Received 18/02/17

SOME GENERALIZED HERMITE-HADAMARD TYPE INEQUALITIES INVOLVING FRACTIONAL INTEGRAL OPERATOR FOR FUNCTIONS WHOSE SECOND DERIVATIVES IN ABSOLUTE VALUE ARE s-CONVEX

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Abstract. In this article, a general integral identity for twice differentiable mapping involving fractional integral operators is derived. As a second, by using this identity we obtained some new generalized Hermite-Hadamards type inequalities for functions whose absolute values of second derivatives are s-convex and concave. The main results generalize the existing Hermite-Hadamard type inequalities involving the Riemann-Liouville fractional integral. Also we pointed out, some results in this study in some special cases, such as setting $s = 1$, $\lambda = \alpha$, $\sigma(0) = 1$ and $w = 0$, more reasonable than those obtained in [10].

1. Introduction and Preliminaries

Let $f : I \subset \mathbb{R} \to \mathbb{R}$ be a convex function on an interval I in the set of real numbers $\mathbb{R}$. Then, for a, $b \in I$ with $a < b$, the following so-called Hermite-Hadamard inequality (see, e.g., [14])

$$f\left(\frac{a+b}{2}\right) \leq \frac{1}{b-a}\int_a^b f(x)\,dx \leq \frac{f(a)+f(b)}{2} \tag{1.1}$$

holds true. Since its discovery in 1983, Hermite-Hadamard's inequality has been considered the most useful inequality in mathematical analysis. A number of the papers have been written on this inequality providing new proofs, noteworthy extensions, generalizations and numerous applications, see and references therein [8, 9, 14, 17].

Two definitions of s-convexity ($0 < s \leq 1$) of real-valued functions are well known in the literature.

Definition 1.1. *Let $0 < s \leq 1$. A function $f : [0, \infty) \to \mathbb{R}$, is said to be s-Orlicz convex or s-convex in the first sense, if for every $x, y \in [0, \infty)$ and $\alpha, \beta \geq 0$ with $\alpha^s + \beta^s = 1$, we have:*

$$f(\alpha x + \beta y) \leq \alpha^s f(x) + \beta^s f(y). \tag{1.2}$$

We denote the set of all s-convex functions in the first sense by K_s^1. This definition of s-convexity was introduced by Orlicz in [16] and was used in the theory of Orlicz spaces. Then, s-convex function in the second sense was introduced in Breckner's paper [6] and a number of properties and connections with s-convexity in the first sense are discussed in paper [12].

Definition 1.2. *[6] A function $f : \mathbb{R}_+ \to \mathbb{R}$ is said to be s-convex in the second sense if*

$$f(\alpha x + \beta y) \leq \alpha^s f(x) + \beta^s f(y)$$

for all $x, y \in \mathbb{R}_+$ and all $\alpha, \beta \geq 0$ with $\alpha + \beta = 1$.

We denote this by K_s^2. It is obvious that the s-convexity means just the convexity when $s = 1$.

In [8] Dragomir and Fitzpatrick proved a variant of Hadamard's inequality which holds for s-convex functions in the second sense as follows:

2010 *Mathematics Subject Classification.* 26A33, 26D10, 26D15, 33B20.

Key words and phrases. Hermite-Hadamard inequality, convex function, Hölder inequality, Riemann-Liouville fractional integral, fractional integral operator.

RGMIA Res. Rep. Coll. 20 (2017), Art. 14, 13 pp.

Theorem 1.1. *Suppose that $f:[0,\infty)\to[0,\infty)$ is an s-convex function in the second sense, where $s\in(0,1]$ and let $a,b\in[0,\infty), a<b$. If $f\in L^1[a,b]$ then the following inequality hold:*

$$2^{s-1}f\left(\frac{a+b}{2}\right)\leq\frac{1}{b-a}\int_a^b f(x)\,dx\leq\frac{f(a)+f(b)}{s+1}. \tag{1.3}$$

The constant $k=\frac{1}{s+1}$ is the best possible in the second inequality in (1.3). For more study, see([4], [5], [8], [13]).

In the following, we will give some necessary definitions and preliminary results which are used and referred to throughout this paper.

Definition 1.3. *Let $f\in L_1[a,b]$. The Riemann-Liouville integrals $J_{a+}^{\alpha}f$ and $J_{b-}^{\alpha}f$ of order $\alpha>0$ with $a\geq 0$ are defined by*

$$J_{a+}^{\alpha}f(x)=\frac{1}{\Gamma(\alpha)}\int_a^x(x-t)^{\alpha-1}f(t)dt,\quad x>a$$

and

$$J_{b-}^{\alpha}f(x)=\frac{1}{\Gamma(\alpha)}\int_x^b(t-x)^{\alpha-1}f(t)dt,\quad x<b$$

respectively where $\Gamma(\alpha)=\int_0^{\infty}e^{-t}u^{\alpha-1}du$. Here is $J_{a+}^{0}f(x)=J_{b-}^{0}f(x)=f(x)$.

In the case of $\alpha=1$, the fractional integral reduces to the classical integral. Some recent results and properties concerning this operator can be found [7, 11, 18, 21–23, 26]. The beta function is defined as follows:

$$B(a,b)=\frac{\Gamma(a)\Gamma(b)}{\Gamma(a+b)}=\int_0^1 t^{a-1}(1-t)^{b-1}dt,\qquad a,b>0,$$

where $\Gamma(\alpha)$ is Gamma function. In [26], Sarıkaya et al. gave a remarkable integral inequalities of Hermite-Hadamard type involving Riemann-Liouville fractional integrals as follows:

Theorem 1.2. *Let $f:[a,b]\to\mathbb{R}$ be a positive function with $0\leq a<b$ and $f\in[a,b]$. If f is convex function on $[a,b]$, then the following inequality for fractional integrals hold:*

$$f\left(\frac{a+b}{2}\right)\leq\frac{\Gamma(\alpha+1)}{2(b-a)^{\alpha}}[(J_{a+}^{\alpha}f)(b)+(J_{b-}^{\alpha}f)(a)]\leq\frac{f(a)+f(b)}{2}. \tag{1.4}$$

It is obviously seen that, if we take $\alpha=1$ in Theorem 1.2, then the inequality (1.4) reduces to well known Hermite-Hadamard's inequality as (1.1).

Hermite Hadamard type inequality for s-convex functions on Riemann-Liouville fractional integral is given in [21] as follows:

Theorem 1.3. *Let $f:[a,b]\to\mathbb{R}$ be a positive function with $0\leq a<b$ and $f\in L_1[a,b]$. If f is s-convex mapping in the second sense on $[a,b]$, then the following inequality for fractional integral with $\alpha>0$ and $s\in(0,1]$ hold:*

$$\begin{aligned}2^{s-1}f\left(\frac{a+b}{2}\right)&\leq\frac{\Gamma(\alpha+1)}{2(b-a)^{\alpha}}[(J_{a+}^{\alpha}f)(b)+(J_{b-}^{\alpha}f)(a)]\\&\leq\alpha\Big[\frac{1}{\alpha+s}+B(\alpha,s+1)\Big]\frac{f(a)+f(b)}{2},\end{aligned} \tag{1.5}$$

where B(a,b) is beta function.

In [10] Dragomir et al proved the following identity and by using this identity they established new results involving Riemann-Liouville fractional integrals for twice differentiable convex mappings.

Lemma 1.1. *[10] Let $f : I \subset \mathbb{R} \to \mathbb{R}$ be a twice differentiable function on I°, the interior of I. Assume that $a, b \in I^\circ$ with $a < b$ and $f^{''} \in L[a,b]$, then the following identity for fractional integral with $\alpha > 0$ holds:*

$$\begin{aligned}
&\frac{f(a)+f(b)}{2} - \frac{\Gamma(\alpha+1)}{2(b-a)^\alpha}\left[J_{b-}^{\alpha} f(a) + J_{a+}^{\alpha} f(b)\right] \qquad (1.6)\\
=\ &\frac{(b-a)^2}{2(\alpha+1)} \int_0^1 t(1-t^\alpha)\left[f''(ta+(1-t)b) + f''(tb+(1-t)a)\right]dt.
\end{aligned}$$

In [24], Raina introduced a class of functions defined formally by

$$\mathcal{F}^{\sigma}_{\rho,\lambda}(x) = \mathcal{F}^{\sigma(0),\sigma(1),\ldots}_{\rho,\lambda}(x) = \sum_{k=0}^{\infty} \frac{\sigma(k)}{\Gamma(\rho k+\lambda)} x^k \quad (\rho,\lambda > 0; |x| < \mathbf{R}), \tag{1.7}$$

where the coefficients $\sigma(k)$ $(k \in \mathbb{N} = \mathbb{N} \cup \{0\})$ is a bounded sequence of positive real numbers and $\mathbf{R}$ is the set of real numbers. With the help of (1.7), Raina [24] and Agarwal *et al.* [3] defined the following left-sided and right-sided fractional integral operators respectively, as follows:

$$\left(\mathcal{J}^{\sigma}_{\rho,\lambda,a+;w}\varphi\right)(x) = \int_a^x (x-t)^{\lambda-1}\mathcal{F}^{\sigma}_{\rho,\lambda}[w(x-t)^\rho]\varphi(t)dt \qquad (x > a > 0), \tag{1.8}$$

$$\left(\mathcal{J}^{\sigma}_{\rho,\lambda,b-;w}\varphi\right)(x) = \int_x^b (t-x)^{\lambda-1}\mathcal{F}^{\sigma}_{\rho,\lambda}[w(t-x)^\rho]\varphi(t)dt \qquad (0 < x < b), \tag{1.9}$$

where $\lambda, \rho > 0$, $w \in \mathbb{R}$ and $\varphi(t)$ is such that the integral on the right side exits. In recently some new integral inequalities this operator involving have appeared in the literature (see, e.g., [1–3, 19, 20, 28]).

It is easy to verify that $\mathcal{J}^{\sigma}_{\rho,\lambda,a+;w}\varphi(x)$ and $\mathcal{J}^{\sigma}_{\rho,\lambda,b-;w}\varphi(x)$ are bounded integral operators on $L(a,b)$, if

$$\mathfrak{M} := \mathcal{F}^{\sigma}_{\rho,\lambda+1}[w(b-a)^\rho] < \infty. \tag{1.10}$$

In fact, for $\varphi \in L(a,b)$, we have

$$||\mathcal{J}^{\sigma}_{\rho,\lambda,a+;w}\varphi(x)||_1 \leq \mathfrak{M}(b-a)^\lambda ||\varphi||_1 \tag{1.11}$$

and

$$||\mathcal{J}^{\sigma}_{\rho,\lambda,b-;w}\varphi(x)||_1 \leq \mathfrak{M}(b-a)^\lambda ||\varphi||_1 \tag{1.12}$$

where

$$||\varphi||_p := \left(\int_a^b |\varphi(t)|^p dt\right)^{\frac{1}{p}}.$$

Here, many useful fractional integral operators can be obtained by specializing the coefficient $\sigma(k)$. For intance the classical Riemann-Liouville fractiona integrals J^α_{a+} and J^α_{b-} of order α follow easily by setting $\lambda = \alpha$, $\sigma(0) = 1$ and $w = 0$ in (1.8) and (1.9).

In [25] generalized Hermite-Hadamard's inequality for s-convex mapping involving fractional integral operators as follows;

Theorem 1.4. *Let $f:[a,b]\to\mathbb{R}$ be a function with $0\leq a<b$ and $f\in L_1[a,b]$. If f is an s-convex function on $[a,b]$ then we have the following inequalities for generalized fractional integral operators:*

$$\begin{aligned} 2^s f\left(\frac{a+b}{2}\right) &\leq \frac{1}{(b-a)^\lambda \mathcal{F}^{\sigma}_{\rho,\lambda}[w(b-a)^\rho]}\left[\left(\mathcal{J}^{\sigma}_{\rho,\lambda,b^-;w}f\right)(a)+\left(\mathcal{J}^{\sigma}_{\rho,\lambda,a^+;w}f\right)(b)\right] \\ &\leq \frac{1}{\mathcal{F}^{\sigma}_{\rho,\lambda+1}[w(b-a)^\rho]}\left[A_1(\lambda,s)+\mathcal{F}^{\sigma_{0,s}}_{\rho,\lambda}[w(b-a)^\rho]\right][f(a)+f(b)], \quad (1.13) \end{aligned}$$

where

$$\sigma_{0,s}(k)=\frac{\sigma(k)}{\lambda+\rho k+s},\ \ k=0,1,2...\ \textit{and}$$

$$A_1(\lambda,s)=\int_0^1 t^{\lambda-1}(1-t)^s\mathcal{F}^{\sigma}_{\rho,\lambda}[w(b-a)^\rho t^\rho]dt.$$

In this paper, first aim is to establish a new integral identity for a twice differentiable function via fractional integral operators. Using this new identity, we next present some Hermite-Hadamard type inequalities for functions whose second-order derivatives absolute values are s-convex and concave in the second sense.

2. Main Results

Lemma 2.1. *Let $f:[a,b]\to\mathbb{R}$ be a twice differentiable mapping on (a,b) with $a<b$ and $\lambda>0$. If $f''\in L[a,b]$, then the following equality for generalized fractional integrals holds:*

$$\begin{aligned} &\mathcal{F}^{\sigma}_{\rho,\lambda+1}[w(b-a)^\rho]\left(\frac{f(a)+f(b)}{2}\right)-\frac{1}{2(b-a)^\lambda}\left[\left(\mathcal{J}^{\sigma}_{\rho,\lambda,b^-;w}f\right)(a)+\left(\mathcal{J}^{\sigma}_{\rho,\lambda,a^+;w}f\right)(b)\right] \quad (2.1) \\ =\ &\frac{(b-a)^2}{2} \\ &\times\int_0^1 t\mathcal{F}^{\sigma}_{\rho,\lambda+2}[(b-a)^\rho]-t^{\lambda+1}\mathcal{F}^{\sigma}_{\rho,\lambda+2}[(b-a)^\rho t^\rho]\left[f''\left(ta+(1-t)b\right)+f''\left(f''\left((1-t)a+tb\right)\right)\right]dt. \end{aligned}$$

Proof. Integrating by parts and changing variables with $x=(ta+(1-t)b)$ we get,

$$\begin{aligned} I_1 &= \mathcal{F}^{\sigma}_{\rho,\lambda+2}[(b-a)^\rho]\int_0^1 tf''\left(ta+(1-t)b\right)dt \quad (2.2) \\ &= \mathcal{F}^{\sigma}_{\rho,\lambda+2}[(b-a)^\rho]\left\{\frac{1}{a-b}tf'\left(ta+(1-t)b\right)\Big|_0^1-\frac{1}{a-b}\int_0^1 f'\left(ta+(1-t)b\right)dt\right\} \\ &= \mathcal{F}^{\sigma}_{\rho,\lambda+2}[(b-a)^\rho]\left\{-\frac{f'(a)}{b-a}-\frac{f(a)-f(b)}{(b-a)^2}\right\}, \end{aligned}$$

by using same method

$$\begin{aligned} I_2 &= \mathcal{F}^{\sigma}_{\rho,\lambda+2}[(b-a)^\rho]\int_0^1 tf''\left((1-t)a+tb\right)dt \quad (2.3) \\ &= \mathcal{F}^{\sigma}_{\rho,\lambda+2}[(b-a)^\rho]\left\{\frac{f'(b)}{b-a}-\frac{f(b)-f(a)}{(b-a)^2}\right\}, \end{aligned}$$

analogously

$$\begin{aligned}
I_3 &= \int_0^1 t^{\lambda+1}\mathcal{F}^{\sigma}_{\rho,\lambda+2}[(b-a)^\rho t^\rho] f^{''}(ta+(1-t)b)\,dt \qquad (2.4)\\
&= t^{\lambda+1}\mathcal{F}^{\sigma}_{\rho,\lambda+2}[(b-a)^\rho t^\rho]\frac{f^{'}(ta+(1-t)b)}{a-b}\Big|_0^1 - \int_0^1 t^{\lambda}\mathcal{F}^{\sigma}_{\rho,\lambda+1}[(b-a)^\rho t^\rho]\frac{f^{'}(ta+(1-t)b)}{a-b}dt\\
&= \mathcal{F}^{\sigma}_{\rho,\lambda+2}[(b-a)^\rho]\frac{f^{'}(a)}{a-b} - \frac{1}{a-b}t^{\lambda}\mathcal{F}^{\sigma}_{\rho,\lambda+1}[(b-a)^\rho t^\rho]\frac{f(ta+(1-t)b)}{a-b}\Big|_0^1\\
&\quad +\frac{1}{a-b}\int_0^1 t^{\lambda-1}\mathcal{F}^{\sigma}_{\rho,\lambda}[(b-a)^\rho t^\rho]\frac{f(ta+(1-t)b)}{a-b}dt\\
&= \frac{\mathcal{F}^{\sigma}_{\rho,\lambda+2}[(b-a)^\rho]f^{'}(a)}{a-b} - \frac{\mathcal{F}^{\sigma}_{\rho,\lambda+1}[(b-a)^\rho]f(a)}{(b-a)^2}\\
&\quad +\frac{1}{(b-a)^2}\int_a^b\left(\frac{b-x}{b-a}\right)^{\lambda-1}\mathcal{F}^{\sigma}_{\rho,\lambda}\left[(b-a)^\rho\left(\frac{b-x}{b-a}\right)^\rho\right]\frac{f(x)}{b-a}dx\\
&= \frac{\mathcal{F}^{\sigma}_{\rho,\lambda+2}[(b-a)^\rho]f^{'}(a)}{a-b} - \frac{\mathcal{F}^{\sigma}_{\rho,\lambda+1}[(b-a)^\rho]f(a)}{(b-a)^2} + \frac{\left(\mathcal{J}^{\sigma}_{\rho,\lambda,a^+;w}f\right)(b)}{(b-a)^{\lambda+2}}
\end{aligned}$$

and

$$\begin{aligned}
I_4 &= \int_0^1 t^{\lambda+1}\mathcal{F}^{\sigma}_{\rho,\lambda+2}[(b-a)^\rho t^\rho] f^{''}((1-t)a+tb)\,dt \qquad (2.5)\\
&= \frac{\mathcal{F}^{\sigma}_{\rho,\lambda+2}[(b-a)^\rho]f^{'}(b)}{b-a} - \frac{\mathcal{F}^{\sigma}_{\rho,\lambda+1}[(b-a)^\rho]f(b)}{(b-a)^2} + \frac{\left(\mathcal{J}^{\sigma}_{\rho,\lambda,b^-;w}f\right)(a)}{(b-a)^{\lambda+2}}.
\end{aligned}$$

Thus combining (2.2), (2.3), (2.4) and (2.5) as $I_1+I_2-I_3-I_4$ and multiplying both sides of the obtained equality with $\frac{(b-a)^2}{2}$, which proof is completed. ☐

Remark 2.1. *Setting $\lambda=\alpha$, $\sigma(0)=1$ and $w=0$ in Lemma 2.1 found to yield the same identity as Lemma 1 in [10].*

Using this lemma, we can get the following results via fractional integral operator for twice differentiable function whose absolute value is s-convex and s-concave.

Theorem 2.1. *Let $f:[a,b]\to\mathbb{R}$ be a twice differentiable function on $[a,b]$ with $a<b$ and $\lambda>0$. If $|f''|$ is s-convex in the seconde sense on (a,b) then the following inequality for generalized fractional integral operators holds:*

$$\begin{aligned}
&\left|\mathcal{F}^{\sigma}_{\rho,\lambda+1}[w(b-a)^\rho]\left(\frac{f(a)+f(b)}{2}\right) - \frac{1}{2(b-a)^\lambda}\left[\left(\mathcal{J}^{\sigma}_{\rho,\lambda,b^-;w}f\right)(a)+\left(\mathcal{J}^{\sigma}_{\rho,\lambda,a^+;w}f\right)(b)\right]\right|\\
\leq\ &\frac{(b-a)^2}{2}\mathcal{F}^{\sigma_{1,s}}_{\rho,\lambda+2}[w(b-a)^\rho]\left[\left|f^{''}b)\right|+\left|f^{''}b)\right|\right]
\end{aligned}$$

where

$$\sigma_{1,s}(k)=\sigma(k)\left[\frac{(\lambda+\rho k)}{(2+s)(\lambda+\rho k+s+2)}+B(2,s+1)-B(\lambda+\rho k+2,s+1)\right],$$

$\rho,\lambda>0$, $w\in\mathbb{R}$, $s\in(0,1]$ and $B(..,..)$, is Euler beta function.

Proof. From Lemma 2.1 with properties of modulus, we get

$$\begin{aligned}
&\left|\mathcal{F}^{\sigma}_{\rho,\lambda+1}[w(b-a)^{\rho}]\left(\frac{f(a)+f(b)}{2}\right)-\frac{1}{2(b-a)^{\lambda}}\left[\left(\mathcal{J}^{\sigma}_{\rho,\lambda,b^{-};w}f\right)(a)+\left(\mathcal{J}^{\sigma}_{\rho,\lambda,a^{+};w}f\right)(b)\right]\right| \\
\leq\ & \frac{(b-a)^2}{2}\int_0^1 \left|t\mathcal{F}^{\sigma}_{\rho,\lambda+2}[(b-a)^{\rho}]-t^{\lambda+1}\mathcal{F}^{\sigma}_{\rho,\lambda+2}[(b-a)^{\rho}t^{\rho}]\right| \qquad (2.6)\\
&\left|\left[f^{''}(ta+(1-t)b)+f^{''}((1-t)a+tb)\right]\right|dt \\
\leq\ & \frac{(b-a)^2}{2}\sum_{k=0}^{\infty}\frac{\sigma(k)|w|^k(b-a)^{\rho k}}{\Gamma(\rho k+\lambda+2)}\int_0^1\left|t-t^{\lambda+\rho k+1}\right|\left|f^{''}(ta+(1-t)b)\right|dt \\
&+\frac{(b-a)^2}{2}\sum_{k=0}^{\infty}\frac{\sigma(k)|w|^k(b-a)^{\rho k}}{\Gamma(\rho k+\lambda+2)}\int_0^1\left|t-t^{\lambda+\rho k+1}\right|\left|f^{''}((1-t)a+tb)\right|dt.
\end{aligned}$$

Since $|f^{''}|$ is s-convex, we have

$$\begin{aligned}
&\int_0^1 (t-t^{\lambda+\rho k+1})\left|f^{''}(ta+(1-t)b)\right|dt+\int_0^1 (t-t^{\lambda+\rho k+1})\left|f^{''}((1-t)a+tb)\right|dt \qquad (2.7)\\
\leq\ & \left[\int_0^1 t^{1+s}(1-t^{\lambda+\rho k})dt+\int_0^1 t(1-t)^s(1-t^{\lambda+\rho k})dt\right]\left[\left|f^{''}a)\right|+\left|f^{''}b)\right|\right] \\
=\ & \left[\frac{(\lambda+\rho k)}{(2+s)(\lambda+\rho k+s+2)}+B(2,s+1)-B(\lambda+\rho k+2,s+1)\right]\left[\left|f^{''}a)\right|+\left|f^{''}b)\right|\right].
\end{aligned}$$

Thus combining the inequalities (2.6) with (2.7), the requested result is obtained. □

Corollary 2.1. *If we take $s=1$ in Theorem 2.1, we get the following inequality;*

$$\begin{aligned}
&\left|\mathcal{F}^{\sigma}_{\rho,\lambda+1}[w(b-a)^{\rho}]\frac{f(a)+f(b)}{2}-\frac{1}{2(b-a)^{\lambda}}\left[\left(\mathcal{J}^{\sigma}_{\rho,\lambda,b^{-};w}f\right)(a)+\left(\mathcal{J}^{\sigma}_{\rho,\lambda,a^{+};w}f\right)(b)\right]\right| \\
\leq\ & \frac{(b-a)^2}{2}\mathcal{F}^{\sigma_{1,1}}_{\rho,\lambda+2}[w(b-a)^{\rho}]\left[\left|f^{''}b)\right|+\left|f^{''}b)\right|\right],
\end{aligned}$$

where

$$\sigma_{1,1}(k)=\sigma(k)\left[\frac{(\lambda+\rho k)}{2(\lambda+\rho k+2)}\right],\ \ \rho,\lambda>0 \quad \textit{and} \ \ w\in\mathbb{R}.$$

Corollary 2.2. *If we take $\lambda=\alpha$, $\sigma(0)=1$ and $w=0$ in Corollary 2.1, we get the following inequality;*

$$\begin{aligned}
&\left|\frac{f(a)+f(b)}{2}-\frac{\Gamma(\alpha+1)}{2(b-a)^{\alpha}}\left[J^{\alpha}_{(b)^{-}}f(a)+J^{\alpha}_{(a)^{+}}f(b)\right]\right| \\
\leq\ & \frac{(b-a)^2\alpha}{4(\alpha+1)(\alpha+2)}\left[\left|f^{''}b)\right|+\left|f^{''}b)\right|\right],
\end{aligned}$$

which is more reasonable than the result obtained Theorem 2 in [10] under the same assumptions.

Remark 2.2. *Setting $s=1$, $\lambda=\alpha=1$, $\sigma(0)=1$ and $w=0$ in Theorem 2.1 found to yield the same result as Proposition 2 in [27].*

Theorem 2.2. *Let $f:[a,b]\to\mathbb{R}$ be a twice differentiable function on (a,b) with $a<b$. If $|f''|^q$ is s-convex in the second sense and $q>1$ with $\frac{1}{p}+\frac{1}{q}=1$, then the following inequality for generalized fractional integral operators holds:*

$$\begin{aligned}&\left|\mathcal{F}^{\sigma}_{\rho,\lambda+1}[w(b-a)^{\rho}]\frac{f(a)+f(b)}{2}-\frac{1}{2(b-a)^{\lambda}}\left[\left(\mathcal{J}^{\sigma}_{\rho,\lambda,b^-;w}f\right)(a)+\left(\mathcal{J}^{\sigma}_{\rho,\lambda,a^+;w}f\right)(b)\right]\right|\\ \leq\;&\frac{(b-a)^2}{2}\mathcal{F}^{\sigma_2}_{\rho,\lambda+2}[w(b-a)^{\rho}]\left[\frac{|f''(a)|^q+|f''(b)|^q}{s+1}\right]^{\frac{1}{q}}\end{aligned}$$

where

$$\sigma_2(k)=2\sigma(k)\left[\frac{1}{\lambda+\rho k}B\left(\frac{p+1}{\lambda+\rho k},p+1\right)\right]^{\frac{1}{p}},$$

$\rho,\lambda>0$, $w\in\mathbb{R}$, $s\in(0,1]$ and $B(..,..)$, is Euler beta function.

Proof. From Lemma 2.1 have,

$$\begin{aligned}&\left|\mathcal{F}^{\sigma}_{\rho,\lambda+1}[w(b-a)^{\rho}]\left(\frac{f(a)+f(b)}{2}\right)-\frac{1}{2(b-a)^{\lambda}}\left[\left(\mathcal{J}^{\sigma}_{\rho,\lambda,b^-;w}f\right)(a)+\left(\mathcal{J}^{\sigma}_{\rho,\lambda,a^+;w}f\right)(b)\right]\right|\\ \leq\;&\frac{(b-a)^2}{2}\int_0^1\left|t\mathcal{F}^{\sigma}_{\rho,\lambda+2}[(b-a)^{\rho}]-t^{\lambda+1}\mathcal{F}^{\sigma}_{\rho,\lambda+2}[(b-a)^{\rho}t^{\rho}]\right| \qquad (2.8)\\ &\left|\left[f''(ta+(1-t)b)+f''((1-t)a+tb)\right]\right|dt\\ \leq\;&\frac{(b-a)^2}{2}\sum_{k=0}^{\infty}\frac{\sigma(k)|w|^k(b-a)^{\rho k}}{\Gamma(\rho k+\lambda+2)}\int_0^1\left|t-t^{\lambda+\rho k+1}\right|\left|f''(ta+(1-t)b)\right|dt\\ &+\frac{(b-a)^2}{2}\sum_{k=0}^{\infty}\frac{\sigma(k)|w|^k(b-a)^{\rho k}}{\Gamma(\rho k+\lambda+2)}\int_0^1\left|t-t^{\lambda+\rho k+1}\right|\left|f''((1-t)a+tb)\right|dt.\end{aligned}$$

Using Hölder Inequality and the s-convexity of $|f''|^q$ we get the following inequality;

$$\begin{aligned}&\int_0^1 t(1-t^{\lambda+\rho k})\left[\left|f''(ta+(1-t)b)\right|+\left|f''((1-t)a+tb)\right|\right]dt \qquad (2.9)\\ \leq\;&\left[\int_0^1\left(t(1-t^{\lambda+\rho k})\right)^p dt\right]^{\frac{1}{p}}\left\{\left[\int_0^1\left|f''(ta+(1-t)b)\right|^q dt\right]^{\frac{1}{q}}+\left[\int_0^1\left|f''((1-t)a+tb)\right|^q dt\right]^{\frac{1}{q}}\right\}\\ \leq\;&\left[\int_0^1 t^p(1-t^{(\lambda+\rho k)})^p dt\right]^{\frac{1}{p}}\left(\frac{1}{s+1}\right)^{\frac{1}{q}}\left[|f''(a)|^q+|f''(b)|^q\right]^{\frac{1}{q}}\\ =\;&2\left[\frac{1}{\lambda+\rho k}B\left(\frac{p+1}{\lambda+\rho k},p+1\right)\right]^{\frac{1}{p}}\left(\frac{1}{s+1}\right)^{\frac{1}{q}}\left[|f''(a)|^q+|f''(b)|^q\right]^{\frac{1}{q}}.\end{aligned}$$

By changing $x=t^{\lambda+\rho k}$ and a simple calculation we get,

$$\int_0^1 t^p(1-t^{(\lambda+\rho k)})^p dt=\frac{1}{\lambda+\rho k}B\left(\frac{p+1}{\lambda+\rho k},p+1\right).$$

Thus combining (2.8) with (2.9), the desired result is obtained. □

Remark 2.3. *Setting $\lambda=\alpha=1$, $\sigma(0)=1$ and $w=0$ in Theorem 2.2 found to yield the same result as Theorem 10 in [13].*

Corollary 2.3. *Taking $s=1$ in Theorem 2.2, the following inequality holds;*

$$\begin{aligned}
&\left|\mathcal{F}^{\sigma}_{\rho,\lambda+1}[w(b-a)^{\rho}]\frac{f(a)+f(b)}{2}-\frac{1}{2(b-a)^{\lambda}}\left[\left(\mathcal{J}^{\sigma}_{\rho,\lambda,b^-;w}f\right)(a)+\left(\mathcal{J}^{\sigma}_{\rho,\lambda,a^+;w}f\right)(b)\right]\right| \\
\leq\;& \frac{(b-a)^2}{2}\mathcal{F}^{\sigma_2}_{\rho,\lambda+2}[w(b-a)^{\rho}]\left[\frac{|f''(a)|^q+|f''(b)|^q}{2}\right]^{\frac{1}{q}}
\end{aligned}$$

where

$$\sigma_2(k)=2\sigma(k)\left[\frac{1}{(\lambda+\rho k)}B\left(\frac{p+1}{\lambda+\rho k},p+1\right)\right]^{\frac{1}{p}},$$

$\rho,\lambda>0$, $w\in\mathbb{R}$ and $B(..,..)$ is Euler Gamma function.

Corollary 2.4. *Taking $\lambda=\alpha$, $\sigma(0)=1$ and $w=0$ in Corollary 2.3, the following inequality holds;*

$$\begin{aligned}
&\left|\frac{f(a)+f(b)}{2}-\frac{\Gamma(\alpha+1)}{2(b-a)^{\alpha}}\left[J^{\alpha}_{(b)^-}f(a)+J^{\alpha}_{(a)^+}f(b)\right]\right| \\
\leq\;& \frac{(b-a)^2}{(\alpha+1)}\left[\frac{1}{\alpha}B\left(\frac{p+1}{\alpha},p+1\right)\right]^{\frac{1}{p}}\left[\frac{|f''(a)|^q+|f''(b)|^q}{2}\right]^{\frac{1}{q}},
\end{aligned}$$

which is more reasonable than obtained Theorem 3 in [10] under the same assumptions.

Theorem 2.3. *Let $f:[a,b]\to\mathbb{R}$ be a twice differentiable function on (a,b) with $a<b$. If $|f''|^q$ is s-convex in the second sense and $q\geq 1$, then the following inequality for generalized fractional integral operators holds:*

$$\begin{aligned}
&\left|\mathcal{F}^{\sigma}_{\rho,\lambda+1}[w(b-a)^{\rho}]\frac{f(a)+f(b)}{2}-\frac{1}{2(b-a)^{\lambda}}\left[\left(\mathcal{J}^{\sigma}_{\rho,\lambda,b^-;w}f\right)(a)+\left(\mathcal{J}^{\sigma}_{\rho,\lambda,a^+;w}f\right)(b)\right]\right| \\
\leq\;& \frac{(b-a)^2}{2}\mathcal{F}^{\sigma_{3,s}}_{\rho,\lambda+2}[w(b-a)^{\rho}]
\end{aligned}$$

where

$$\begin{aligned}
\sigma_{3,s}(k) &= \sigma(k)\left[\frac{\lambda+\rho k}{2(\lambda+\rho k+2)}\right]^{1-\frac{1}{q}} \\
&\times\left\{\left[\frac{\lambda+\rho k}{(s+2)(\lambda+\rho k+s+2)}|f''(a)|^q+\left(B(2,s+1)-B(\lambda+\rho k+2,s+1)\right)|f''(b)|^q\right]^{\frac{1}{q}}\right. \\
&\left.+\left[\left(B(2,s+1)-B(\lambda+\rho k+2,s+1)\right)|f''(a)|^q+\frac{\lambda+\rho k}{(s+2)(\lambda+\rho k+s+2)}|f''(b)|^q\right]^{\frac{1}{q}}\right\},
\end{aligned}$$

$\rho,\lambda>0$, $w\in\mathbb{R}$, $s\in(0,1]$ and $B(..,..)$, is Euler beta function.

Proof. From Lemma 2.1 with properties of modulus we get,

$$\begin{aligned}
&\left|\mathcal{F}^{\sigma}_{\rho,\lambda+1}[w(b-a)^{\rho}]\left(\frac{f(a)+f(b)}{2}\right)-\frac{1}{2(b-a)^{\lambda}}\left[\left(\mathcal{J}^{\sigma}_{\rho,\lambda,b^-;w}f\right)(a)+\left(\mathcal{J}^{\sigma}_{\rho,\lambda,a^+;w}f\right)(b)\right]\right| \\
\leq\ & \frac{(b-a)^2}{2}\int_0^1\left|t\mathcal{F}^{\sigma}_{\rho,\lambda+2}[(b-a)^{\rho}]-t^{\lambda+1}\mathcal{F}^{\sigma}_{\rho,\lambda+2}[(b-a)^{\rho}t^{\rho}]\right| \qquad (2.10)\\
&\left|\left[f^{''}(ta+(1-t)b)+f^{''}((1-t)a+tb)\right]\right|dt \\
\leq\ & \frac{(b-a)^2}{2}\sum_{k=0}^{\infty}\frac{\sigma(k)|w|^k(b-a)^{\rho k}}{\Gamma(\rho k+\lambda+2)}\int_0^1\left|t-t^{\lambda+\rho k+1}\right|\left|f^{''}(ta+(1-t)b)\right|dt \\
&+\frac{(b-a)^2}{2}\sum_{k=0}^{\infty}\frac{\sigma(k)|w|^k(b-a)^{\rho k}}{\Gamma(\rho k+\lambda+2)}\int_0^1\left|t-t^{\lambda+\rho k+1}\right|\left|f^{''}((1-t)a+tb)\right|dt.
\end{aligned}$$

Using Power-mean Inequality and s-convexity of $|f^{''}|^q$, we obtain the following inequality;

$$\begin{aligned}
&\int_0^1 t(1-t^{\lambda+\rho k})\left[\left|f^{''}(ta+(1-t)b)\right|+\left|f^{''}((1-t)a+tb)\right|\right]dt \qquad (2.11)\\
\leq\ & \left[\int_0^1(t-t^{\lambda+\rho k+1})dt\right]^{1-\frac{1}{q}}\left[\int_0^1(t-t^{\lambda+\rho k+1})\left|f^{''}(ta+(1-t)b)\right|^q dt\right]^{\frac{1}{q}} \\
&+\left[\int_0^1(t-t^{\lambda+\rho k+1})dt\right]^{1-\frac{1}{q}}\left[\int_0^1(t-t^{\lambda+\rho k+1})\left|f^{''}((1-t)a+tb)\right|^q dt\right]^{\frac{1}{q}} \\
\leq\ & \left[\frac{\lambda+\rho k}{2(\lambda+\rho k+2)}\right]^{1-\frac{1}{q}} \\
&\times\left\{\left[\frac{\lambda+\rho k}{(s+2)(\lambda+\rho k+s+2)}|f^{''}(a)|^q+\left(B(2,s+1)-B(\lambda+\rho k+2,s+1)\right)|f^{''}(b)|^q\right]^{\frac{1}{q}}\right. \\
&\left.+\left[\left(B(2,s+1)-B(\lambda+\rho k+2,s+1)\right)|f^{''}(a)|^q+\frac{\lambda+\rho k}{(s+2)(\lambda+\rho k+s+2)}|f^{''}(b)|^q\right]^{\frac{1}{q}}\right\}.
\end{aligned}$$

Combining the inequalities (2.10) with (2.11) we have

$$\begin{aligned}
&\left|\mathcal{F}^{\sigma}_{\rho,\lambda+1}[w(b-a)^{\rho}]\frac{f(a)+f(b)}{2}-\frac{1}{2(b-a)^{\lambda}}\left[\left(\mathcal{J}^{\sigma}_{\rho,\lambda,b^-;w}f\right)(a)+\left(\mathcal{J}^{\sigma}_{\rho,\lambda,a^+;w}f\right)(b)\right]\right| \\
\leq\ & \frac{(b-a)^2}{2}\sum_{k=0}^{\infty}\frac{\sigma(k)|w|^k(b-a)^{\rho k}}{\Gamma(\rho k+\lambda+2)}\left[\frac{\lambda+\rho k}{2(\lambda+\rho k+2)}\right]^{1-\frac{1}{q}} \\
&\times\left\{\left[\frac{\lambda+\rho k}{(s+2)(\lambda+\rho k+s+2)}|f^{''}(a)|^q+\left(B(2,s+1)-B(\lambda+\rho k+2,s+1)\right)|f^{''}(b)|^q\right]^{\frac{1}{q}}\right. \\
&\left.+\left[\left(B(2,s+1)-B(\lambda+\rho k+2,s+1)\right)|f^{''}(a)|^q+\frac{\lambda+\rho k}{(s+2)(\lambda+\rho k+s+2)}|f^{''}(b)|^q\right]^{\frac{1}{q}}\right\} \\
=\ & \frac{(b-a)^2}{2}\mathcal{F}^{\sigma_{3,s}}_{\rho,\lambda+2}[w(b-a)^{\rho}].
\end{aligned}$$

Thus the proof is completed. □

Corollary 2.5. *Taking $s=1$ with $\rho,\lambda>0$ and $w\in\mathbb{R}$ in Theorem 2.3, the following inequality holds;*

$$
\begin{aligned}
&\left|\mathcal{F}^{\sigma}_{\rho,\lambda+1}[w(b-a)^{\rho}]\frac{f(a)+f(b)}{2}-\frac{1}{2(b-a)^{\lambda}}\left[\left(\mathcal{J}^{\sigma}_{\rho,\lambda,b^{-};w}f\right)(a)+\left(\mathcal{J}^{\sigma}_{\rho,\lambda,a^{+};w}f\right)(b)\right]\right|\\
\leq\ &\frac{(b-a)^2}{2}\mathcal{F}^{\sigma_{3,1}}_{\rho,\lambda+2}[w(b-a)^{\rho}]
\end{aligned}
$$

where

$$
\begin{aligned}
\sigma_{3,1}(k) &= \sigma(k)\left[\frac{\lambda+\rho k}{2(\lambda+\rho k+2)}\right]^{1-\frac{1}{q}}\\
&\times\left\{\left[\frac{\lambda+\rho k}{3(\lambda+\rho k+3)}|f^{''}(a)|^q+\frac{(\lambda+\rho k)(\lambda+\rho k+5)}{6(\lambda+\rho k+2)(\lambda+\rho k+3)}|f^{''}(b)|^q\right]^{\frac{1}{q}}\right.\\
&\left.+\left[\frac{(\lambda+\rho k)(\lambda+\rho k+5)}{6(\lambda+\rho k+2)(\lambda+\rho k+3)}|f^{''}(a)|^q+\frac{\lambda+\rho k}{3(\lambda+\rho k+3)}|f^{''}(b)|^q\right]^{\frac{1}{q}}\right\}
\end{aligned}
$$

Remark 2.4. *Setting $\lambda=\alpha=1$, $\sigma(0)=1$ and $w=0$ in Theorem 2.3 founds to yield the same result as Theorem 8 in [13]*

Remark 2.5. *Setting $s=1$, $\lambda=\alpha$, $\sigma(0)=1$ and $w=0$ in Theorem 2.3 founds to yield the same result as Theorem 4 in [10]*

Theorem 2.4. *Let $f:[a,b]\to\mathbb{R}$ be a twice differentiable function on (a,b) with $a<b$. If $|f^{''}|^q$ is s-concave in the second sense and $q>1$ with $\frac{1}{p}+\frac{1}{q}=1$, then the following inequality for generalized fractional integral operators holds:*

$$
\begin{aligned}
&\left|\mathcal{F}^{\sigma}_{\rho,\lambda+1}[w(b-a)^{\rho}]\frac{f(a)+f(b)}{2}-\frac{1}{2(b-a)^{\lambda}}\left[\left(\mathcal{J}^{\sigma}_{\rho,\lambda,b^{-};w}f\right)(a)+\left(\mathcal{J}^{\sigma}_{\rho,\lambda,a^{+};w}f\right)(b)\right]\right|\\
\leq\ &\frac{(b-a)^2}{2}\mathcal{F}^{\sigma_{4,s}}_{\rho,\lambda+2}[w(b-a)^{\rho}]|f^{''}(\frac{a+b}{2})|
\end{aligned}
$$

where $\sigma_{4,s}(k)=\sigma(k)2^{\frac{s}{q}}\left[\frac{2}{\lambda+\rho k}B\left(\frac{p+1}{\lambda+\rho k},p+1\right)\right]^{\frac{1}{p}}$, $\rho,\lambda>0$, $s\in(0,1]$, *and* $w\in\mathbb{R}$, *Proof.* From Lemma 2.1 and Hölder inequality with properties of modulus, we have

$$\begin{aligned}
&\left|\mathcal{F}^{\sigma}_{\rho,\lambda+1}[w(b-a)^{\rho}]\frac{f(a)+f(b)}{2}-\frac{1}{2(b-a)^{\lambda}}\left[\left(\mathcal{J}^{\sigma}_{\rho,\lambda,b^-;w}f\right)(a)+\left(\mathcal{J}^{\sigma}_{\rho,\lambda,a^+;w}f\right)(b)\right]\right|\\
\leq\;& \frac{(b-a)^2}{2}\int_0^1\left|t\mathcal{F}^{\sigma}_{\rho,\lambda+2}[(b-a)^{\rho}]-t^{\lambda+1}\mathcal{F}^{\sigma}_{\rho,\lambda+2}[(b-a)^{\rho}t^{\rho}]\right| \qquad (2.12)\\
&\left|\left[f^{''}(ta+(1-t)b)+f^{''}((1-t)a+tb)\right]\right|dt\\
\leq\;& \frac{(b-a)^2}{2}\sum_{k=0}^{\infty}\frac{\sigma(k)|w|^k(b-a)^{\rho k}}{\Gamma(\rho k+\lambda+2)}\\
&\times\int_0^1\left|t-t^{\lambda+\rho k+1}\right|\left[\left|f^{''}(ta+(1-t)b)\right|+\left|f^{''}((1-t)a+tb)\right|\right]dt\\
\leq\;& \frac{(b-a)^2}{2}\sum_{k=0}^{\infty}\frac{\sigma(k)|w|^k(b-a)^{\rho k}}{\Gamma(\rho k+\lambda+2)}\\
&\times\left[\int_0^1(t-t^{\lambda+\rho k+1})^p dt\right]^{\frac{1}{p}}\left\{\left[\int_0^1\left|f^{''}(ta+(1-t)b)\right|^q dt\right]^{\frac{1}{q}}+\left[\int_0^1\left|f^{''}((1-t)a+tb)\right|^q dt\right]^{\frac{1}{q}}\right\}.
\end{aligned}$$

Since $|f^{''}|^q$ is s-concave, we can write

$$\begin{aligned}
\int_0^1\left|f^{''}((1-t)a+tb)\right|^q dt &\leq 2^{s-1}|f^{''}(\frac{a+b}{2})|^q \qquad (2.13)\\
\textit{and}\quad \int_0^1\left|f^{''}(ta+(1-t)b)\right|^q dt &\leq 2^{s-1}|f^{''}(\frac{a+b}{2})|^q,
\end{aligned}$$

On the other hand, by simple calculating we establish,

$$\int_0^1(t-t^{\lambda+\rho k+1})^p dt = \frac{1}{\lambda+\rho k}B\left(\frac{p+1}{\lambda+\rho k},p+1\right). \qquad (2.14)$$

Thus combining (2.13) and (2.14) with (2.12) the requested result is obtained. □

Corollary 2.6. *Taking $s=1$ in Theorem 2.4, the following inequality holds;*

$$\begin{aligned}
&\left|\mathcal{F}^{\sigma}_{\rho,\lambda+1}[w(b-a)^{\rho}]\frac{f(a)+f(b)}{2}-\frac{1}{2(b-a)^{\lambda}}\left[\left(\mathcal{J}^{\sigma}_{\rho,\lambda,b^-;w}f\right)(a)+\left(\mathcal{J}^{\sigma}_{\rho,\lambda,a^+;w}f\right)(b)\right]\right|\\
\leq\;& \frac{(b-a)^2}{2}\mathcal{F}^{\sigma_{4,1}}_{\rho,\lambda+2}[w(b-a)^{\rho}]|f^{''}(\frac{a+b}{2})|
\end{aligned}$$

where $\rho,\lambda>0$ and $w\in\mathbb{R}$,

$$\sigma_{4,1}(k)=\sigma(k)2^{\frac{1}{q}}\left[\frac{2}{\lambda+\rho k}B\left(\frac{p+1}{\lambda+\rho k},p+1\right)\right]^{\frac{1}{p}}.$$

Corollary 2.7. *If we take $\lambda=\alpha$, $\sigma(0)=1$ and $w=0$ in Corollary 2.6, the following inequality holds;*

$$\begin{aligned}
&\left|\frac{f(a)+f(b)}{2}-\frac{\Gamma(\alpha+1)}{2(b-a)^{\lambda}}\left[J^{\alpha}_{(b)^-}f(a)+J^{\alpha}_{(a)^+}f(b)\right]\right|\\
\leq\;& \frac{(b-a)^2}{(\alpha+1)}\left[\frac{1}{\alpha}B\left(\frac{p+1}{\alpha},p+1\right)\right]^{\frac{1}{p}}|f^{''}(\frac{a+b}{2})|,
\end{aligned}$$

which is more reasonable than Theorem 5 in [10] under the same assumptions.

Remark 2.6. *Setting $s = 1$, $\lambda = \alpha = 1$, $\sigma(0) = 1$ and $w = 0$ in Theorem 2.3 found to yield the same result as Theorem 9 in [13].*

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