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Block-Wise Equal Sums Magic Squares of Order 3k

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Abstract

This paper brings ***block-wise*** *magic squares multiple of 3 from orders 9 to 36 for the consecutive positive natural numbers entries. All the cases are of* ***pan magic squares*** *except the orders 18 and 30. The constructions are in such a way that in each case the* ***sub-blocks*** *are of same magic sum or sum of all entries are of equal sums. The study of magic squares of orders multiples of 3 and 4 simultaneously, such as magic squares of orders 12, 24 and 36 are considered as multiple of* $4k$*. The work on* ***pan magic squares*** *of orders multiple of* $4k$ *are already studied by author [27]. In these cases, all the sub-blocks are* ***pan magic square*** *of order 4 with same magic squares sums. In case of order 30, two different ways of constructions are given. The* ***pan magic square*** *of order 36 is done as 81 sub-blocks of* ***pan magic squares*** *of order 4. The magic squares of higher orders follows similar lines.*

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1 Introduction

Author [27], worked with **pan magic squares** of orders multiple of $4k$ in such a way that all the **sub-blocks** of order 4 in each case are of equal sum, and also are **pan magic squares** of order 4. The work done is for the magic squares of orders 8, 12, 16, 20, 24, 28 and 32. It is observed that for writing these magic squares just the knowledge of magic square of order 4 is sufficient. The magic square of order 4 used is Khajuraho's **pan magic square** of order 4.

In this paper, we worked on magic squares of orders multiple of $3k$, such as, of orders 9, 12, 15, etc. There are two types of orders $3k$. One is odd orders, such as, orders 9, 15, 21, etc. The second is of even orders, such as, orders 12, 18, 24, etc. The work is divided in two separate sections. One section is on odd order magic squares, and another on even orders, both multiples of 3. The even order cases, 12 and 24 are already done in previous paper [**?**]. We have repeated them here just to complete the list. It is understood that the entries for each magic square are consecutive natural numbers starting from 1.

2 Odd Orders Magic Squares of Order $3k$

This section brings **block-wise pan magic squares** of order $3k$, $k \geq 3$. The constructions are in such a way that **sub-blocks** are of same magic sums. In some cases, the magic squares are **pan magic squares**. The example of odd order are from orders 9 to 33. The work is based on triples resulting in equal sums. For simiplicyt, let's consider a magic square of order 3:

Example 1. *A magic square of order 3 is given by*

			15
8	1	6	15
3	5	7	15
4	9	2	15
15	15	15	15

Let's organize the numbers of above Example 1 in increasing order and consider the following distribution:

Distribution 1. *Let's reorganize the members of above magic square in an increasing way:*

(1)	1	6	8	15
(2)	3	5	7	15
(3)	2	4	9	15

Structure 1. *Let's put the rows (1), (2) and (3) according to following table:*

(11)	(12)	(13)
(21)	(22)	(23)
(31)	(32)	(33)

Above distribution is useful to construct **pan magic square** of order 9. See the example below:

2.1 Pan Magic Square of Order 9

Example 2. *Let's consider the following* ***pan magic square*** *of order 9 is given by*

		369	369	369	369	369	369	369	369	369
	8	49	66	61	24	38	36	77	10	369
369	37	63	23	12	35	76	65	7	51	369
369	78	11	34	50	64	9	22	39	62	369
369	14	28	81	67	3	53	42	56	25	369
369	52	69	2	27	41	55	80	13	30	369
369	57	26	40	29	79	15	1	54	68	369
369	20	43	60	73	18	32	48	71	4	369
369	31	75	17	6	47	70	59	19	45	369
369	72	5	46	44	58	21	16	33	74	369
	369	369	369	369	369	369	369	369	369	369

In this case, the magic sum is $S_{9\times9} = 369$. The sum of all the entries of each 3×3 blocks are the same sums as of magic square, i.e., $S_9 = 369$. The middle block of order 3 is a magic square, and other 8 subgroups are semi-magic squares of order 3.

The magic square given in Example 2 is a combination of two **mutually orthogonal diagonal Latin squares**:

Example 3. *Two mutually orthogonal Latin squares composing magic square of order 9 given in Example 2 are given by*

A		45	45	45	45	45	45	45	45	45
	1	6	8	7	3	5	4	9	2	45
45	5	7	3	2	4	9	8	1	6	45
45	9	2	4	6	8	1	3	5	7	45
45	2	4	9	8	1	6	5	7	3	45
45	6	8	1	3	5	7	9	2	4	45
45	7	3	5	4	9	2	1	6	8	45
45	3	5	7	9	2	4	6	8	1	45
45	4	9	2	1	6	8	7	3	5	45
45	8	1	6	5	7	3	2	4	9	45
	45	45	45	45	45	45	45	45	45	45

B		45	45	45	45	45	45	45	45	45
	8	4	3	7	6	2	9	5	1	45
45	1	9	5	3	8	4	2	7	6	45
45	6	2	7	5	1	9	4	3	8	45
45	5	1	9	4	3	8	6	2	7	45
45	7	6	2	9	5	1	8	4	3	45
45	3	8	4	2	7	6	1	9	5	45
45	2	7	6	1	9	5	3	8	4	45
45	4	3	8	6	2	7	5	1	9	45
45	9	5	1	8	4	3	7	6	2	45
	45	45	45	45	45	45	45	45	45	45

Making the operation $9\times(A-1)+B$ *we get the magic square given in Example 2*

2.2 Pan Magic Square of Order 15

Distribution 2. *Let's consider the following distribution to construct* ***pan magic square*** *of order 15:*

1	1	6	8	12	13	40
2	3	5	7	11	14	40
3	2	4	9	10	15	40

The construction of **pan magic square** of order 15 with each sub-block of same sum is based on the following pan diagonal magic square of order 5 constructed using a pair of mutually orthogonal diagonal Latin squares:

Example 4. *The **pan magic square** of order 5 is given by*

A		15	15	15	15	15
	1	2	3	4	5	15
15	4	5	1	2	3	15
15	2	3	4	5	1	15
15	5	1	2	3	4	15
15	3	4	5	1	2	15
	15	15	15	15	15	15

B		15	15	15	15	15
	1	4	2	5	3	15
15	2	5	3	1	4	15
15	3	1	4	2	5	15
15	4	2	5	3	1	15
15	5	3	1	4	2	15
	15	15	15	15	15	15

AB		65	65	65	65	65
	1	9	12	20	23	65
65	17	25	3	6	14	65
65	8	11	19	22	5	65
65	24	2	10	13	16	65
65	15	18	21	4	7	65
	65	65	65	65	65	65

The **pan magic square** appearing in ***AB*** is constructed according to the following formula:

$$AB := 5 \times (A-1) + B$$

where A and B are the **mutually orthogonal diagonal Latin squares** of order 5.

We shall construct 9 magic squares of order 5 using the Distribution 2 and Example 4, and put them according to Structure 1. In this case, the composition considered is as follows:

$$AB := 15 \times (A-1) + B$$

i.e., instead of 5, the multiplications is with 15.

Construction 1. *Below are 9 **pan magic squares** of order 5 constructed according to Example 4 using the Distribution 2:*

- **Block 11**

1		40	40	40	40	40
	1	6	8	12	13	40
40	12	13	1	6	8	40
40	6	8	12	13	1	40
40	13	1	6	8	12	40
40	8	12	13	1	6	40
	40	40	40	40	40	40

1		40	40	40	40	40
	1	12	6	13	8	40
40	6	13	8	1	12	40
40	8	1	12	6	13	40
40	12	6	13	8	1	40
40	13	8	1	12	6	40
	40	40	40	40	40	40

11		565	565	565	565	565
	1	87	111	178	188	565
565	171	193	8	76	117	565
565	83	106	177	186	13	565
565	192	6	88	113	166	565
565	118	173	181	12	81	565
	565	565	565	565	565	565

- **Block 12**

1		40	40	40	40	40
	1	6	8	12	13	40
40	12	13	1	6	8	40
40	6	8	12	13	1	40
40	13	1	6	8	12	40
40	8	12	13	1	6	40
	40	40	40	40	40	40

2		40	40	40	40	40
	3	11	5	14	7	40
40	5	14	7	3	11	40
40	7	3	11	5	14	40
40	11	5	14	7	3	40
40	14	7	3	11	5	40
	40	40	40	40	40	40

12		565	565	565	565	565
	3	86	110	179	187	565
565	170	194	7	78	116	565
565	82	108	176	185	14	565
565	191	5	89	112	168	565
565	119	172	183	11	80	565
	565	565	565	565	565	565

- **Block 13**

1		40	40	40	40	40
	1	6	8	12	13	40
40	12	13	1	6	8	40
40	6	8	12	13	1	40
40	13	1	6	8	12	40
40	8	12	13	1	6	40
	40	40	40	40	40	40

3		40	40	40	40	40
	2	10	4	15	9	40
40	4	15	9	2	10	40
40	9	2	10	4	15	40
40	10	4	15	9	2	40
40	15	9	2	10	4	40
	40	40	40	40	40	40

13		565	565	565	565	565
	2	85	109	180	189	565
565	169	195	9	77	115	565
565	84	107	175	184	15	565
565	190	4	90	114	167	565
565	120	174	182	10	79	565
	565	565	565	565	565	565

- **Block 21**

2		40	40	40	40	40
	3	5	7	11	14	40
40	11	14	3	5	7	40
40	5	7	11	14	3	40
40	14	3	5	7	11	40
40	7	11	14	3	5	40
	40	40	40	40	40	40

1		40	40	40	40	40
	1	12	6	13	8	40
40	6	13	8	1	12	40
40	8	1	12	6	13	40
40	12	6	13	8	1	40
40	13	8	1	12	6	40
	40	40	40	40	40	40

21		565	565	565	565	565
	31	72	96	163	203	565
565	156	208	38	61	102	565
565	68	91	162	201	43	565
565	207	36	73	98	151	565
565	103	158	196	42	66	565
	565	565	565	565	565	565

- **Block 22**

2		40	40	40	40	40
	3	5	7	11	14	40
40	11	14	3	5	7	40
40	5	7	11	14	3	40
40	14	3	5	7	11	40
40	7	11	14	3	5	40
	40	40	40	40	40	40

2		40	40	40	40	40
	3	11	5	14	7	40
40	5	14	7	3	11	40
40	7	3	11	5	14	40
40	11	5	14	7	3	40
40	14	7	3	11	5	40
	40	40	40	40	40	40

22		565	565	565	565	565
	33	71	95	164	202	565
565	155	209	37	63	101	565
565	67	93	161	200	44	565
565	206	35	74	97	153	565
565	104	157	198	41	65	565
	565	565	565	565	565	565

- **Block 23**

2		40	40	40	40	40
	3	5	7	11	14	40
40	11	14	3	5	7	40
40	5	7	11	14	3	40
40	14	3	5	7	11	40
40	7	11	14	3	5	40
	40	40	40	40	40	40

3		40	40	40	40	40
	2	10	4	15	9	40
40	4	15	9	2	10	40
40	9	2	10	4	15	40
40	10	4	15	9	2	40
40	15	9	2	10	4	40
	40	40	40	40	40	40

23		565	565	565	565	565
	32	70	94	165	204	565
565	154	210	39	62	100	565
565	69	92	160	199	45	565
565	205	34	75	99	152	565
565	105	159	197	40	64	565
	565	565	565	565	565	565

- **Block 31**

3		40	40	40	40	40
	2	4	9	10	15	40
40	10	15	2	4	9	40
40	4	9	10	15	2	40
40	15	2	4	9	10	40
40	9	10	15	2	4	40
	40	40	40	40	40	40

1		40	40	40	40	40
	1	12	6	13	8	40
40	6	13	8	1	12	40
40	8	1	12	6	13	40
40	12	6	13	8	1	40
40	13	8	1	12	6	40
	40	40	40	40	40	40

31		565	565	565	565	565
	16	57	126	148	218	565
565	141	223	23	46	132	565
565	53	121	147	216	28	565
565	222	21	58	128	136	565
565	133	143	211	27	51	565
	565	565	565	565	565	565

- **Block 32**

3		40	40	40	40	40
	2	4	9	10	15	40
40	10	15	2	4	9	40
40	4	9	10	15	2	40
40	15	2	4	9	10	40
40	9	10	15	2	4	40
	40	40	40	40	40	40

2		40	40	40	40	40
	3	11	5	14	7	40
40	5	14	7	3	11	40
40	7	3	11	5	14	40
40	11	5	14	7	3	40
40	14	7	3	11	5	40
	40	40	40	40	40	40

32		565	565	565	565	565
	18	56	125	149	217	565
565	140	224	22	48	131	565
565	52	123	146	215	29	565
565	221	20	59	127	138	565
565	134	142	213	26	50	565
	565	565	565	565	565	565

- **Block 33**

3		40	40	40	40	40
	2	4	9	10	15	40
40	10	15	2	4	9	40
40	4	9	10	15	2	40
40	15	2	4	9	10	40
40	9	10	15	2	4	40
	40	40	40	40	40	40

3		40	40	40	40	40
	2	10	4	15	9	40
40	4	15	9	2	10	40
40	9	2	10	4	15	40
40	10	4	15	9	2	40
40	15	9	2	10	4	40
	40	40	40	40	40	40

33		565	565	565	565	565
	17	55	124	150	219	565
565	139	225	24	47	130	565
565	54	122	145	214	30	565
565	220	19	60	129	137	565
565	135	144	212	25	49	565
	565	565	565	565	565	565

In all the 9 blocks, the composition is applied by use of following formula:

$$AB := 15 \times (A-1) + B$$

Combining the above 9 **pan magic squares** of order 5 according to structure 1 we get a **pan magic square** of order 15 given in the example below.

Example 5. *According to Distribution 2, Structure 1 and 9* ***pan magic squares*** *given above, we have a* ***pan magic square*** *of order 15 given by*

		1695	1695	1695	1695	1695	1695	1695	1695	1695	1695	1695	1695	1695	1695	1695
	1	87	111	178	188	3	86	110	179	187	2	85	109	180	189	1695
1695	171	193	8	76	117	170	194	7	78	116	169	195	9	77	115	1695
1695	83	106	177	186	13	82	108	176	185	14	84	107	175	184	15	1695
1695	192	6	88	113	166	191	5	89	112	168	190	4	90	114	167	1695
1695	118	173	181	12	81	119	172	183	11	80	120	174	182	10	79	1695
1695	31	72	96	163	203	33	71	95	164	202	32	70	94	165	204	1695
1695	156	208	38	61	102	155	209	37	63	101	154	210	39	62	100	1695
1695	68	91	162	201	43	67	93	161	200	44	69	92	160	199	45	1695
1695	207	36	73	98	151	206	35	74	97	153	205	34	75	99	152	1695
1695	103	158	196	42	66	104	157	198	41	65	105	159	197	40	64	1695
1695	16	57	126	148	218	18	56	125	149	217	17	55	124	150	219	1695
1695	141	223	23	46	132	140	224	22	48	131	139	225	24	47	130	1695
1695	53	121	147	216	28	52	123	146	215	29	54	122	145	214	30	1695
1695	222	21	58	128	136	221	20	59	127	138	220	19	60	129	137	1695
1695	133	143	211	27	51	134	142	213	26	50	135	144	212	25	49	1695
	1695	1695	1695	1695	1695	1695	1695	1695	1695	1695	1695	1695	1695	1695	1695	1695

In this case, the magic sum is $S_{15\times 15} = 1695$. Each 5×5 block is a **pan magic square** of order 5 with the same magic sum $S_{5\times 5} = 565$.

2.3 Pan Magic Square of Order 21

Distribution 3. *Let's consider the following distribution to construct* ***pan magic square*** *of order 21:*

1	1	6	8	12	13	182	19	77
2	3	5	7	11	14	17	20	77
3	2	4	9	10	15	16	21	77

The construction of **pan magic square** of order 21 with each sub-block of same sum is based on the following pan diagonal magic square of order 7 constructed using a pair of mutually orthogonal diagonal Latin squares:

Example 6. *The* ***pan diagonal magic square of order 7*** *is given by*

A								
		28	28	28	28	28	28	28
	1	2	3	4	5	6	7	28
28	6	7	1	2	3	4	5	28
28	4	5	6	7	1	2	3	28
28	2	3	4	5	6	7	1	28
28	7	1	2	3	4	5	6	28
28	5	6	7	1	2	3	4	28
28	3	4	5	6	7	1	2	28
	28	28	28	28	28	28	28	28

B								
		28	28	28	28	28	28	28
	1	2	3	4	5	6	7	28
28	5	6	7	1	2	3	4	28
28	2	3	4	5	6	7	1	28
28	6	7	1	2	3	4	5	28
28	3	4	5	6	7	1	2	28
28	7	1	2	3	4	5	6	28
28	4	5	6	7	1	2	3	28
	28	28	28	28	28	28	28	28

(AB)		175	175	175	175	175	175	175
	1	9	17	25	33	41	49	175
175	40	48	7	8	16	24	32	175
175	23	31	39	47	6	14	15	175
175	13	21	22	30	38	46	5	175
175	45	4	12	20	28	29	37	175
175	35	36	44	3	11	19	27	175
175	18	26	34	42	43	2	10	175
	175	175	175	175	175	175	175	175

The **pan magic square** appearing in ***AB*** is constructed according to the following formula:

$$AB := 7 \times (A-1) + B$$

where A and B are the **mutually orthogonal diagonal Latin squares** of order 7.

We shall construct 9 magic squares of order 7 using the Distribution 3 and Example 6, and put them according to Structure 1. In this case, the composition considered is as follows:

$$AB := 21 \times (A-1) + B$$

i.e., instead of 7, the multiplications is with 21.

Construction 2. *Below are 9* ***pan magic squares*** *of order 7 constructed according to Example 6 using the Distribution 3:*

- **Block 11**

(1)		77	77	77	77	77	77	77
	1	6	8	12	13	18	19	77
77	18	19	1	6	8	12	13	77
77	12	13	18	19	1	6	8	77
77	6	8	12	13	18	19	1	77
77	19	1	6	8	12	13	18	77
77	13	18	19	1	6	8	12	77
77	8	12	13	18	19	1	6	77
	77	77	77	77	77	77	77	77

(1)		77	77	77	77	77	77	77
	1	6	8	12	13	18	19	77
77	13	18	19	1	6	8	12	77
77	6	8	12	13	18	19	1	77
77	18	19	1	6	8	12	13	77
77	8	12	13	18	19	1	6	77
77	19	1	6	8	12	13	18	77
77	12	13	18	19	1	6	8	77
	77	77	77	77	77	77	77	77

(11)		1547	1547	1547	1547	1547	1547	1547
	1	111	155	243	265	375	397	1547
1547	370	396	19	106	153	239	264	1547
1547	237	260	369	391	18	124	148	1547
1547	123	166	232	258	365	390	13	1547
1547	386	12	118	165	250	253	363	1547
1547	271	358	384	8	117	160	249	1547
1547	159	244	270	376	379	6	113	1547
	1547	1547	1547	1547	1547	1547	1547	1547

- **Block 12**

(1)		77	77	77	77	77	77	77
	1	6	8	12	13	18	19	77
77	18	19	1	6	8	12	13	77
77	12	13	18	19	1	6	8	77
77	6	8	12	13	18	19	1	77
77	19	1	6	8	12	13	18	77
77	13	18	19	1	6	8	12	77
77	8	12	13	18	19	1	6	77
	77	77	77	77	77	77	77	77

(2)		77	77	77	77	77	77	77
	3	5	7	11	14	17	20	77
77	14	17	20	3	5	7	11	77
77	5	7	11	14	17	20	3	77
77	17	20	3	5	7	11	14	77
77	7	11	14	17	20	3	5	77
77	20	3	5	7	11	14	17	77
77	11	14	17	20	3	5	7	77
	77	77	77	77	77	77	77	77

(12)		1547	1547	1547	1547	1547	1547	1547
	3	110	154	242	266	374	398	1547
1547	371	395	20	108	152	238	263	1547
1547	236	259	368	392	17	125	150	1547
1547	122	167	234	257	364	389	14	1547
1547	385	11	119	164	251	255	362	1547
1547	272	360	383	7	116	161	248	1547
1547	158	245	269	377	381	5	112	1547
	1547	1547	1547	1547	1547	1547	1547	1547

- **Block 13**

(1)		77	77	77	77	77	77	77
	1	6	8	12	13	18	19	77
77	18	19	1	6	8	12	13	77
77	12	13	18	19	1	6	8	77
77	6	8	12	13	18	19	1	77
77	19	1	6	8	12	13	18	77
77	13	18	19	1	6	8	12	77
77	8	12	13	18	19	1	6	77
	77	77	77	77	77	77	77	77

(3)		77	77	77	77	77	77	77
	2	4	9	10	15	16	21	77
77	15	16	21	2	4	9	10	77
77	4	9	10	15	16	21	2	77
77	16	21	2	4	9	10	15	77
77	9	10	15	16	21	2	4	77
77	21	2	4	9	10	15	16	77
77	10	15	16	21	2	4	9	77
	77	77	77	77	77	77	77	77

(13)		1547	1547	1547	1547	1547	1547	1547
	2	109	156	241	267	373	399	1547
1547	372	394	21	107	151	240	262	1547
1547	235	261	367	393	16	126	149	1547
1547	121	168	233	256	366	388	15	1547
1547	387	10	120	163	252	254	361	1547
1547	273	359	382	9	115	162	247	1547
1547	157	246	268	378	380	4	114	1547
	1547	1547	1547	1547	1547	1547	1547	1547

- **Block 21**

②		77	77	77	77	77	77	77
	3	5	7	11	14	17	20	77
77	17	20	3	5	7	11	14	77
77	11	14	17	20	3	5	7	77
77	5	7	11	14	17	20	3	77
77	20	3	5	7	11	14	17	77
77	14	17	20	3	5	7	11	77
77	7	11	14	17	20	3	5	77
	77	77	77	77	77	77	77	77

①		77	77	77	77	77	77	77
	1	6	8	12	13	18	19	77
77	13	18	19	1	6	8	12	77
77	6	8	12	13	18	19	1	77
77	18	19	1	6	8	12	13	77
77	8	12	13	18	19	1	6	77
77	19	1	6	8	12	13	18	77
77	12	13	18	19	1	6	8	77
	77	77	77	77	77	77	77	77

(21)		1547	1547	1547	1547	1547	1547	1547
	43	90	134	222	286	354	418	1547
1547	349	417	61	85	132	218	285	1547
1547	216	281	348	412	60	103	127	1547
1547	102	145	211	279	344	411	55	1547
1547	407	54	97	144	229	274	342	1547
1547	292	337	405	50	96	139	228	1547
1547	138	223	291	355	400	48	92	1547
	1547	1547	1547	1547	1547	1547	1547	1547

- **Block 22**

②		77	77	77	77	77	77	77
	3	5	7	11	14	17	20	77
77	17	20	3	5	7	11	14	77
77	11	14	17	20	3	5	7	77
77	5	7	11	14	17	20	3	77
77	20	3	5	7	11	14	17	77
77	14	17	20	3	5	7	11	77
77	7	11	14	17	20	3	5	77
	77	77	77	77	77	77	77	77

②		77	77	77	77	77	77	77
	3	5	7	11	14	17	20	77
77	14	17	20	3	5	7	11	77
77	5	7	11	14	17	20	3	77
77	17	20	3	5	7	11	14	77
77	7	11	14	17	20	3	5	77
77	20	3	5	7	11	14	17	77
77	11	14	17	20	3	5	7	77
	77	77	77	77	77	77	77	77

(22)		1547	1547	1547	1547	1547	1547	1547
	45	89	133	221	287	353	419	1547
1547	350	416	62	87	131	217	284	1547
1547	215	280	347	413	59	104	129	1547
1547	101	146	213	278	343	410	56	1547
1547	406	53	98	143	230	276	341	1547
1547	293	339	404	49	95	140	227	1547
1547	137	224	290	356	402	47	91	1547
	1547	1547	1547	1547	1547	1547	1547	1547

- **Block 23**

2		77	77	77	77	77	77	77
	3	5	7	11	14	17	20	77
77	17	20	3	5	7	11	14	77
77	11	14	17	20	3	5	7	77
77	5	7	11	14	17	20	3	77
77	20	3	5	7	11	14	17	77
77	14	17	20	3	5	7	11	77
77	7	11	14	17	20	3	5	77
	77	77	77	77	77	77	77	77

3		77	77	77	77	77	77	77
	2	4	9	10	15	16	21	77
77	15	16	21	2	4	9	10	77
77	4	9	10	15	16	21	2	77
77	16	21	2	4	9	10	15	77
77	9	10	15	16	21	2	4	77
77	21	2	4	9	10	15	16	77
77	10	15	16	21	2	4	9	77
	77	77	77	77	77	77	77	77

23		1547	1547	1547	1547	1547	1547	1547
	44	88	135	220	288	352	420	1547
1547	351	415	63	86	130	219	283	1547
1547	214	282	346	414	58	105	128	1547
1547	100	147	212	277	345	409	57	1547
1547	408	52	99	142	231	275	340	1547
1547	294	338	403	51	94	141	226	1547
1547	136	225	289	357	401	46	93	1547
	1547	1547	1547	1547	1547	1547	1547	1547

- **Block 31**

3		77	77	77	77	77	77	77
	2	4	9	10	15	16	21	77
77	16	21	2	4	9	10	15	77
77	10	15	16	21	2	4	9	77
77	4	9	10	15	16	21	2	77
77	21	2	4	9	10	15	16	77
77	15	16	21	2	4	9	10	77
77	9	10	15	16	21	2	4	77
	77	77	77	77	77	77	77	77

1		77	77	77	77	77	77	77
	1	6	8	12	13	18	19	77
77	13	18	19	1	6	8	12	77
77	6	8	12	13	18	19	1	77
77	18	19	1	6	8	12	13	77
77	8	12	13	18	19	1	6	77
77	19	1	6	8	12	13	18	77
77	12	13	18	19	1	6	8	77
	77	77	77	77	77	77	77	77

31		1547	1547	1547	1547	1547	1547	1547
	22	69	176	201	307	333	439	1547
1547	328	438	40	64	174	197	306	1547
1547	195	302	327	433	39	82	169	1547
1547	81	187	190	300	323	432	34	1547
1547	428	33	76	186	208	295	321	1547
1547	313	316	426	29	75	181	207	1547
1547	180	202	312	334	421	27	71	1547
	1547	1547	1547	1547	1547	1547	1547	1547

- **Block 32**

(3)		77	77	77	77	77	77	77
	2	4	9	10	15	16	21	77
77	16	21	2	4	9	10	15	77
77	10	15	16	21	2	4	9	77
77	4	9	10	15	16	21	2	77
77	21	2	4	9	10	15	16	77
77	15	16	21	2	4	9	10	77
77	9	10	15	16	21	2	4	77
	77	77	77	77	77	77	77	77

(2)		77	77	77	77	77	77	77
	3	5	7	11	14	17	20	77
77	14	17	20	3	5	7	11	77
77	5	7	11	14	17	20	3	77
77	17	20	3	5	7	11	14	77
77	7	11	14	17	20	3	5	77
77	20	3	5	7	11	14	17	77
77	11	14	17	20	3	5	7	77
	77	77	77	77	77	77	77	77

(32)		1547	1547	1547	1547	1547	1547	1547
	24	68	175	200	308	332	440	1547
1547	329	437	41	66	173	196	305	1547
1547	194	301	326	434	38	83	171	1547
1547	80	188	192	299	322	431	35	1547
1547	427	32	77	185	209	297	320	1547
1547	314	318	425	28	74	182	206	1547
1547	179	203	311	335	423	26	70	1547
	1547	1547	1547	1547	1547	1547	1547	1547

- **Block 33**

(3)		77	77	77	77	77	77	77
	2	4	9	10	15	16	21	77
77	16	21	2	4	9	10	15	77
77	10	15	16	21	2	4	9	77
77	4	9	10	15	16	21	2	77
77	21	2	4	9	10	15	16	77
77	15	16	21	2	4	9	10	77
77	9	10	15	16	21	2	4	77
	77	77	77	77	77	77	77	77

(3)		77	77	77	77	77	77	77
	2	4	9	10	15	16	21	77
77	15	16	21	2	4	9	10	77
77	4	9	10	15	16	21	2	77
77	16	21	2	4	9	10	15	77
77	9	10	15	16	21	2	4	77
77	21	2	4	9	10	15	16	77
77	10	15	16	21	2	4	9	77
	77	77	77	77	77	77	77	77

(33)		1547	1547	1547	1547	1547	1547	1547
	23	67	177	199	309	331	441	1547
1547	330	436	42	65	172	198	304	1547
1547	193	303	325	435	37	84	170	1547
1547	79	189	191	298	324	430	36	1547
1547	429	31	78	184	210	296	319	1547
1547	315	317	424	30	73	183	205	1547
1547	178	204	310	336	422	25	72	1547
	1547	1547	1547	1547	1547	1547	1547	1547

In all the 9 blocks, the composition is applied by use of following formula:

$$AB := 15 \times (A-1) + B$$

Combining the above 9 **pan magic squares** of order 7 according to structure 1 we get a **pan magic square** of order 21 given in the example below.

Example 7. *According to Distribution 3, Structure 1 and 9* ***pan magic squares*** *given above, we have a* ***pan magic square*** *of order 21 given by*

		4641	4641	4641	4641	4641	4641	4641	4641	4641	4641	4641	4641	4641	4641	4641	4641	4641	4641	4641	4641	4641
	1	111	155	243	265	375	397	3	110	154	242	266	374	398	2	109	156	241	267	373	399	4641
4641	370	396	19	106	153	239	264	371	395	20	108	152	238	263	372	394	21	107	151	240	262	4641
4641	237	260	369	391	18	124	148	236	259	368	392	17	125	150	235	261	367	393	16	126	149	4641
4641	123	166	232	258	365	390	13	122	167	234	257	364	389	14	121	168	233	256	366	388	15	4641
4641	386	12	118	165	250	253	363	385	11	119	164	251	255	362	387	10	120	163	252	254	361	4641
4641	271	358	384	8	117	160	249	272	360	383	7	116	161	248	273	359	382	9	115	162	247	4641
4641	159	244	270	376	379	6	113	158	245	269	377	381	5	112	157	246	268	378	380	4	114	4641
4641	43	90	134	222	286	354	418	45	89	133	221	287	353	419	44	88	135	220	288	352	420	4641
4641	349	417	61	85	132	218	285	350	416	62	87	131	217	284	351	415	63	86	130	219	283	4641
4641	216	281	348	412	60	103	127	215	280	347	413	59	104	129	214	282	346	414	58	105	128	4641
4641	102	145	211	279	344	411	55	101	146	213	278	343	410	56	100	147	212	277	345	409	57	4641
4641	407	54	97	144	229	274	342	406	53	98	143	230	276	341	408	52	99	142	231	275	340	4641
4641	292	337	405	50	96	139	228	293	339	404	49	95	140	227	294	338	403	51	94	141	226	4641
4641	138	223	291	355	400	48	92	137	224	290	356	402	47	91	136	225	289	357	401	46	93	4641
4641	22	69	176	201	307	333	439	24	68	175	200	308	332	440	23	67	177	199	309	331	441	4641
4641	328	438	40	64	174	197	306	329	437	41	66	173	196	305	330	436	42	65	172	198	304	4641
4641	195	302	327	433	39	82	169	194	301	326	434	38	83	171	193	303	325	435	37	84	170	4641
4641	81	187	190	300	323	432	34	80	188	192	299	322	431	35	79	189	191	298	324	430	36	4641
4641	428	33	76	186	208	295	321	427	32	77	185	209	297	320	429	31	78	184	210	296	319	4641
4641	313	316	426	29	75	181	207	314	318	425	28	74	182	206	315	317	424	30	73	183	205	4641
4641	180	202	312	334	421	27	71	179	203	311	335	423	26	70	178	204	310	336	422	25	72	4641
	4641	4641	4641	4641	4641	4641	4641	4641	4641	4641	4641	4641	4641	4641	4641	4641	4641	4641	4641	4641	4641	4641

2.4 Pan Magic Square of Order 27

Distribution 4. *Let's consider the following distribution to construct* ***pan magic square*** *of order 27:*

1	1	6	8	12	13	18	19	24	25	126
2	3	5	7	11	14	17	20	23	26	126
3	2	4	9	10	15	16	21	22	27	126

The construction of **pan magic square** of order 21 with each sub-block of same sum is based on the following pan diagonal magic square of order 7 constructed using a pair of mutually orthogonal diagonal Latin squares:

Example 8. *The* ***pan diagonal magic square of order 9*** *is given by*

A		45	45	45	45	45	45	45	45	45
	1	6	8	7	3	5	4	9	2	45
45	5	7	3	2	4	9	8	1	6	45
45	9	2	4	6	8	1	3	5	7	45
45	2	4	9	8	1	6	5	7	3	45
45	6	8	1	3	5	7	9	2	4	45
45	7	3	5	4	9	2	1	6	8	45
45	3	5	7	9	2	4	6	8	1	45
45	4	9	2	1	6	8	7	3	5	45
45	8	1	6	5	7	3	2	4	9	45
	45	45	45	45	45	45	45	45	45	45

B		45	45	45	45	45	45	45	45	45
	8	4	3	7	6	2	9	5	1	45
45	1	9	5	3	8	4	2	7	6	45
45	6	2	7	5	1	9	4	3	8	45
45	5	1	9	4	3	8	6	2	7	45
45	7	6	2	9	5	1	8	4	3	45
45	3	8	4	2	7	6	1	9	5	45
45	2	7	6	1	9	5	3	8	4	45
45	4	3	8	6	2	7	5	1	9	45
45	9	5	1	8	4	3	7	6	2	45
	45	45	45	45	45	45	45	45	45	45

AB		369	369	369	369	369	369	369	369	369
	8	49	66	61	24	38	36	77	10	369
369	37	63	23	12	35	76	65	7	51	369
369	78	11	34	50	64	9	22	39	62	369
369	14	28	81	67	3	53	42	56	25	369
369	52	69	2	27	41	55	80	13	30	369
369	57	26	40	29	79	15	1	54	68	369
369	20	43	60	73	18	32	48	71	4	369
369	31	75	17	6	47	70	59	19	45	369
369	72	5	46	44	58	21	16	33	74	369
	369	369	369	369	369	369	369	369	369	369

The **pan magic square** appearing in ***AB*** is constructed according to the following formula:

$$AB := 9 \times (A - 1) + B$$

where A and B are the **mutually orthogonal diagonal Latin squares** of order 9. This is the same Example 2 as given above. We have written it again to bring pan magic of order 27.

We shall construct 9 magic squares of order 9 using the distribution 4 and Example 8, and put them according to Structure 1. In this case the composition considered is as follows:

$$AB := 27 \times (A - 1) + B$$

i.e., instead of 9, the multiplications is with 27.

Construction 3. *Below are 9 magic squares of order 9 constructed according to Example 8 using the Distribution 4:*

- **Block 11**

(1)									126
1	18	22	19	8	13	12	27	6	126
13	19	8	6	12	27	22	1	18	126
27	6	12	18	22	1	8	13	19	126
6	12	27	22	1	18	13	19	8	126
18	22	1	8	13	19	27	6	12	126
19	8	13	12	27	6	1	18	22	126
8	13	19	27	6	12	18	22	1	126
12	27	6	1	18	22	19	8	13	126
22	1	18	13	19	8	6	12	27	126
126	126	126	126	126	126	126	126	126	126

(1)									126
22	12	8	19	18	6	27	13	1	126
1	27	13	8	22	12	6	19	18	126
18	6	19	13	1	27	12	8	22	126
13	1	27	12	8	22	18	6	19	126
19	18	6	27	13	1	22	12	8	126
8	22	12	6	19	18	1	27	13	126
6	19	18	1	27	13	8	22	12	126
12	8	22	18	6	19	13	1	27	126
27	13	1	22	12	8	19	18	6	126
126	126	126	126	126	126	126	126	126	126

(11)									3285
22	471	575	505	207	330	324	715	136	3285
325	513	202	143	319	714	573	19	477	3285
720	141	316	472	568	27	201	332	508	3285
148	298	729	579	8	481	342	492	208	3285
478	585	6	216	337	487	724	147	305	3285
494	211	336	303	721	153	1	486	580	3285
195	343	504	703	162	310	467	589	12	3285
309	710	157	18	465	586	499	190	351	3285
594	13	460	346	498	197	154	315	708	3285
3285	3285	3285	3285	3285	3285	3285	3285	3285	3285

- **Block 12**

(1)									126
1	18	22	19	8	13	12	27	6	126
13	19	8	6	12	27	22	1	18	126
27	6	12	18	22	1	8	13	19	126
6	12	27	22	1	18	13	19	8	126
18	22	1	8	13	19	27	6	12	126
19	8	13	12	27	6	1	18	22	126
8	13	19	27	6	12	18	22	1	126
12	27	6	1	18	22	19	8	13	126
22	1	18	13	19	8	6	12	27	126
126	126	126	126	126	126	126	126	126	126

(2)									126
23	11	7	20	17	5	26	14	3	126
3	26	14	7	23	11	5	20	17	126
17	5	20	14	3	26	11	7	23	126
14	3	26	11	7	23	17	5	20	126
20	17	5	26	14	3	23	11	7	126
7	23	11	5	20	17	3	26	14	126
5	20	17	3	26	14	7	23	11	126
11	7	23	17	5	20	14	3	26	126
26	14	3	23	11	7	20	17	5	126
126	126	126	126	126	126	126	126	126	126

12									3285
23	470	574	506	206	329	323	716	138	3285
327	512	203	142	320	713	572	20	476	3285
719	140	317	473	570	26	200	331	509	3285
149	300	728	578	7	482	341	491	209	3285
479	584	5	215	338	489	725	146	304	3285
493	212	335	302	722	152	3	485	581	3285
194	344	503	705	161	311	466	590	11	3285
308	709	158	17	464	587	500	192	350	3285
593	14	462	347	497	196	155	314	707	3285
3285	3285	3285	3285	3285	3285	3285	3285	3285	3285

- **Block 13**

1									126
1	18	22	19	8	13	12	27	6	126
13	19	8	6	12	27	22	1	18	126
27	6	12	18	22	1	8	13	19	126
6	12	27	22	1	18	13	19	8	126
18	22	1	8	13	19	27	6	12	126
19	8	13	12	27	6	1	18	22	126
8	13	19	27	6	12	18	22	1	126
12	27	6	1	18	22	19	8	13	126
22	1	18	13	19	8	6	12	27	126
126	126	126	126	126	126	126	126	126	126

3									126
24	10	9	21	16	4	25	15	2	126
2	25	15	9	24	10	4	21	16	126
16	4	21	15	2	25	10	9	24	126
15	2	25	10	9	24	16	4	21	126
21	16	4	25	15	2	24	10	9	126
9	24	10	4	21	16	2	25	15	126
4	21	16	2	25	15	9	24	10	126
10	9	24	16	4	21	15	2	25	126
25	15	2	24	10	9	21	16	4	126
126	126	126	126	126	126	126	126	126	126

13									3285
24	469	576	507	205	328	322	717	137	3285
326	511	204	144	321	712	571	21	475	3285
718	139	318	474	569	25	199	333	510	3285
150	299	727	577	9	483	340	490	210	3285
480	583	4	214	339	488	726	145	306	3285
495	213	334	301	723	151	2	484	582	3285
193	345	502	704	160	312	468	591	10	3285
307	711	159	16	463	588	501	191	349	3285
592	15	461	348	496	198	156	313	706	3285
3285	3285	3285	3285	3285	3285	3285	3285	3285	3285

- **Block 21**

(2)									126
3	17	23	20	7	14	11	26	5	126
14	20	7	5	11	26	23	3	17	126
26	5	11	17	23	3	7	14	20	126
5	11	26	23	3	17	14	20	7	126
17	23	3	7	14	20	26	5	11	126
20	7	14	11	26	5	3	17	23	126
7	14	20	26	5	11	17	23	3	126
11	26	5	3	17	23	20	7	14	126
23	3	17	14	20	7	5	11	26	126
126	126	126	126	126	126	126	126	126	126

(1)									126
22	12	8	19	18	6	27	13	1	126
1	27	13	8	22	12	6	19	18	126
18	6	19	13	1	27	12	8	22	126
13	1	27	12	8	22	18	6	19	126
19	18	6	27	13	1	22	12	8	126
8	22	12	6	19	18	1	27	13	126
6	19	18	1	27	13	8	22	12	126
12	8	22	18	6	19	13	1	27	126
27	13	1	22	12	8	19	18	6	126
126	126	126	126	126	126	126	126	126	126

(21)									3285
76	444	602	532	180	357	297	688	109	3285
352	540	175	116	292	687	600	73	450	3285
693	114	289	445	595	81	174	359	535	3285
121	271	702	606	62	454	369	519	181	3285
451	612	60	189	364	514	697	120	278	3285
521	184	363	276	694	126	55	459	607	3285
168	370	531	676	135	283	440	616	66	3285
282	683	130	72	438	613	526	163	378	3285
621	67	433	373	525	170	127	288	681	3285
3285	3285	3285	3285	3285	3285	3285	3285	3285	3285

- **Block 22**

(2)									126
3	17	23	20	7	14	11	26	5	126
14	20	7	5	11	26	23	3	17	126
26	5	11	17	23	3	7	14	20	126
5	11	26	23	3	17	14	20	7	126
17	23	3	7	14	20	26	5	11	126
20	7	14	11	26	5	3	17	23	126
7	14	20	26	5	11	17	23	3	126
11	26	5	3	17	23	20	7	14	126
23	3	17	14	20	7	5	11	26	126
126	126	126	126	126	126	126	126	126	126

(2)									126
23	11	7	20	17	5	26	14	3	126
3	26	14	7	23	11	5	20	17	126
17	5	20	14	3	26	11	7	23	126
14	3	26	11	7	23	17	5	20	126
20	17	5	26	14	3	23	11	7	126
7	23	11	5	20	17	3	26	14	126
5	20	17	3	26	14	7	23	11	126
11	7	23	17	5	20	14	3	26	126
26	14	3	23	11	7	20	17	5	126
126	126	126	126	126	126	126	126	126	126

(22)									3285
77	443	601	533	179	356	296	689	111	3285
354	539	176	115	293	686	599	74	449	3285
692	113	290	446	597	80	173	358	536	3285
122	273	701	605	61	455	368	518	182	3285
452	611	59	188	365	516	698	119	277	3285
520	185	362	275	695	125	57	458	608	3285
167	371	530	678	134	284	439	617	65	3285
281	682	131	71	437	614	527	165	377	3285
620	68	435	374	524	169	128	287	680	3285
3285	3285	3285	3285	3285	3285	3285	3285	3285	3285

- **Block 23**

(2)									126
3	17	23	20	7	14	11	26	5	126
14	20	7	5	11	26	23	3	17	126
26	5	11	17	23	3	7	14	20	126
5	11	26	23	3	17	14	20	7	126
17	23	3	7	14	20	26	5	11	126
20	7	14	11	26	5	3	17	23	126
7	14	20	26	5	11	17	23	3	126
11	26	5	3	17	23	20	7	14	126
23	3	17	14	20	7	5	11	26	126
126	126	126	126	126	126	126	126	126	126

(2)									126
24	10	9	21	16	4	25	15	2	126
2	25	15	9	24	10	4	21	16	126
16	4	21	15	2	25	10	9	24	126
15	2	25	10	9	24	16	4	21	126
21	16	4	25	15	2	24	10	9	126
9	24	10	4	21	16	2	25	15	126
4	21	16	2	25	15	9	24	10	126
10	9	24	16	4	21	15	2	25	126
25	15	2	24	10	9	21	16	4	126
126	126	126	126	126	126	126	126	126	126

(23)									3285
78	442	603	534	178	355	295	690	110	3285
353	538	177	117	294	685	598	75	448	3285
691	112	291	447	596	79	172	360	537	3285
123	272	700	604	63	456	367	517	183	3285
453	610	58	187	366	515	699	118	279	3285
522	186	361	274	696	124	56	457	609	3285
166	372	529	677	133	285	441	618	64	3285
280	684	132	70	436	615	528	164	376	3285
619	69	434	375	523	171	129	286	679	3285
3285	3285	3285	3285	3285	3285	3285	3285	3285	3285

- **Block 31**

(3)									126
2	16	24	21	9	15	10	25	4	126
15	21	9	4	10	25	24	2	16	126
25	4	10	16	24	2	9	15	21	126
4	10	25	24	2	16	15	21	9	126
16	24	2	9	15	21	25	4	10	126
21	9	15	10	25	4	2	16	24	126
9	15	21	25	4	10	16	24	2	126
10	25	4	2	16	24	21	9	15	126
24	2	16	15	21	9	4	10	25	126
126	126	126	126	126	126	126	126	126	126

(1)									126
22	12	8	19	18	6	27	13	1	126
1	27	13	8	22	12	6	19	18	126
18	6	19	13	1	27	12	8	22	126
13	1	27	12	8	22	18	6	19	126
19	18	6	27	13	1	22	12	8	126
8	22	12	6	19	18	1	27	13	126
6	19	18	1	27	13	8	22	12	126
12	8	22	18	6	19	13	1	27	126
27	13	1	22	12	8	19	18	6	126
126	126	126	126	126	126	126	126	126	126

(31)									3285
49	417	629	559	234	384	270	661	82	3285
379	567	229	89	265	660	627	46	423	3285
666	87	262	418	622	54	228	386	562	3285
94	244	675	633	35	427	396	546	235	3285
424	639	33	243	391	541	670	93	251	3285
548	238	390	249	667	99	28	432	634	3285
222	397	558	649	108	256	413	643	39	3285
255	656	103	45	411	640	553	217	405	3285
648	40	406	400	552	224	100	261	654	3285
3285	3285	3285	3285	3285	3285	3285	3285	3285	3285

- **Block 32**

(3)									126
2	16	24	21	9	15	10	25	4	126
15	21	9	4	10	25	24	2	16	126
25	4	10	16	24	2	9	15	21	126
4	10	25	24	2	16	15	21	9	126
16	24	2	9	15	21	25	4	10	126
21	9	15	10	25	4	2	16	24	126
9	15	21	25	4	10	16	24	2	126
10	25	4	2	16	24	21	9	15	126
24	2	16	15	21	9	4	10	25	126
126	126	126	126	126	126	126	126	126	126

(2)									126
23	11	7	20	17	5	26	14	3	126
3	26	14	7	23	11	5	20	17	126
17	5	20	14	3	26	11	7	23	126
14	3	26	11	7	23	17	5	20	126
20	17	5	26	14	3	23	11	7	126
7	23	11	5	20	17	3	26	14	126
5	20	17	3	26	14	7	23	11	126
11	7	23	17	5	20	14	3	26	126
26	14	3	23	11	7	20	17	5	126
126	126	126	126	126	126	126	126	126	126

(32)									3285
50	416	628	560	233	383	269	662	84	3285
381	566	230	88	266	659	626	47	422	3285
665	86	263	419	624	53	227	385	563	3285
95	246	674	632	34	428	395	545	236	3285
425	638	32	242	392	543	671	92	250	3285
547	239	389	248	668	98	30	431	635	3285
221	398	557	651	107	257	412	644	38	3285
254	655	104	44	410	641	554	219	404	3285
647	41	408	401	551	223	101	260	653	3285
3285	3285	3285	3285	3285	3285	3285	3285	3285	3285

- **Block 33**

(3)									126
2	16	24	21	9	15	10	25	4	126
15	21	9	4	10	25	24	2	16	126
25	4	10	16	24	2	9	15	21	126
4	10	25	24	2	16	15	21	9	126
16	24	2	9	15	21	25	4	10	126
21	9	15	10	25	4	2	16	24	126
9	15	21	25	4	10	16	24	2	126
10	25	4	2	16	24	21	9	15	126
24	2	16	15	21	9	4	10	25	126
126	126	126	126	126	126	126	126	126	126

(3)									126
24	10	9	21	16	4	25	15	2	126
2	25	15	9	24	10	4	21	16	126
16	4	21	15	2	25	10	9	24	126
15	2	25	10	9	24	16	4	21	126
21	16	4	25	15	2	24	10	9	126
9	24	10	4	21	16	2	25	15	126
4	21	16	2	25	15	9	24	10	126
10	9	24	16	4	21	15	2	25	126
25	15	2	24	10	9	21	16	4	126
126	126	126	126	126	126	126	126	126	126

(33)									3285
51	415	630	561	232	382	268	663	83	3285
380	565	231	90	267	658	625	48	421	3285
664	85	264	420	623	52	226	387	564	3285
96	245	673	631	36	429	394	544	237	3285
426	637	31	241	393	542	672	91	252	3285
549	240	388	247	669	97	29	430	636	3285
220	399	556	650	106	258	414	645	37	3285
253	657	105	43	409	642	555	218	403	3285
646	42	407	402	550	225	102	259	652	3285
3285	3285	3285	3285	3285	3285	3285	3285	3285	3285

In all the 9 blocks, the composition is applied by use of following formula:

$$AB := 27 \times (A-1) + B$$

Combining the above 9 magic squares of order 9 according to structure 1 we get a **pan magic square** of order 27 given in the example below.

Example 9. *According to Distribution 4, Structure 1 and 9 magic squares given above , we have a* ***pan magic square*** *of order 27 given by*

I	1	2	3	4	5	6	7	8	9	10	11	12	13
		9855	9855	9855	9855	9855	9855	9855	9855	9855	9855	9855	9855
	22	471	575	505	207	330	324	715	136	23	470	574	506
9855	325	513	202	143	319	714	573	19	477	327	512	203	142
9855	720	141	316	472	568	27	201	332	508	719	140	317	473
9855	148	298	729	579	8	481	342	492	208	149	300	728	578
9855	478	585	6	216	337	487	724	147	305	479	584	5	215
9855	494	211	336	303	721	153	1	486	580	493	212	335	302
9855	195	343	504	703	162	310	467	589	12	194	344	503	705
9855	309	710	157	18	465	586	499	190	351	308	709	158	17
9855	594	13	460	346	498	197	154	315	708	593	14	462	347
9855	76	444	602	532	180	357	297	688	109	77	443	601	533
9855	352	540	175	116	292	687	600	73	450	354	539	176	115
9855	693	114	289	445	595	81	174	359	535	692	113	290	446
9855	121	271	702	606	62	454	369	519	181	122	273	701	605
9855	451	612	60	189	364	514	697	120	278	452	611	59	188
9855	521	184	363	276	694	126	55	459	607	520	185	362	275
9855	168	370	531	676	135	283	440	616	66	167	371	530	678
9855	282	683	130	72	438	613	526	163	378	281	682	131	71
9855	621	67	433	373	525	170	127	288	681	620	68	435	374
9855	49	417	629	559	234	384	270	661	82	50	416	628	560
9855	379	567	229	89	265	660	627	46	423	381	566	230	88
9855	666	87	262	418	622	54	228	386	562	665	86	263	419
9855	94	244	675	633	35	427	396	546	235	95	246	674	632
9855	424	639	33	243	391	541	670	93	251	425	638	32	242
9855	548	238	390	249	667	99	28	432	634	547	239	389	248
9855	222	397	558	649	108	256	413	643	39	221	398	557	651
9855	255	656	103	45	411	640	553	217	405	254	655	104	44
9855	648	40	406	400	552	224	100	261	654	647	41	408	401
	9855	9855	9855	9855	9855	9855	9855	9855	9855	9855	9855	9855	9855

14	15	16	17	18	19	20	21	22	23	24	25	26	27	(II)
9855	9855	9855	9855	9855	9855	9855	9855	9855	9855	9855	9855	9855	9855	9855
206	329	323	716	138	24	469	576	507	205	328	322	717	137	9855
320	713	572	20	476	326	511	204	144	321	712	571	21	475	9855
570	26	200	331	509	718	139	318	474	569	25	199	333	510	9855
7	482	341	491	209	150	299	727	577	9	483	340	490	210	9855
338	489	725	146	304	480	583	4	214	339	488	726	145	306	9855
722	152	3	485	581	495	213	334	301	723	151	2	484	582	9855
161	311	466	590	11	193	345	502	704	160	312	468	591	10	9855
464	587	500	192	350	307	711	159	16	463	588	501	191	349	9855
497	196	155	314	707	592	15	461	348	496	198	156	313	706	9855
179	356	296	689	111	78	442	603	534	178	355	295	690	110	9855
293	686	599	74	449	353	538	177	117	294	685	598	75	448	9855
597	80	173	358	536	691	112	291	447	596	79	172	360	537	9855
61	455	368	518	182	123	272	700	604	63	456	367	517	183	9855
365	516	698	119	277	453	610	58	187	366	515	699	118	279	9855
695	125	57	458	608	522	186	361	274	696	124	56	457	609	9855
134	284	439	617	65	166	372	529	677	133	285	441	618	64	9855
437	614	527	165	377	280	684	132	70	436	615	528	164	376	9855
524	169	128	287	680	619	69	434	375	523	171	129	286	679	9855
233	383	269	662	84	51	415	630	561	232	382	268	663	83	9855
266	659	626	47	422	380	565	231	90	267	658	625	48	421	9855
624	53	227	385	563	664	85	264	420	623	52	226	387	564	9855
34	428	395	545	236	96	245	673	631	36	429	394	544	237	9855
392	543	671	92	250	426	637	31	241	393	542	672	91	252	9855
668	98	30	431	635	549	240	388	247	669	97	29	430	636	9855
107	257	412	644	38	220	399	556	650	106	258	414	645	37	9855
410	641	554	219	404	253	657	105	43	409	642	555	218	403	9855
551	223	101	260	653	646	42	407	402	550	225	102	259	652	9855
9855	9855	9855	9855	9855	9855	9855	9855	9855	9855	9855	9855	9855	9855	9855

Combining parts (I) and (II) we get a **pan magic square** of order 27 with each block of order 9 a magic squares of equal sums. In this case, the magic sum is $S_{27\times 27}=9855$. Each 9×9 block is a **magic square** of order 9 with equal magic sum $S_{9\times 9}=3285$.

We observed that in the above example we get a **pan magic square** of order 27. But each block of order 9 is a magic square, but not pan. Below is example due to Dwane [6], where the each block of order 9 is also a pan diagonal.

Example 10. ***Dwane's Construction [6]:*** *The following example of* ***pan magic squares*** *of order 27 with each sub-block of order 9 also a* ***pan magic square****:*

I	1	2	3	4	5	6	7	8	9	10	11	12	13
		9855	9855	9855	9855	9855	9855	9855	9855	9855	9855	9855	9855
	22	471	575	505	207	330	324	715	136	23	470	574	506
9855	325	513	202	143	319	714	573	19	477	327	512	203	142
9855	720	141	316	472	568	27	201	332	508	719	140	317	473
9855	148	298	729	579	8	481	342	492	208	149	300	728	578
9855	478	585	6	216	337	487	724	147	305	479	584	5	215
9855	494	211	336	303	721	153	1	486	580	493	212	335	302
9855	195	343	504	703	162	310	467	589	12	194	344	503	705
9855	309	710	157	18	465	586	499	190	351	308	709	158	17
9855	594	13	460	346	498	197	154	315	708	593	14	462	347
9855	76	444	602	532	180	357	297	688	109	77	443	601	533
9855	352	540	175	116	292	687	600	73	450	354	539	176	115
9855	693	114	289	445	595	81	174	359	535	692	113	290	446
9855	121	271	702	606	62	454	369	519	181	122	273	701	605
9855	451	612	60	189	364	514	697	120	278	452	611	59	188
9855	521	184	363	276	694	126	55	459	607	520	185	362	275
9855	168	370	531	676	135	283	440	616	66	167	371	530	678
9855	282	683	130	72	438	613	526	163	378	281	682	131	71
9855	621	67	433	373	525	170	127	288	681	620	68	435	374
9855	49	417	629	559	234	384	270	661	82	50	416	628	560
9855	379	567	229	89	265	660	627	46	423	381	566	230	88
9855	666	87	262	418	622	54	228	386	562	665	86	263	419
9855	94	244	675	633	35	427	396	546	235	95	246	674	632
9855	424	639	33	243	391	541	670	93	251	425	638	32	242
9855	548	238	390	249	667	99	28	432	634	547	239	389	248
9855	222	397	558	649	108	256	413	643	39	221	398	557	651
9855	255	656	103	45	411	640	553	217	405	254	655	104	44
9855	648	40	406	400	552	224	100	261	654	647	41	408	401
	9855	9855	9855	9855	9855	9855	9855	9855	9855	9855	9855	9855	9855

14	15	16	17	18	19	20	21	22	23	24	25	26	27	(II)
9855	9855	9855	9855	9855	9855	9855	9855	9855	9855	9855	9855	9855	9855	9855
206	329	323	716	138	24	469	576	507	205	328	322	717	137	9855
320	713	572	20	476	326	511	204	144	321	712	571	21	475	9855
570	26	200	331	509	718	139	318	474	569	25	199	333	510	9855
7	482	341	491	209	150	299	727	577	9	483	340	490	210	9855
338	489	725	146	304	480	583	4	214	339	488	726	145	306	9855
722	152	3	485	581	495	213	334	301	723	151	2	484	582	9855
161	311	466	590	11	193	345	502	704	160	312	468	591	10	9855
464	587	500	192	350	307	711	159	16	463	588	501	191	349	9855
497	196	155	314	707	592	15	461	348	496	198	156	313	706	9855
179	356	296	689	111	78	442	603	534	178	355	295	690	110	9855
293	686	599	74	449	353	538	177	117	294	685	598	75	448	9855
597	80	173	358	536	691	112	291	447	596	79	172	360	537	9855
61	455	368	518	182	123	272	700	604	63	456	367	517	183	9855
365	516	698	119	277	453	610	58	187	366	515	699	118	279	9855
695	125	57	458	608	522	186	361	274	696	124	56	457	609	9855
134	284	439	617	65	166	372	529	677	133	285	441	618	64	9855
437	614	527	165	377	280	684	132	70	436	615	528	164	376	9855
524	169	128	287	680	619	69	434	375	523	171	129	286	679	9855
233	383	269	662	84	51	415	630	561	232	382	268	663	83	9855
266	659	626	47	422	380	565	231	90	267	658	625	48	421	9855
624	53	227	385	563	664	85	264	420	623	52	226	387	564	9855
34	428	395	545	236	96	245	673	631	36	429	394	544	237	9855
392	543	671	92	250	426	637	31	241	393	542	672	91	252	9855
668	98	30	431	635	549	240	388	247	669	97	29	430	636	9855
107	257	412	644	38	220	399	556	650	106	258	414	645	37	9855
410	641	554	219	404	253	657	105	43	409	642	555	218	403	9855
551	223	101	260	653	646	42	407	402	550	225	102	259	652	9855
9855	9855	9855	9855	9855	9855	9855	9855	9855	9855	9855	9855	9855	9855	9855

Combining parts (I) and (II) we get a **pan magic square** of order 27 with each block of order 9 **pan magic squares** of equal sums. In this case, the magic sum is $S_{27\times 27} = 9855$. Each 9×9 block is also **pan magic square** of order 9 with equal magic sum $S_{9\times 9} = 3285$. 3×3 block also sums to same values, i.e., $S_9 = 3285$. This example is Dwane's construction. More details can be seen in his site [6].

2.5 Pan Magic Square of Order 33

Distribution 5. *Let's consider the following distribution to construct* ***pan magic square*** *of order 33:*

(1)	1	6	8	12	13	18	19	24	25	30	31	187
(2)	3	5	7	11	14	17	20	23	26	29	32	187
(3)	2	4	9	10	15	16	21	22	27	28	33	187

The construction of **pan magic square** of order 33 with each sub-block of same sum is based on the following pan diagonal magic square of order 11 constructed using a pair of mutually orthogonal diagonal Latin squares:

Example 11. *Let's consider the following* ***pan diagonal magic square*** *of order 11:*

AB		671	671	671	671	671	671	671	671	671	671	671
	1	13	25	37	49	61	73	85	97	109	121	671
671	108	120	11	12	24	36	48	60	72	84	96	671
671	83	95	107	119	10	22	23	35	47	59	71	671
671	58	70	82	94	106	118	9	21	33	34	46	671
671	44	45	57	69	81	93	105	117	8	20	32	671
671	19	31	43	55	56	68	80	92	104	116	7	671
671	115	6	18	30	42	54	66	67	79	91	103	671
671	90	102	114	5	17	29	41	53	65	77	78	671
671	76	88	89	101	113	4	16	28	40	52	64	671
671	51	63	75	87	99	100	112	3	15	27	39	671
671	26	38	50	62	74	86	98	110	111	2	14	671
	671	671	671	671	671	671	671	671	671	671	671	671

Example 12. *The above* ***pan magic square*** *of order 11 is constructed based on pair of* ***mutually diagonal Latin squares*** *A and B given by*

A		66	66	66	66	66	66	66	66	66	66	66
	1	2	3	4	5	6	7	8	9	10	11	66
66	10	11	1	2	3	4	5	6	7	8	9	66
66	8	9	10	11	1	2	3	4	5	6	7	66
66	6	7	8	9	10	11	1	2	3	4	5	66
66	4	5	6	7	8	9	10	11	1	2	3	66
66	2	3	4	5	6	7	8	9	10	11	1	66
66	11	1	2	3	4	5	6	7	8	9	10	66
66	9	10	11	1	2	3	4	5	6	7	8	66
66	7	8	9	10	11	1	2	3	4	5	6	66
66	5	6	7	8	9	10	11	1	2	3	4	66
66	3	4	5	6	7	8	9	10	11	1	2	66
	66	66	66	66	66	66	66	66	66	66	66	66

B		66	66	66	66	66	66	66	66	66	66	66
	1	2	3	4	5	6	7	8	9	10	11	66
66	9	10	11	1	2	3	4	5	6	7	8	66
66	6	7	8	9	10	11	1	2	3	4	5	66
66	3	4	5	6	7	8	9	10	11	1	2	66
66	11	1	2	3	4	5	6	7	8	9	10	66
66	8	9	10	11	1	2	3	4	5	6	7	66
66	5	6	7	8	9	10	11	1	2	3	4	66
66	2	3	4	5	6	7	8	9	10	11	1	66
66	10	11	1	2	3	4	5	6	7	8	9	66
66	7	8	9	10	11	1	2	3	4	5	6	66
66	4	5	6	7	8	9	10	11	1	2	3	66
	66	66	66	66	66	66	66	66	66	66	66	66

The **pan magic square** appearing in ***AB*** is constructed according to the following formula:

$$AB := 11 \times (A-1) + B$$

where A and B are the **mutually orthogonal diagonal Latin squares** of order 11 as given above.

We shall construct 9 magic squares of order 11 using the distribution 5 and Example 11, and put them according to Structure 1. In this case, the composition considered is as follows:

$$AB := 33 \times (A-1) + B$$

i.e., instead of 11, the multiplications is with 33.

In the Examples of orders 15, 21 and 27 respectively given in 5, 7 and 9, all the 9 blocks are constructed based on a pair of **mutual orthogonal diagonal Latin squares**. The same process is applied for the **pan magic square** of order 33. In this case, we won't write the Latin squares, such as A and B. Only composite magic squares of type AB are written to construct **pan magic square** of order 33.

Construction 4. *Below are 9* ***pan magic squares*** *of order 11 constructed according to Example 11 using the Distribution 5:*

- **Block 11**

11		5995	5995	5995	5995	5995	5995	5995	5995	5995	5995	5995
	1	171	239	375	409	579	613	783	817	987	1021	5995
5995	982	1020	31	166	237	371	408	574	612	778	816	5995
5995	777	811	981	1015	30	196	232	369	404	573	607	5995
5995	569	606	772	810	976	1014	25	195	262	364	402	5995
5995	394	397	567	602	771	805	975	1009	24	190	261	5995
5995	189	256	393	427	562	600	767	804	970	1008	19	5995
5995	1003	18	184	255	388	426	592	595	765	800	969	5995
5995	798	965	1002	13	183	250	387	421	591	625	760	5995
5995	624	790	793	963	998	12	178	249	382	420	586	5995
5995	415	585	619	789	823	958	996	8	177	244	381	5995
5995	243	376	414	580	618	784	822	988	991	6	173	5995
	5995	5995	5995	5995	5995	5995	5995	5995	5995	5995	5995	5995

- **Block 12**

12		5995	5995	5995	5995	5995	5995	5995	5995	5995	5995	5995
	3	170	238	374	410	578	614	782	818	986	1022	5995
5995	983	1019	32	168	236	370	407	575	611	779	815	5995
5995	776	812	980	1016	29	197	234	368	403	572	608	5995
5995	568	605	773	809	977	1013	26	194	263	366	401	5995
5995	395	399	566	601	770	806	974	1010	23	191	260	5995
5995	188	257	392	428	564	599	766	803	971	1007	20	5995
5995	1004	17	185	254	389	425	593	597	764	799	968	5995
5995	797	964	1001	14	182	251	386	422	590	626	762	5995
5995	623	791	795	962	997	11	179	248	383	419	587	5995
5995	416	584	620	788	824	960	995	7	176	245	380	5995
5995	242	377	413	581	617	785	821	989	993	5	172	5995
	5995	5995	5995	5995	5995	5995	5995	5995	5995	5995	5995	5995

- **Block 13**

13		5995	5995	5995	5995	5995	5995	5995	5995	5995	5995	5995
	2	169	240	373	411	577	615	781	819	985	1023	5995
5995	984	1018	33	167	235	372	406	576	610	780	814	5995
5995	775	813	979	1017	28	198	233	367	405	571	609	5995
5995	570	604	774	808	978	1012	27	193	264	365	400	5995
5995	396	398	565	603	769	807	973	1011	22	192	259	5995
5995	187	258	391	429	563	598	768	802	972	1006	21	5995
5995	1005	16	186	253	390	424	594	596	763	801	967	5995
5995	796	966	1000	15	181	252	385	423	589	627	761	5995
5995	622	792	794	961	999	10	180	247	384	418	588	5995
5995	417	583	621	787	825	959	994	9	175	246	379	5995
5995	241	378	412	582	616	786	820	990	992	4	174	5995
	5995	5995	5995	5995	5995	5995	5995	5995	5995	5995	5995	5995

- **Block 21**

21		5995	5995	5995	5995	5995	5995	5995	5995	5995	5995	5995
	67	138	206	342	442	546	646	750	850	954	1054	5995
5995	949	1053	97	133	204	338	441	541	645	745	849	5995
5995	744	844	948	1048	96	163	199	336	437	540	640	5995
5995	536	639	739	843	943	1047	91	162	229	331	435	5995
5995	361	430	534	635	738	838	942	1042	90	157	228	5995
5995	156	223	360	460	529	633	734	837	937	1041	85	5995
5995	1036	84	151	222	355	459	559	628	732	833	936	5995
5995	831	932	1035	79	150	217	354	454	558	658	727	5995
5995	657	757	826	930	1031	78	145	216	349	453	553	5995
5995	448	552	652	756	856	925	1029	74	144	211	348	5995
5995	210	343	447	547	651	751	855	955	1024	72	140	5995
	5995	5995	5995	5995	5995	5995	5995	5995	5995	5995	5995	5995

- **Block 22**

22		5995	5995	5995	5995	5995	5995	5995	5995	5995	5995	5995
	69	137	205	341	443	545	647	749	851	953	1055	5995
5995	950	1052	98	135	203	337	440	542	644	746	848	5995
5995	743	845	947	1049	95	164	201	335	436	539	641	5995
5995	535	638	740	842	944	1046	92	161	230	333	434	5995
5995	362	432	533	634	737	839	941	1043	89	158	227	5995
5995	155	224	359	461	531	632	733	836	938	1040	86	5995
5995	1037	83	152	221	356	458	560	630	731	832	935	5995
5995	830	931	1034	80	149	218	353	455	557	659	729	5995
5995	656	758	828	929	1030	77	146	215	350	452	554	5995
5995	449	551	653	755	857	927	1028	73	143	212	347	5995
5995	209	344	446	548	650	752	854	956	1026	71	139	5995
	5995	5995	5995	5995	5995	5995	5995	5995	5995	5995	5995	5995

- **Block 23**

23		5995	5995	5995	5995	5995	5995	5995	5995	5995	5995	5995
	68	136	207	340	444	544	648	748	852	952	1056	5995
5995	951	1051	99	134	202	339	439	543	643	747	847	5995
5995	742	846	946	1050	94	165	200	334	438	538	642	5995
5995	537	637	741	841	945	1045	93	160	231	332	433	5995
5995	363	431	532	636	736	840	940	1044	88	159	226	5995
5995	154	225	358	462	530	631	735	835	939	1039	87	5995
5995	1038	82	153	220	357	457	561	629	730	834	934	5995
5995	829	933	1033	81	148	219	352	456	556	660	728	5995
5995	655	759	827	928	1032	76	147	214	351	451	555	5995
5995	450	550	654	754	858	926	1027	75	142	213	346	5995
5995	208	345	445	549	649	753	853	957	1025	70	141	5995
	5995	5995	5995	5995	5995	5995	5995	5995	5995	5995	5995	5995

- **Block 31**

31		5995	5995	5995	5995	5995	5995	5995	5995	5995	5995	5995
	34	105	272	309	475	513	679	717	883	921	1087	5995
5995	916	1086	64	100	270	305	474	508	678	712	882	5995
5995	711	877	915	1081	63	130	265	303	470	507	673	5995
5995	503	672	706	876	910	1080	58	129	295	298	468	5995
5995	328	463	501	668	705	871	909	1075	57	124	294	5995
5995	123	289	327	493	496	666	701	870	904	1074	52	5995
5995	1069	51	118	288	322	492	526	661	699	866	903	5995
5995	864	899	1068	46	117	283	321	487	525	691	694	5995
5995	690	724	859	897	1064	45	112	282	316	486	520	5995
5995	481	519	685	723	889	892	1062	41	111	277	315	5995
5995	276	310	480	514	684	718	888	922	1057	39	107	5995
	5995	5995	5995	5995	5995	5995	5995	5995	5995	5995	5995	5995

- **Block 32**

32		5995	5995	5995	5995	5995	5995	5995	5995	5995	5995	5995
	36	104	271	308	476	512	680	716	884	920	1088	5995
5995	917	1085	65	102	269	304	473	509	677	713	881	5995
5995	710	878	914	1082	62	131	267	302	469	506	674	5995
5995	502	671	707	875	911	1079	59	128	296	300	467	5995
5995	329	465	500	667	704	872	908	1076	56	125	293	5995
5995	122	290	326	494	498	665	700	869	905	1073	53	5995
5995	1070	50	119	287	323	491	527	663	698	865	902	5995
5995	863	898	1067	47	116	284	320	488	524	692	696	5995
5995	689	725	861	896	1063	44	113	281	317	485	521	5995
5995	482	518	686	722	890	894	1061	40	110	278	314	5995
5995	275	311	479	515	683	719	887	923	1059	38	106	5995
	5995	5995	5995	5995	5995	5995	5995	5995	5995	5995	5995	5995

- **Block 33**

(33)		5995	5995	5995	5995	5995	5995	5995	5995	5995	5995	5995
	35	103	273	307	477	511	681	715	885	919	1089	5995
5995	918	1084	66	101	268	306	472	510	676	714	880	5995
5995	709	879	913	1083	61	132	266	301	471	505	675	5995
5995	504	670	708	874	912	1078	60	127	297	299	466	5995
5995	330	464	499	669	703	873	907	1077	55	126	292	5995
5995	121	291	325	495	497	664	702	868	906	1072	54	5995
5995	1071	49	120	286	324	490	528	662	697	867	901	5995
5995	862	900	1066	48	115	285	319	489	523	693	695	5995
5995	688	726	860	895	1065	43	114	280	318	484	522	5995
5995	483	517	687	721	891	893	1060	42	109	279	313	5995
5995	274	312	478	516	682	720	886	924	1058	37	108	5995
	5995	5995	5995	5995	5995	5995	5995	5995	5995	5995	5995	5995

Combining the above 9 magic squares of order 11 according to structure 1 we get a **pan magic square** of order 33 given in the example below.

Example 13. *According to Distribution 5, Structure 1 and 9 magic squares given above, a* ***pan magic square*** *of order 33 is given in two parts* (I) *and* (II) *is as follows:*

(I)	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
		17985	17985	17985	17985	17985	17985	17985	17985	17985	17985	17985	17985	17985	17985	17985
17985	1	171	239	375	409	579	613	783	817	987	1021	3	170	238	374	410
17985	982	1020	31	166	237	371	408	574	612	778	816	983	1019	32	168	236
17985	777	811	981	1015	30	196	232	369	404	573	607	776	812	980	1016	29
17985	569	606	772	810	976	1014	25	195	262	364	402	568	605	773	809	977
17985	394	397	567	602	771	805	975	1009	24	190	261	395	399	566	601	770
17985	189	256	393	427	562	600	767	804	970	1008	19	188	257	392	428	564
17985	1003	18	184	255	388	426	592	595	765	800	969	1004	17	185	254	389
17985	798	965	1002	13	183	250	387	421	591	625	760	797	964	1001	14	182
17985	624	790	793	963	998	12	178	249	382	420	586	623	791	795	962	997
17985	415	585	619	789	823	958	996	8	177	244	381	416	584	620	788	824
17985	243	376	414	580	618	784	822	988	991	6	173	242	377	413	581	617
17985	67	138	206	342	442	546	646	750	850	954	1054	69	137	205	341	443
17985	949	1053	97	133	204	338	441	541	645	745	849	950	1052	98	135	203
17985	744	844	948	1048	96	163	199	336	437	540	640	743	845	947	1049	95
17985	536	639	739	843	943	1047	91	162	229	331	435	535	638	740	842	944
17985	361	430	534	635	738	838	942	1042	90	157	228	362	432	533	634	737
17985	156	223	360	460	529	633	734	837	937	1041	85	155	224	359	461	531
17985	1036	84	151	222	355	459	559	628	732	833	936	1037	83	152	221	356
17985	831	932	1035	79	150	217	354	454	558	658	727	830	931	1034	80	149
17985	657	757	826	930	1031	78	145	216	349	453	553	656	758	828	929	1030
17985	448	552	652	756	856	925	1029	74	144	211	348	449	551	653	755	857
17985	210	343	447	547	651	751	855	955	1024	72	140	209	344	446	548	650
17985	34	105	272	309	475	513	679	717	883	921	1087	36	104	271	308	476
17985	916	1086	64	100	270	305	474	508	678	712	882	917	1085	65	102	269
17985	711	877	915	1081	63	130	265	303	470	507	673	710	878	914	1082	62
17985	503	672	706	876	910	1080	58	129	295	298	468	502	671	707	875	911
17985	328	463	501	668	705	871	909	1075	57	124	294	329	465	500	667	704
17985	123	289	327	493	496	666	701	870	904	1074	52	122	290	326	494	498
17985	1069	51	118	288	322	492	526	661	699	866	903	1070	50	119	287	323
17985	864	899	1068	46	117	283	321	487	525	691	694	863	898	1067	47	116
17985	690	724	859	897	1064	45	112	282	316	486	520	689	725	861	896	1063
17985	481	519	685	723	889	892	1062	41	111	277	315	482	518	686	722	890
	276	310	480	514	684	718	888	922	1057	39	107	275	311	479	515	683
	17985	17985	17985	17985	17985	17985	17985	17985	17985	17985	17985	17985	17985	17985	17985	17985

17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	(II)
17985	17985	17985	17985	17985	17985	17985	17985	17985	17985	17985	17985	17985	17985	17985	17985	17985	17985
578	614	782	818	986	1022	2	169	240	373	411	577	615	781	819	985	1023	17985
370	407	575	611	779	815	984	1018	33	167	235	372	406	576	610	780	814	17985
197	234	368	403	572	608	775	813	979	1017	28	198	233	367	405	571	609	17985
1013	26	194	263	366	401	570	604	774	808	978	1012	27	193	264	365	400	17985
806	974	1010	23	191	260	396	398	565	603	769	807	973	1011	22	192	259	17985
599	766	803	971	1007	20	187	258	391	429	563	598	768	802	972	1006	21	17985
425	593	597	764	799	968	1005	16	186	253	390	424	594	596	763	801	967	17985
251	386	422	590	626	762	796	966	1000	15	181	252	385	423	589	627	761	17985
11	179	248	383	419	587	622	792	794	961	999	10	180	247	384	418	588	17985
960	995	7	176	245	380	417	583	621	787	825	959	994	9	175	246	379	17985
785	821	989	993	5	172	241	378	412	582	616	786	820	990	992	4	174	17985
545	647	749	851	953	1055	68	136	207	340	444	544	648	748	852	952	1056	17985
337	440	542	644	746	848	951	1051	99	134	202	339	439	543	643	747	847	17985
164	201	335	436	539	641	742	846	946	1050	94	165	200	334	438	538	642	17985
1046	92	161	230	333	434	537	637	741	841	945	1045	93	160	231	332	433	17985
839	941	1043	89	158	227	363	431	532	636	736	840	940	1044	88	159	226	17985
632	733	836	938	1040	86	154	225	358	462	530	631	735	835	939	1039	87	17985
458	560	630	731	832	935	1038	82	153	220	357	457	561	629	730	834	934	17985
218	353	455	557	659	729	829	933	1033	81	148	219	352	456	556	660	728	17985
77	146	215	350	452	554	655	759	827	928	1032	76	147	214	351	451	555	17985
927	1028	73	143	212	347	450	550	654	754	858	926	1027	75	142	213	346	17985
752	854	956	1026	71	139	208	345	445	549	649	753	853	957	1025	70	141	17985
512	680	716	884	920	1088	35	103	273	307	477	511	681	715	885	919	1089	17985
304	473	509	677	713	881	918	1084	66	101	268	306	472	510	676	714	880	17985
131	267	302	469	506	674	709	879	913	1083	61	132	266	301	471	505	675	17985
1079	59	128	296	300	467	504	670	708	874	912	1078	60	127	297	299	466	17985
872	908	1076	56	125	293	330	464	499	669	703	873	907	1077	55	126	292	17985
665	700	869	905	1073	53	121	291	325	495	497	664	702	868	906	1072	54	17985
491	527	663	698	865	902	1071	49	120	286	324	490	528	662	697	867	901	17985
284	320	488	524	692	696	862	900	1066	48	115	285	319	489	523	693	695	17985
44	113	281	317	485	521	688	726	860	895	1065	43	114	280	318	484	522	17985
894	1061	40	110	278	314	483	517	687	721	891	893	1060	42	109	279	313	17985
719	887	923	1059	38	106	274	312	478	516	682	720	886	924	1058	37	108	17985
17985	17985	17985	17985	17985	17985	17985	17985	17985	17985	17985	17985	17985	17985	17985	17985	17985	17985

Combining parts (I) and (II) we get required result. In this case, the magic sum is $S_{33\times 33} = 17985$. Each 11×11 block is a **pan magic square** of order 11 with equal magic sums $S_{11\times 11} = 5995$.

3 Even Orders Magic Squares of Order $3k$

This section brings even order magic squares of type $3k$. In case of even order the distribution is much simple than what we did in the previous section on odd order. This distribution is given in each case separately.

3.1 Pan Magic Square of Order 12

In order to construct pan magic of order 12, we need a pan magic of order 4 given in example below:

Example 14. *Let us consider a* ***pan magic square*** *of order 4.*

		34	34	34	34
	7	12	1	14	34
34	2	13	8	11	34
34	16	3	10	5	34
34	9	6	15	4	34
	34	34	34	34	34

Let's distribute total 144 numbers from 1-144 in 9 blocks 16 giving equal sum. See the distribution below.

Distribution 6. *Let's distribute the 144 numbers from 1 to 144 in 9 blocks resulting in equal sums:*

A1	1	18	19	36	37	54	55	72	73	90	91	108	109	126	127	144	1160
A2	2	17	20	35	38	53	56	71	74	89	92	107	110	125	128	143	1160
A3	3	16	21	34	39	52	57	70	75	88	93	106	111	124	129	142	1160
A4	4	15	22	33	40	51	58	69	76	87	94	105	112	123	130	141	1160
A5	5	14	23	32	41	50	59	68	77	86	95	104	113	122	131	140	1160
A6	6	13	24	31	42	49	60	67	78	85	96	103	114	121	132	139	1160
A7	7	12	25	30	43	48	61	66	79	84	97	102	115	120	133	138	1160
A8	8	11	26	29	44	47	62	65	80	83	98	101	116	119	134	137	1160
A9	9	10	27	28	45	46	63	64	81	82	99	100	117	118	135	136	1160

Above distribution is written in increasing and decreasing orders, such as, [1, 2, 3, 4, 5, 6, 7, 8, 9], [18, 17, 16, 15, 14, 13, 12, 11, 10], [19, 20, 21, 22, 23, 24, 25, 26, 27], etc. In order to construct a **pan magic square** of order 12, we shall use the following structure.

Structure 2. *Let's consider 9 blocks of order 4 given as below:*

A1	A2	A3
A4	A5	A6
A7	A8	A9

Example 15. *9 block of magic squares of order 4 constructed according to Example 14, using data given in distribution 6, and putting them according to structure 2, we get a* ***pan magic square*** *of order 12 is given by*

		870	870	870	870	870	870	870	870	870	870	870	870
	55	108	1	126	56	107	2	125	57	106	3	124	870
870	18	109	72	91	17	110	71	92	16	111	70	93	870
870	144	19	90	37	143	20	89	38	142	21	88	39	870
870	73	54	127	36	74	53	128	35	75	52	129	34	870
870	58	105	4	123	59	104	5	122	60	103	6	121	870
870	15	112	69	94	14	113	68	95	13	114	67	96	870
870	141	22	87	40	140	23	86	41	139	24	85	42	870
870	76	51	130	33	77	50	131	32	78	49	132	31	870
870	61	102	7	120	62	101	8	119	63	100	9	118	870
870	12	115	66	97	11	116	65	98	10	117	64	99	870
870	138	25	84	43	137	26	83	44	136	27	82	45	870
870	79	48	133	30	80	47	134	29	81	46	135	28	870
	870	870	870	870	870	870	870	870	870	870	870	870	870

In this case the magic sum is $S_{12\times 12} = 870$. Each 4×4 block is a **pan magic square** of order 4 with the same magic sum $S_{4\times 4} = 290$.

3.2 Magic Square of Order 18

In order to construct a magic square of order 18, we shall use a similar procedures as of order 12. Instead of pan magic of order 4, we shall use only a magic square of order 6 as given below:

Example 16. *Let us consider a magic square of order 6.*

AB						111
1	23	28	34	17	8	111
29	7	35	14	21	5	111
12	6	13	27	31	22	111
32	16	4	24	10	25	111
19	33	11	3	30	15	111
18	26	20	9	2	36	111
111	111	111	111	111	111	111

Let us divide the above magic square in two Latin squares given by

Example 17. *The Latin squares decomposition of magic square of order 6 given in Example 16 is given by*

A						21
1	4	5	6	3	2	21
5	2	6	3	4	1	21
2	1	3	5	6	4	21
6	3	1	4	2	5	21
4	6	2	1	5	3	21
3	5	4	2	1	6	21
21	21	21	21	21	21	21

B						21
1	5	4	4	5	2	21
5	1	5	2	3	5	21
6	6	1	3	1	4	21
2	4	4	6	4	1	21
1	3	5	3	6	3	21
6	2	2	3	2	6	21
21	21	21	21	21	21	21

The **magic square** of order 6 given in Example 16 is obtained as

$$AB := 6 \times (A - 1) + B$$

where A is a diagonal Latin square of order 6, B just as simple distribution of numbers of order 6.

We shall use the Example 17 to construct magic square of order 18. For this let's distribute the number 1 to 18 in three groups of 6 each with equal sums as

Distribution 7. *Let's consider the following distribution to construct* ***magic square*** *of order 18:*

1	1	6	7	12	13	18	57
2	2	5	8	11	14	17	57
3	3	4	9	10	15	16	57

Since 18 is an even number, the Distribution 7 is much simpler to write. Just put each column in increasing/decreasing orders. In order to write a magic square of order 18, we shall use Distribution 7 over example Example 17 and put them according to Structure 1. Let's see it again:

11	12	13
21	22	23
31	32	33

Example 18. *According to the values given in equation, the magic square of order 18 is given by*

																		2925
1	211	228	318	121	96	2	212	227	317	122	95	3	213	226	316	123	94	2925
229	91	319	114	205	13	230	92	320	113	206	14	231	93	321	112	207	15	2925
108	18	109	223	307	210	107	17	110	224	308	209	106	16	111	225	309	208	2925
312	120	12	216	102	217	311	119	11	215	101	218	310	118	10	214	100	219	2925
199	313	103	7	234	115	200	314	104	8	233	116	201	315	105	9	232	117	2925
126	222	204	97	6	324	125	221	203	98	5	323	124	220	202	99	4	322	2925
19	193	246	300	139	78	20	194	245	299	140	77	21	195	244	298	141	76	2925
247	73	301	132	187	31	248	74	302	131	188	32	249	75	303	130	189	33	2925
90	36	127	241	289	192	89	35	128	242	290	191	88	34	129	243	291	190	2925
294	138	30	198	84	235	293	137	29	197	83	236	292	136	28	196	82	237	2925
181	295	85	25	252	133	182	296	86	26	251	134	183	297	87	27	250	135	2925
144	240	186	79	24	306	143	239	185	80	23	305	142	238	184	81	22	304	2925
37	175	264	282	157	60	38	176	263	281	158	59	39	177	262	280	159	58	2925
265	55	283	150	169	49	266	56	284	149	170	50	267	57	285	148	171	51	2925
72	54	145	259	271	174	71	53	146	260	272	173	70	52	147	261	273	172	2925
276	156	48	180	66	253	275	155	47	179	65	254	274	154	46	178	64	255	2925
163	277	67	43	270	151	164	278	68	44	269	152	165	279	69	45	268	153	2925
162	258	168	61	42	288	161	257	167	62	41	287	160	256	166	63	40	286	2925
2925	2925	2925	2925	2925	2925	2925	2925	2925	2925	2925	2925	2925	2925	2925	2925	2925	2925	2925

The above magic square is with magic sum $S_{18\times18} = 2925$. All the entries in each 6×6 block are with same sum, i.e. $S_{36} = 5895$. Still there are three blocks in the middle column are magic squares of order 6 with magic sum $S_{6\times6} = 975$

3.3 Pan Magic Square of Order 24

In order to construct pan magic of order 24, we shall apply the procedure similar to order 12. Below is a distribution of 576 numbers in 36 blocks of equal sums.

Distribution 8. *Let's distribute the 576 numbers from 1 to 576 in 36 blocks resulting in equal sums:*

A1	1	72	73	144	145	216	217	288	289	360	361	432	433	504	505	576	4616
A2	2	71	74	143	146	215	218	287	290	359	362	431	434	503	506	575	4616
A3	3	70	75	142	147	214	219	286	291	358	363	430	435	502	507	574	4616
A4	4	69	76	141	148	213	220	285	292	357	364	429	436	501	508	573	4616
A5	5	68	77	140	149	212	221	284	293	356	365	428	437	500	509	572	4616
A6	6	67	78	139	150	211	222	283	294	355	366	427	438	499	510	571	4616
A7	7	66	79	138	151	210	223	282	295	354	367	426	439	498	511	570	4616
A8	8	65	80	137	152	209	224	281	296	353	368	425	440	497	512	569	4616
A9	9	64	81	136	153	208	225	280	297	352	369	424	441	496	513	568	4616
A10	10	63	82	135	154	207	226	279	298	351	370	423	442	495	514	567	4616
A11	11	62	83	134	155	206	227	278	299	350	371	422	443	494	515	566	4616
A12	12	61	84	133	156	205	228	277	300	349	372	421	444	493	516	565	4616
A13	13	60	85	132	157	204	229	276	301	348	373	420	445	492	517	564	4616
A14	14	59	86	131	158	203	230	275	302	347	374	419	446	491	518	563	4616
A15	15	58	87	130	159	202	231	274	303	346	375	418	447	490	519	562	4616
A16	16	57	88	129	160	201	232	273	304	345	376	417	448	489	520	561	4616
A17	17	56	89	128	161	200	233	272	305	344	377	416	449	488	521	560	4616
A18	18	55	90	127	162	199	234	271	306	343	378	415	450	487	522	559	4616
A19	19	54	91	126	163	198	235	270	307	342	379	414	451	486	523	558	4616
A20	20	53	92	125	164	197	236	269	308	341	380	413	452	485	524	557	4616
A21	21	52	93	124	165	196	237	268	309	340	381	412	453	484	525	556	4616
A22	22	51	94	123	166	195	238	267	310	339	382	411	454	483	526	555	4616
A23	23	50	95	122	167	194	239	266	311	338	383	410	455	482	527	554	4616
A24	24	49	96	121	168	193	240	265	312	337	384	409	456	481	528	553	4616
A25	25	48	97	120	169	192	241	264	313	336	385	408	457	480	529	552	4616
A26	26	47	98	119	170	191	242	263	314	335	386	407	458	479	530	551	4616
A27	27	46	99	118	171	190	243	262	315	334	387	406	459	478	531	550	4616
A28	28	45	100	117	172	189	244	261	316	333	388	405	460	477	532	549	4616
A29	29	44	101	116	173	188	245	260	317	332	389	404	461	476	533	548	4616
A30	30	43	102	115	174	187	246	259	318	331	390	403	462	475	534	547	4616
A31	31	42	103	114	175	186	247	258	319	330	391	402	463	474	535	546	4616
A32	32	41	104	113	176	185	248	257	320	329	392	401	464	473	536	545	4616
A33	33	40	105	112	177	184	249	256	321	328	393	400	465	472	537	544	4616
A34	34	39	106	111	178	183	250	255	322	327	394	399	466	471	538	543	4616
A35	35	38	107	110	179	182	251	254	323	326	395	398	467	470	539	542	4616
A36	36	37	108	109	180	181	252	253	324	325	396	397	468	469	540	541	4616

Above distribution is similar to 6, where the columns are written in increasing and decreasing orders, such as, $[1,2,\ldots,36]$, $[72,49,\ldots,37]$, $[73,74,\ldots,108]$, etc. Each block results in a **pan magic square** of order 4 constructed according to Example 14. In order to construct **pan magic square** of order 24, we shall use the following structure.

Structure 3. *Let's consider 36 blocks of order 4 given as below:*

A1	*A2*	*A3*	*A4*	*A5*	*A6*
A7	*A8*	*A9*	*A10*	*A11*	*A12*
A13	*A14*	*A15*	*A16*	*A17*	*A18*
A19	*A20*	*A21*	*A22*	*A23*	*A24*
A25	*A26*	*A27*	*A28*	*A29*	*A30*
A31	*A32*	*A33*	*A34*	*A35*	*A36*

Putting **pan magic squares** constructed according to Example 14 according to above Structure 3, lead us to a pan magic of order 24 given in example below.

Example 19. *36 blocks of magic squares of order 4 constructed according to Example 14 using data given in distribution 8 and putting them according to structure 3, we get a* ***pan magic square*** *of order 24 is given in parts (I) and (I):*

(I)	1	2	3	4	5	6	7	8	9	10	11	12
		6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924
	217	432	1	504	218	431	2	503	219	430	3	502
6924	72	433	288	361	71	434	287	362	70	435	286	363
6924	576	73	360	145	575	74	359	146	574	75	358	147
6924	289	216	505	144	290	215	506	143	291	214	507	142
6924	223	426	7	498	224	425	8	497	225	424	9	496
6924	66	439	282	367	65	440	281	368	64	441	280	369
6924	570	79	354	151	569	80	353	152	568	81	352	153
6924	295	210	511	138	296	209	512	137	297	208	513	136
6924	229	420	13	492	230	419	14	491	231	418	15	490
6924	60	445	276	373	59	446	275	374	58	447	274	375
6924	564	85	348	157	563	86	347	158	562	87	346	159
6924	301	204	517	132	302	203	518	131	303	202	519	130
6924	235	414	19	486	236	413	20	485	237	412	21	484
6924	54	451	270	379	53	452	269	380	52	453	268	381
6924	558	91	342	163	557	92	341	164	556	93	340	165
6924	307	198	523	126	308	197	524	125	309	196	525	124
6924	241	408	25	480	242	407	26	479	243	406	27	478
6924	48	457	264	385	47	458	263	386	46	459	262	387
6924	552	97	336	169	551	98	335	170	550	99	334	171
6924	313	192	529	120	314	191	530	119	315	190	531	118
6924	247	402	31	474	248	401	32	473	249	400	33	472
6924	42	463	258	391	41	464	257	392	40	465	256	393
6924	546	103	330	175	545	104	329	176	544	105	328	177
6924	319	186	535	114	320	185	536	113	321	184	537	112
	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924

13	14	15	16	17	18	19	20	21	22	23	24	(II)
6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924
220	429	4	501	221	428	5	500	222	427	6	499	6924
69	436	285	364	68	437	284	365	67	438	283	366	6924
573	76	357	148	572	77	356	149	571	78	355	150	6924
292	213	508	141	293	212	509	140	294	211	510	139	6924
226	423	10	495	227	422	11	494	228	421	12	493	6924
63	442	279	370	62	443	278	371	61	444	277	372	6924
567	82	351	154	566	83	350	155	565	84	349	156	6924
298	207	514	135	299	206	515	134	300	205	516	133	6924
232	417	16	489	233	416	17	488	234	415	18	487	6924
57	448	273	376	56	449	272	377	55	450	271	378	6924
561	88	345	160	560	89	344	161	559	90	343	162	6924
304	201	520	129	305	200	521	128	306	199	522	127	6924
238	411	22	483	239	410	23	482	240	409	24	481	6924
51	454	267	382	50	455	266	383	49	456	265	384	6924
555	94	339	166	554	95	338	167	553	96	337	168	6924
310	195	526	123	311	194	527	122	312	193	528	121	6924
244	405	28	477	245	404	29	476	246	403	30	475	6924
45	460	261	388	44	461	260	389	43	462	259	390	6924
549	100	333	172	548	101	332	173	547	102	331	174	6924
316	189	532	117	317	188	533	116	318	187	534	115	6924
250	399	34	471	251	398	35	470	252	397	36	469	6924
39	466	255	394	38	467	254	395	37	468	253	396	6924
543	106	327	178	542	107	326	179	541	108	325	180	6924
322	183	538	111	323	182	539	110	324	181	540	109	6924
6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924	6924

Combining the partes (I) and (II) we get the required result. In this case the magic sum is $S_{24\times 24} = 6924$. Each 4×4 block is a **pan magic square** of order 4 with the same magic sum $S_{4\times 4} = 1154$.

Note 1. *The **pan magic squares** of orders 12 and 24 are already studied by author in [27]. Here we have re-written to complete the list.*

3.4 Magic Square of Order 30

There are many ways of writing block-wise magic square of order 30 with same magic sum in each sub-block. One is considering 9 blocks of order 10. This way we have given in two different ways. Another way is to write sub-blocks of order 5. In this case we have to divide the 900 numbers in 36 blocks. The sum of all the 900 numbers is 405450. Dividing in 36 parts of equal sums we get a fractionary number, i.e., $\frac{22525}{2}$. It means that it is impossible to get each block of order 5 a magic square with equal sum. We can get with different sums, but is is not a part of study here. We shall work only in 9 blocks of order 10 with equal magic square sums. This process is done in two different ways.

3.4.1 First Approach: 9 Blocks of Order 10

Example 20. *Let's consider a magic square of order 10 is given by*

										505
1	80	65	97	39	22	48	86	53	14	505
98	12	9	66	90	74	55	33	41	27	505
47	81	23	79	16	35	94	60	62	8	505
70	57	88	34	2	91	29	15	76	43	505
84	99	52	11	45	68	73	7	30	36	505
13	38	44	10	77	56	82	21	95	69	505
75	46	40	83	28	19	67	92	4	51	505
59	24	96	42	61	3	20	78	37	85	505
26	5	17	58	93	50	31	64	89	72	505
32	63	71	25	54	87	6	49	18	100	505
505	505	505	505	505	505	505	505	505	505	505

Let's distribute 900 numbers from 1 to 900 in 9 groups of 100 each giving equal sums.

Distribution 9. *Let's distribute the 900 numbers from 1 to 900 in 9 blocks resulting in equal sums:*

	1	1	2	3	...	...	97	98	99	100	Total
A1	1	18	19	36	...	...	865	882	883	900	45050
A2	2	17	20	35	...	...	866	881	884	899	45050
A3	3	16	21	34	...	...	867	880	885	898	45050
A4	4	15	22	33	...	...	868	879	886	897	45050
A5	5	14	23	32	...	...	869	878	887	896	45050
A6	6	13	24	31	...	...	870	877	888	895	45050
A7	7	12	25	30	...	...	871	876	889	894	45050
A8	8	11	26	29	...	...	872	875	890	893	45050
A9	9	10	27	28	...	...	873	874	891	892	45050

Above distribution is written in increasing and decreasing orders, such as, [1, 2, 3, 4, 5, 6, 7, 8, 9], [18, 17, 16, 15, 14, 13, 12, 11, 10], [19, 20, 21, 22, 23, 24, 25, 26, 27], etc.

Applying the Distribution **??** over magic square of order 10 given in Example 20, we get 9 magic squares of order 10 as below:

- **Block A1**

A1										4505
1	720	577	865	343	198	432	774	469	126	4505
882	108	73	594	810	666	487	289	361	235	4505
415	721	199	703	144	307	846	540	558	72	4505
630	4505	792	306	18	811	253	127	684	379	4505
756	883	468	91	397	612	649	55	270	324	4505
109	342	396	90	685	504	738	181	847	613	4505
667	414	360	739	252	163	595	828	36	451	4505
523	216	864	378	541	19	180	702	325	757	4505
234	37	145	522	829	450	271	576	793	648	4505
288	559	631	217	486	775	54	433	162	900	4505
4505	4505	4505	4505	4505	4505	4505	4505	4505	4505	4505

- **Block A2**

A2										*4505*
2	719	578	866	344	197	431	773	470	125	*4505*
881	107	74	593	809	665	488	290	362	236	*4505*
416	722	200	704	143	308	845	539	557	71	*4505*
629	506	791	305	17	812	254	128	683	380	*4505*
755	884	467	92	398	611	650	56	269	323	*4505*
110	341	395	89	686	503	737	182	848	614	*4505*
668	413	359	740	251	164	596	827	35	452	*4505*
524	215	863	377	542	20	179	701	326	758	*4505*
233	38	146	521	830	449	272	575	794	647	*4505*
287	560	632	218	485	776	53	434	161	899	*4505*
4505	*4505*	*4505*	*4505*	*4505*	*4505*	*4505*	*4505*	*4505*	*4505*	*4505*

- **Block A3**

A3										*4505*
3	718	579	867	345	196	430	772	471	124	*4505*
880	106	75	592	808	664	489	291	363	237	*4505*
417	723	201	705	142	309	844	538	556	70	*4505*
628	507	790	304	16	813	255	129	682	381	*4505*
754	885	466	93	399	610	651	57	268	322	*4505*
111	340	394	88	687	502	736	183	849	615	*4505*
669	412	358	741	250	165	597	826	34	453	*4505*
525	214	862	376	543	21	178	700	327	759	*4505*
232	39	147	520	831	448	273	574	795	646	*4505*
286	561	633	219	484	777	52	435	160	898	*4505*
4505	*4505*	*4505*	*4505*	*4505*	*4505*	*4505*	*4505*	*4505*	*4505*	*4505*

- **Block A4**

A4										*4505*
4	717	580	868	346	195	429	771	472	123	*4505*
879	105	76	591	807	663	490	292	364	238	*4505*
418	724	202	706	141	310	843	537	555	69	*4505*
627	508	789	303	15	814	256	130	681	382	*4505*
753	886	465	94	400	609	652	58	267	321	*4505*
112	339	393	87	688	501	735	184	850	616	*4505*
670	411	357	742	249	166	598	825	33	454	*4505*
526	213	861	375	544	22	177	699	328	760	*4505*
231	40	148	519	832	447	274	573	796	645	*4505*
285	562	634	220	483	778	51	436	159	897	*4505*
4505	*4505*	*4505*	*4505*	*4505*	*4505*	*4505*	*4505*	*4505*	*4505*	*4505*

- **Block A5**

A5										*4505*
5	716	581	869	347	194	428	770	473	122	*4505*
878	104	77	590	806	662	491	293	365	239	*4505*
419	725	203	707	140	311	842	536	554	68	*4505*
626	509	788	302	14	815	257	131	680	383	*4505*
752	887	464	95	401	608	653	59	266	320	*4505*
113	338	392	86	689	500	734	185	851	617	*4505*
671	410	356	743	248	167	599	824	32	455	*4505*
527	212	860	374	545	23	176	698	329	761	*4505*
230	41	149	518	833	446	275	572	797	644	*4505*
284	563	635	221	482	779	50	437	158	896	*4505*
4505	*4505*	*4505*	*4505*	*4505*	*4505*	*4505*	*4505*	*4505*	*4505*	*4505*

- **Block A6**

A6										*4505*
6	715	582	870	348	193	427	769	474	121	*4505*
877	103	78	589	805	661	492	294	366	240	*4505*
420	726	204	708	139	312	841	535	553	67	*4505*
625	510	787	301	13	816	258	132	679	384	*4505*
751	888	463	96	402	607	654	60	265	319	*4505*
114	337	391	85	690	499	733	186	852	618	*4505*
672	409	355	744	247	168	600	823	31	456	*4505*
528	211	859	373	546	24	175	697	330	762	*4505*
229	42	150	517	834	445	276	571	798	643	*4505*
283	564	636	222	481	780	49	438	157	895	*4505*
4505	*4505*	*4505*	*4505*	*4505*	*4505*	*4505*	*4505*	*4505*	*4505*	*4505*

- **Block A7**

A7										*4505*
7	714	583	871	349	192	426	768	475	120	*4505*
876	102	79	588	804	660	493	295	367	241	*4505*
421	727	205	709	138	313	840	534	552	66	*4505*
624	511	786	300	12	817	259	133	678	385	*4505*
750	889	462	97	403	606	655	61	264	318	*4505*
115	336	390	84	691	498	732	187	853	619	*4505*
673	408	354	745	246	169	601	822	30	457	*4505*
529	210	858	372	547	25	174	696	331	763	*4505*
228	43	151	516	835	444	277	570	799	642	*4505*
282	565	637	223	480	781	48	439	156	894	*4505*
4505	*4505*	*4505*	*4505*	*4505*	*4505*	*4505*	*4505*	*4505*	*4505*	*4505*

- **Block A8**

A8										*4505*
8	713	584	872	350	191	425	767	476	119	*4505*
875	101	80	587	803	659	494	296	368	242	*4505*
422	728	206	710	137	314	839	533	551	65	*4505*
623	512	785	299	11	818	260	134	677	386	*4505*
749	890	461	98	404	605	656	62	263	317	*4505*
116	335	389	83	692	497	731	188	854	620	*4505*
674	407	353	746	245	170	602	821	29	458	*4505*
530	209	857	371	548	26	173	695	332	764	*4505*
227	44	152	515	836	443	278	569	800	641	*4505*
281	566	638	224	479	782	47	440	155	893	*4505*
4505	*4505*	*4505*	*4505*	*4505*	*4505*	*4505*	*4505*	*4505*	*4505*	*4505*

- **Block A9**

A9										*4505*
9	712	585	873	351	190	424	766	477	118	*4505*
874	100	81	586	802	658	495	297	369	243	*4505*
423	729	207	711	136	315	838	532	550	64	*4505*
622	513	784	298	10	819	261	135	676	387	*4505*
748	891	460	99	405	604	657	63	262	316	*4505*
117	334	388	82	693	496	730	189	855	621	*4505*
675	406	352	747	244	171	603	820	28	459	*4505*
531	208	856	370	549	27	172	694	333	765	*4505*
226	45	153	514	837	442	279	568	801	640	*4505*
280	567	639	225	478	783	46	441	154	892	*4505*
4505	*4505*	*4505*	*4505*	*4505*	*4505*	*4505*	*4505*	*4505*	*4505*	*4505*

In order to construct a magic square of order 30, let us put the above 9 magic squares of order 10 as given below:

A1	A2	A3
A4	A5	A6
A7	A8	A9

Writing the above 9 magic squares of order 10 according to Structure 1 we get a **magic square** of order 30 given in the example below.

Example 21. *According to Distribution* **??**, *Structure 1 and 9 magic squares of order 10 given above , we have a* ***magic square*** *of order 30 given by*

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
(I)														
1	720	577	865	343	198	432	774	469	126	2	719	578	866	344
882	108	73	594	810	666	487	289	361	235	881	107	74	593	809
415	721	199	703	144	307	846	540	558	72	416	722	200	704	143
630	505	792	306	18	811	253	127	684	379	629	506	791	305	17
756	883	468	91	397	612	649	55	270	324	755	884	467	92	398
109	342	396	90	685	504	738	181	847	613	110	341	395	89	686
667	414	360	739	252	163	595	828	36	451	668	413	359	740	251
523	216	864	378	541	19	180	702	325	757	524	215	863	377	542
234	37	145	522	829	450	271	576	793	648	233	38	146	521	830
288	559	631	217	486	775	54	433	162	900	287	560	632	218	485
4	717	580	868	346	195	429	771	472	123	5	716	581	869	347
879	105	76	591	807	663	490	292	364	238	878	104	77	590	806
418	724	202	706	141	310	843	537	555	69	419	725	203	707	140
627	508	789	303	15	814	256	130	681	382	626	509	788	302	14
753	886	465	94	400	609	652	58	267	321	752	887	464	95	401
112	339	393	87	688	501	735	184	850	616	113	338	392	86	689
670	411	357	742	249	166	598	825	33	454	671	410	356	743	248
526	213	861	375	544	22	177	699	328	760	527	212	860	374	545
231	40	148	519	832	447	274	573	796	645	230	41	149	518	833
285	562	634	220	483	778	51	436	159	897	284	563	635	221	482
7	714	583	871	349	192	426	768	475	120	8	713	584	872	350
876	102	79	588	804	660	493	295	367	241	875	101	80	587	803
421	727	205	709	138	313	840	534	552	66	422	728	206	710	137
624	511	786	300	12	817	259	133	678	385	623	512	785	299	11
750	889	462	97	403	606	655	61	264	318	749	890	461	98	404
115	336	390	84	691	498	732	187	853	619	116	335	389	83	692
673	408	354	745	246	169	601	822	30	457	674	407	353	746	245
529	210	858	372	547	25	174	696	331	763	530	209	857	371	548
228	43	151	516	835	444	277	570	799	642	227	44	152	515	836
282	565	637	223	480	781	48	439	156	894	281	566	638	224	479
13515	**13515**	**13515**	**13515**	**13515**	**13515**	**13515**	**13515**	**13515**	**13515**	**13515**	**13515**	**13515**	**13515**	**13515**

16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	
(II)															**13515**
197	431	773	470	125	3	718	579	867	345	196	430	772	471	124	**13515**
665	488	290	362	236	880	106	75	592	808	664	489	291	363	237	**13515**
308	845	539	557	71	417	723	201	705	142	309	844	538	556	70	**13515**
812	254	128	683	380	628	507	790	304	16	813	255	129	682	381	**13515**
611	650	56	269	323	754	885	466	93	399	610	651	57	268	322	**13515**
503	737	182	848	614	111	340	394	88	687	502	736	183	849	615	**13515**
164	596	827	35	452	669	412	358	741	250	165	597	826	34	453	**13515**
20	179	701	326	758	525	214	862	376	543	21	178	700	327	759	**13515**
449	272	575	794	647	232	39	147	520	831	448	273	574	795	646	**13515**
776	53	434	161	899	286	561	633	219	484	777	52	435	160	898	**13515**
194	428	770	473	122	6	715	582	870	348	193	427	769	474	121	**13515**
662	491	293	365	239	877	103	78	589	805	661	492	294	366	240	**13515**
311	842	536	554	68	420	726	204	708	139	312	841	535	553	67	**13515**
815	257	131	680	383	625	510	787	301	13	816	258	132	679	384	**13515**
608	653	59	266	320	751	888	463	96	402	607	654	60	265	319	**13515**
500	734	185	851	617	114	337	391	85	690	499	733	186	852	618	**13515**
167	599	824	32	455	672	409	355	744	247	168	600	823	31	456	**13515**
23	176	698	329	761	528	211	859	373	546	24	175	697	330	762	**13515**
446	275	572	797	644	229	42	150	517	834	445	276	571	798	643	**13515**
779	50	437	158	896	283	564	636	222	481	780	49	438	157	895	**13515**
191	425	767	476	119	9	712	585	873	351	190	424	766	477	118	**13515**
659	494	296	368	242	874	100	81	586	802	658	495	297	369	243	**13515**
314	839	533	551	65	423	729	207	711	136	315	838	532	550	64	**13515**
818	260	134	677	386	622	513	784	298	10	819	261	135	676	387	**13515**
605	656	62	263	317	748	891	460	99	405	604	657	63	262	316	**13515**
497	731	188	854	620	117	334	388	82	693	496	730	189	855	621	**13515**
170	602	821	29	458	675	406	352	747	244	171	603	820	28	459	**13515**
26	173	695	332	764	531	208	856	370	549	27	172	694	333	765	**13515**
443	278	569	800	641	226	45	153	514	837	442	279	568	801	640	**13515**
782	47	440	155	893	280	567	639	225	478	783	46	441	154	892	**13515**
13515	**13515**	**13515**	**13515**	**13515**	**13515**	**13515**	**13515**	**13515**	**13515**	**13515**	**13515**	**13515**	**13515**	**13515**	**13515**

Combining Parts (I) and (II) we get the required result. In this case, the magic square sum is $S_{30\times30} = 13515$. Each 10×10 block is a **magic square** of order 10 are with equal magic sums $S_{10\times10} = 4505$.

3.4.2 Second Approach: 30 blocks of Order 6

In the previous subsection, we gave a construction of magic square of order 30 using 9 blocks of magic squares of order 10 with equal magic sums. Below is another possibility of writing a magic square of order 30, where sum of 36 entries of each sub-block of order 6 are equal. First, let us distribute the numbers 1-900 according to following table.

Distribution 10. *Let's consider the following distribution to construct **magic square** of order 36:*

	1	2	3	4	5	6	…	…	31	32	33	34	35	36	Total
A1	1	50	51	100	101	150	…	…	751	800	801	850	851	900	16218
A2	2	49	52	99	102	149	…	…	752	799	802	849	852	899	16218
A3	3	48	53	98	103	148	…	…	753	798	803	848	853	898	16218
A4	4	47	54	97	104	147	…	…	754	797	804	847	854	897	16218
A5	5	46	55	96	105	146	…	…	755	796	805	846	855	896	16218
A6	6	45	56	95	106	145	…	…	756	795	806	845	856	895	16218
A7	7	44	57	94	107	144	…	…	757	794	807	844	857	894	16218
A8	8	43	58	93	108	143	…	…	758	793	808	843	858	893	16218
A9	9	42	59	92	109	142	…	…	759	792	809	842	859	892	16218
A10	10	41	60	91	110	141	…	…	760	791	810	841	860	891	16218
A11	11	40	61	90	111	140	…	…	761	790	811	840	861	890	16218
A12	12	39	62	89	112	139	…	…	762	789	812	839	862	889	16218
A13	13	38	63	88	113	138	…	…	763	788	813	838	863	888	16218
A14	14	37	64	87	114	137	…	…	764	787	814	837	864	887	16218
A15	15	36	65	86	115	136	…	…	765	786	815	836	865	886	16218
A16	16	35	66	85	116	135	…	…	766	785	816	835	866	885	16218
A17	17	34	67	84	117	134	…	…	767	784	817	834	867	884	16218
A18	18	33	68	83	118	133	…	…	768	783	818	833	868	883	16218
A19	19	32	69	82	119	132	…	…	769	782	819	832	869	882	16218
A20	20	31	70	81	120	131	…	…	770	781	820	831	870	881	16218
A21	21	30	71	80	121	130	…	…	771	780	821	830	871	880	16218
A22	22	29	72	79	122	129	…	…	772	779	822	829	872	879	16218
A23	23	28	73	78	123	128	…	…	773	778	823	828	873	878	16218
A24	24	27	74	77	124	127	…	…	774	777	824	827	874	877	16218
A25	25	26	75	76	125	126	…	…	775	776	825	826	875	876	16218

Let's construct 25 blocks according to magic square of order 6 given in Example 16. In this case, all the 25 blocks are just distributions of 36 entries of 6×6, except one a magic square. Put these 6×6 blocks according to a**pan magic square** of order 5 given in Example 4. See below the structure.

Structure 4. *Let's consider 36 blocks of type* 6×6 *constructed according to Example 16 and put them as*

A1	A9	A12	A20	A23
A17	A25	A3	A6	A14
A8	A11	A19	A22	A5
A24	A2	A10	A13	A16
A15	A18	A21	A4	A7

Example 22. *According to Distribution 10 and Structure 4 we have a* ***magic square*** *of order 30 given by*

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
(I)														
1	551	700	850	401	200	9	559	692	842	409	192	12	562	689
701	151	851	350	501	101	709	159	859	342	509	109	712	162	862
300	150	301	651	751	550	292	142	309	659	759	542	289	139	312
800	400	100	600	250	601	792	392	92	592	242	609	789	389	89
451	801	251	51	750	351	459	809	259	59	742	359	462	812	262
450	650	500	201	50	900	442	642	492	209	42	892	439	639	489
17	567	684	834	417	184	25	575	676	826	425	176	3	553	698
717	167	867	334	517	117	725	175	875	326	525	125	703	153	853
284	134	317	667	767	534	276	126	325	675	775	526	298	148	303
784	384	84	584	234	617	776	376	76	576	226	625	798	398	98
467	817	267	67	734	367	475	825	275	75	726	375	453	803	253
434	634	484	217	34	884	426	626	476	225	26	876	448	648	498
8	558	693	843	408	193	11	561	690	840	411	190	19	569	682
708	158	858	343	508	108	711	161	861	340	511	111	719	169	869
293	143	308	658	758	543	290	140	311	661	761	540	282	132	319
793	393	93	593	243	608	790	390	90	590	240	611	782	382	82
458	808	258	58	743	358	461	811	261	61	740	361	469	819	269
443	643	493	208	43	893	440	640	490	211	40	890	432	632	482
24	574	677	827	424	177	2	552	699	849	402	199	10	560	691
724	174	874	327	524	124	702	152	852	349	502	102	710	160	860
277	127	324	674	774	527	299	149	302	652	752	549	291	141	310
777	377	77	577	227	624	799	399	99	599	249	602	791	391	91
474	824	274	74	727	374	452	802	252	52	749	352	460	810	260
427	627	477	224	27	877	449	649	499	202	49	899	441	641	491
15	565	686	836	415	186	18	568	683	833	418	183	21	571	680
715	165	865	336	515	115	718	168	868	333	518	118	721	171	871
286	136	315	665	765	536	283	133	318	668	768	533	280	130	321
786	386	86	586	236	615	783	383	83	583	233	618	780	380	80
465	815	265	65	736	365	468	818	268	68	733	368	471	821	271
436	636	486	215	36	886	433	633	483	218	33	883	430	630	480
13515	**13515**	**13515**	**13515**	**13515**	**13515**	**13515**	**13515**	**13515**	**13515**	**13515**	**13515**	**13515**	**13515**	**13515**

16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	
(II)															13515
839	412	189	20	570	681	831	420	181	23	573	678	828	423	178	13515
339	512	112	720	170	870	331	520	120	723	173	873	328	523	123	13515
662	762	539	281	131	320	670	770	531	278	128	323	673	773	528	13515
589	239	612	781	381	81	581	231	620	778	378	78	578	228	623	13515
62	739	362	470	820	270	70	731	370	473	823	273	73	728	373	13515
212	39	889	431	631	481	220	31	881	428	628	478	223	28	878	13515
848	403	198	6	556	695	845	406	195	14	564	687	837	414	187	13515
348	503	103	706	156	856	345	506	106	714	164	864	337	514	114	13515
653	753	548	295	145	306	656	756	545	287	137	314	664	764	537	13515
598	248	603	795	395	95	595	245	606	787	387	87	587	237	614	13515
53	748	353	456	806	256	56	745	356	464	814	264	64	737	364	13515
203	48	898	445	645	495	206	45	895	437	637	487	214	37	887	13515
832	419	182	22	572	679	829	422	179	5	555	696	846	405	196	13515
332	519	119	722	172	872	329	522	122	705	155	855	346	505	105	13515
669	769	532	279	129	322	672	772	529	296	146	305	655	755	546	13515
582	232	619	779	379	79	579	229	622	796	396	96	596	246	605	13515
69	732	369	472	822	272	72	729	372	455	805	255	55	746	355	13515
219	32	882	429	629	479	222	29	879	446	646	496	205	46	896	13515
841	410	191	13	563	688	838	413	188	16	566	685	835	416	185	13515
341	510	110	713	163	863	338	513	113	716	166	866	335	516	116	13515
660	760	541	288	138	313	663	763	538	285	135	316	666	766	535	13515
591	241	610	788	388	88	588	238	613	785	385	85	585	235	616	13515
60	741	360	463	813	263	63	738	363	466	816	266	66	735	366	13515
210	41	891	438	638	488	213	38	888	435	635	485	216	35	885	13515
830	421	180	4	554	697	847	404	197	7	557	694	844	407	194	13515
330	521	121	704	154	854	347	504	104	707	157	857	344	507	107	13515
671	771	530	297	147	304	654	754	547	294	144	307	657	757	544	13515
580	230	621	797	397	97	597	247	604	794	394	94	594	244	607	13515
71	730	371	454	804	254	54	747	354	457	807	257	57	744	357	13515
221	30	880	447	647	497	204	47	897	444	644	494	207	44	894	13515
13515	13515	13515	13515	13515	13515	13515	13515	13515	13515	13515	13515	13515	13515	13515	13515

Combining Parts (I) and (II) we get the required result. In this case, the magic sum is $S_{30\times 30} = 13515$. Each 6×6 block formed by 36 entries are of same sum $S_{36} = 16218$. There is one block, a magic square of order 6 given in **black color** with magic sum $S_{6\times 6} = 2703$.

3.5 Pan Magic Square of Order 36

This subsection brings block-wise construction of **pan magic square** of order 36. Instead working with 12x3 we shall work with 4x9, i.e., 81 sub-blocks of order 4 having equal **pan magic square** sums. It is constructed by considering 81 sub-blocks of **pan magic square** of order 4 with equal magic sums, resulting in a magic square of order 36. The total 1296 numbers from 1 tov1296 are divided in 81 blocks of 16 each according to distribution below.

Distribution 11. *Let's distribute the 1296 numbers from 1 to 1296 in 81 blocks resulting in equal sums:*

A1	*1*	*162*	*163*	*324*	*325*	*486*	*487*	*648*	*649*	*810*	*811*	*972*	*973*	*1134*	*1135*	*1296*	*10376*
A2	*2*	*161*	*164*	*323*	*326*	*485*	*488*	*647*	*650*	*809*	*812*	*971*	*974*	*1133*	*1136*	*1295*	*10376*
A3	*3*	*160*	*165*	*322*	*327*	*484*	*489*	*646*	*651*	*808*	*813*	*970*	*975*	*1132*	*1137*	*1294*	*10376*
A4	*4*	*159*	*166*	*321*	*328*	*483*	*490*	*645*	*652*	*807*	*814*	*969*	*976*	*1131*	*1138*	*1293*	*10376*
A5	*5*	*158*	*167*	*320*	*329*	*482*	*491*	*644*	*653*	*806*	*815*	*968*	*977*	*1130*	*1139*	*1292*	*10376*
A6	*6*	*157*	*168*	*319*	*330*	*481*	*492*	*643*	*654*	*805*	*816*	*967*	*978*	*1129*	*1140*	*1291*	*10376*
A7	*7*	*156*	*169*	*318*	*331*	*480*	*493*	*642*	*655*	*804*	*817*	*966*	*979*	*1128*	*1141*	*1290*	*10376*
A8	*8*	*155*	*170*	*317*	*332*	*479*	*494*	*641*	*656*	*803*	*818*	*965*	*980*	*1127*	*1142*	*1289*	*10376*
A9	*9*	*154*	*171*	*316*	*333*	*478*	*495*	*640*	*657*	*802*	*819*	*964*	*981*	*1126*	*1143*	*1288*	*10376*
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
A73	*73*	*90*	*235*	*252*	*397*	*414*	*559*	*576*	*721*	*738*	*883*	*900*	*1045*	*1062*	*1207*	*1224*	*10376*
A74	*74*	*89*	*236*	*251*	*398*	*413*	*560*	*575*	*722*	*737*	*884*	*899*	*1046*	*1061*	*1208*	*1223*	*10376*
A75	*75*	*88*	*237*	*250*	*399*	*412*	*561*	*574*	*723*	*736*	*885*	*898*	*1047*	*1060*	*1209*	*1222*	*10376*
A76	*76*	*87*	*238*	*249*	*400*	*411*	*562*	*573*	*724*	*735*	*886*	*897*	*1048*	*1059*	*1210*	*1221*	*10376*
A77	*77*	*86*	*239*	*248*	*401*	*410*	*563*	*572*	*725*	*734*	*887*	*896*	*1049*	*1058*	*1211*	*1220*	*10376*
A78	*78*	*85*	*240*	*247*	*402*	*409*	*564*	*571*	*726*	*733*	*888*	*895*	*1050*	*1057*	*1212*	*1219*	*10376*
A79	*79*	*84*	*241*	*246*	*403*	*408*	*565*	*570*	*727*	*732*	*889*	*894*	*1051*	*1056*	*1213*	*1218*	*10376*
A80	*80*	*83*	*242*	*245*	*404*	*407*	*566*	*569*	*728*	*731*	*890*	*893*	*1052*	*1055*	*1214*	*1217*	*10376*
A81	*81*	*82*	*243*	*244*	*405*	*406*	*567*	*568*	*729*	*730*	*891*	*892*	*1053*	*1054*	*1215*	*1216*	*10376*

Above distribution is similar to 6 or 8, where the columns are written in increasing and decreasing orders, such as, $[1,2,\ldots,81]$, $[162,161,\ldots,82]$, $[83,84,\ldots,243]$, etc. Each block results in a **pan magic square** of order 4 constructed according to Example 14. To construct **pan magic square** of order 36, we shall use the following structure.

Structure 5. *Let's consider 81 blocks of order 4 given as below:*

A1	*A2*	*A3*	*A4*	*A5*	*A6*	*A7*	*A8*	*A9*
A10	*A11*	*A12*	*A13*	*A14*	*A15*	*A16*	*A17*	*A18*
A19	*A20*	*A21*	*A22*	*A23*	*A24*	*A25*	*A26*	*A27*
A28	*A29*	*A30*	*A31*	*A32*	*A33*	*A34*	*A35*	*A36*
A37	*A38*	*A39*	*A40*	*A41*	*A42*	*A43*	*A44*	*A45*
A46	*A47*	*A48*	*A49*	*A50*	*A51*	*A52*	*A53*	*A54*
A55	*A56*	*A57*	*A58*	*A59*	*A60*	*A61*	*A62*	*A63*
A64	*A65*	*A66*	*A67*	*A68*	*A69*	*A70*	*A71*	*A72*
A73	*A74*	*A75*	*A76*	*A77*	*A78*	*A79*	*A80*	*A81*

According to above Structure 5, where each sub-block is a pan magic of order 4 constructed according to Example 14 give the following magic square of order 36.

Example 23. *A **pan magic square** of order 36 with each sub block of order 4 a pan magic of order 4 is given by*

(I)	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18
		23346	23346	23346	23346	23346	23346	23346	23346	23346	23346	23346	23346	23346	23346	23346	23346	23346
	487	972	1	1134	488	971	2	1133	489	970	3	1132	490	969	4	1131	491	968
23346	162	973	648	811	161	974	647	812	160	975	646	813	159	976	645	814	158	977
23346	1296	163	810	325	1295	164	809	326	1294	165	808	327	1293	166	807	328	1292	167
23346	649	486	1135	324	650	485	1136	323	651	484	1137	322	652	483	1138	321	653	482
23346	496	963	10	1125	497	962	11	1124	498	961	12	1123	499	960	13	1122	500	959
23346	153	982	639	820	152	983	638	821	151	984	637	822	150	985	636	823	149	986
23346	1287	172	801	334	1286	173	800	335	1285	174	799	336	1284	175	798	337	1283	176
23346	658	477	1144	315	659	476	1145	314	660	475	1146	313	661	474	1147	312	662	473
23346	505	954	19	1116	506	953	20	1115	507	952	21	1114	508	951	22	1113	509	950
23346	144	991	630	829	143	992	629	830	142	993	628	831	141	994	627	832	140	995
23346	1278	181	792	343	1277	182	791	344	1276	183	790	345	1275	184	789	346	1274	185
23346	667	468	1153	306	668	467	1154	305	669	466	1155	304	670	465	1156	303	671	464
23346	514	945	28	1107	515	944	29	1106	516	943	30	1105	517	942	31	1104	518	941
23346	135	1000	621	838	134	1001	620	839	133	1002	619	840	132	1003	618	841	131	1004
23346	1269	190	783	352	1268	191	782	353	1267	192	781	354	1266	193	780	355	1265	194
23346	676	459	1162	297	677	458	1163	296	678	457	1164	295	679	456	1165	294	680	455
23346	523	936	37	1098	524	935	38	1097	525	934	39	1096	526	933	40	1095	527	932
23346	126	1009	612	847	125	1010	611	848	124	1011	610	849	123	1012	609	850	122	1013
23346	1260	199	774	361	1259	200	773	362	1258	201	772	363	1257	202	771	364	1256	203
23346	685	450	1171	288	686	449	1172	287	687	448	1173	286	688	447	1174	285	689	446
23346	532	927	46	1089	533	926	47	1088	534	925	48	1087	535	924	49	1086	536	923
23346	117	1018	603	856	116	1019	602	857	115	1020	601	858	114	1021	600	859	113	1022
23346	1251	208	765	370	1250	209	764	371	1249	210	763	372	1248	211	762	373	1247	212
23346	694	441	1180	279	695	440	1181	278	696	439	1182	277	697	438	1183	276	698	437
23346	541	918	55	1080	542	917	56	1079	543	916	57	1078	544	915	58	1077	545	914
23346	108	1027	594	865	107	1028	593	866	106	1029	592	867	105	1030	591	868	104	1031
23346	1242	217	756	379	1241	218	755	380	1240	219	754	381	1239	220	753	382	1238	221
23346	703	432	1189	270	704	431	1190	269	705	430	1191	268	706	429	1192	267	707	428
23346	550	909	64	1071	551	908	65	1070	552	907	66	1069	553	906	67	1068	554	905
23346	99	1036	585	874	98	1037	584	875	97	1038	583	876	96	1039	582	877	95	1040
23346	1233	226	747	388	1232	227	746	389	1231	228	745	390	1230	229	744	391	1229	230
23346	712	423	1198	261	713	422	1199	260	714	421	1200	259	715	420	1201	258	716	419
23346	559	900	73	1062	560	899	74	1061	561	898	75	1060	562	897	76	1059	563	896
23346	90	1045	576	883	89	1046	575	884	88	1047	574	885	87	1048	573	886	86	1049
23346	1224	235	738	397	1223	236	737	398	1222	237	736	399	1221	238	735	400	1220	239
23346	721	414	1207	252	722	413	1208	251	723	412	1209	250	724	411	1210	249	725	410
	23346	23346	23346	23346	23346	23346	23346	23346	23346	23346	23346	23346	23346	23346	23346	23346	23346	23346

19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	(II)
23346	23346	23346	23346	23346	23346	23346	23346	23346	23346	23346	23346	23346	23346	23346	23346	23346	23346	23346
5	1130	492	967	6	1129	493	966	7	1128	494	965	8	1127	495	964	9	1126	23346
644	815	157	978	643	816	156	979	642	817	155	980	641	818	154	981	640	819	23346
806	329	1291	168	805	330	1290	169	804	331	1289	170	803	332	1288	171	802	333	23346
1139	320	654	481	1140	319	655	480	1141	318	656	479	1142	317	657	478	1143	316	23346
14	1121	501	958	15	1120	502	957	16	1119	503	956	17	1118	504	955	18	1117	23346
635	824	148	987	634	825	147	988	633	826	146	989	632	827	145	990	631	828	23346
797	338	1282	177	796	339	1281	178	795	340	1280	179	794	341	1279	180	793	342	23346
1148	311	663	472	1149	310	664	471	1150	309	665	470	1151	308	666	469	1152	307	23346
23	1112	510	949	24	1111	511	948	25	1110	512	947	26	1109	513	946	27	1108	23346
626	833	139	996	625	834	138	997	624	835	137	998	623	836	136	999	622	837	23346
788	347	1273	186	787	348	1272	187	786	349	1271	188	785	350	1270	189	784	351	23346
1157	302	672	463	1158	301	673	462	1159	300	674	461	1160	299	675	460	1161	298	23346
32	1103	519	940	33	1102	520	939	34	1101	521	938	35	1100	522	937	36	1099	23346
617	842	130	1005	616	843	129	1006	615	844	128	1007	614	845	127	1008	613	846	23346
779	356	1264	195	778	357	1263	196	777	358	1262	197	776	359	1261	198	775	360	23346
1166	293	681	454	1167	292	682	453	1168	291	683	452	1169	290	684	451	1170	289	23346
41	1094	528	931	42	1093	529	930	43	1092	530	929	44	1091	531	928	45	1090	23346
608	851	121	1014	607	852	120	1015	606	853	119	1016	605	854	118	1017	604	855	23346
770	365	1255	204	769	366	1254	205	768	367	1253	206	767	368	1252	207	766	369	23346
1175	284	690	445	1176	283	691	444	1177	282	692	443	1178	281	693	442	1179	280	23346
50	1085	537	922	51	1084	538	921	52	1083	539	920	53	1082	540	919	54	1081	23346
599	860	112	1023	598	861	111	1024	597	862	110	1025	596	863	109	1026	595	864	23346
761	374	1246	213	760	375	1245	214	759	376	1244	215	758	377	1243	216	757	378	23346
1184	275	699	436	1185	274	700	435	1186	273	701	434	1187	272	702	433	1188	271	23346
59	1076	546	913	60	1075	547	912	61	1074	548	911	62	1073	549	910	63	1072	23346
590	869	103	1032	589	870	102	1033	588	871	101	1034	587	872	100	1035	586	873	23346
752	383	1237	222	751	384	1236	223	750	385	1235	224	749	386	1234	225	748	387	23346
1193	266	708	427	1194	265	709	426	1195	264	710	425	1196	263	711	424	1197	262	23346
68	1067	555	904	69	1066	556	903	70	1065	557	902	71	1064	558	901	72	1063	23346
581	878	94	1041	580	879	93	1042	579	880	92	1043	578	881	91	1044	577	882	23346
743	392	1228	231	742	393	1227	232	741	394	1226	233	740	395	1225	234	739	396	23346
1202	257	717	418	1203	256	718	417	1204	255	719	416	1205	254	720	415	1206	253	23346
77	1058	564	895	78	1057	565	894	79	1056	566	893	80	1055	567	892	81	1054	23346
572	887	85	1050	571	888	84	1051	570	889	83	1052	569	890	82	1053	568	891	23346
734	401	1219	240	733	402	1218	241	732	403	1217	242	731	404	1216	243	730	405	23346
1211	248	726	409	1212	247	727	408	1213	246	728	407	1214	245	729	406	1215	244	23346
23346	23346	23346	23346	23346	23346	23346	23346	23346	23346	23346	23346	23346	23346	23346	23346	23346	23346	23346

In this case, the magic sum is $S_{36\times 36} = 23346$. Each 4×4 block is a **pan magic square** of order 4 with equal magic sums $S_{4\times 4} = 2594$.

4 Final Comments

In this work we tried to bring block-wise equal sums pan diagonal magic squares of type 3k by use of idea of triples. Most the of the case we got satisfactory results except the cases of order 18, 27 and 30. The triples used in respective cases ares

4.1 Triples

Below are triples used to construct magic squares with equal sums.

- **Order** 9×9

①	1	6	8	15
②	3	5	7	15
③	2	4	9	15

- **Order** 12×12

①	1	6	7	12	28
②	2	5	8	11	28
③	3	4	9	10	28

- **Order** 15×15

①	1	6	8	12	13	40
②	3	5	7	11	14	40
③	2	4	9	10	15	40

- **Order** 18×18

①	1	6	7	12	13	18	57
②	2	5	8	11	14	17	57
③	3	4	9	10	15	16	57

- **Order** 21×21

①	1	6	8	12	13	182	19	77
②	3	5	7	11	14	17	20	77
③	2	4	9	10	15	16	21	77

- **Order** 24×24

①	1	6	7	12	13	18	19	24	100
②	2	5	8	11	14	17	20	23	100
③	3	4	9	10	15	16	21	22	100

- **Order** 27×27

①	1	6	8	12	13	18	19	24	25	126
②	3	5	7	11	14	17	20	23	26	126
③	2	4	9	10	15	16	21	22	27	126

- **Order** 30×30

①	1	6	7	12	13	18	19	24	25	30	155
②	2	5	8	11	14	17	20	23	26	29	155
③	3	4	9	10	15	16	21	22	27	28	155

- **Order** 33×33

①	1	6	8	12	13	18	19	24	25	30	31	187
②	3	5	7	11	14	17	20	23	26	29	32	187
③	2	4	9	10	15	16	21	22	27	28	33	187

- **Order** 36×36

①	1	6	7	12	13	18	19	24	25	30	31	36	222
②	2	5	8	11	14	17	20	23	26	29	32	35	222
③	3	4	9	10	15	16	21	22	27	28	33	34	222

4.2 Analysis

The idea of triples is from Aale [1] through online discussions. The triples for odd order are adjusted according to first three rows and columns. The even orders triples are just written in simple increasing and decreasing order of numbers. The magic squares of orders 12, 24 and 36 are simultaneously multiples of $3k$ and $4k$. In theses cases, the triples are not used. Just simple idea of **pan magic square** of order 4 is applied to bring these magic squares resulting in block-wise equal sums magic squares. The orders 12 and 24 are also studied in author's previous work [27]. Since the orders 18 and 30 are of type $4p+2$, p positive integers, and it is well-known that in these cases the pan diagonal magic squares don't exists [11]. The block-wise construction of order 18 is with 9 blocks of order 6 with entries sums entires, and out of them three are magic squares of order 6. The order 30 is compose of 9 magic squares of order 10 resulting in a magic square of order 30. This is by direct substition of distributed numbers in a magic square of order 10. The second way is given having 25 blocks of order 6 with equal sums entries. According to our study, we don't have pan diagonal magic square of order 27, but the sub-blocks are of pan diagonal magic squares of order 9. There is an interesting Example 10 due to Dwane [6] of pan diagonal magic square of order 27 with each block of order 9 a **pan magic square**. Still, it is with each block of order 3 a semi-magic square (in rows and columns equal sums, except diagonal). Some ideas of block-wise constructions of magic squares can seen in a classical old book by Candy [2, 3]. The block-wise construction of magic squares not necessarily equal sums is undertaken by author in [21, 22, 23].

4.3 Further Orders

The next magic square multiple of 3 is of order 39. It is follows the similar lines of order 33, as it a multiple of prime number, i.e., $39 = 3 \times 13$. In this case we have 9 blocks of pan diagonal magic square of order 13 resulting in a **pan magic square** of order 39. The next comes the order 42. It is similar to order 30, and is of type $4p+2$. In this case, the magic square of order 42 is formed by 9 blocks of equal sums magic squares of order 14. The construction of magic square of order 14 is given in author's work [22], pp 6, in two different ways. The next is of order 45. It follows the similar lines of order 15. In this case, we can have 81 blocks of pan diagonal magic square of order 5 with equal magic sums, resulting in a pan diagonal magic square of order 45. The next order 48, follows the lines of order $4k$. It gives 144 blocks of pan diagonal magic square of equal sums of order 4, resulting in a **pan magic square** of order 48. The further orders continues.....

- **Author's Contributions**

The item-wise author's work on magic squares is as follows:

(i) **Digital numbers** magic squares - [12, 13, 14, 15, 16, 17];
(ii) **Block-wise construction** of **bimagic squares** - [18];
(iii) Connections with **genetic tables** and **Shannon's entropy** - [19];
(iv) **Selfie** and **palindromic-type** magic squares - [20];
(v) **Intervally distributed** and **block-wise** magic squares - [21, 22, 23];
(vi) **Multi-digits** magic squares - [24];
(vii) **Perfect square sum** magic squares with uniformity and minimum sum - [25, 26];
(viii) **Pythagorean triples** to generate **perfect square sum** magic squares - [26];
(ix) **Block-wise** equal sums **pan magic squares of order 4k** - [27];
(x) **Block-wise** equal sums **magic squares of order 3k** - This work;
(xi) **Block-wise** unequal sums **magic squares** - [28].

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