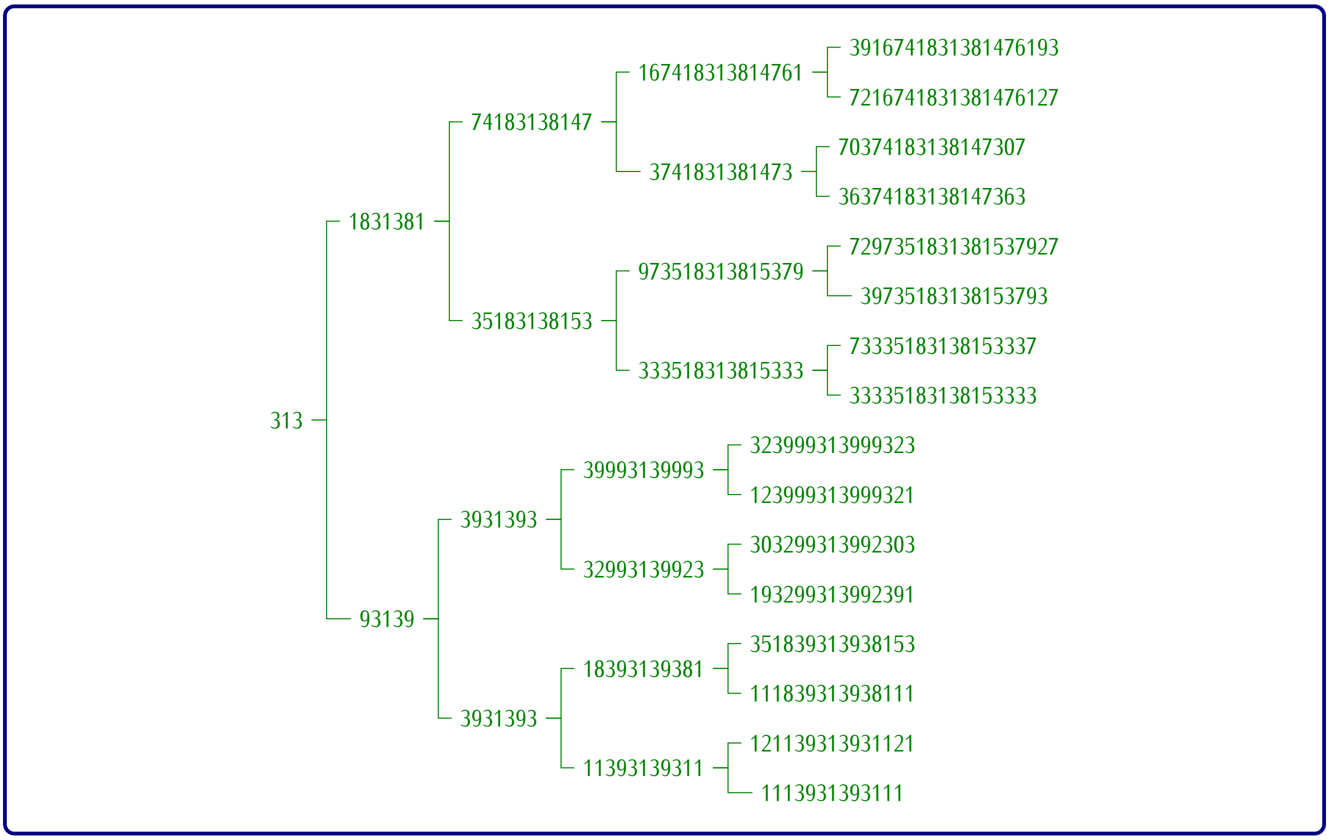
3.2.3 Embedded Tree with 181



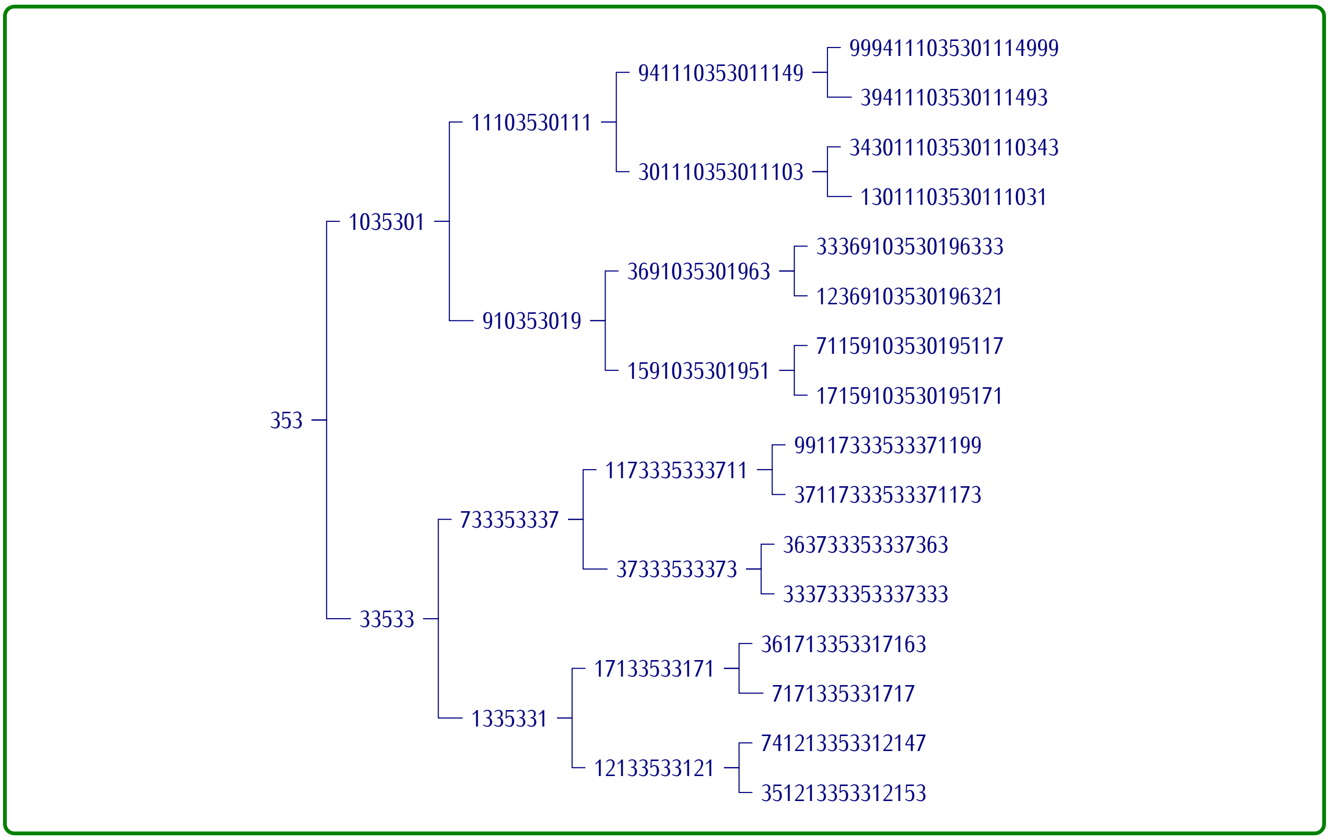
3.2.4 Embedded Tree with 191



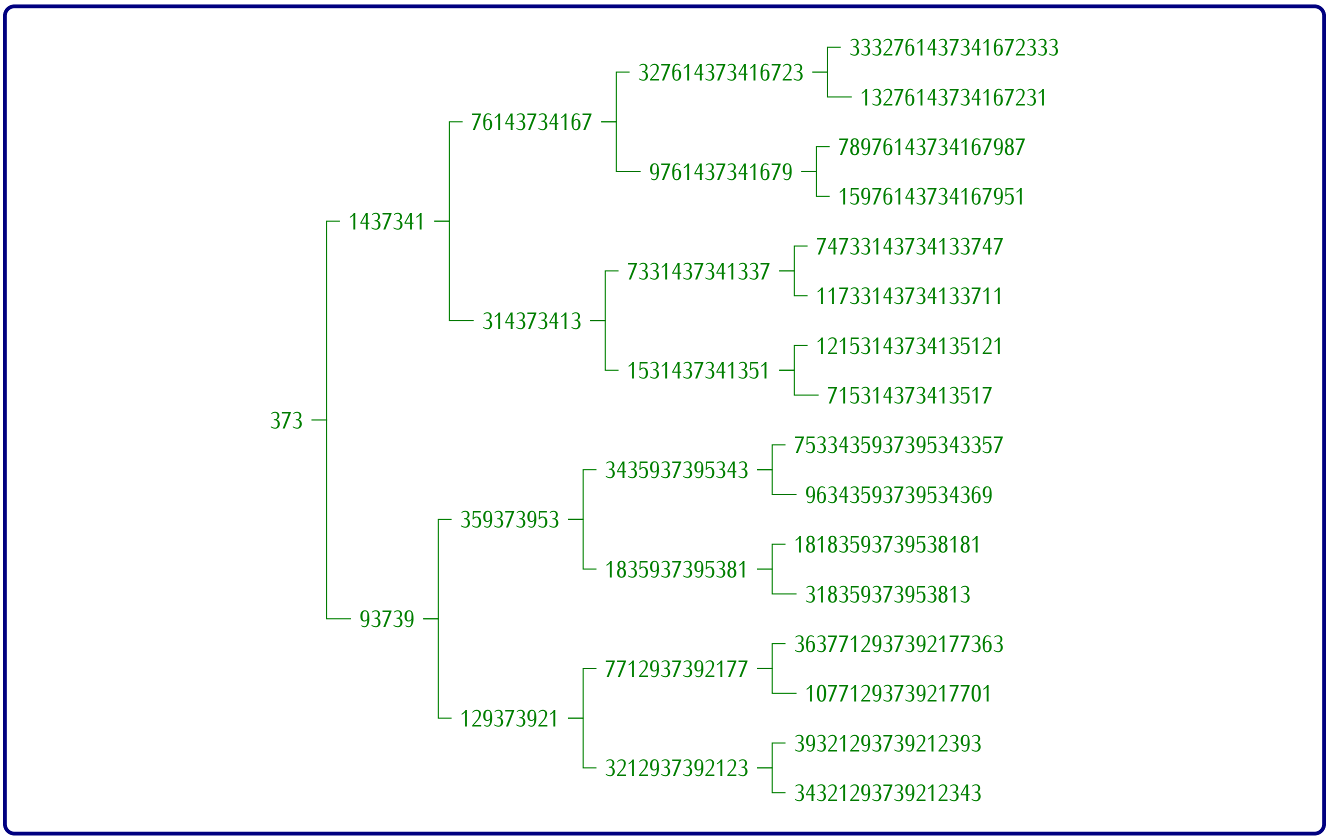
3.2.5 Embedded Tree with 313



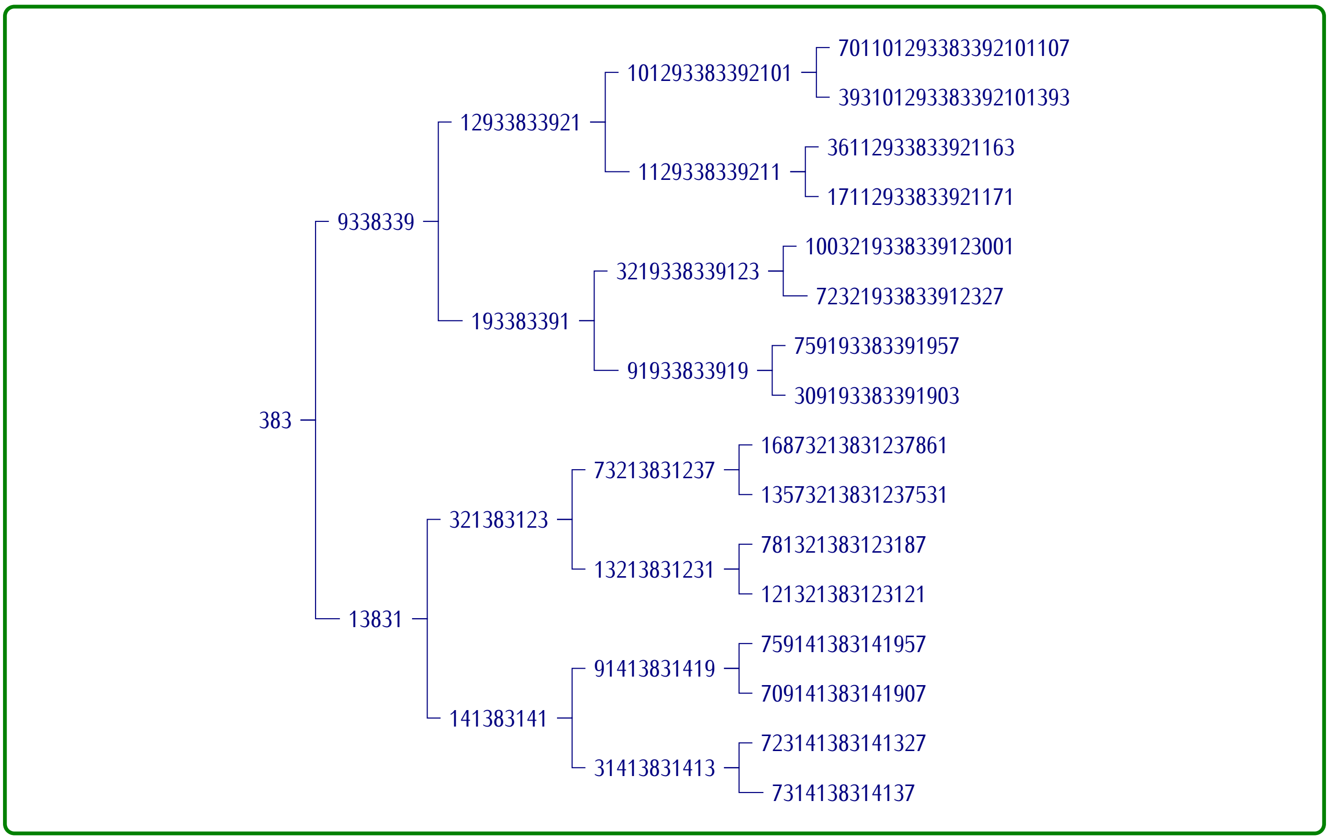
3.2.6 Embedded Tree with 353



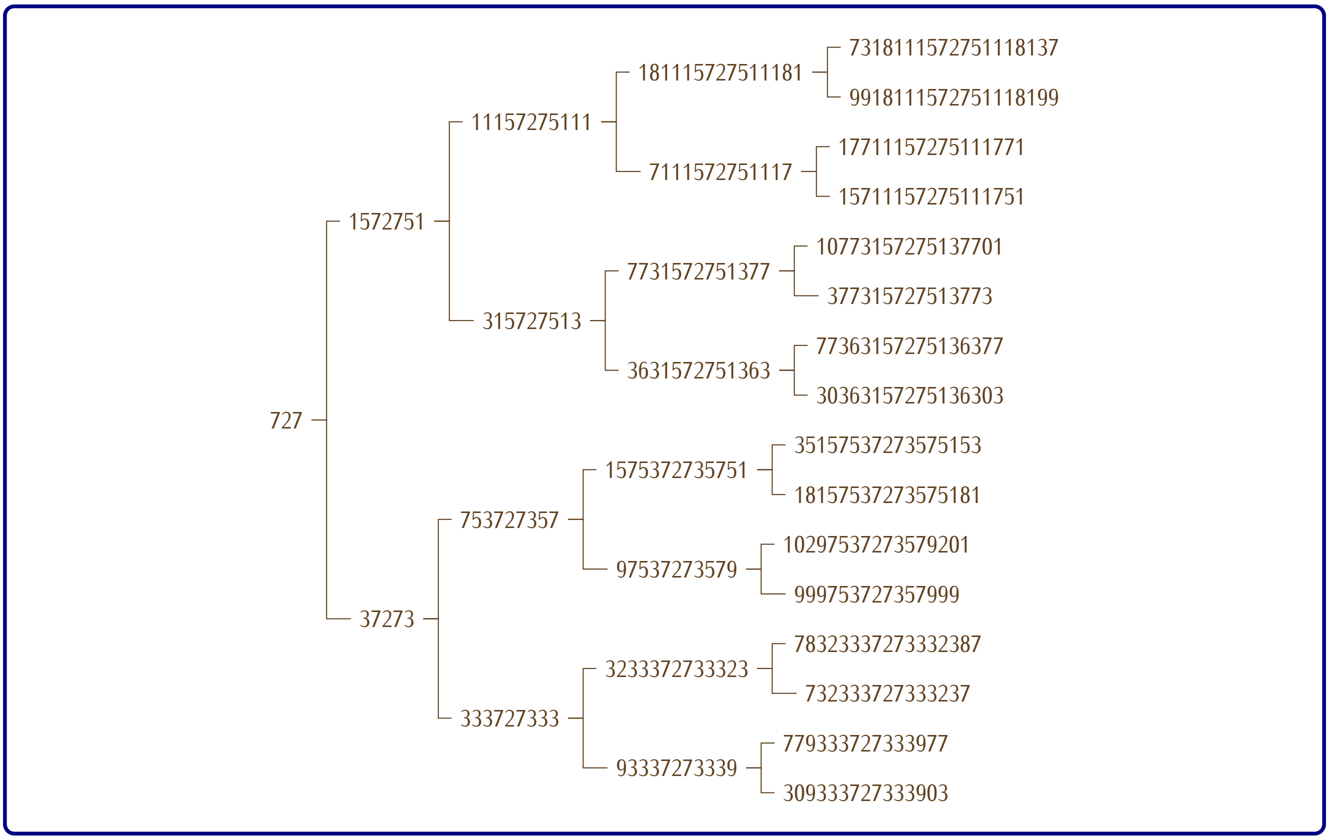
3.2.7 Embedded Tree with 373



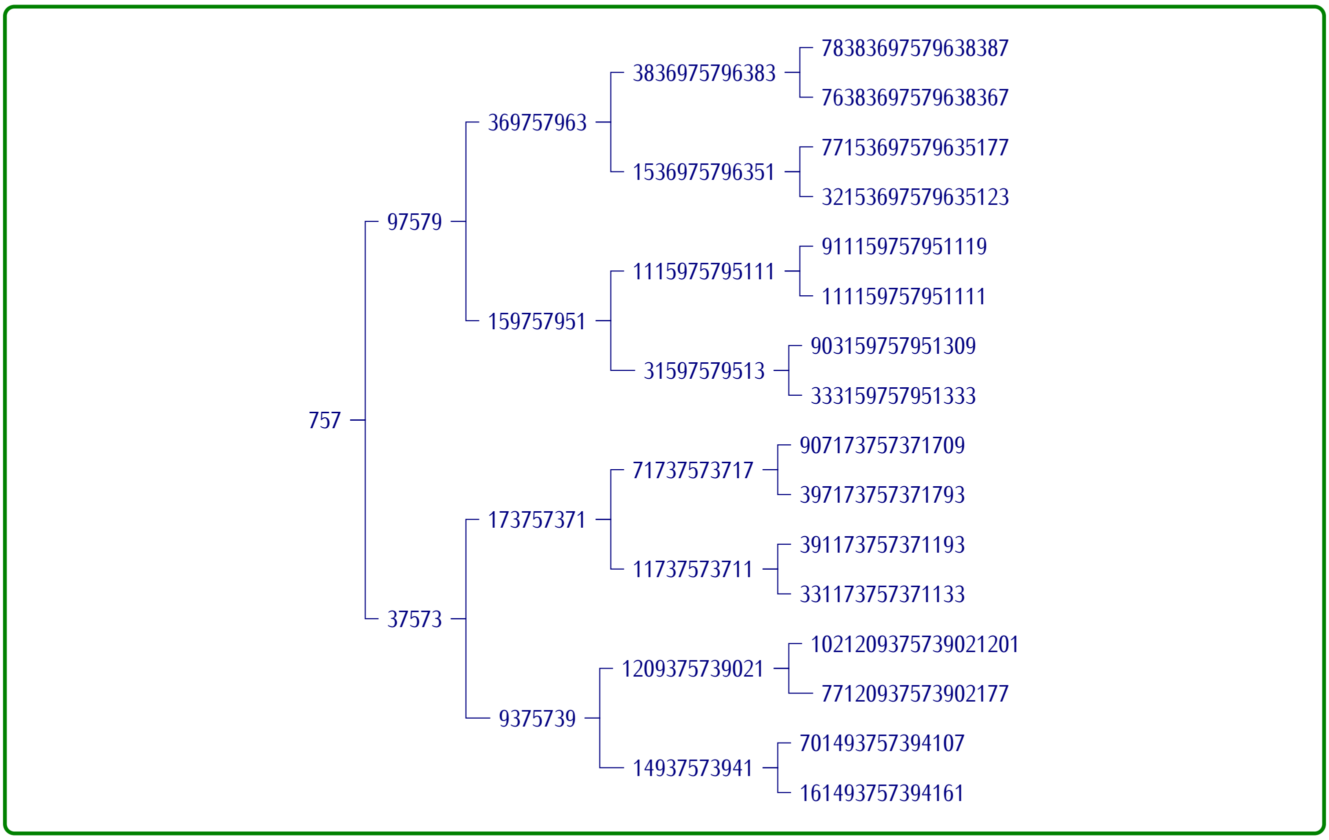
3.2.8 Embedded Tree with 383



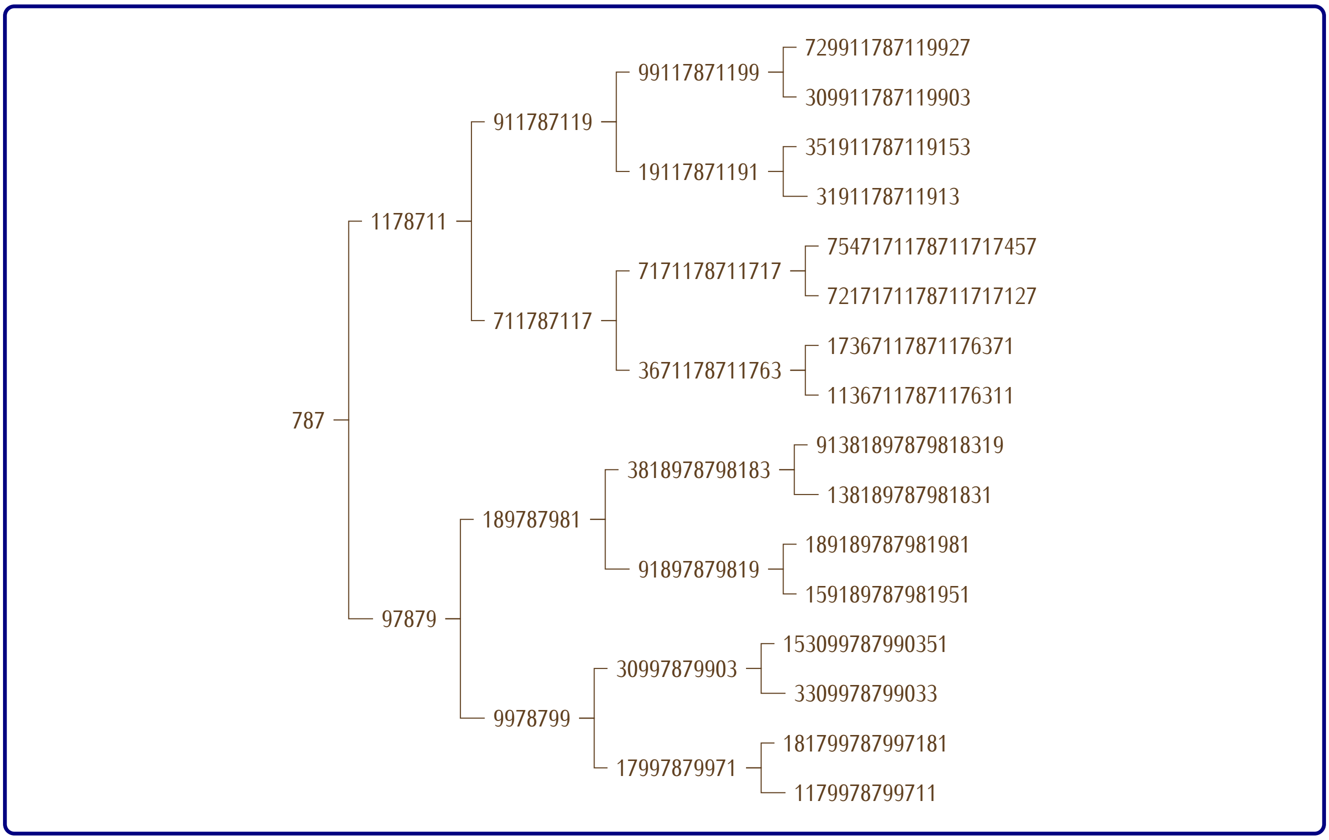
3.2.9 Embedded Tree with 727



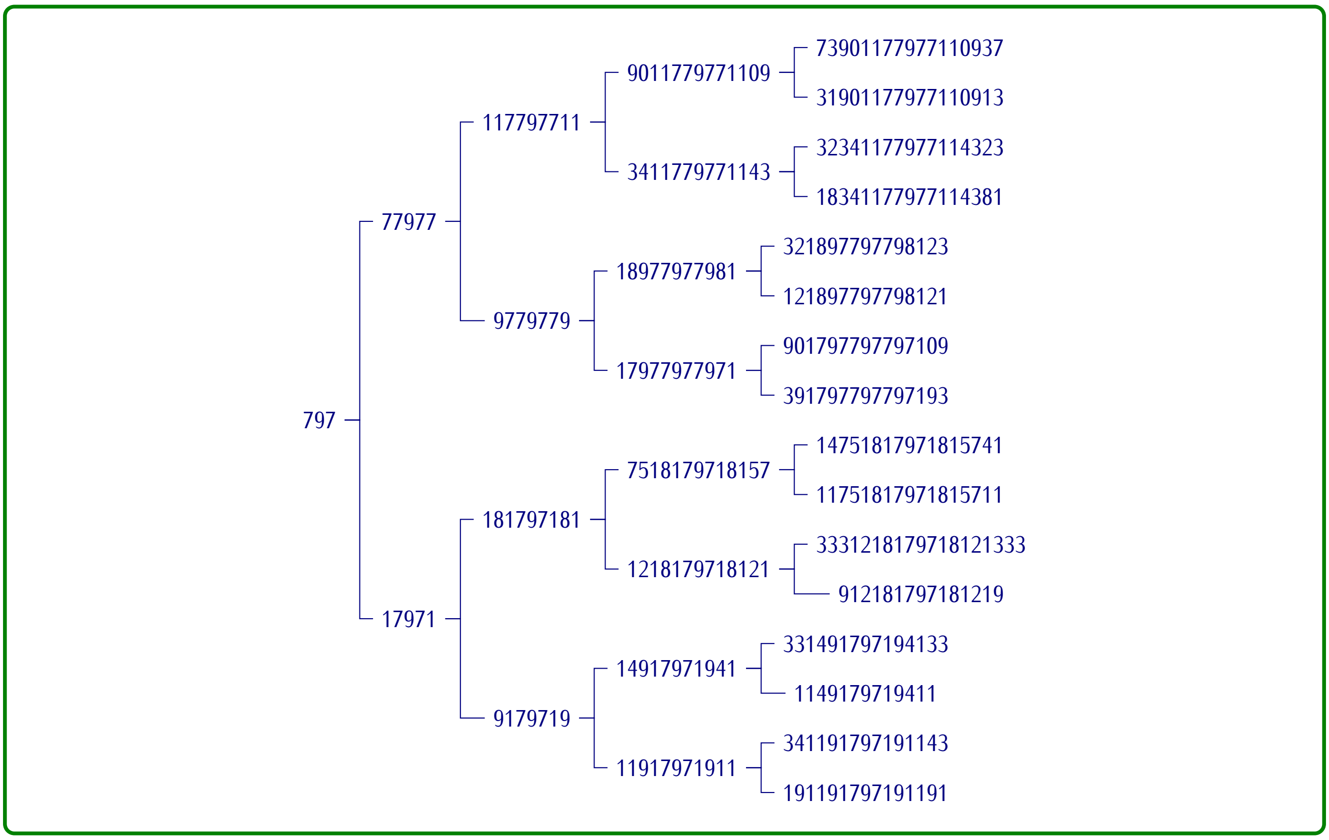
3.2.10 Embedded Tree with 757



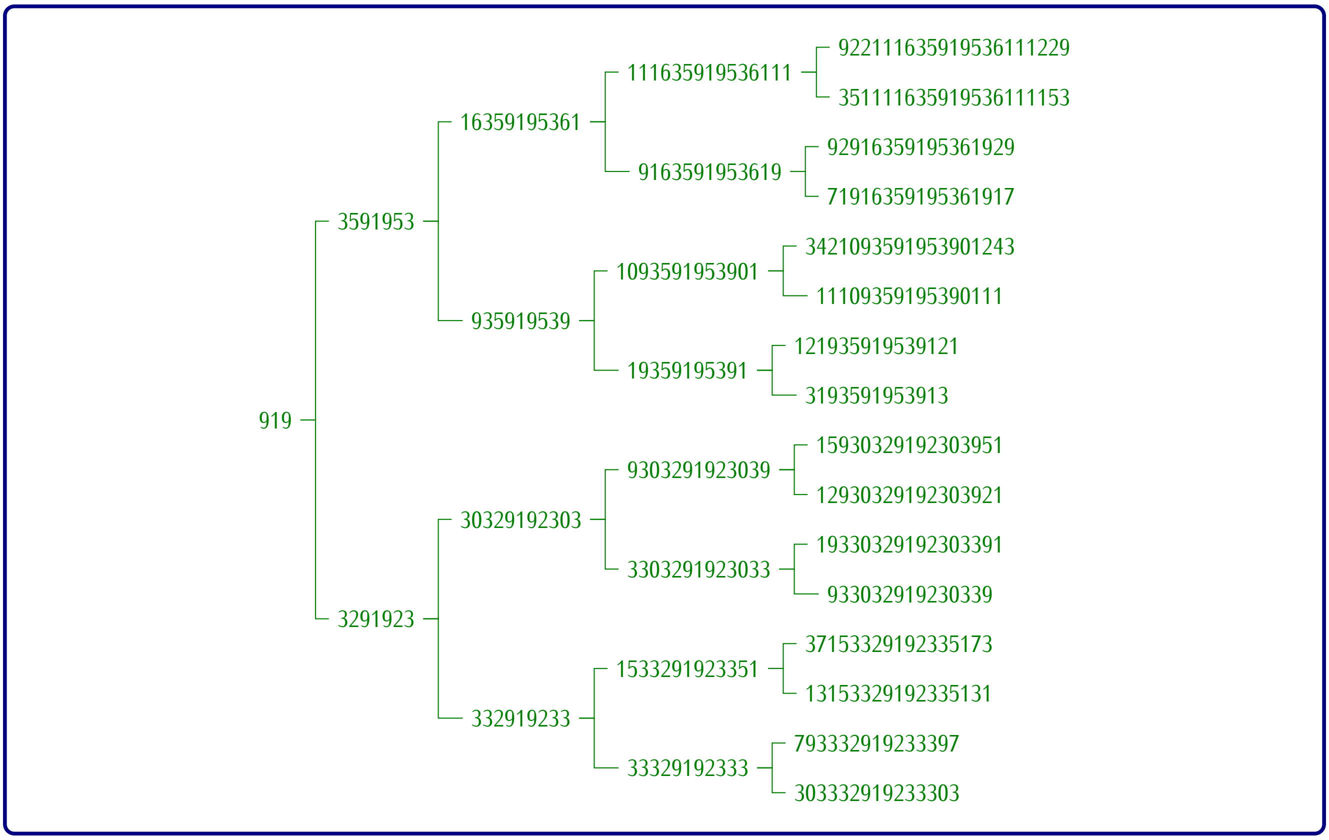
3.2.11 Embedded Tree with 787



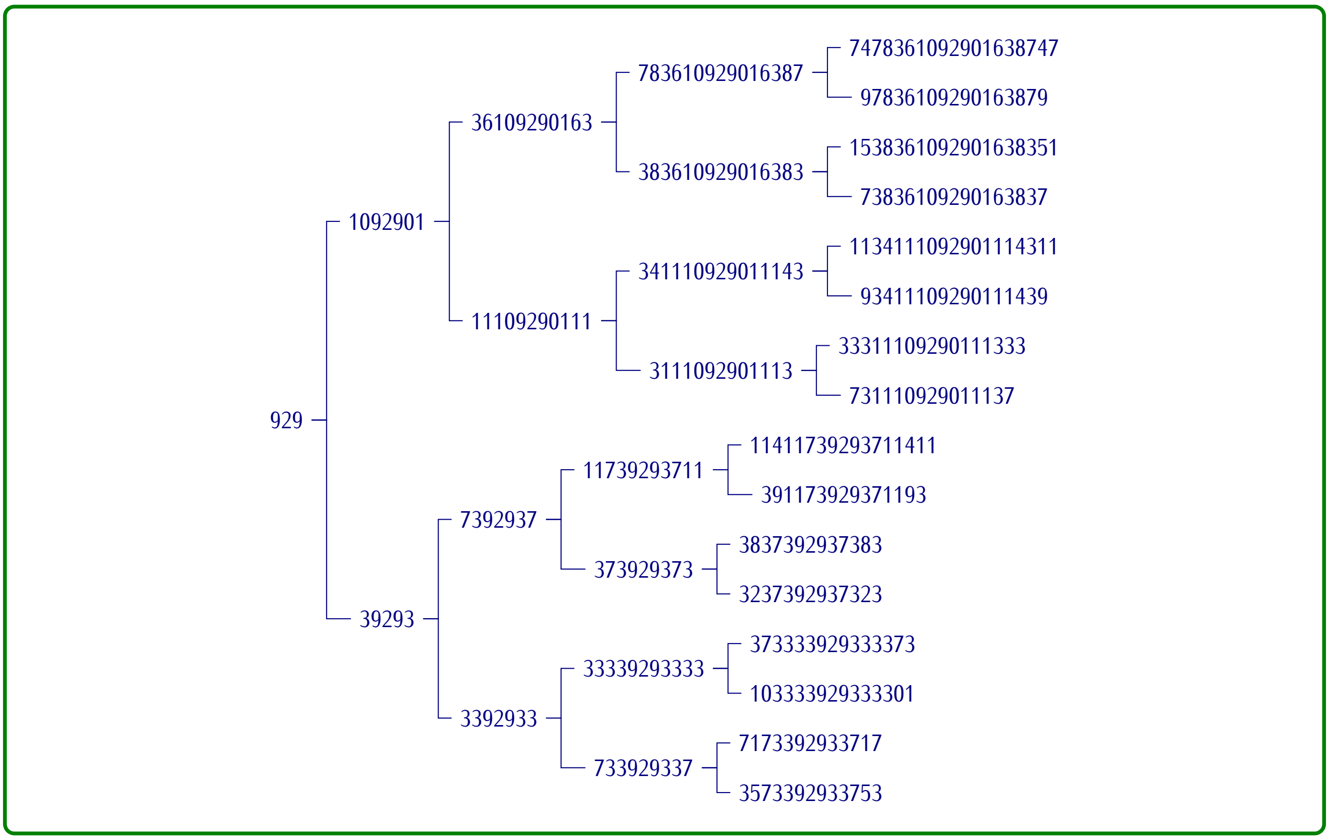
3.2.12 Embedded Tree with 797



3.2.13 Embedded Tree with 919



3.2.14 Embedded Tree with 929



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