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Block-Wise Equal Sums Pan Magic Squares of Order $4k$

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Abstract

This paper brings block-wise pan magic squares multiple of 4. The representations are in such a way that each block of order 4 is a perfect magic square of order 4 with equal sums. The work is written for the magic square of order 4, 8, 12, ...,32. Applying the same process the results can be extended for further orders. The way these magic squares are constructed, just the knowledge of order 4 magic square is sufficient

An equation means nothing to me unless it expresses a thought of God.
– S. Ramanujan

On a special day:
September 29, 17 (1729: Hardy-Ramanujan Number)

Contents

1 Introduction

Magic squares are know in the literature for a long time. Lot of work has been done in this direction. There are lot of sites on internet bring about magic squares. This paper work with magic squares of order $4k$, $k \geq 2$. A systematic way is created in such a way that magic squares are represented block-wise, with each $4\times$ block a pan magic square of order 4 with same sum. Before let's consider a perfect pan magic square of order 4. This will serve a guide to construct other order magic squares.

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Example 1.1. *Let us consider a pan magic square of order 4.*

		34	34	34	34
	7	12	1	14	34
34	2	13	8	11	34
34	16	3	10	5	34
34	9	6	15	4	34
	34	34	34	34	34

It is one of the most **perfect pan magic square of order 4**. Below are some properties in colors resulting magic square sum for each color:

7	12	1	14
2	13	8	11
16	3	10	5
9	6	15	4

7	12	1	14
2	13	8	11
16	3	10	5
9	6	15	4

7	12	1	14
2	13	8	11
16	3	10	5
9	6	15	4

7	12	1	14
2	13	8	11
16	3	10	5
9	6	15	4

7	12	1	14
2	13	8	11
16	3	10	5
9	6	15	4

7	12	1	14
2	13	8	11
16	3	10	5
9	6	15	4

7	12	1	14
2	13	8	11
16	3	10	5
9	6	15	4

7	12	1	14
2	13	8	11
16	3	10	5
9	6	15	4

It is also known by **compact magic square**. More studies can be seen at Taneja [10].

The aim of this paper is to bring **block-wise pan magic square multiple of 4**, i.e., $4k$, $k \geq 2$, such as, of order 8, 12, 16, ... etc. It is done in such way that sum of each 4×4 block is of same sum, and the resulting magic square is pan diagonal.

1.1 History and Dedication

The magic square given in Example 1.1 is one of the most **perfect magic square** of order 4 studied around 10th century. It is famous as **Khajuraho magic square**. Below is original plate of above magic square seen at **Parshvanath Jain temple in Khajuraho** – (*Link: Wikipedia – https://goo.gl/nsYn2j*):

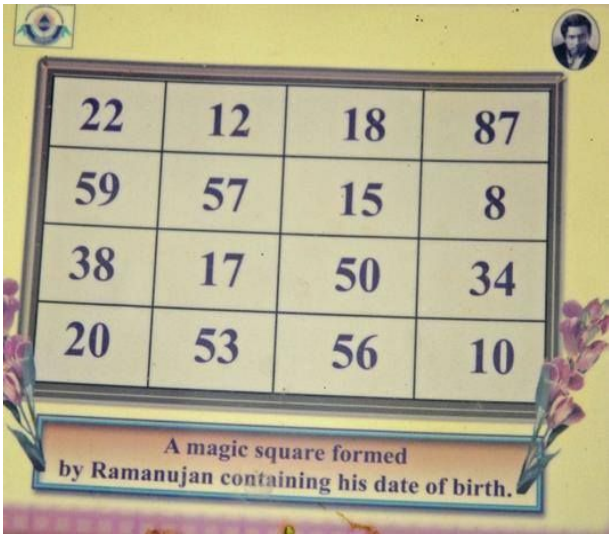
७	१२	१	१४
२	१३	८	११
१६	३	१०	५
९	६	१५	४

7	12	1	14
2	13	8	11
16	3	10	5
9	6	15	4

transcription of the indian numerals

Most-perfect magic square from the Parshvanath Jain temple in Khajuraho

Ramanujan (1887-1920) constructed a magic square of order 4 containing his date of birth. See below:



https://br.pinterest.com/AbitExtraApps/magic-square-word-and-number-puzzles/?lp=true

The first row of above magic square is formed by his date of birth, i.e., 22.12.1887. Below is a **perfect pan magic square** of order 4 constructed by author [21] containing more Ramanujan's details:

Date of Birth : 22.12.1887

Date of Death : 26.04.1920

Hardy-Ramanujan Number : 1729.

		8432	*8432*	*8432*	*8432*
	2212	**1887**	**1729**	**2604**	*8432*
8432	1696	2637	2179	**1920**	*8432*
8432	2487	1612	2004	2329	*8432*
8432	2037	2296	2520	1579	*8432*
	8432	*8432*	*8432*	*8432*	*8432*

Let's consider the numbers 8432 in two parts as 84 and 32. The number 84 is 84-gonal 7th value is 1729, i.e., $P_{84}(7) := 1729$ and 32 the age Ramanujan died. According to Hindu philosophy *"The number 84 is a "whole" or "perfect" number. Thus the eighty-four siddhas can be seen as archetypes representing the thousands of exemplars and adept of the tantric way"*. The idea of S-gonal numbers refer author's work [22].

http://keithdowman.net/essays/introduction-mahasiddhas-and-tantra.html

The number "1729" appearing above is a famous **Hardy-Ramamujan TaxiCab number**. This year this number appear 11 times as date 29, 17, i.e., January to December except February. The same number will appear again in 2029 twelve times with same order as 17-29. For some studies on this number refer to author's work [21].

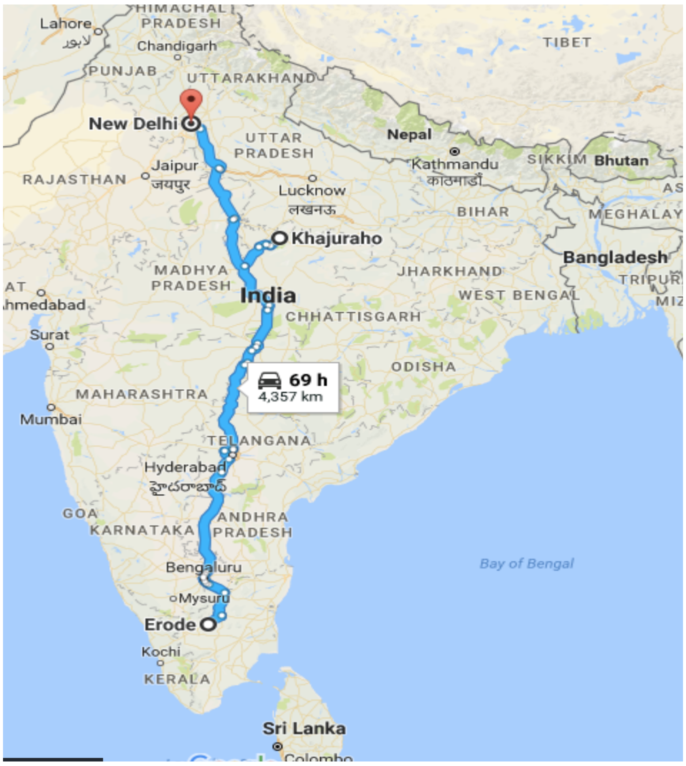
This work connects three places:

(i) **New Delhi** – Author's birth place;

(ii) **Khajuraho** – Place of perfect magic square of order 4;

(iii) **Erode** – Birth place of Ramanujan.

See these three cities almost in a row using Google Maps:



This work is dedicated to **Ramanujan's contribution to Mathematics** specially on a day September 29, 17 (17-29). Since, he died at the age of 32, we shall work up to order 32 perfect pan magic squares with block-wise equal sums of order 4 magic squares.

2 Pan Magic Squares of Order $4k$

This section brings **block-wise pan magic squares** of order $4k$, $k \geq 2$. The construction is in such a way that each 4×4 block is of same sum and a perfect magic square of order 4 as given in Example 1.1. The work is done for orders 8 to 32. We shall use the following **general way** to distribute the entries to bring equal sum pan magic squares.

Procedure 2.1. *Let's distribute the total entries of a magic square of type $4k$ according to following table:*

	C1	C2	C3	C46	C5	C6	C7	C8	C9	C10	C11	C12	C13	C14	C15	C16	Sum
A1	↓	↑	↓	↑	↓	↑	↓	↑	↓	↑	↓	↑	↓	↑	↓	↑	T
A2	↓	↑	↓	↑	↓	↑	↓	↑	↓	↑	↓	↑	↓	↑	↓	↑	T
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
An	↓	↑	↓	↑	↓	↑	↓	↑	↓	↑	↓	↑	↓	↑	↓	↑	T

where $n = p^2/16$, p order of a magic square of type $4k$, $k \geq 2$. T is the sum of the elements of each row. The symbol ↓ represents column-wise numbers in increasing order, i.e., [1, 2, 3, 4], and the symbol ↑ represents column-wise numbers in decreasing order, such as, [8, 7, 6, 5], etc.

The above Procedure 2.1 shall be used to distribute entries of each order magic squares studied in this work.It will follow the following path to get magic square

Path

Procedure 2.1 → Distribution → Structure → Magic Square

2.1 Pan Magic Square of Order 8

Distribution 2.1. *According to Procedure 2.1, let's distribute the 64 numbers from 1 to 64 in 4 blocks of 16 each resulting in equal sums:*

A1	1	8	9	16	17	24	25	32	33	40	41	48	49	56	57	64	520
A2	2	7	10	15	18	23	26	31	34	39	42	47	50	55	58	63	520
A3	3	6	11	14	19	22	27	30	35	38	43	46	51	54	59	62	520
A4	4	5	12	13	20	21	28	29	36	37	44	45	52	53	60	61	520

The above distribution of 64 numbers are divided in 4×4 blocks with columns following increasing and decreasing orders according to Procedure 2.1, i.e., [1, 2, 3, 4], [8, 7, 6, 5], [9, 10, 11, 12], [16, 15, 14, 13], etc.

Structure 2.1. *We shall use the following Structure to construct pan magic square of order 8:*

A1	A2
A3	A4

Example 2.1. *4 blocks of magic squares of order 4 constructed according to Example 1.1 using data given in distribution 2.1, and putting them according to structure 2.1, we get a pan magic square of order 8 is given by*

		260	260	260	260	260	260	260	260
	25	48	1	56	26	47	2	55	260
260	8	49	32	41	7	50	31	42	260
260	64	9	40	17	63	10	39	18	260
260	33	24	57	16	34	23	58	15	260
260	27	46	3	54	28	45	4	53	260
260	6	51	30	43	5	52	29	44	260
260	62	11	38	19	61	12	37	20	260
260	35	22	59	14	36	21	60	13	260
	260	260	260	260	260	260	260	260	260

In this case the magic sum is $S_{8\times8} = 260$. Each 4×4 block is a perfect **pan magic square** of order 4 with the same magic sum $S_{4\times4} = 130$.

This is not the only way write pan magic square of order 8. Below are some other types of distributions:

Distribution 2.2. *Below is a distribution due to Aale [1]:*

A1	1	4	5	8	25	28	29	32	33	36	37	40	57	60	61	64	520
A2	9	12	13	16	17	20	21	24	41	44	45	48	49	52	53	56	520
A3	2	3	6	7	26	27	30	31	34	35	38	39	58	59	62	63	520
A4	10	11	14	15	18	19	22	23	42	43	46	47	50	51	54	55	520

Distribution 2.3. *The following distribution is due to Willem Barink [5]:*

A1	3	4	5	6	11	12	13	14	51	52	53	54	59	60	61	62	520
A2	19	20	21	22	27	28	29	30	35	36	37	38	43	44	45	46	520
A3	1	2	7	8	9	10	15	16	49	50	55	56	57	58	63	64	520
A4	17	18	23	24	25	26	31	32	33	34	39	40	41	42	47	48	520

For more studies on magic squares in this direction refer [1, 2, 3, 4, 5].

2.2 Pan Magic Square of Order 12

In order to construct pan magic of order 12, we shall apply the procedure similar to as of order 8. The total number 1-144 are distributed 9 blocks of order 4 as below:

Distribution 2.4. *According to Procedure 2.1, let's distribute the 144 numbers from 1 to 144 in 9 blocks resulting in equal sums:*

A1	1	18	19	36	37	54	55	72	73	90	91	108	109	126	127	144	1160
A2	2	17	20	35	38	53	56	71	74	89	92	107	110	125	128	143	1160
A3	3	16	21	34	39	52	57	70	75	88	93	106	111	124	129	142	1160
A4	4	15	22	33	40	51	58	69	76	87	94	105	112	123	130	141	1160
A5	5	14	23	32	41	50	59	68	77	86	95	104	113	122	131	140	1160
A6	6	13	24	31	42	49	60	67	78	85	96	103	114	121	132	139	1160
A7	7	12	25	30	43	48	61	66	79	84	97	102	115	120	133	138	1160
A8	8	11	26	29	44	47	62	65	80	83	98	101	116	119	134	137	1160
A9	9	10	27	28	45	46	63	64	81	82	99	100	117	118	135	136	1160

Above distribution is according to Procedure 2.1,where columns are written in increasing and decreasing orders, such as, [1, 2, 3, 4, 5, 6, 7, 8, 9], [18, 17, 16, 15, 14, 13, 12, 11, 10], [19, 20, 21, 22, 23, 24, 25, 26, 27], etc. In order to construct pan magic square of order 12, we shall use the following structure.

Structure 2.2. *Let's consider 9 blocks of order 4 given as below:*

A1	A2	A3
A4	A5	A6
A7	A8	A9

Example 2.2. *9 block of magic squares of order 4 constructed according to Example 1.1, using data given in distribution 2.4, and putting them according to structure 2.2, we get a pan magic square of order 12 is given by*

		870	870	870	870	870	870	870	870	870	870	870	870
	55	108	1	126	56	107	2	125	57	106	3	124	870
870	18	109	72	91	17	110	71	92	16	111	70	93	870
870	144	19	90	37	143	20	89	38	142	21	88	39	870
870	73	54	127	36	74	53	128	35	75	52	129	34	870
870	58	105	4	123	59	104	5	122	60	103	6	121	870
870	15	112	69	94	14	113	68	95	13	114	67	96	870
870	141	22	87	40	140	23	86	41	139	24	85	42	870
870	76	51	130	33	77	50	131	32	78	49	132	31	870
870	61	102	7	120	62	101	8	119	63	100	9	118	870
870	12	115	66	97	11	116	65	98	10	117	64	99	870
870	138	25	84	43	137	26	83	44	136	27	82	45	870
870	79	48	133	30	80	47	134	29	81	46	135	28	870
	870	870	870	870	870	870	870	870	870	870	870	870	870

In this case the magic sum is $S_{12\times12} = 870$. Each 4×4 block is a perfect **pan magic square** of order 4 with the same magic sum $S_{4\times4} = 290$.

2.3 Pan Magic Square of Order 16

In order to construct pan magic square of order 16, we shall apply procedure similar to as of order 8. The total numbers 1–256 are distributed 16 blocks of order 4 according to following distribution.

Distribution 2.5. *According to Procedure 2.1, let's distribute the 256 numbers from 1 to 256 in 16 blocks resulting in equal sums:*

A1	1	32	33	64	65	96	97	128	129	160	161	192	193	224	225	256	2056
A2	2	31	34	63	66	95	98	127	130	159	162	191	194	223	226	255	2056
A3	3	30	35	62	67	94	99	126	131	158	163	190	195	222	227	254	2056
A4	4	29	36	61	68	93	100	125	132	157	164	189	196	221	228	253	2056
A5	5	28	37	60	69	92	101	124	133	156	165	188	197	220	229	252	2056
A6	6	27	38	59	70	91	102	123	134	155	166	187	198	219	230	251	2056
A7	7	26	39	58	71	90	103	122	135	154	167	186	199	218	231	250	2056
A8	8	25	40	57	72	89	104	121	136	153	168	185	200	217	232	249	2056
A9	9	24	41	56	73	88	105	120	137	152	169	184	201	216	233	248	2056
A10	10	23	42	55	74	87	106	119	138	151	170	183	202	215	234	247	2056
A11	11	22	43	54	75	86	107	118	139	150	171	182	203	214	235	246	2056
A12	12	21	44	53	76	85	108	117	140	149	172	181	204	213	236	245	2056
A13	13	20	45	52	77	84	109	116	141	148	173	180	205	212	237	244	2056
A14	14	19	46	51	78	83	110	115	142	147	174	179	206	211	238	243	2056
A15	15	18	47	50	79	82	111	114	143	146	175	178	207	210	239	242	2056
A16	16	17	48	49	80	81	112	113	144	145	176	177	208	209	240	241	2056

Above distribution is according to Procedure 2.1,where columns are written in increasing and decreasing orders, such as, $[1, 2, \ldots, 16]$, $[32, 31, \ldots, 17]$, $[33, 34, \ldots, 48]$, etc. In order to construct pan magic square of order 16, we shall use the following structure.

Structure 2.3. *Let's consider 16 blocks of order 4 as given below:*

A1	A2	A3	A4
A5	A6	A7	A8
A9	A10	A11	A12
A13	A14	A15	A16

Example 2.3. *16 blocks of magic squares of order 4 constructed according to Example 1.1 using data given in distribution 2.5, and putting them according to structure 2.3, we get a pan magic square of order 16 is given by*

		2056	2056	2056	2056	2056	2056	2056	2056	2056	2056	2056	2056	2056	2056	2056	2056
	97	192	1	224	98	191	2	223	99	190	3	222	100	189	4	221	2056
2056	32	193	128	161	31	194	127	162	30	195	126	163	29	196	125	164	2056
2056	256	33	160	65	255	34	159	66	254	35	158	67	253	36	157	68	2056
2056	129	96	225	64	130	95	226	63	131	94	227	62	132	93	228	61	2056
2056	101	188	5	220	102	187	6	219	103	186	7	218	104	185	8	217	2056
2056	28	197	124	165	27	198	123	166	26	199	122	167	25	200	121	168	2056
2056	252	37	156	69	251	38	155	70	250	39	154	71	249	40	153	72	2056
2056	133	92	229	60	134	91	230	59	135	90	231	58	136	89	232	57	2056
2056	105	184	9	216	106	183	10	215	107	182	11	214	108	181	12	213	2056
2056	24	201	120	169	23	202	119	170	22	203	118	171	21	204	117	172	2056
2056	248	41	152	73	247	42	151	74	246	43	150	75	245	44	149	76	2056
2056	137	88	233	56	138	87	234	55	139	86	235	54	140	85	236	53	2056
2056	109	180	13	212	110	179	14	211	111	178	15	210	112	177	16	209	2056
2056	20	205	116	173	19	206	115	174	18	207	114	175	17	208	113	176	2056
2056	244	45	148	77	243	46	147	78	242	47	146	79	241	48	145	80	2056
2056	141	84	237	52	142	83	238	51	143	82	239	50	144	81	240	49	2056
	2056	2056	2056	2056	2056	2056	2056	2056	2056	2056	2056	2056	2056	2056	2056	2056	2056

In this case, the magic sum is $S_{16\times16} = 2056$. Each 4×4 block is a perfect **pan magic square** of order 4 with the same magic sum $S_{4\times4} = 514$.

2.4 Pan Magic Square of Order 20

In order to construct pan magic of order 20, we shall apply the similar procedure as of order 8. The total number 1–400 are distributed 25 blocks of order 4 as below:

Distribution 2.6. *According to Procedure 2.1, let's distribute the 400 numbers from 1 to 400 in 25 blocks resulting in equal sums:*

A1	1	50	51	100	101	150	151	200	201	250	251	300	301	350	351	400	3208
A2	2	49	52	99	102	149	152	199	202	249	252	299	302	349	352	399	3208
A3	3	48	53	98	103	148	153	198	203	248	253	298	303	348	353	398	3208
A4	4	47	54	97	104	147	154	197	204	247	254	297	304	347	354	397	3208
A5	5	46	55	96	105	146	155	196	205	246	255	296	305	346	355	396	3208
A6	6	45	56	95	106	145	156	195	206	245	256	295	306	345	356	395	3208
A7	7	44	57	94	107	144	157	194	207	244	257	294	307	344	357	394	3208
A8	8	43	58	93	108	143	158	193	208	243	258	293	308	343	358	393	3208
A9	9	42	59	92	109	142	159	192	209	242	259	292	309	342	359	392	3208
A10	10	41	60	91	110	141	160	191	210	241	260	291	310	341	360	391	3208
A11	11	40	61	90	111	140	161	190	211	240	261	290	311	340	361	390	3208
A12	12	39	62	89	112	139	162	189	212	239	262	289	312	339	362	389	3208
A13	13	38	63	88	113	138	163	188	213	238	263	288	313	338	363	388	3208
A14	14	37	64	87	114	137	164	187	214	237	264	287	314	337	364	387	3208
A15	15	36	65	86	115	136	165	186	215	236	265	286	315	336	365	386	3208
A16	16	35	66	85	116	135	166	185	216	235	266	285	316	335	366	385	3208

A17	17	34	67	84	117	134	167	184	217	234	267	284	317	334	367	384	3208
A18	18	33	68	83	118	133	168	183	218	233	268	283	318	333	368	383	3208
A19	19	32	69	82	119	132	169	182	219	232	269	282	319	332	369	382	3208
A20	20	31	70	81	120	131	170	181	220	231	270	281	320	331	370	381	3208
A21	21	30	71	80	121	130	171	180	221	230	271	280	321	330	371	380	3208
A22	22	29	72	79	122	129	172	179	222	229	272	279	322	329	372	379	3208
A23	23	28	73	78	123	128	173	178	223	228	273	278	323	328	373	378	3208
A24	24	27	74	77	124	127	174	177	224	227	274	277	324	327	374	377	3208
A25	25	26	75	76	125	126	175	176	225	226	275	276	325	326	375	376	3208

Above distribution is according to Procedure 2.1,where columns are written in increasing and decreasing orders, such as, $[1, 2, \dots, 25]$, $[50, 49, \dots, 24]$, $[51, 52, \dots, 75]$, etc. In order to construct pan magic square of order 20, we shall use the following structure.

Structure 2.4. *Let's consider 25 blocks of order 4 given as below:*

A1	A2	A3	A4	A5
A6	A7	A8	A9	A10
A11	A12	A13	A14	A15
A16	A17	A18	A19	A20
A21	A22	A23	A24	A25

Example 2.4. *25 blocks of magic squares of order 4 constructed according to Example 1.1 using data given in distribution 2.6 and putting them according to structure 2.4, we get a pan magic square of order 20 is given by*

		4010	4010	4010	4010	4010	4010	4010	4010	4010	4010	4010	4010	4010	4010	4010	4010	4010	4010	4010	4010
	151	300	1	350	152	299	2	349	153	298	3	348	154	297	4	347	155	296	5	346	4010
4010	50	301	200	251	49	302	199	252	48	303	198	253	47	304	197	254	46	305	196	255	4010
4010	400	51	250	101	399	52	249	102	398	53	248	103	397	54	247	104	396	55	246	105	4010
4010	201	150	351	100	202	149	352	99	203	148	353	98	204	147	354	97	205	146	355	96	4010
4010	156	295	6	345	157	294	7	344	158	293	8	343	159	292	9	342	160	291	10	341	4010
4010	45	306	195	256	44	307	194	257	43	308	193	258	42	309	192	259	41	310	191	260	4010
4010	395	56	245	106	394	57	244	107	393	58	243	108	392	59	242	109	391	60	241	110	4010
4010	206	145	356	95	207	144	357	94	208	143	358	93	209	142	359	92	210	141	360	91	4010
4010	161	290	11	340	162	289	12	339	163	288	13	338	164	287	14	337	165	286	15	336	4010
4010	40	311	190	261	39	312	189	262	38	313	188	263	37	314	187	264	36	315	186	265	4010
4010	390	61	240	111	389	62	239	112	388	63	238	113	387	64	237	114	386	65	236	115	4010
4010	211	140	361	90	212	139	362	89	213	138	363	88	214	137	364	87	215	136	365	86	4010
4010	166	285	16	335	167	284	17	334	168	283	18	333	169	282	19	332	170	281	20	331	4010
4010	35	316	185	266	34	317	184	267	33	318	183	268	32	319	182	269	31	320	181	270	4010
4010	385	66	235	116	384	67	234	117	383	68	233	118	382	69	232	119	381	70	231	120	4010
4010	216	135	366	85	217	134	367	84	218	133	368	83	219	132	369	82	220	131	370	81	4010
4010	171	280	21	330	172	279	22	329	173	278	23	328	174	277	24	327	175	276	25	326	4010
4010	30	321	180	271	29	322	179	272	28	323	178	273	27	324	177	274	26	325	176	275	4010
4010	380	71	230	121	379	72	229	122	378	73	228	123	377	74	227	124	376	75	226	125	4010
4010	221	130	371	80	222	129	372	79	223	128	373	78	224	127	374	77	225	126	375	76	4010
	4010	4010	4010	4010	4010	4010	4010	4010	4010	4010	4010	4010	4010	4010	4010	4010	4010	4010	4010	4010	4010

In this case the magic sum is $S_{20\times 20} = 4010$. Each 4×4 block is a perfect **pan magic square** of order 4 with the same magic sum $S_{4\times 4} = 802$.

2.5 Pan Magic Square of Order 24

In order to construct pan magic of order 24, we shall apply the procedure similar to order 8.

Distribution 2.7. *According to Procedure 2.1, let's distribute the 576 numbers from 1 to 576 in 36 blocks resulting in equal sums:*

A1	1	72	73	144	145	216	217	288	289	360	361	432	433	504	505	576	4616
A2	2	71	74	143	146	215	218	287	290	359	362	431	434	503	506	575	4616
A3	3	70	75	142	147	214	219	286	291	358	363	430	435	502	507	574	4616
A4	4	69	76	141	148	213	220	285	292	357	364	429	436	501	508	573	4616
A5	5	68	77	140	149	212	221	284	293	356	365	428	437	500	509	572	4616
A6	6	67	78	139	150	211	222	283	294	355	366	427	438	499	510	571	4616
A7	7	66	79	138	151	210	223	282	295	354	367	426	439	498	511	570	4616
A8	8	65	80	137	152	209	224	281	296	353	368	425	440	497	512	569	4616
A9	9	64	81	136	153	208	225	280	297	352	369	424	441	496	513	568	4616
A10	10	63	82	135	154	207	226	279	298	351	370	423	442	495	514	567	4616
A11	11	62	83	134	155	206	227	278	299	350	371	422	443	494	515	566	4616
A12	12	61	84	133	156	205	228	277	300	349	372	421	444	493	516	565	4616
A13	13	60	85	132	157	204	229	276	301	348	373	420	445	492	517	564	4616
A14	14	59	86	131	158	203	230	275	302	347	374	419	446	491	518	563	4616
A15	15	58	87	130	159	202	231	274	303	346	375	418	447	490	519	562	4616
A16	16	57	88	129	160	201	232	273	304	345	376	417	448	489	520	561	4616
A17	17	56	89	128	161	200	233	272	305	344	377	416	449	488	521	560	4616
A18	18	55	90	127	162	199	234	271	306	343	378	415	450	487	522	559	4616
A19	19	54	91	126	163	198	235	270	307	342	379	414	451	486	523	558	4616
A20	20	53	92	125	164	197	236	269	308	341	380	413	452	485	524	557	4616
A21	21	52	93	124	165	196	237	268	309	340	381	412	453	484	525	556	4616
A22	22	51	94	123	166	195	238	267	310	339	382	411	454	483	526	555	4616
A23	23	50	95	122	167	194	239	266	311	338	383	410	455	482	527	554	4616
A24	24	49	96	121	168	193	240	265	312	337	384	409	456	481	528	553	4616
A25	25	48	97	120	169	192	241	264	313	336	385	408	457	480	529	552	4616
A26	26	47	98	119	170	191	242	263	314	335	386	407	458	479	530	551	4616
A27	27	46	99	118	171	190	243	262	315	334	387	406	459	478	531	550	4616
A28	28	45	100	117	172	189	244	261	316	333	388	405	460	477	532	549	4616
A29	29	44	101	116	173	188	245	260	317	332	389	404	461	476	533	548	4616
A30	30	43	102	115	174	187	246	259	318	331	390	403	462	475	534	547	4616
A31	31	42	103	114	175	186	247	258	319	330	391	402	463	474	535	546	4616
A32	32	41	104	113	176	185	248	257	320	329	392	401	464	473	536	545	4616
A33	33	40	105	112	177	184	249	256	321	328	393	400	465	472	537	544	4616
A34	34	39	106	111	178	183	250	255	322	327	394	399	466	471	538	543	4616
A35	35	38	107	110	179	182	251	254	323	326	395	398	467	470	539	542	4616
A36	36	37	108	109	180	181	252	253	324	325	396	397	468	469	540	541	4616

Above distribution is according to Procedure 2.1, where columns are written in increasing and decreasing orders, such as, $[1, 2, \ldots, 36]$, $[72, 49, \ldots, 37]$, $[73, 74, \ldots, 108]$, etc. In order to construct pan magic square of order 24, we shall use the following structure.

Structure 2.5. *Let's consider 36 blocks of order 4 given as below:*

A1	*A2*	*A3*	*A4*	*A5*	*A6*
A7	*A8*	*A9*	*A10*	*A11*	*A12*
A13	*A14*	*A15*	*A16*	*A17*	*A18*
A19	*A20*	*A21*	*A22*	*A23*	*A24*
A25	*A26*	*A27*	*A28*	*A29*	*A30*
A31	*A32*	*A33*	*A34*	*A35*	*A36*

Example 2.5. *36 blocks of magic squares of order 4 constructed according to Example 1.1 using data given in distribution 2.7 and putting them according to structure 2.5, we get a pan magic square of order 24 is given by*

		6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924
	217	432	1	504	218	431	2	503	219	430	3	502	220	429	4	501	221	428	5	500	222	427	6	499	6924
6924	72	433	288	361	71	434	287	362	70	435	286	363	69	436	285	364	68	437	284	365	67	438	283	366	6924
6924	576	73	360	145	575	74	359	146	574	75	358	147	573	76	357	148	572	77	356	149	571	78	355	150	6924
6924	289	216	505	144	290	215	506	143	291	214	507	142	292	213	508	141	293	212	509	140	294	211	510	139	6924
6924	223	426	7	498	224	425	8	497	225	424	9	496	226	423	10	495	227	422	11	494	228	421	12	493	6924
6924	66	439	282	367	65	440	281	368	64	441	280	369	63	442	279	370	62	443	278	371	61	444	277	372	6924
6924	570	79	354	151	569	80	353	152	568	81	352	153	567	82	351	154	566	83	350	155	565	84	349	156	6924
6924	295	210	511	138	296	209	512	137	297	208	513	136	298	207	514	135	299	206	515	134	300	205	516	133	6924
6924	229	420	13	492	230	419	14	491	231	418	15	490	232	417	16	489	233	416	17	488	234	415	18	487	6924
6924	60	445	276	373	59	446	275	374	58	447	274	375	57	448	273	376	56	449	272	377	55	450	271	378	6924
6924	564	85	348	157	563	86	347	158	562	87	346	159	561	88	345	160	560	89	344	161	559	90	343	162	6924
6924	301	204	517	132	302	203	518	131	303	202	519	130	304	201	520	129	305	200	521	128	306	199	522	127	6924
6924	235	414	19	486	236	413	20	485	237	412	21	484	238	411	22	483	239	410	23	482	240	409	24	481	6924
6924	54	451	270	379	53	452	269	380	52	453	268	381	51	454	267	382	50	455	266	383	49	456	265	384	6924
6924	558	91	342	163	557	92	341	164	556	93	340	165	555	94	339	166	554	95	338	167	553	96	337	168	6924
6924	307	198	523	126	308	197	524	125	309	196	525	124	310	195	526	123	311	194	527	122	312	193	528	121	6924
6924	241	408	25	480	242	407	26	479	243	406	27	478	244	405	28	477	245	404	29	476	246	403	30	475	6924
6924	48	457	264	385	47	458	263	386	46	459	262	387	45	460	261	388	44	461	260	389	43	462	259	390	6924
6924	552	97	336	169	551	98	335	170	550	99	334	171	549	100	333	172	548	101	332	173	547	102	331	174	6924
6924	313	192	529	120	314	191	530	119	315	190	531	118	316	189	532	117	317	188	533	116	318	187	534	115	6924
6924	247	402	31	474	248	401	32	473	249	400	33	472	250	399	34	471	251	398	35	470	252	397	36	469	6924
6924	42	463	258	391	41	464	257	392	40	465	256	393	39	466	255	394	38	467	254	395	37	468	253	396	6924
6924	546	103	330	175	545	104	329	176	544	105	328	177	543	106	327	178	542	107	326	179	541	108	325	180	6924
6924	319	186	535	114	320	185	536	113	321	184	537	112	322	183	538	111	323	182	539	110	324	181	540	109	6924
	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924

In this case the magic sum is $S_{24\times 24} = 6924$. Each 4×4 block is a perfect **pan magic square** of order 4 with the same magic sum $S_{4\times 4} = 1154$.

2.6 Pan Magic Square of Order 28

In order to construct pan magic of order 28, we shall apply the similar procedure as of order 8. The total number 1–784 are distributed 49 blocks of order 4 as below:

Distribution 2.8. *According to Procedure 2.1, let's distribute the 784 numbers from 1 to 784 in 49 blocks resulting in equal sums:*

A1	1	98	99	196	197	294	295	392	393	490	491	588	589	686	687	784	6280
A2	2	97	100	195	198	293	296	391	394	489	492	587	590	685	688	783	6280
A3	3	96	101	194	199	292	297	390	395	488	493	586	591	684	689	782	6280
A4	4	95	102	193	200	291	298	389	396	487	494	585	592	683	690	781	6280
A5	5	94	103	192	201	290	299	388	397	486	495	584	593	682	691	780	6280
A6	6	93	104	191	202	289	300	387	398	485	496	583	594	681	692	779	6280
A7	7	92	105	190	203	288	301	386	399	484	497	582	595	680	693	778	6280
A8	8	91	106	189	204	287	302	385	400	483	498	581	596	679	694	777	6280
A9	9	90	107	188	205	286	303	384	401	482	499	580	597	678	695	776	6280
A10	10	89	108	187	206	285	304	383	402	481	500	579	598	677	696	775	6280
A11	11	88	109	186	207	284	305	382	403	480	501	578	599	676	697	774	6280
A12	12	87	110	185	208	283	306	381	404	479	502	577	600	675	698	773	6280
A13	13	86	111	184	209	282	307	380	405	478	503	576	601	674	699	772	6280
A14	14	85	112	183	210	281	308	379	406	477	504	575	602	673	700	771	6280
A15	15	84	113	182	211	280	309	378	407	476	505	574	603	672	701	770	6280
A16	16	83	114	181	212	279	310	377	408	475	506	573	604	671	702	769	6280
A17	17	82	115	180	213	278	311	376	409	474	507	572	605	670	703	768	6280
A18	18	81	116	179	214	277	312	375	410	473	508	571	606	669	704	767	6280
A19	19	80	117	178	215	276	313	374	411	472	509	570	607	668	705	766	6280
A20	20	79	118	177	216	275	314	373	412	471	510	569	608	667	706	765	6280
A21	21	78	119	176	217	274	315	372	413	470	511	568	609	666	707	764	6280
A22	22	77	120	175	218	273	316	371	414	469	512	567	610	665	708	763	6280
A23	23	76	121	174	219	272	317	370	415	468	513	566	611	664	709	762	6280
A24	24	75	122	173	220	271	318	369	416	467	514	565	612	663	710	761	6280
A25	25	74	123	172	221	270	319	368	417	466	515	564	613	662	711	760	6280
A26	26	73	124	171	222	269	320	367	418	465	516	563	614	661	712	759	6280
A27	27	72	125	170	223	268	321	366	419	464	517	562	615	660	713	758	6280
A28	28	71	126	169	224	267	322	365	420	463	518	561	616	659	714	757	6280
A29	29	70	127	168	225	266	323	364	421	462	519	560	617	658	715	756	6280
A30	30	69	128	167	226	265	324	363	422	461	520	559	618	657	716	755	6280
A31	31	68	129	166	227	264	325	362	423	460	521	558	619	656	717	754	6280
A32	32	67	130	165	228	263	326	361	424	459	522	557	620	655	718	753	6280
A33	33	66	131	164	229	262	327	360	425	458	523	556	621	654	719	752	6280
A34	34	65	132	163	230	261	328	359	426	457	524	555	622	653	720	751	6280
A35	35	64	133	162	231	260	329	358	427	456	525	554	623	652	721	750	6280
A36	36	63	134	161	232	259	330	357	428	455	526	553	624	651	722	749	6280
A37	37	62	135	160	233	258	331	356	429	454	527	552	625	650	723	748	6280
A38	38	61	136	159	234	257	332	355	430	453	528	551	626	649	724	747	6280
A39	39	60	137	158	235	256	333	354	431	452	529	550	627	648	725	746	6280
A40	40	59	138	157	236	255	334	353	432	451	530	549	628	647	726	745	6280
A41	41	58	139	156	237	254	335	352	433	450	531	548	629	646	727	744	6280
A42	42	57	140	155	238	253	336	351	434	449	532	547	630	645	728	743	6280
A43	43	56	141	154	239	252	337	350	435	448	533	546	631	644	729	742	6280
A44	44	55	142	153	240	251	338	349	436	447	534	545	632	643	730	741	6280
A45	45	54	143	152	241	250	339	348	437	446	535	544	633	642	731	740	6280
A46	46	53	144	151	242	249	340	347	438	445	536	543	634	641	732	739	6280
A47	47	52	145	150	243	248	341	346	439	444	537	542	635	640	733	738	6280
A48	48	51	146	149	244	247	342	345	440	443	538	541	636	639	734	737	6280
A49	49	50	147	148	245	246	343	344	441	442	539	540	637	638	735	736	6280

Above distribution is according to Procedure 2.1,where columns are written in increasing and decreasing orders, such as, $[1, 2, \ldots, 49]$, $[98, 97, \ldots, 50]$, $[99, 100, \ldots, 147]$, etc. In order to construct pan magic square of order 28, we shall use the following structure.

Structure 2.6. *Let's consider 49 blocks of order 4 given as below:*

A1	A2	A3	A4	A5	A6	A7
A8	A9	A10	A11	A12	A13	A14
A15	A16	A17	A18	A19	A20	A21
A22	A23	A24	A25	A26	A27	A28
A29	A30	A31	A32	A33	A34	A35
A36	A37	A38	A39	A40	A41	A42
A43	A44	A45	A46	A47	A48	A49

Example 2.6. *49 blocks of magic squares of order 4 constructed according to Example 1.1 using data given in distribution 2.8 and putting them according to structure 2.6, we get a pan magic square of order 28 is given by*

		10990	10990	10990	10990	10990	10990	10990	10990	10990	10990	10990	10990	10990	10990	10990	10990	10990	10990	10990	10990	10990	10990	10990	10990	10990	10990	10990	10990
	295	588	1	686	296	587	2	685	297	586	3	684	298	585	4	683	299	584	5	682	300	583	6	681	301	582	7	680	10990
10990	98	589	392	491	97	590	391	492	96	591	390	493	95	592	389	494	94	593	388	495	93	594	387	496	92	595	386	497	10990
10990	784	99	490	197	783	100	489	198	782	101	488	199	781	102	487	200	780	103	486	201	779	104	485	202	778	105	484	203	10990
10990	393	294	687	196	394	293	688	195	395	292	689	194	396	291	690	193	397	290	691	192	398	289	692	191	399	288	693	190	10990
10990	302	581	8	679	303	580	9	678	304	579	10	677	305	578	11	676	306	577	12	675	307	576	13	674	308	575	14	673	10990
10990	91	596	385	498	90	597	384	499	89	598	383	500	88	599	382	501	87	600	381	502	86	601	380	503	85	602	379	504	10990
10990	777	106	483	204	776	107	482	205	775	108	481	206	774	109	480	207	773	110	479	208	772	111	478	209	771	112	477	210	10990
10990	400	287	694	189	401	286	695	188	402	285	696	187	403	284	697	186	404	283	698	185	405	282	699	184	406	281	700	183	10990
10990	309	574	15	672	310	573	16	671	311	572	17	670	312	571	18	669	313	570	19	668	314	569	20	667	315	568	21	666	10990
10990	84	603	378	505	83	604	377	506	82	605	376	507	81	606	375	508	80	607	374	509	79	608	373	510	78	609	372	511	10990
10990	770	113	476	211	769	114	475	212	768	115	474	213	767	116	473	214	766	117	472	215	765	118	471	216	764	119	470	217	10990
10990	407	280	701	182	408	279	702	181	409	278	703	180	410	277	704	179	411	276	705	178	412	275	706	177	413	274	707	176	10990
10990	316	567	22	665	317	566	23	664	318	565	24	663	319	564	25	662	320	563	26	661	321	562	27	660	322	561	28	659	10990
10990	77	610	371	512	76	611	370	513	75	612	369	514	74	613	368	515	73	614	367	516	72	615	366	517	71	616	365	518	10990
10990	763	120	469	218	762	121	468	219	761	122	467	220	760	123	466	221	759	124	465	222	758	125	464	223	757	126	463	224	10990
10990	414	273	708	175	415	272	709	174	416	271	710	173	417	270	711	172	418	269	712	171	419	268	713	170	420	267	714	169	10990
10990	323	560	29	658	324	559	30	657	325	558	31	656	326	557	32	655	327	556	33	654	328	555	34	653	329	554	35	652	10990
10990	70	617	364	519	69	618	363	520	68	619	362	521	67	620	361	522	66	621	360	523	65	622	359	524	64	623	358	525	10990
10990	756	127	462	225	755	128	461	226	754	129	460	227	753	130	459	228	752	131	458	229	751	132	457	230	750	133	456	231	10990
10990	421	266	715	168	422	265	716	167	423	264	717	166	424	263	718	165	425	262	719	164	426	261	720	163	427	260	721	162	10990
10990	330	553	36	651	331	552	37	650	332	551	38	649	333	550	39	648	334	549	40	647	335	548	41	646	336	547	42	645	10990
10990	63	624	357	526	62	625	356	527	61	626	355	528	60	627	354	529	59	628	353	530	58	629	352	531	57	630	351	532	10990
10990	749	134	455	232	748	135	454	233	747	136	453	234	746	137	452	235	745	138	451	236	744	139	450	237	743	140	449	238	10990
10990	428	259	722	161	429	258	723	160	430	257	724	159	431	256	725	158	432	255	726	157	433	254	727	156	434	253	728	155	10990
10990	337	546	43	644	338	545	44	643	339	544	45	642	340	543	46	641	341	542	47	640	342	541	48	639	343	540	49	638	10990
10990	56	631	350	533	55	632	349	534	54	633	348	535	53	634	347	536	52	635	346	537	51	636	345	538	50	637	344	539	10990
10990	742	141	448	239	741	142	447	240	740	143	446	241	739	144	445	242	738	145	444	243	737	146	443	244	736	147	442	245	10990
10990	435	252	729	154	436	251	730	153	437	250	731	152	438	249	732	151	439	248	733	150	440	247	734	149	441	246	735	148	10990
	10990	10990	10990	10990	10990	10990	10990	10990	10990	10990	10990	10990	10990	10990	10990	10990	10990	10990	10990	10990	10990	10990	10990	10990	10990	10990	10990	10990	10990

In this case the magic sum is $S_{28\times 28} = 10990$. Each 4×4 block is a perfect **pan magic square** of order 4 with the same magic sum $S_{4\times 4} = 1570$.

2.7 Pan Magic Square of Order 32

In order to construct pan magic of order 32, we shall apply the similar procedure as of order 8. The total number 1–1024 are distributed 64 blocks of order 4 as below:

Distribution 2.9. *According to Procedure 2.1, let's distribute the 1024 numbers from 1 to 1024 in 64 blocks resulting in equal sums:*

A1	1	128	129	256	257	384	385	512	513	640	641	768	769	896	897	1024	8200
A2	2	127	130	255	258	383	386	511	514	639	642	767	770	895	898	1023	8200
A3	3	126	131	254	259	382	387	510	515	638	643	766	771	894	899	1022	8200
A4	4	125	132	253	260	381	388	509	516	637	644	765	772	893	900	1021	8200
A5	5	124	133	252	261	380	389	508	517	636	645	764	773	892	901	1020	8200
A6	6	123	134	251	262	379	390	507	518	635	646	763	774	891	902	1019	8200
A7	7	122	135	250	263	378	391	506	519	634	647	762	775	890	903	1018	8200
A8	8	121	136	249	264	377	392	505	520	633	648	761	776	889	904	1017	8200
A9	9	120	137	248	265	376	393	504	521	632	649	760	777	888	905	1016	8200
A10	10	119	138	247	266	375	394	503	522	631	650	759	778	887	906	1015	8200
A11	11	118	139	246	267	374	395	502	523	630	651	758	779	886	907	1014	8200
A12	12	117	140	245	268	373	396	501	524	629	652	757	780	885	908	1013	8200
A13	13	116	141	244	269	372	397	500	525	628	653	756	781	884	909	1012	8200
A14	14	115	142	243	270	371	398	499	526	627	654	755	782	883	910	1011	8200
A15	15	114	143	242	271	370	399	498	527	626	655	754	783	882	911	1010	8200
A16	16	113	144	241	272	369	400	497	528	625	656	753	784	881	912	1009	8200
A17	17	112	145	240	273	368	401	496	529	624	657	752	785	880	913	1008	8200
A18	18	111	146	239	274	367	402	495	530	623	658	751	786	879	914	1007	8200
A19	19	110	147	238	275	366	403	494	531	622	659	750	787	878	915	1006	8200
A20	20	109	148	237	276	365	404	493	532	621	660	749	788	877	916	1005	8200
A21	21	108	149	236	277	364	405	492	533	620	661	748	789	876	917	1004	8200
A22	22	107	150	235	278	363	406	491	534	619	662	747	790	875	918	1003	8200
A23	23	106	151	234	279	362	407	490	535	618	663	746	791	874	919	1002	8200
A24	24	105	152	233	280	361	408	489	536	617	664	745	792	873	920	1001	8200
A25	25	104	153	232	281	360	409	488	537	616	665	744	793	872	921	1000	8200
A26	26	103	154	231	282	359	410	487	538	615	666	743	794	871	922	999	8200
A27	27	102	155	230	283	358	411	486	539	614	667	742	795	870	923	998	8200
A28	28	101	156	229	284	357	412	485	540	613	668	741	796	869	924	997	8200
A29	29	100	157	228	285	356	413	484	541	612	669	740	797	868	925	996	8200
A30	30	99	158	227	286	355	414	483	542	611	670	739	798	867	926	995	8200
A31	31	98	159	226	287	354	415	482	543	610	671	738	799	866	927	994	8200
A32	32	97	160	225	288	353	416	481	544	609	672	737	800	865	928	993	8200

A33	33	96	161	224	289	352	417	480	545	608	673	736	801	864	929	992	8200
A34	34	95	162	223	290	351	418	479	546	607	674	735	802	863	930	991	8200
A35	35	94	163	222	291	350	419	478	547	606	675	734	803	862	931	990	8200
A36	36	93	164	221	292	349	420	477	548	605	676	733	804	861	932	989	8200
A37	37	92	165	220	293	348	421	476	549	604	677	732	805	860	933	988	8200
A38	38	91	166	219	294	347	422	475	550	603	678	731	806	859	934	987	8200
A39	39	90	167	218	295	346	423	474	551	602	679	730	807	858	935	986	8200
A40	40	89	168	217	296	345	424	473	552	601	680	729	808	857	936	985	8200
A41	41	88	169	216	297	344	425	472	553	600	681	728	809	856	937	984	8200
A42	42	87	170	215	298	343	426	471	554	599	682	727	810	855	938	983	8200
A43	43	86	171	214	299	342	427	470	555	598	683	726	811	854	939	982	8200
A44	44	85	172	213	300	341	428	469	556	597	684	725	812	853	940	981	8200
A45	45	84	173	212	301	340	429	468	557	596	685	724	813	852	941	980	8200
A46	46	83	174	211	302	339	430	467	558	595	686	723	814	851	942	979	8200
A47	47	82	175	210	303	338	431	466	559	594	687	722	815	850	943	978	8200
A48	48	81	176	209	304	337	432	465	560	593	688	721	816	849	944	977	8200
A49	49	80	177	208	305	336	433	464	561	592	689	720	817	848	945	976	8200
A50	50	79	178	207	306	335	434	463	562	591	690	719	818	847	946	975	8200
A51	51	78	179	206	307	334	435	462	563	590	691	718	819	846	947	974	8200
A52	52	77	180	205	308	333	436	461	564	589	692	717	820	845	948	973	8200
A53	53	76	181	204	309	332	437	460	565	588	693	716	821	844	949	972	8200
A54	54	75	182	203	310	331	438	459	566	587	694	715	822	843	950	971	8200
A55	55	74	183	202	311	330	439	458	567	586	695	714	823	842	951	970	8200
A56	56	73	184	201	312	329	440	457	568	585	696	713	824	841	952	969	8200
A57	57	72	185	200	313	328	441	456	569	584	697	712	825	840	953	968	8200
A58	58	71	186	199	314	327	442	455	570	583	698	711	826	839	954	967	8200
A59	59	70	187	198	315	326	443	454	571	582	699	710	827	838	955	966	8200
A60	60	69	188	197	316	325	444	453	572	581	700	709	828	837	956	965	8200
A61	61	68	189	196	317	324	445	452	573	580	701	708	829	836	957	964	8200
A62	62	67	190	195	318	323	446	451	574	579	702	707	830	835	958	963	8200
A63	63	66	191	194	319	322	447	450	575	578	703	706	831	834	959	962	8200
A64	64	65	192	193	320	321	448	449	576	577	704	705	832	833	960	961	8200

Above distribution is according to Procedure 2.1,where columns are written in increasing and decreasing orders, such as, $[1, 2, \ldots, 64]$, $[128, 127, \ldots, 65]$, $[129, 130, \ldots, 192]$, etc. In order to construct pan magic square of order 28, we shall use the following structure.

Structure 2.7. *Let's consider 64 blocks of order 4 given as below:*

A1	A2	A3	A4	A5	A6	A7	A8
A9	A10	A11	A12	A13	A14	A15	A16
A17	A18	A19	A20	A21	A22	A23	A24
A25	A26	A27	A28	A29	A30	A31	A32
A33	A34	A35	A36	A37	A38	A39	A40
A41	A42	A43	A44	A45	A46	A47	A48
A49	A50	A51	A52	A53	A54	A55	A56
A57	A58	A59	A60	A61	A62	A63	A64

Example 2.7. *64 four blocks of magic squares of order 4 constructed according to Example 1.1 using data given in distribution 2.8 and putting them according to structure 2.7, we get a pan magic square of order 32 is given by*

		16400	16400	16400	16400	16400	16400	16400	16400	16400	16400	16400	16400	16400	16400	16400	16400	16400	16400	16400	16400	16400	16400	16400	16400	16400	16400	16400	16400	16400	16400	16400	16400
	385	768	1	896	386	767	2	895	387	766	3	894	388	765	4	893	389	764	5	892	390	763	6	891	391	762	7	890	392	761	8	889	16400
16400	128	769	512	641	127	770	511	642	126	771	510	643	125	772	509	644	124	773	508	645	123	774	507	646	122	775	506	647	121	776	505	648	16400
16400	1024	129	640	257	1023	130	639	258	1022	131	638	259	1021	132	637	260	1020	133	636	261	1019	134	635	262	1018	135	634	263	1017	136	633	264	16400
16400	513	384	897	256	514	383	898	255	515	382	899	254	516	381	900	253	517	380	901	252	518	379	902	251	519	378	903	250	520	377	904	249	16400
16400	393	760	9	888	394	759	10	887	395	758	11	886	396	757	12	885	397	756	13	884	398	755	14	883	399	754	15	882	400	753	16	881	16400
16400	120	777	504	649	119	778	503	650	118	779	502	651	117	780	501	652	116	781	500	653	115	782	499	654	114	783	498	655	113	784	497	656	16400
16400	1016	137	632	265	1015	138	631	266	1014	139	630	267	1013	140	629	268	1012	141	628	269	1011	142	627	270	1010	143	626	271	1009	144	625	272	16400
16400	521	376	905	248	522	375	906	247	523	374	907	246	524	373	908	245	525	372	909	244	526	371	910	243	527	370	911	242	528	369	912	241	16400
16400	401	752	17	880	402	751	18	879	403	750	19	878	404	749	20	877	405	748	21	876	406	747	22	875	407	746	23	874	408	745	24	873	16400
16400	112	785	496	657	111	786	495	658	110	787	494	659	109	788	493	660	108	789	492	661	107	790	491	662	106	791	490	663	105	792	489	664	16400
16400	1008	145	624	273	1007	146	623	274	1006	147	622	275	1005	148	621	276	1004	149	620	277	1003	150	619	278	1002	151	618	279	1001	152	617	280	16400
16400	529	368	913	240	530	367	914	239	531	366	915	238	532	365	916	237	533	364	917	236	534	363	918	235	535	362	919	234	536	361	920	233	16400
16400	409	744	25	872	410	743	26	871	411	742	27	870	412	741	28	869	413	740	29	868	414	739	30	867	415	738	31	866	416	737	32	865	16400
16400	104	793	488	665	103	794	487	666	102	795	486	667	101	796	485	668	100	797	484	669	99	798	483	670	98	799	482	671	97	800	481	672	16400
16400	1000	153	616	281	999	154	615	282	998	155	614	283	997	156	613	284	996	157	612	285	995	158	611	286	994	159	610	287	993	160	609	288	16400
16400	537	360	921	232	538	359	922	231	539	358	923	230	540	357	924	229	541	356	925	228	542	355	926	227	543	354	927	226	544	353	928	225	16400
16400	417	736	33	864	418	735	34	863	419	734	35	862	420	733	36	861	421	732	37	860	422	731	38	859	423	730	39	858	424	729	40	857	16400
16400	96	801	480	673	95	802	479	674	94	803	478	675	93	804	477	676	92	805	476	677	91	806	475	678	90	807	474	679	89	808	473	680	16400
16400	992	161	608	289	991	162	607	290	990	163	606	291	989	164	605	292	988	165	604	293	987	166	603	294	986	167	602	295	985	168	601	296	16400
16400	545	352	929	224	546	351	930	223	547	350	931	222	548	349	932	221	549	348	933	220	550	347	934	219	551	346	935	218	552	345	936	217	16400
16400	425	728	41	856	426	727	42	855	427	726	43	854	428	725	44	853	429	724	45	852	430	723	46	851	431	722	47	850	432	721	48	849	16400
16400	88	809	472	681	87	810	471	682	86	811	470	683	85	812	469	684	84	813	468	685	83	814	467	686	82	815	466	687	81	816	465	688	16400
16400	984	169	600	297	983	170	599	298	982	171	598	299	981	172	597	300	980	173	596	301	979	174	595	302	978	175	594	303	977	176	593	304	16400
16400	553	344	937	216	554	343	938	215	555	342	939	214	556	341	940	213	557	340	941	212	558	339	942	211	559	338	943	210	560	337	944	209	16400
16400	433	720	49	848	434	719	50	847	435	718	51	846	436	717	52	845	437	716	53	844	438	715	54	843	439	714	55	842	440	713	56	841	16400
16400	80	817	464	689	79	818	463	690	78	819	462	691	77	820	461	692	76	821	460	693	75	822	459	694	74	823	458	695	73	824	457	696	16400
16400	976	177	592	305	975	178	591	306	974	179	590	307	973	180	589	308	972	181	588	309	971	182	587	310	970	183	586	311	969	184	585	312	16400
16400	561	336	945	208	562	335	946	207	563	334	947	206	564	333	948	205	565	332	949	204	566	331	950	203	567	330	951	202	568	329	952	201	16400
16400	441	712	57	840	442	711	58	839	443	710	59	838	444	709	60	837	445	708	61	836	446	707	62	835	447	706	63	834	448	705	64	833	16400
16400	72	825	456	697	71	826	455	698	70	827	454	699	69	828	453	700	68	829	452	701	67	830	451	702	66	831	450	703	65	832	449	704	16400
16400	968	185	584	313	967	186	583	314	966	187	582	315	965	188	581	316	964	189	580	317	963	190	579	318	962	191	578	319	961	192	577	320	16400
16400	569	328	953	200	570	327	954	199	571	326	955	198	572	325	956	197	573	324	957	196	574	323	958	195	575	322	959	194	576	321	960	193	16400
	16400	16400	16400	16400	16400	16400	16400	16400	16400	16400	16400	16400	16400	16400	16400	16400	16400	16400	16400	16400	16400	16400	16400	16400	16400	16400	16400	16400	16400	16400	16400	16400	16400

In this case the magic sum is $S_{32\times 32} = 16400$. Each 4×4 block is a perfect **pan magic square** of order 4 with the same magic sum $S_{4\times 4} = 2050$.

3 Final Comments

This paper brings a systematic way to construct block-wise pan magic squares of order $4k$. The construction is very simple and it requires only the knowledge of magic square of order 4. The work is for the orders 8 to 32. In each case, the sums are equal for each block of order 4. Below are item-wise details of author's work on magic squares:

(i) **Digital numbers** magic squares – [6, 7, 8, 9, 10, 11];

(ii) **Block-wise construction** of **bimagic squares** – [12];

(iii) Connections with **genetic tables** and **Shannon's entropy** – [13];

(iv) **Selfie** and **palindromic-type** magic squares – [14];

(v) **Intervally distributed** and **block-wise** magic squares – [15, 16, 17];

(vi) **Multi-digits** magic squares – [18];

(vii) **Perfect square sum** magic squares with uniformity and minimum sum – [19, 20];

(viii) **Pythagorean triples** to generate **perfect square sum** magic squares – [20].

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References

[1] Aale de Winkel, Online discussion, The Magic Encyclopedia, *http://magichypercubes.com/Encyclopedia/*.

[2] Dwane H. Campbell and Keith A. Campbell, Online discussion, WELCOME TO MAGIC CUBE GENERATOR, *http://magictesseract.com*.

[3] Francis Gaspalou, Magic Squares, *http://www.gaspalou.fr/magic-squares/*

[4] Walter Trump, Notes on Magic Squares, *http://www.trump.de/magic-squares/index.html*.

[5] Willem Barink, The construction of perfect panmagic squares of order $4k$, ($k \geq 2$), *www.allergrootste.com/friends/wba/magic-squares.html*

[6] I.J. TANEJA, Digital Era: Magic Squares and 8th May 2010 (08.05.2010), May, 2010, pp. 1-4, *https://arxiv.org/abs/1005.1384 - https://goo.gl/XpyvWu*.

[7] I.J. TANEJA, Universal Bimagic Squares and the day 10th October 2010 (10.10.10), Oct, 2010, pp. 1-5, *https://arxiv.org/abs/1010.2083 - https://goo.gl/TtrP9B*.

[8] I.J. TANEJA, DIGITAL ERA: Universal Bimagic Squares, Oct, 2010, pp. 1-8, *https://arxiv.org/abs/1010.2541*; *https://goo.gl/MQWgiw*.

[9] I.J. TANEJA, Upside Down Numerical Equation, Bimagic Squares, and the day September 11, Oct. 2010, pp. 1-7, *https://arxiv.org/abs/1010.4186*; *https://goo.gl/kdBEbk*.

[10] I.J. TANEJA, Equivalent Versions of "Khajuraho" and "Lo-Shu" Magic Squares and the day 1st October 2010 (01.10.2010), Nov. 2010, pp. 1-7, *https://arxiv.org/abs/1011.0451*; *https://goo.gl/vnJxoX*.

[11] I.J. TANEJA, Upside Down Magic, Bimagic, Palindromic Squares and Pythagoras Theorem on a Palindromic Day - 11.02.2011, Feb. 2011, pp.1-9, *https://arxiv.org/abs/1102.2394*; *https://goo.gl/dPLezL*.

[12] I.J. TANEJA, Bimagic Squares of Bimagic Squares and an Open Problem, Feb. 2011, pp. 1-14, *https://arxiv.org/abs/1102.3052*; *https://goo.gl/4fuvqs*.

[13] I.J. TANEJA, Representations of Genetic Tables, Bimagic Squares, Hamming Distances and Shannon Entropy, Jun. 2012, pp. 1-19, *https://arxiv.org/abs/1206.2220*; *https://goo.gl/Jd4JXc*.

[14] I.J. TANEJA, Selfie Palindromic Magic Squares, RGMIA Research Report Collection, **18**(2015), Art. 98, pp. 1-15. *http://rgmia.org/papers/v18/v18a98.pdf - https://goo.gl/n3mhe5*.

[15] I.J. TANEJA, Intervally Distributed, Palindromic, Selfie Magic Squares, and Double Colored Patterns, RGMIA Research Report Collection, **18**(2015), Art. 127, pp. 1-45. *http://rgmia.org/papers/v18/v18a127.pdf - https://goo.gl/yzcRWa*.

[16] I.J. TANEJA, Intervally Distributed, Palindromic and Selfie Magic Squares: Genetic Table and Colored Pattern – Orders 11 to 20, RGMIA Research Report Collection, **18**(2015), Art. 140, pp. 1-43. *http://rgmia.org/papers/v18/v18a140.pdf - https://goo.gl/DE1iyK*.

[17] I.J. TANEJA, Intervally Distributed, Palindromic and Selfie Magic Squares – Orders 21 to 25 , **18**(2015), Art. 151, pp. 1-33. *http://rgmia.org/papers/v18/v18a151.pdf* - *https://goo.gl/rzJYuG*.

[18] I.J. TANEJA, Multi-Digits Magic Squares, RGMIA Research Report Collection, **18**(2015), Art. 159, pp. 1-22. *http://rgmia.org/papers/v18/v18a159.pdf* - *https://goo.gl/rw13Dw*.

[19] I.J. TANEJA, Magic Squares with Perfect Square Number Sums, Research Report Collection, **20**(2017), Article 11, pp. 1-24, *http://rgmia.org/papers/v20/v20a11.pdf* - https://goo.gl/JFLEZJ.

[20] I.J. TANEJA, Pythagorean Triples and Perfect Square Sum Magic Squares, RGMIA Research Report Collection, **20**(2017), Art. 128, pp. 1-22, *http://rgmia.org/papers/v20/v20a128.pdf* - https://goo.gl/qUPV66.

[21] I.J. TANEJA, Hardy-Ramanujan Number - 1729, RGMIA Research Report Collection, **20**(2017), Article 06, pp. 1-50, *http://rgmia.org/papers/v20/v20a06.pdf* *https://goo.gl/3LNf35*.

[22] I.J. TANEJA, I.J. TANEJA, S-gonal and Centered Polygonal Selfie Numbers, and Connections with Binomials Coefficients, RGMIA Research Report Collection, **20**(2017), Art. 43, pp. 1-42, *http://rgmia.org/papers/v20/v20a43.pdf* - *https://goo.gl/73o86h*.