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Block-Wise Unequal Sums Magic Squares

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Abstract

There are magic square where it impossible to divide in sub-blocks with equal sums magic squares. For example, in case of order 12, it is impossible to divide in equal sums magic squares of order 3. In case of order 20, it is impossible to divide equal sums magic squares of order 5. In case of order 28, it is impossible to divide equal sums magic squares of order 7, etc. This paper brings constructions of magic squares with sub-blocks of unequal sums magic squares. The work is for the magic squares of orders, 12, 18, 20, 30 and 36. Even though, the magic square of order 18 can be divided in 6 equal parts, still, we have constructed it as 9 sub-blocks of order 6 with different magic squares sums.

Contents

1 Introduction

In previous works [26, 27], **block-wise** magic squares are constructed with equal sums blocks for the magic squares of orders 8 to 36. These results are in two different situations. One when orders are multiples of 4 and another with orders multiples of 3. In case of orders of type $4k$ [26], the magic squares obtained are **pan magic squares**, where blocks are of equal sums **pan magic squares** of order 4. The work done is for the magic squares of orders 8, 12, 16, 20, 24, 28 and 32. It is observed that for writing these magic squares just the knowledge of magic square of order 4 is sufficient. The second work [27], the magic square of order 4 used is **Khajuraho's pan magic square** of order 4, generally famous as **most-perfect magic square**.In case of orders of type $3k$ (excluding multiples of orders $4k$), the results obtained are with equal sub-blocks sums, except the order 18. In this case, the sums of sub-blocks entries are of equal. This work brings magic squares, where it

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is impossible to divide in equal sums sub-blocks. For example, in case of order 12, it is impossible to divide equal sums of magic squares of order 3. In case of order 20, it is impossible to divide equal sums sub-blocks of order 5, etc. This paper brings block-wise unequal sums magic squares. The results are for orders 12 to 36.

In this paper, we brings block-wise constructions of magic squares, where sub-blocks are with different magic sums. This we have done for magic squares of orders, 12 (16-blocks of order 3), 18 (9-blocks of order 6), 20 (16-blocks of order 5), 30 (25-blocks of order 6), 30 (36 blocks of order 5), and 36 (16-blocks of order 9).

2 Basic Magic Squares

Below are magic squares of orders 3, 4, 5 and 6. We shall use them in subsequent section. The magic square of orders 4 and 5 are pan magic squares.

Example 1. *Let's consider a magic square of order 3 is given by*

			15
8	1	6	15
3	5	7	15
4	9	2	15
15	15	15	15

Example 2. *Let's consider a **pan magic square** of order 4.*

		34	34	34	34
	7	12	1	14	34
34	2	13	8	11	34
34	16	3	10	5	34
34	9	6	15	4	34
	34	34	34	34	34

Example 3. *Let's consider a **pan magic square** of order 5 is given by*

		65	65	65	65	65
	1	9	12	20	23	65
65	17	25	3	6	14	65
65	8	11	19	22	5	65
65	24	2	10	13	16	65
65	15	18	21	4	7	65
	65	65	65	65	65	65

Example 4. *Let's consider a magic square of order 6.*

						111
1	23	28	34	17	8	111
29	7	35	14	21	5	111
12	6	13	27	31	22	111
32	16	4	24	10	25	111
19	33	11	3	30	15	111
18	26	20	9	2	36	111
111	111	111	111	111	111	111

Example 5. *Let's consider a* ***pan magic square*** *of order 7 is given by*

		175	175	175	175	175	175	175
	1	9	17	25	33	41	49	175
175	40	48	7	8	16	24	32	175
175	23	31	39	47	6	14	15	175
175	13	21	22	30	38	46	5	175
175	45	4	12	20	28	29	37	175
175	35	36	44	3	11	19	27	175
175	18	26	34	42	43	2	10	175
	175	175	175	175	175	175	175	175

Example 6. *Let's consider a* ***pan magic square*** *of order 9 is given by*

		369	369	369	369	369	369	369	369	369
	8	49	66	61	24	38	36	77	10	369
369	37	63	23	12	35	76	65	7	51	369
369	78	11	34	50	64	9	22	39	62	369
369	14	28	81	67	3	53	42	56	25	369
369	52	69	2	27	41	55	80	13	30	369
369	57	26	40	29	79	15	1	54	68	369
369	20	43	60	73	18	32	48	71	4	369
369	31	75	17	6	47	70	59	19	45	369
369	72	5	46	44	58	21	16	33	74	369
	369	369	369	369	369	369	369	369	369	369

Additionally it has property that each 3×3 block is of same sum as of magic square, i.e., $S_9=369$. Also each 3×3 blocks are semi-magic square of order 3 (equal sums only in rows and columns).

3 Block-Wise Unequal Magic Squares Sums

3.1 Magic Square of Order 12

Below are two different approaches to construct magic square of order 12 with different sub-blocks sums of magic squares of order 3. The approach is a general magic square of order 12. The second approach give **pan magic square** of order 12.

3.1.1 First Approach

In [26] author constructed a **pan magic square** of order 12, where each block of order 4 are **pan magic squares** with equal sums. Now the question is if we want to construct a magic square of order 3, with each block of order 3 is of same sum. It impossible to construct as the magic sums of order 12, $S_{12\times 12}=870$ is not division by 4, i.e., we have $\frac{870}{4}=\frac{425}{2}=212.5$. In this the other possibility is to construct with different sub-blocks sums.

Distribution 1. *Let's divide the numbers 144, i.e., from 1 to 144 in 16 blcoks of 9 each according to following table:*

	1	2	3	4	5	6	7	8	9	Total
A1	1	17	33	49	65	81	97	113	129	585
A2	2	18	34	50	66	82	98	114	130	594
A3	3	19	35	51	67	83	99	115	131	603
A4	4	20	36	52	68	84	100	116	132	612
A5	5	21	37	53	69	85	101	117	133	621
A6	6	22	38	54	70	86	102	118	134	630
A7	7	23	39	55	71	87	103	119	135	639
A8	8	24	40	56	72	88	104	120	136	648
A9	9	25	41	57	73	89	105	121	137	657
A10	10	26	42	58	74	90	106	122	138	666
A11	11	27	43	59	75	91	107	123	139	675
A12	12	28	44	60	76	92	108	124	140	684
A13	13	29	45	61	77	93	109	125	141	693
A14	14	30	46	62	78	94	110	126	142	702
A15	15	31	47	63	79	95	111	127	143	711
A16	16	32	48	64	80	96	112	128	144	720

Construction 1. *Below are 16 blocks of order 3 magic squares constructed according to Example 1 based on the values given in Distribution 1:*

A1			195
113	1	81	195
33	65	97	195
49	129	17	195
195	195	195	195

A2			198
114	2	82	198
34	66	98	198
50	130	18	198
198	198	198	198

A3			201
115	3	83	201
35	67	99	201
51	131	19	201
201	201	201	201

A4			204
116	4	84	204
36	68	100	204
52	132	20	204
204	204	204	204

A5			207
117	5	85	207
37	69	101	207
53	133	21	207
207	207	207	207

A6			210
118	6	86	210
38	70	102	210
54	134	22	210
210	210	210	210

A7			213
119	7	87	213
39	71	103	213
55	135	23	213
213	213	213	213

A8			216
120	8	88	216
40	72	104	216
56	136	24	216
216	216	216	216

A9			219
121	9	89	219
41	73	105	219
57	137	25	219
219	219	219	219

A10			222
122	10	90	222
42	74	106	222
58	138	26	222
222	222	222	222

A11			225
123	11	91	225
43	75	107	225
59	139	27	225
225	225	225	225

A12			228
124	12	92	228
44	76	108	228
60	140	28	228
228	228	228	228

A13			231
125	13	93	231
45	77	109	231
61	141	29	231
231	231	231	231

A14			234
126	14	94	234
46	78	110	234
62	142	30	234
234	234	234	234

A15			237
127	15	95	237
47	79	111	237
63	143	31	237
237	237	237	237

A16			240
128	16	96	240
48	80	112	240
64	144	32	240
240	240	240	240

Structure 1. *Let's put the Distribution 1 according to magic square of order 4 given in Example 2:*

A7	A12	A1	A14
A2	A13	A8	A11
A16	A3	A10	A5
A9	A6	A15	A4

According to Structure 1 we have two magic squares. One is of order 4 formed by magic sum of each 3×3 block and another a normal magic square of order 12.

Example 7. *Pan magic square of order 4, formed by 16 magic sums of each* 3×3 *block is given by*

		870	870	870	870
	213	228	195	234	870
870	198	231	216	225	870
870	240	201	222	207	870
870	219	210	237	204	870
	870	870	870	870	870

Example 8. *16 block of magic squares of order 3 constructed according to Example 2, using data given in distribution 1, and putting them according to Structure 1, we get a magic square of order 12 is given by*

												870
119	7	87	124	12	92	113	1	81	126	14	94	870
39	71	103	44	76	108	33	65	97	46	78	110	870
55	135	23	60	140	28	49	129	17	62	142	30	870
114	2	82	125	13	93	120	8	88	123	11	91	870
34	66	98	45	77	109	40	72	104	43	75	107	870
50	130	18	61	141	29	56	136	24	59	139	27	870
128	16	96	115	3	83	122	10	90	117	5	85	870
48	80	112	35	67	99	42	74	106	37	69	101	870
64	144	32	51	131	19	58	138	26	53	133	21	870
121	9	89	118	6	86	127	15	95	116	4	84	870
41	73	105	38	70	102	47	79	111	36	68	100	870
57	137	25	54	134	22	63	143	31	52	132	20	870
870	870	870	870	870	870	870	870	870	870	870	870	870

In this case, the magic sum is $S_{12\times 12} := 870$. Each block of order 3 a magic square with different magic sums given respectively in each cell of Example 7.

3.1.2 Second Approach

We observe that the simple construction done above don't give a pan magic square. It is just a magic square of order 12. Below is an alternative way to write a pan magic square of order 12. In this case, use decompositions of magic squares of order 3 and two Latin squares. See below:

Example 9. *Let's consider following decomposition of magic square of order 3 given in Example 1:*

(A)			6
2	3	1	6
1	2	3	6
3	1	2	6
6	6	6	6

(B)			6
1	3	2	6
3	2	1	6
2	1	3	6
6	6	6	6

(AB)			15
4	9	2	15
3	5	7	15
8	1	6	15
15	15	15	15

The magic square 3 (AB) of order 3 is obtained as

$$AB := 3 \times (A-1) + B$$

Example 10. *Let's consider a **pan magic square** of order 4.*

(A)		10	10	10	10
	2	3	1	4	10
10	1	4	2	3	10
10	4	1	3	2	10
10	3	2	4	1	10
	10	10	10	10	10

(B)		10	10	10	10
	3	4	1	2	10
10	2	1	4	3	10
10	4	3	2	1	10
10	1	2	3	4	10
	10	10	10	10	10

(AB)		34	34	34	34
	7	12	1	14	34
34	2	13	8	11	34
34	16	3	10	5	34
34	9	6	15	4	34
	34	34	34	34	34

The magic square (AB) of order 4 is obtained as

$$AB := 4 \times (A-1) + B$$

Let's write composite magic square of order 4 based on the Latin squares distributions A and B:

Example 11. *The composite magic square of order 4 using the operation* $C := 10 \times A + B$ *is given by*

(C)		110	110	110	110
	23	34	11	42	110
110	12	41	24	33	110
110	44	13	32	21	110
110	31	22	43	14	110
	110	110	110	110	110

Let's consider following distribution of 12 numbers 1 to 12:

Distribution 2. *Let's consider following distribution of 12 numbers 1-12 given below*

				Total
A1	1	2	3	6
A2	6	5	4	15
A3	7	8	9	24
A4	12	11	10	33
Total	26	26	26	

Let's construct 16 composite magic squares of order 3 using the Latin square distribution of magic square of order 3 given in Example 9:

Construction 2. *Applying Example 9 over the Distribution 2 we get following 16 blocks of magic square of order 3:*

(11)			42
13	27	2	42
3	14	25	42
26	1	15	42
42	42	42	42

(12)			51
18	28	5	51
4	17	30	51
29	6	16	51
51	51	51	51

(13)			60
19	33	8	60
9	20	31	60
32	7	21	60
60	60	60	60

(14)			69
24	34	11	69
10	23	36	69
35	12	22	69
69	69	69	69

(21)			150
49	39	62	150
63	50	37	150
38	61	51	150
150	150	150	150

(22)			159
54	40	65	159
64	53	42	159
41	66	52	159
159	159	159	159

(23)			168
55	45	68	168
69	56	43	168
44	67	57	168
168	168	168	168

(24)			177
60	46	71	177
70	59	48	177
47	72	58	177
177	177	177	177

(31)			258
85	99	74	258
75	86	97	258
98	73	87	258
258	258	258	258

(32)			267
90	100	77	267
76	89	102	267
101	78	88	267
267	267	267	267

(33)			276
91	105	80	276
81	92	103	276
104	79	93	276
276	276	276	276

(34)			285
96	106	83	285
82	95	108	285
107	84	94	285
285	285	285	285

(41)			366
121	111	134	366
135	122	109	366
110	133	123	366
366	366	366	366

(42)			375
126	112	137	375
136	125	114	375
113	138	124	375
375	375	375	375

(43)			384
127	117	140	384
141	128	115	384
116	139	129	384
384	384	384	384

(44)			393
132	118	143	393
142	131	120	393
119	144	130	393
393	393	393	393

Above 16 blocks of magic squares of order 3 lead us to two pan magic squares. One is of order 4 formed by sums of each block and second a general magic square.

Example 12. . *The* ***pan magic square*** *of order 4 constructed according to sums of each block of order 3 and put according to Exemple 11, we get following pan magic square of order 4:*

		870	870	870	870
	168	285	42	375	870
870	51	366	177	276	870
870	393	60	267	150	870
870	258	159	384	69	870
	870	870	870	870	870

Example 13. . *Let's put above 16 blocks of magic squares of order 3 according to Exemple 11, we get a* ***pan magic square*** *of order 12 given by*

		870	870	870	870	870	870	870	870	870	870	870	870
	55	45	68	96	106	83	13	27	2	126	112	137	870
870	69	56	43	82	95	108	3	14	25	136	125	114	870
870	44	67	57	107	84	94	26	1	15	113	138	124	870
870	18	28	5	121	111	134	60	46	71	91	105	80	870
870	4	17	30	135	122	109	70	59	48	81	92	103	870
870	29	6	16	110	133	123	47	72	58	104	79	93	870
870	132	118	143	19	33	8	90	100	77	49	39	62	870
870	142	131	120	9	20	31	76	89	102	63	50	37	870
870	119	144	130	32	7	21	101	78	88	38	61	51	870
870	85	99	74	54	40	65	127	117	140	24	34	11	870
870	75	86	97	64	53	42	141	128	115	10	23	36	870
870	98	73	87	41	66	52	116	139	129	35	12	22	870
	870	870	870	870	870	870	870	870	870	870	870	870	870

Dwane [6] gave interesting examples of **pan magic squares** of order 12 constructed on similar lines. See below two of examples.

Example 14. ***Dwane's Construction [6]: Pan magic square*** *of order 12, where each block of order 3 are magic squares with different magic sums is given by*

		870	870	870	870	870	870	870	870	870	870	870	870
	8	1	6	69	64	71	87	82	89	134	127	132	870
870	3	5	7	70	68	66	88	86	84	129	131	133	870
870	4	9	2	65	72	67	83	90	85	130	135	128	870
870	107	100	105	114	109	116	24	19	26	53	46	51	870
870	102	104	106	115	113	111	25	23	21	48	50	52	870
870	103	108	101	110	117	112	20	27	22	49	54	47	870
870	58	63	56	11	18	13	137	144	139	76	81	74	870
870	57	59	61	16	14	12	142	140	138	75	77	79	870
870	62	55	60	15	10	17	141	136	143	80	73	78	870
870	121	126	119	92	99	94	38	45	40	31	36	29	870
870	120	122	124	97	95	93	43	41	39	30	32	34	870
870	125	118	123	96	91	98	42	37	44	35	28	33	870
	870	870	870	870	870	870	870	870	870	870	870	870	870

Example 15. ***Dwane's Construction [6]: Pan magic square*** *of order 12, where each block of order 3 are magic squares with different magic sums is given by*

		870	870	870	870	870	870	870	870	870	870	870	870
	98	1	51	72	22	119	78	28	125	140	43	93	870
870	3	50	97	118	71	24	124	77	30	45	92	139	870
870	49	99	2	23	120	70	29	126	76	91	141	44	870
870	131	34	84	87	37	134	57	7	104	113	16	66	870
870	36	83	130	133	86	39	103	56	9	18	65	112	870
870	82	132	35	38	135	85	8	105	55	64	114	17	870
870	67	117	20	5	102	52	47	144	94	73	123	26	870
870	21	68	115	100	53	6	142	95	48	27	74	121	870
870	116	19	69	54	4	101	96	46	143	122	25	75	870
870	88	138	41	32	129	79	14	111	61	58	108	11	870
870	42	89	136	127	80	33	109	62	15	12	59	106	870
870	137	40	90	81	31	128	63	13	110	107	10	60	870
	870	870	870	870	870	870	870	870	870	870	870	870	870

More details on Examples 14 and 15 can be seen in Dwane's web-site [6].

3.2 Magic Square of Order 18

We observed from previous work [27] that the construction of magic square of order 18 is based on block-wise equal sums entries of order 6. All the 9 blocks of order 6 are not magic square. Here we shall bring magic square of order 18, where each block a magic square of order 6 with different magic square sums resulting in a magic square of order 18.

Distribution 3. *Let's consider the following distribution of 1 to 324 numbers in 9 blocks to construct* ***magic square*** *of order 18:*

	1	2	3	4	5	6	...	...	31	32	33	34	35	36	Total
A1	1	10	19	28	37	46	...	...	271	280	289	298	307	316	5706
A2	2	11	20	29	38	47	...	...	272	281	290	299	308	317	5742
A3	3	12	21	30	39	48	...	...	273	282	291	300	309	318	5778
A4	4	13	22	31	40	49	...	...	274	283	292	301	310	319	5814
A5	5	14	23	32	41	50	...	...	275	284	293	302	311	320	5850
A6	6	15	24	33	42	51	...	...	276	285	294	303	312	321	5886
A7	7	16	25	34	43	52	...	...	277	286	295	304	313	322	5922
A8	8	17	26	35	44	53	...	...	278	287	296	305	314	323	5958
A9	9	18	27	36	45	54	...	...	279	288	297	306	315	324	5994

The above Distribution 3 is done in such a way that we have equal difference from one number to another. i.e., 1, 10, 19, etc. In this case, the difference is of 9 numbers.

Construction 3. *Below are 9 blocks of order 6 magic squares constructed according to Example 4 based on the values given in Distribution 3:*

A1						951
1	199	244	298	145	64	951
253	55	307	118	181	37	951
100	46	109	235	271	190	951
280	136	28	208	82	217	951
163	289	91	19	262	127	951
154	226	172	73	10	316	951
951	951	951	951	951	951	951

A2						957
2	200	245	299	146	65	957
254	56	308	119	182	38	957
101	47	110	236	272	191	957
281	137	29	209	83	218	957
164	290	92	20	263	128	957
155	227	173	74	11	317	957
957	957	957	957	957	957	957

A3						963
3	201	246	300	147	66	963
255	57	309	120	183	39	963
102	48	111	237	273	192	963
282	138	30	210	84	219	963
165	291	93	21	264	129	963
156	228	174	75	12	318	963
963	963	963	963	963	963	963

A4						969
4	202	247	301	148	67	969
256	58	310	121	184	40	969
103	49	112	238	274	193	969
283	139	31	211	85	220	969
166	292	94	22	265	130	969
157	229	175	76	13	319	969
969	969	969	969	969	969	969

A5						975
5	203	248	302	149	68	975
257	59	311	122	185	41	975
104	50	113	239	275	194	975
284	140	32	212	86	221	975
167	293	95	23	266	131	975
158	230	176	77	14	320	975
975	975	975	975	975	975	975

A6						981
6	204	249	303	150	69	981
258	60	312	123	186	42	981
105	51	114	240	276	195	981
285	141	33	213	87	222	981
168	294	96	24	267	132	981
159	231	177	78	15	321	981
981	981	981	981	981	981	981

A7						987
7	205	250	304	151	70	987
259	61	313	124	187	43	987
106	52	115	241	277	196	987
286	142	34	214	88	223	987
169	295	97	25	268	133	987
160	232	178	79	16	322	987
987	987	987	987	987	987	987

A8						993
8	206	251	305	152	71	993
260	62	314	125	188	44	993
107	53	116	242	278	197	993
287	143	35	215	89	224	993
170	296	98	26	269	134	993
161	233	179	80	17	323	993
993	993	993	993	993	993	993

A9						999
9	207	252	306	153	72	999
261	63	315	126	189	45	999
108	54	117	243	279	198	999
288	144	36	216	90	225	999
171	297	99	27	270	135	999
162	234	180	81	18	324	999
999	999	999	999	999	999	999

Structure 2. *Let's put the above 9 blocks of magic squares of order 6 according to magic square of order 3 as given below:*

A8	A1	A6
A3	A5	A7
A4	A9	A2

The above Structure 2 lead us to two magic squares. One is of order 3 based on sum of each block and another a magic square of order 18.

Example 16. *A magic square of order 3 constructed according Structure 2 is given by*

			2925
993	951	981	2925
963	975	987	2925
969	999	957	2925
2925	2925	2925	2925

Example 17. *A magic square of order 18 constructed according Structure 2 is given by*

																		2925
8	206	251	305	152	71	1	199	244	298	145	64	6	204	249	303	150	69	2925
260	62	314	125	188	44	253	55	307	118	181	37	258	60	312	123	186	42	2925
107	53	116	242	278	197	100	46	109	235	271	190	105	51	114	240	276	195	2925
287	143	35	215	89	224	280	136	28	208	82	217	285	141	33	213	87	222	2925
170	296	98	26	269	134	163	289	91	19	262	127	168	294	96	24	267	132	2925
161	233	179	80	17	323	154	226	172	73	10	316	159	231	177	78	15	321	2925
3	201	246	300	147	66	5	203	248	302	149	68	7	205	250	304	151	70	2925
255	57	309	120	183	39	257	59	311	122	185	41	259	61	313	124	187	43	2925
102	48	111	237	273	192	104	50	113	239	275	194	106	52	115	241	277	196	2925
282	138	30	210	84	219	284	140	32	212	86	221	286	142	34	214	88	223	2925
165	291	93	21	264	129	167	293	95	23	266	131	169	295	97	25	268	133	2925
156	228	174	75	12	318	158	230	176	77	14	320	160	232	178	79	16	322	2925
4	202	247	301	148	67	9	207	252	306	153	72	2	200	245	299	146	65	2925
256	58	310	121	184	40	261	63	315	126	189	45	254	56	308	119	182	38	2925
103	49	112	238	274	193	108	54	117	243	279	198	101	47	110	236	272	191	2925
283	139	31	211	85	220	288	144	36	216	90	225	281	137	29	209	83	218	2925
166	292	94	22	265	130	171	297	99	27	270	135	164	290	92	20	263	128	2925
157	229	175	76	13	319	162	234	180	81	18	324	155	227	173	74	11	317	2925
2925	2925	2925	2925	2925	2925	2925	2925	2925	2925	2925	2925	2925	2925	2925	2925	2925	2925	2925

The above magic square is with magic sum $S_{18\times18} = 2925$. The 9 blocks of order 6 are magic squares with different magic sums forming a magic square of order 3 given in Example 16.

3.3 Magic Square of Order 20

In [26] author constructed a **pan magic square** of order 20, where each block of order 4 are **pan magic squares** with equal sums. Now the question is if we want to construct a magic square of order 20, with each block of order 5 of same sum. It impossible to construct as the magic sums of order 20, $S_{20\times20} = 4010$ is not division by 4, i.e., we have $\frac{4010}{4} = \frac{2005}{2} = 1002.5$. In this the other possibility is to construct with different sub-blocks sums. Let's divide the numbers 1-400 according to distribution given below.

Distribution 4. . *Let's divide the numbers 400, i.e., from 1 to 400 in 16 blocks of 25 each according to following table:*

	1	2	3	4	5	6	...	...	20	21	22	23	24	25	Total
A1	1	17	33	49	65	81	...	...	305	321	337	353	369	385	4825
A2	2	18	34	50	66	82	...	...	306	322	338	354	370	386	4850
A3	3	19	35	51	67	83	...	...	307	323	339	355	371	387	4875
A4	4	20	36	52	68	84	...	...	308	324	340	356	372	388	4900
A5	5	21	37	53	69	85	...	...	309	325	341	357	373	389	4925
A6	6	22	38	54	70	86	...	...	310	326	342	358	374	390	4950
A7	7	23	39	55	71	87	...	...	311	327	343	359	375	391	4975
A8	8	24	40	56	72	88	...	...	312	328	344	360	376	392	5000
A9	9	25	41	57	73	89	...	...	313	329	345	361	377	393	5025
A10	10	26	42	58	74	90	...	...	314	330	346	362	378	394	5050
A11	11	27	43	59	75	91	...	...	315	331	347	363	379	395	5075
A12	12	28	44	60	76	92	...	...	316	332	348	364	380	396	5100
A13	13	29	45	61	77	93	...	...	317	333	349	365	381	397	5125
A14	14	30	46	62	78	94	...	...	318	334	350	366	382	398	5150
A15	15	31	47	63	79	95	...	...	319	335	351	367	383	399	5175
A16	16	32	48	64	80	96	...	...	320	336	352	368	384	400	5200

According to Structure 1 we have two magic squares. One is of order 4 formed by magic sum of each 5×5 block and another a normal magic square of order 20.

Construction 4. *Below are 16 blocks of order 5 pan magic squares constructed according to Example 3 based on the values given in Distribution 4:*

A1		965	965	965	965	965
	1	129	177	305	353	965
965	257	385	33	81	209	965
965	113	161	289	337	65	965
965	369	17	145	193	241	965
965	225	273	321	49	97	965
	965	965	965	965	965	965

A2		970	970	970	970	970
	2	130	178	306	354	970
970	258	386	34	82	210	970
970	114	162	290	338	66	970
970	370	18	146	194	242	970
970	226	274	322	50	98	970
	970	970	970	970	970	970

A3		975	975	975	975	975
	3	131	179	307	355	975
975	259	387	35	83	211	975
975	115	163	291	339	67	975
975	371	19	147	195	243	975
975	227	275	323	51	99	975
	975	975	975	975	975	975

A4		980	980	980	980	980
	4	132	180	308	356	980
980	260	388	36	84	212	980
980	116	164	292	340	68	980
980	372	20	148	196	244	980
980	228	276	324	52	100	980
	980	980	980	980	980	980

A5		985	985	985	985	985
	5	133	181	309	357	985
985	261	389	37	85	213	985
985	117	165	293	341	69	985
985	373	21	149	197	245	985
985	229	277	325	53	101	985
	985	985	985	985	985	985

A6		990	990	990	990	990
	6	134	182	310	358	990
990	262	390	38	86	214	990
990	118	166	294	342	70	990
990	374	22	150	198	246	990
990	230	278	326	54	102	990
	990	990	990	990	990	990

A7		995	995	995	995	995
	7	135	183	311	359	995
995	263	391	39	87	215	995
995	119	167	295	343	71	995
995	375	23	151	199	247	995
995	231	279	327	55	103	995
	995	995	995	995	995	995

A8		1000	1000	1000	1000	1000
	8	136	184	312	360	1000
1000	264	392	40	88	216	1000
1000	120	168	296	344	72	1000
1000	376	24	152	200	248	1000
1000	232	280	328	56	104	1000
	1000	1000	1000	1000	1000	1000

A9		1005	1005	1005	1005	1005
	9	137	185	313	361	1005
1005	265	393	41	89	217	1005
1005	121	169	297	345	73	1005
1005	377	25	153	201	249	1005
1005	233	281	329	57	105	1005
	1005	1005	1005	1005	1005	1005

A10		1010	1010	1010	1010	1010
	10	138	186	314	362	1010
1010	266	394	42	90	218	1010
1010	122	170	298	346	74	1010
1010	378	26	154	202	250	1010
1010	234	282	330	58	106	1010
	1010	1010	1010	1010	1010	1010

A11		1015	1015	1015	1015	1015
	11	139	187	315	363	1015
1015	267	395	43	91	219	1015
1015	123	171	299	347	75	1015
1015	379	27	155	203	251	1015
1015	235	283	331	59	107	1015
	1015	1015	1015	1015	1015	1015

A12		1020	1020	1020	1020	1020
	12	140	188	316	364	1020
1020	268	396	44	92	220	1020
1020	124	172	300	348	76	1020
1020	380	28	156	204	252	1020
1020	236	284	332	60	108	1020
	1020	1020	1020	1020	1020	1020

A13		1025	1025	1025	1025	1025
	13	141	189	317	365	1025
1025	269	397	45	93	221	1025
1025	125	173	301	349	77	1025
1025	381	29	157	205	253	1025
1025	237	285	333	61	109	1025
	1025	1025	1025	1025	1025	1025

A14		1030	1030	1030	1030	1030
	14	142	190	318	366	1030
1030	270	398	46	94	222	1030
1030	126	174	302	350	78	1030
1030	382	30	158	206	254	1030
1030	238	286	334	62	110	1030
	1030	1030	1030	1030	1030	1030

A15		1035	1035	1035	1035	1035
	15	143	191	319	367	1035
1035	271	399	47	95	223	1035
1035	127	175	303	351	79	1035
1035	383	31	159	207	255	1035
1035	239	287	335	63	111	1035
	1035	1035	1035	1035	1035	1035

A16		1040	1040	1040	1040	1040
	16	144	192	320	368	1040
1040	272	400	48	96	224	1040
1040	128	176	304	352	80	1040
1040	384	32	160	208	256	1040
1040	240	288	336	64	112	1040
	1040	1040	1040	1040	1040	1040

Example 18. ***Pan magic square*** *of order 4, formed by 16 magic square sums of each* 5×5 *block is given by*

		4010	4010	4010	4010
	995	1020	965	1030	4010
4010	970	1025	1000	1015	4010
4010	1040	975	1010	985	4010
4010	1005	990	1035	980	4010
	4010	4010	4010	4010	4010

Example 19. *16 blocks of magic squares of order 5 constructed according to Example 2, using data given in Distribution 4, and putting them according to Structure 1, we get a* ***pan magic square*** *of order 20 is given by*

		4010	4010	4010	4010	4010	4010	4010	4010	4010	4010	4010	4010	4010	4010	4010	4010	4010	4010	4010	4010
	7	135	183	311	359	12	140	188	316	364	1	129	177	305	353	14	142	190	318	366	4010
4010	263	391	39	87	215	268	396	44	92	220	257	385	33	81	209	270	398	46	94	222	4010
4010	119	167	295	343	71	124	172	300	348	76	113	161	289	337	65	126	174	302	350	78	4010
4010	375	23	151	199	247	380	28	156	204	252	369	17	145	193	241	382	30	158	206	254	4010
4010	231	279	327	55	103	236	284	332	60	108	225	273	321	49	97	238	286	334	62	110	4010
4010	2	130	178	306	354	13	141	189	317	365	8	136	184	312	360	11	139	187	315	363	4010
4010	258	386	34	82	210	269	397	45	93	221	264	392	40	88	216	267	395	43	91	219	4010
4010	114	162	290	338	66	125	173	301	349	77	120	168	296	344	72	123	171	299	347	75	4010
4010	370	18	146	194	242	381	29	157	205	253	376	24	152	200	248	379	27	155	203	251	4010
4010	226	274	322	50	98	237	285	333	61	109	232	280	328	56	104	235	283	331	59	107	4010
4010	16	144	192	320	368	3	131	179	307	355	10	138	186	314	362	5	133	181	309	357	4010
4010	272	400	48	96	224	259	387	35	83	211	266	394	42	90	218	261	389	37	85	213	4010
4010	128	176	304	352	80	115	163	291	339	67	122	170	298	346	74	117	165	293	341	69	4010
4010	384	32	160	208	256	371	19	147	195	243	378	26	154	202	250	373	21	149	197	245	4010
4010	240	288	336	64	112	227	275	323	51	99	234	282	330	58	106	229	277	325	53	101	4010
4010	9	137	185	313	361	6	134	182	310	358	15	143	191	319	367	4	132	180	308	356	4010
4010	265	393	41	89	217	262	390	38	86	214	271	399	47	95	223	260	388	36	84	212	4010
4010	121	169	297	345	73	118	166	294	342	70	127	175	303	351	79	116	164	292	340	68	4010
4010	377	25	153	201	249	374	22	150	198	246	383	31	159	207	255	372	20	148	196	244	4010
4010	233	281	329	57	105	230	278	326	54	102	239	287	335	63	111	228	276	324	52	100	4010
	4010	4010	4010	4010	4010	4010	4010	4010	4010	4010	4010	4010	4010	4010	4010	4010	4010	4010	4010	4010	4010

In this case, the magic sum is $S_{20\times 20} := 4010$. Each block of order 5 a magic square with different magic sums given respectively in each cell of Example 18.

3.4 Magic Square of Order 28

In [26] author constructed a **pan magic square** of order 28, where each block of order 4 are **pan magic squares** with equal sums. Now the question is if we want to construct a magic square of order 28, with each block of order 7 of same sum. It impossible to construct as the magic sums of order 28, $S_{28\times 28} = 4010$ is not division by 4, i.e., we have $\frac{10990}{4} = \frac{5495}{2} = 2747.5$. In this the other possibility is to construct with different sub-blocks sums. Let's divide the numbers 1-784 according to distribution given below.

Distribution 5. . *Let's divide the numbers 784, i.e., from 1 to 784 in 16 blocks of 49 each according to following table:*

	1	2	3	4	5	6	7	...	...	43	44	45	46	47	48	49	Total
A1	1	17	33	49	65	81	97	...	...	673	689	705	721	737	753	769	18865
A2	2	18	34	50	66	82	98	...	...	674	690	706	722	738	754	770	18914
A3	3	19	35	51	67	83	99	...	...	675	691	707	723	739	755	771	18963
A4	4	20	36	52	68	84	100	...	...	676	692	708	724	740	756	772	19012
A5	5	21	37	53	69	85	101	...	...	677	693	709	725	741	757	773	19061
A6	6	22	38	54	70	86	102	...	...	678	694	710	726	742	758	774	19110
A7	7	23	39	55	71	87	103	...	...	679	695	711	727	743	759	775	19159
A8	8	24	40	56	72	88	104	...	...	680	696	712	728	744	760	776	19208
A9	9	25	41	57	73	89	105	...	...	681	697	713	729	745	761	777	19257
A10	10	26	42	58	74	90	106	...	...	682	698	714	730	746	762	778	19306
A11	11	27	43	59	75	91	107	...	...	683	699	715	731	747	763	779	19355
A12	12	28	44	60	76	92	108	...	...	684	700	716	732	748	764	780	19404
A13	13	29	45	61	77	93	109	...	...	685	701	717	733	749	765	781	19453
A14	14	30	46	62	78	94	110	...	...	686	702	718	734	750	766	782	19502
A15	15	31	47	63	79	95	111	...	...	687	703	719	735	751	767	783	19551
A16	16	32	48	64	80	96	112	...	...	688	704	720	736	752	768	784	19600

Construction 5. *Below are 16 blocks of order 7* ***pan magic squares*** *constructed according to Example 5 based on the values given in Distribution 5:*

A1		2695	2695	2695	2695	2695	2695	2695
	1	129	257	385	513	641	769	2695
2695	625	753	97	113	241	369	497	2695
2695	353	481	609	737	81	209	225	2695
2695	193	321	337	465	593	721	65	2695
2695	705	49	177	305	433	449	577	2695
2695	545	561	689	33	161	289	417	2695
2695	273	401	529	657	673	17	145	2695
	2695	2695	2695	2695	2695	2695	2695	2695

A2		2702	2702	2702	2702	2702	2702	2702
	2	130	258	386	514	642	770	2702
2702	626	754	98	114	242	370	498	2702
2702	354	482	610	738	82	210	226	2702
2702	194	322	338	466	594	722	66	2702
2702	706	50	178	306	434	450	578	2702
2702	546	562	690	34	162	290	418	2702
2702	274	402	530	658	674	18	146	2702
	2702	2702	2702	2702	2702	2702	2702	2702

A3		2709	2709	2709	2709	2709	2709	2709
	3	131	259	387	515	643	771	2709
2709	627	755	99	115	243	371	499	2709
2709	355	483	611	739	83	211	227	2709
2709	195	323	339	467	595	723	67	2709
2709	707	51	179	307	435	451	579	2709
2709	547	563	691	35	163	291	419	2709
2709	275	403	531	659	675	19	147	2709
	2709	2709	2709	2709	2709	2709	2709	2709

A4		2716	2716	2716	2716	2716	2716	2716
	4	132	260	388	516	644	772	2716
2716	628	756	100	116	244	372	500	2716
2716	356	484	612	740	84	212	228	2716
2716	196	324	340	468	596	724	68	2716
2716	708	52	180	308	436	452	580	2716
2716	548	564	692	36	164	292	420	2716
2716	276	404	532	660	676	20	148	2716
	2716	2716	2716	2716	2716	2716	2716	2716

A5		2723	2723	2723	2723	2723	2723	2723
	5	133	261	389	517	645	773	2723
2723	629	757	101	117	245	373	501	2723
2723	357	485	613	741	85	213	229	2723
2723	197	325	341	469	597	725	69	2723
2723	709	53	181	309	437	453	581	2723
2723	549	565	693	37	165	293	421	2723
2723	277	405	533	661	677	21	149	2723
	2723	2723	2723	2723	2723	2723	2723	2723

A6		2730	2730	2730	2730	2730	2730	2730
	6	134	262	390	518	646	774	2730
2730	630	758	102	118	246	374	502	2730
2730	358	486	614	742	86	214	230	2730
2730	198	326	342	470	598	726	70	2730
2730	710	54	182	310	438	454	582	2730
2730	550	566	694	38	166	294	422	2730
2730	278	406	534	662	678	22	150	2730
	2730	2730	2730	2730	2730	2730	2730	2730

A7		2737	2737	2737	2737	2737	2737	2737
	7	135	263	391	519	647	775	2737
2737	631	759	103	119	247	375	503	2737
2737	359	487	615	743	87	215	231	2737
2737	199	327	343	471	599	727	71	2737
2737	711	55	183	311	439	455	583	2737
2737	551	567	695	39	167	295	423	2737
2737	279	407	535	663	679	23	151	2737
	2737	2737	2737	2737	2737	2737	2737	2737

A8		2744	2744	2744	2744	2744	2744	2744
	8	136	264	392	520	648	776	2744
2744	632	760	104	120	248	376	504	2744
2744	360	488	616	744	88	216	232	2744
2744	200	328	344	472	600	728	72	2744
2744	712	56	184	312	440	456	584	2744
2744	552	568	696	40	168	296	424	2744
2744	280	408	536	664	680	24	152	2744
	2744	2744	2744	2744	2744	2744	2744	2744

A9		2751	2751	2751	2751	2751	2751	2751
	9	137	265	393	521	649	777	2751
2751	633	761	105	121	249	377	505	2751
2751	361	489	617	745	89	217	233	2751
2751	201	329	345	473	601	729	73	2751
2751	713	57	185	313	441	457	585	2751
2751	553	569	697	41	169	297	425	2751
2751	281	409	537	665	681	25	153	2751
	2751	2751	2751	2751	2751	2751	2751	2751

A10		2758	2758	2758	2758	2758	2758	2758
	10	138	266	394	522	650	778	2758
2758	634	762	106	122	250	378	506	2758
2758	362	490	618	746	90	218	234	2758
2758	202	330	346	474	602	730	74	2758
2758	714	58	186	314	442	458	586	2758
2758	554	570	698	42	170	298	426	2758
2758	282	410	538	666	682	26	154	2758
	2758	2758	2758	2758	2758	2758	2758	2758

A11		2765	2765	2765	2765	2765	2765	2765
	11	139	267	395	523	651	779	2765
2765	635	763	107	123	251	379	507	2765
2765	363	491	619	747	91	219	235	2765
2765	203	331	347	475	603	731	75	2765
2765	715	59	187	315	443	459	587	2765
2765	555	571	699	43	171	299	427	2765
2765	283	411	539	667	683	27	155	2765
	2765	2765	2765	2765	2765	2765	2765	2765

A12		2772	2772	2772	2772	2772	2772	2772
	12	140	268	396	524	652	780	2772
2772	636	764	108	124	252	380	508	2772
2772	364	492	620	748	92	220	236	2772
2772	204	332	348	476	604	732	76	2772
2772	716	60	188	316	444	460	588	2772
2772	556	572	700	44	172	300	428	2772
2772	284	412	540	668	684	28	156	2772
	2772	2772	2772	2772	2772	2772	2772	2772

A13		2779	2779	2779	2779	2779	2779	2779
	13	141	269	397	525	653	781	2779
2779	637	765	109	125	253	381	509	2779
2779	365	493	621	749	93	221	237	2779
2779	205	333	349	477	605	733	77	2779
2779	717	61	189	317	445	461	589	2779
2779	557	573	701	45	173	301	429	2779
2779	285	413	541	669	685	29	157	2779
	2779	2779	2779	2779	2779	2779	2779	2779

A14		2786	2786	2786	2786	2786	2786	2786
	14	142	270	398	526	654	782	2786
2786	638	766	110	126	254	382	510	2786
2786	366	494	622	750	94	222	238	2786
2786	206	334	350	478	606	734	78	2786
2786	718	62	190	318	446	462	590	2786
2786	558	574	702	46	174	302	430	2786
2786	286	414	542	670	686	30	158	2786
	2786	2786	2786	2786	2786	2786	2786	2786

A15		2793	2793	2793	2793	2793	2793	2793
	15	143	271	399	527	655	783	2793
2793	639	767	111	127	255	383	511	2793
2793	367	495	623	751	95	223	239	2793
2793	207	335	351	479	607	735	79	2793
2793	719	63	191	319	447	463	591	2793
2793	559	575	703	47	175	303	431	2793
2793	287	415	543	671	687	31	159	2793
	2793	2793	2793	2793	2793	2793	2793	2793

A16		2800	2800	2800	2800	2800	2800	2800
	16	144	272	400	528	656	784	2800
2800	640	768	112	128	256	384	512	2800
2800	368	496	624	752	96	224	240	2800
2800	208	336	352	480	608	736	80	2800
2800	720	64	192	320	448	464	592	2800
2800	560	576	704	48	176	304	432	2800
2800	288	416	544	672	688	32	160	2800
	2800	2800	2800	2800	2800	2800	2800	2800

According to Structure 1 we have two magic squares. One is of order 4, formed by magic square sums of each 7×7 block and another a normal magic square of order 28.

Example 20. *Pan magic square of order 4, formed by 16 magic square sums of each* 7×7 *block is given by*

		10990	10990	10990	10990
	2737	2772	2695	2786	10990
10990	2702	2779	2744	2765	10990
10990	2800	2709	2758	2723	10990
10990	2751	2730	2793	2716	10990
	10990	10990	10990	10990	10990

Example 21. *16 block of magic squares of order 7 constructed according to Example 2, using data given in distribution 5, and putting them according to Structure 1, we get a* ***pan magic square*** *of order 28 is given by*

I	1	2	3	4	5	6	7	8	9	10	11	12	13	14
		10990	**10990**	**10990**	**10990**	**10990**	**10990**	**10990**	**10990**	**10990**	**10990**	**10990**	**10990**	**10990**
	7	135	263	391	519	647	775	12	140	268	396	524	652	780
10990	631	759	103	119	247	375	503	636	764	108	124	252	380	508
10990	359	487	615	743	87	215	231	364	492	620	748	92	220	236
10990	199	327	343	471	599	727	71	204	332	348	476	604	732	76
10990	711	55	183	311	439	455	583	716	60	188	316	444	460	588
10990	551	567	695	39	167	295	423	556	572	700	44	172	300	428
10990	279	407	535	663	679	23	151	284	412	540	668	684	28	156
10990	2	130	258	386	514	642	770	13	141	269	397	525	653	781
10990	626	754	98	114	242	370	498	637	765	109	125	253	381	509
10990	354	482	610	738	82	210	226	365	493	621	749	93	221	237
10990	194	322	338	466	594	722	66	205	333	349	477	605	733	77
10990	706	50	178	306	434	450	578	717	61	189	317	445	461	589
10990	546	562	690	34	162	290	418	557	573	701	45	173	301	429
10990	274	402	530	658	674	18	146	285	413	541	669	685	29	157
10990	16	144	272	400	528	656	784	3	131	259	387	515	643	771
10990	640	768	112	128	256	384	512	627	755	99	115	243	371	499
10990	368	496	624	752	96	224	240	355	483	611	739	83	211	227
10990	208	336	352	480	608	736	80	195	323	339	467	595	723	67
10990	720	64	192	320	448	464	592	707	51	179	307	435	451	579
10990	560	576	704	48	176	304	432	547	563	691	35	163	291	419
10990	288	416	544	672	688	32	160	275	403	531	659	675	19	147
10990	9	137	265	393	521	649	777	6	134	262	390	518	646	774
10990	633	761	105	121	249	377	505	630	758	102	118	246	374	502
10990	361	489	617	745	89	217	233	358	486	614	742	86	214	230
10990	201	329	345	473	601	729	73	198	326	342	470	598	726	70
10990	713	57	185	313	441	457	585	710	54	182	310	438	454	582
10990	553	569	697	41	169	297	425	550	566	694	38	166	294	422
10990	281	409	537	665	681	25	153	278	406	534	662	678	22	150
	10990	**10990**	**10990**	**10990**	**10990**	**10990**	**10990**	**10990**	**10990**	**10990**	**10990**	**10990**	**10990**	**10990**

15	16	17	18	19	20	21	22	23	24	25	26	27	28	(II)
10990	**10990**	**10990**	**10990**	**10990**	**10990**	**10990**	**10990**	**10990**	**10990**	**10990**	**10990**	**10990**	**10990**	**10990**
1	129	257	385	513	641	769	14	142	270	398	526	654	782	**10990**
625	753	97	113	241	369	497	638	766	110	126	254	382	510	**10990**
353	481	609	737	81	209	225	366	494	622	750	94	222	238	**10990**
193	321	337	465	593	721	65	206	334	350	478	606	734	78	**10990**
705	49	177	305	433	449	577	718	62	190	318	446	462	590	**10990**
545	561	689	33	161	289	417	558	574	702	46	174	302	430	**10990**
273	401	529	657	673	17	145	286	414	542	670	686	30	158	**10990**
8	136	264	392	520	648	776	11	139	267	395	523	651	779	**10990**
632	760	104	120	248	376	504	635	763	107	123	251	379	507	**10990**
360	488	616	744	88	216	232	363	491	619	747	91	219	235	**10990**
200	328	344	472	600	728	72	203	331	347	475	603	731	75	**10990**
712	56	184	312	440	456	584	715	59	187	315	443	459	587	**10990**
552	568	696	40	168	296	424	555	571	699	43	171	299	427	**10990**
280	408	536	664	680	24	152	283	411	539	667	683	27	155	**10990**
10	138	266	394	522	650	778	5	133	261	389	517	645	773	**10990**
634	762	106	122	250	378	506	629	757	101	117	245	373	501	**10990**
362	490	618	746	90	218	234	357	485	613	741	85	213	229	**10990**
202	330	346	474	602	730	74	197	325	341	469	597	725	69	**10990**
714	58	186	314	442	458	586	709	53	181	309	437	453	581	**10990**
554	570	698	42	170	298	426	549	565	693	37	165	293	421	**10990**
282	410	538	666	682	26	154	277	405	533	661	677	21	149	**10990**
15	143	271	399	527	655	783	4	132	260	388	516	644	772	**10990**
639	767	111	127	255	383	511	628	756	100	116	244	372	500	**10990**
367	495	623	751	95	223	239	356	484	612	740	84	212	228	**10990**
207	335	351	479	607	735	79	196	324	340	468	596	724	68	**10990**
719	63	191	319	447	463	591	708	52	180	308	436	452	580	**10990**
559	575	703	47	175	303	431	548	564	692	36	164	292	420	**10990**
287	415	543	671	687	31	159	276	404	532	660	676	20	148	**10990**
10990	**10990**	**10990**	**10990**	**10990**	**10990**	**10990**	**10990**	**10990**	**10990**	**10990**	**10990**	**10990**	**10990**	**10990**

Combining Parts (I) and (II) we get the required result. In this case, the magic square sum is $S_{28\times 28} = 10990$. Each 7×7 block is a **pan magic square** of order 7 with different magic square sums given in Example 20.

3.5 Magic Square of Order 30

We observed from previous work [27] that the construction of magic square of order 30 is given in two different ways. One when each block of order 10 a magic square of with equal sums. The second way is based on 25 blocks of order 6 with equal sums of 36 entries in each case. Here the aim is little different, i.e., to present magic square of order 30 with blocks of magic square of orders 6 and 5 with different magic sums resulting in magic squares of order 30. In case of each block of order 5, it is impossible to construct a magic square of order 30, with each block of order 5 of same sums. The reason is the total magic square sum is not divisible by 6, i.e., $S_{30\times 30} = 13515$ is not division by 6. In this case, we have $\frac{13515}{6} = 2252.5$. In this case, the possiblity is to construct with different sub-blocks sums of order 5.

3.5.1 First Approach

Let's distribute 900 numbers from 1 to 900 in 36 blocks with each block 25 numbers separated by equal differences, such as 1, 37, 73, etc.

Distribution 6. *Let's consider the following distribution of 324 numbers in 9 blocks to construct **magic square** of order 30:*

	1	2	3	4	5	6	…	…	20	21	22	23	24	25	Total
A1	1	37	73	109	145	181	…	…	685	721	757	793	829	865	10825
A2	2	38	74	110	146	182	…	…	686	722	758	794	830	866	10850
A3	3	39	75	111	147	183	…	…	687	723	759	795	831	867	10875
A4	4	40	76	112	148	184	…	…	688	724	760	796	832	868	10900
A5	5	41	77	113	149	185	…	…	689	725	761	797	833	869	10925
A6	6	42	78	114	150	186	…	…	690	726	762	798	834	870	10950
A7	7	43	79	115	151	187	…	…	691	727	763	799	835	871	10975
A8	8	44	80	116	152	188	…	…	692	728	764	800	836	872	11000
A9	9	45	81	117	153	189	…	…	693	729	765	801	837	873	11025
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
A28	28	64	100	136	172	208	…	…	712	748	784	820	856	892	11500
A29	29	65	101	137	173	209	…	…	713	749	785	821	857	893	11525
A30	30	66	102	138	174	210	…	…	714	750	786	822	858	894	11550
A31	31	67	103	139	175	211	…	…	715	751	787	823	859	895	11575
A32	32	68	104	140	176	212	…	…	716	752	788	824	860	896	11600
A33	33	69	105	141	177	213	…	…	717	753	789	825	861	897	11625
A34	34	70	106	142	178	214	…	…	718	754	790	826	862	898	11650
A35	35	71	107	143	179	215	…	…	719	755	791	827	863	899	11675
A36	36	72	108	144	180	216	…	…	720	756	792	828	864	900	11700

The above distribution is done in such a way that we have equal difference from one number to another, i.e., 1, 37, 73, etc. In this case, the difference is of 36 numbers. Let's construct 36 **pan magic squares** of order 5 according to magic square 3 using the values given in Distribution 6. These 36 magic squares of order 5 are given below.

Construction 6. *Below are 36 pan magic squares of order 5 constructed according to Example 3 using the values given in Distribution 6:*

A1		2165	2165	2165	2165	2165
	1	289	397	685	793	2165
2165	577	865	73	181	469	2165
2165	253	361	649	757	145	2165
2165	829	37	325	433	541	2165
2165	505	613	721	109	217	2165
	2165	2165	2165	2165	2165	2165

A2		2170	2170	2170	2170	2170
	2	290	398	686	794	2170
2170	578	866	74	182	470	2170
2170	254	362	650	758	146	2170
2170	830	38	326	434	542	2170
2170	506	614	722	110	218	2170
	2170	2170	2170	2170	2170	2170

A3		2175	2175	2175	2175	2175
	3	291	399	687	795	2175
2175	579	867	75	183	471	2175
2175	255	363	651	759	147	2175
2175	831	39	327	435	543	2175
2175	507	615	723	111	219	2175
	2175	2175	2175	2175	2175	2175

A4		2180	2180	2180	2180	2180
	4	292	400	688	796	2180
2180	580	868	76	184	472	2180
2180	256	364	652	760	148	2180
2180	832	40	328	436	544	2180
2180	508	616	724	112	220	2180
	2180	2180	2180	2180	2180	2180

A5		2185	2185	2185	2185	2185
	5	293	401	689	797	2185
2185	581	869	77	185	473	2185
2185	257	365	653	761	149	2185
2185	833	41	329	437	545	2185
2185	509	617	725	113	221	2185
	2185	2185	2185	2185	2185	2185

A6		2190	2190	2190	2190	2190
	6	294	402	690	798	2190
2190	582	870	78	186	474	2190
2190	258	366	654	762	150	2190
2190	834	42	330	438	546	2190
2190	510	618	726	114	222	2190
	2190	2190	2190	2190	2190	2190

A7		2195	2195	2195	2195	2195
	7	295	403	691	799	2195
2195	583	871	79	187	475	2195
2195	259	367	655	763	151	2195
2195	835	43	331	439	547	2195
2195	511	619	727	115	223	2195
	2195	2195	2195	2195	2195	2195

A8		2200	2200	2200	2200	2200
	8	296	404	692	800	2200
2200	584	872	80	188	476	2200
2200	260	368	656	764	152	2200
2200	836	44	332	440	548	2200
2200	512	620	728	116	224	2200
	2200	2200	2200	2200	2200	2200

A9		2205	2205	2205	2205	2205
	9	297	405	693	801	2205
2205	585	873	81	189	477	2205
2205	261	369	657	765	153	2205
2205	837	45	333	441	549	2205
2205	513	621	729	117	225	2205
	2205	2205	2205	2205	2205	2205

A10		2210	2210	2210	2210	2210
	10	298	406	694	802	2210
2210	586	874	82	190	478	2210
2210	262	370	658	766	154	2210
2210	838	46	334	442	550	2210
2210	514	622	730	118	226	2210
	2210	2210	2210	2210	2210	2210

A11		2215	2215	2215	2215	2215
	11	299	407	695	803	2215
2215	587	875	83	191	479	2215
2215	263	371	659	767	155	2215
2215	839	47	335	443	551	2215
2215	515	623	731	119	227	2215
	2215	2215	2215	2215	2215	2215

A12		2220	2220	2220	2220	2220
	12	300	408	696	804	2220
2220	588	876	84	192	480	2220
2220	264	372	660	768	156	2220
2220	840	48	336	444	552	2220
2220	516	624	732	120	228	2220
	2220	2220	2220	2220	2220	2220

A13		2225	2225	2225	2225	2225
	13	301	409	697	805	2225
2225	589	877	85	193	481	2225
2225	265	373	661	769	157	2225
2225	841	49	337	445	553	2225
2225	517	625	733	121	229	2225
	2225	2225	2225	2225	2225	2225

A14		2230	2230	2230	2230	2230
	14	302	410	698	806	2230
2230	590	878	86	194	482	2230
2230	266	374	662	770	158	2230
2230	842	50	338	446	554	2230
2230	518	626	734	122	230	2230
	2230	2230	2230	2230	2230	2230

A15		2235	2235	2235	2235	2235
	15	303	411	699	807	2235
2235	591	879	87	195	483	2235
2235	267	375	663	771	159	2235
2235	843	51	339	447	555	2235
2235	519	627	735	123	231	2235
	2235	2235	2235	2235	2235	2235

A16		2240	2240	2240	2240	2240
	16	304	412	700	808	2240
2240	592	880	88	196	484	2240
2240	268	376	664	772	160	2240
2240	844	52	340	448	556	2240
2240	520	628	736	124	232	2240
	2240	2240	2240	2240	2240	2240

A17		2245	2245	2245	2245	2245
	17	305	413	701	809	2245
2245	593	881	89	197	485	2245
2245	269	377	665	773	161	2245
2245	845	53	341	449	557	2245
2245	521	629	737	125	233	2245
	2245	2245	2245	2245	2245	2245

A18		2250	2250	2250	2250	2250
	18	306	414	702	810	2250
2250	594	882	90	198	486	2250
2250	270	378	666	774	162	2250
2250	846	54	342	450	558	2250
2250	522	630	738	126	234	2250
	2250	2250	2250	2250	2250	2250

A19		2255	2255	2255	2255	2255
	19	307	415	703	811	2255
2255	595	883	91	199	487	2255
2255	271	379	667	775	163	2255
2255	847	55	343	451	559	2255
2255	523	631	739	127	235	2255
	2255	2255	2255	2255	2255	2255

A20		2260	2260	2260	2260	2260
	20	308	416	704	812	2260
2260	596	884	92	200	488	2260
2260	272	380	668	776	164	2260
2260	848	56	344	452	560	2260
2260	524	632	740	128	236	2260
	2260	2260	2260	2260	2260	2260

A21		2265	2265	2265	2265	2265
	21	309	417	705	813	2265
2265	597	885	93	201	489	2265
2265	273	381	669	777	165	2265
2265	849	57	345	453	561	2265
2265	525	633	741	129	237	2265
	2265	2265	2265	2265	2265	2265

A22		2270	2270	2270	2270	2270
	22	310	418	706	814	2270
2270	598	886	94	202	490	2270
2270	274	382	670	778	166	2270
2270	850	58	346	454	562	2270
2270	526	634	742	130	238	2270
	2270	2270	2270	2270	2270	2270

A23		2275	2275	2275	2275	2275
	23	311	419	707	815	2275
2275	599	887	95	203	491	2275
2275	275	383	671	779	167	2275
2275	851	59	347	455	563	2275
2275	527	635	743	131	239	2275
	2275	2275	2275	2275	2275	2275

A24		2280	2280	2280	2280	2280
	24	312	420	708	816	2280
2280	600	888	96	204	492	2280
2280	276	384	672	780	168	2280
2280	852	60	348	456	564	2280
2280	528	636	744	132	240	2280
	2280	2280	2280	2280	2280	2280

A25		2285	2285	2285	2285	2285
	25	313	421	709	817	2285
2285	601	889	97	205	493	2285
2285	277	385	673	781	169	2285
2285	853	61	349	457	565	2285
2285	529	637	745	133	241	2285
	2285	2285	2285	2285	2285	2285

A26		2290	2290	2290	2290	2290
	26	314	422	710	818	2290
2290	602	890	98	206	494	2290
2290	278	386	674	782	170	2290
2290	854	62	350	458	566	2290
2290	530	638	746	134	242	2290
	2290	2290	2290	2290	2290	2290

A27		2295	2295	2295	2295	2295
	27	315	423	711	819	2295
2295	603	891	99	207	495	2295
2295	279	387	675	783	171	2295
2295	855	63	351	459	567	2295
2295	531	639	747	135	243	2295
	2295	2295	2295	2295	2295	2295

A28		2300	2300	2300	2300	2300
	28	316	424	712	820	2300
2300	604	892	100	208	496	2300
2300	280	388	676	784	172	2300
2300	856	64	352	460	568	2300
2300	532	640	748	136	244	2300
	2300	2300	2300	2300	2300	2300

A29		2305	2305	2305	2305	2305
	29	317	425	713	821	2305
2305	605	893	101	209	497	2305
2305	281	389	677	785	173	2305
2305	857	65	353	461	569	2305
2305	533	641	749	137	245	2305
	2305	2305	2305	2305	2305	2305

A30		2310	2310	2310	2310	2310
	30	318	426	714	822	2310
2310	606	894	102	210	498	2310
2310	282	390	678	786	174	2310
2310	858	66	354	462	570	2310
2310	534	642	750	138	246	2310
	2310	2310	2310	2310	2310	2310

A31		2315	2315	2315	2315	2315
	31	319	427	715	823	2315
2315	607	895	103	211	499	2315
2315	283	391	679	787	175	2315
2315	859	67	355	463	571	2315
2315	535	643	751	139	247	2315
	2315	2315	2315	2315	2315	2315

A32		2320	2320	2320	2320	2320
	32	320	428	716	824	2320
2320	608	896	104	212	500	2320
2320	284	392	680	788	176	2320
2320	860	68	356	464	572	2320
2320	536	644	752	140	248	2320
	2320	2320	2320	2320	2320	2320

A33		2325	2325	2325	2325	2325
	33	321	429	717	825	2325
2325	609	897	105	213	501	2325
2325	285	393	681	789	177	2325
2325	861	69	357	465	573	2325
2325	537	645	753	141	249	2325
	2325	2325	2325	2325	2325	2325

A34		2330	2330	2330	2330	2330
	34	322	430	718	826	2330
2330	610	898	106	214	502	2330
2330	286	394	682	790	178	2330
2330	862	70	358	466	574	2330
2330	538	646	754	142	250	2330
	2330	2330	2330	2330	2330	2330

A35		2335	2335	2335	2335	2335
	35	323	431	719	827	2335
2335	611	899	107	215	503	2335
2335	287	395	683	791	179	2335
2335	863	71	359	467	575	2335
2335	539	647	755	143	251	2335
	2335	2335	2335	2335	2335	2335

A36		2340	2340	2340	2340	2340
	36	324	432	720	828	2340
2340	612	900	108	216	504	2340
2340	288	396	684	792	180	2340
2340	864	72	360	468	576	2340
2340	540	648	756	144	252	2340
	2340	2340	2340	2340	2340	2340

Structure 3. *Let's put 36 blocks of magic square given Construction 6 according to following Structure:*

A1	A23	A28	A34	A17	A8
A29	A7	A35	A14	A21	A5
A12	A6	A13	A27	A31	A22
A32	A16	A4	A24	A10	A25
A19	A33	A11	A3	A30	A15
A18	A26	A20	A9	A2	A36

The above Structure 3 lead us to two magic squares. One is of order 6 based on sum of each block of order 5 given in Construction 6 and second a general magic square of order 30 with all the entries.

Example 22. *A magic square of order 6 constructed according Structure 3 is given by*

						13515
2165	*2275*	*2300*	*2330*	*2245*	*2200*	*13515*
2305	*2195*	*2335*	*2230*	*2265*	*2185*	*13515*
2220	*2190*	*2225*	*2295*	*2315*	*2270*	*13515*
2320	*2240*	*2180*	*2280*	*2210*	*2285*	*13515*
2255	*2325*	*2215*	*2175*	*2310*	*2235*	*13515*
2250	*2290*	*2260*	*2205*	*2170*	*2340*	*13515*
13515	*13515*	*13515*	*13515*	*13515*	*13515*	*13515*

Example 23. *A magic square of order 30 constructed according Structure 3 is given by*

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
(I)														
1	289	397	685	793	23	311	419	707	815	28	316	424	712	820
577	865	73	181	469	599	887	95	203	491	604	892	100	208	496
253	361	649	757	145	275	383	671	779	167	280	388	676	784	172
829	37	325	433	541	851	59	347	455	563	856	64	352	460	568
505	613	721	109	217	527	635	743	131	239	532	640	748	136	244
29	317	425	713	821	7	295	403	691	799	35	323	431	719	827
605	893	101	209	497	583	871	79	187	475	611	899	107	215	503
281	389	677	785	173	259	367	655	763	151	287	395	683	791	179
857	65	353	461	569	835	43	331	439	547	863	71	359	467	575
533	641	749	137	245	511	619	727	115	223	539	647	755	143	251
12	300	408	696	804	6	294	402	690	798	13	301	409	697	805
588	876	84	192	480	582	870	78	186	474	589	877	85	193	481
264	372	660	768	156	258	366	654	762	150	265	373	661	769	157
840	48	336	444	552	834	42	330	438	546	841	49	337	445	553
516	624	732	120	228	510	618	726	114	222	517	625	733	121	229
32	320	428	716	824	16	304	412	700	808	4	292	400	688	796
608	896	104	212	500	592	880	88	196	484	580	868	76	184	472
284	392	680	788	176	268	376	664	772	160	256	364	652	760	148
860	68	356	464	572	844	52	340	448	556	832	40	328	436	544
536	644	752	140	248	520	628	736	124	232	508	616	724	112	220
19	307	415	703	811	33	321	429	717	825	11	299	407	695	803
595	883	91	199	487	609	897	105	213	501	587	875	83	191	479
271	379	667	775	163	285	393	681	789	177	263	371	659	767	155
847	55	343	451	559	861	69	357	465	573	839	47	335	443	551
523	631	739	127	235	537	645	753	141	249	515	623	731	119	227
18	306	414	702	810	26	314	422	710	818	20	308	416	704	812
594	882	90	198	486	602	890	98	206	494	596	884	92	200	488
270	378	666	774	162	278	386	674	782	170	272	380	668	776	164
846	54	342	450	558	854	62	350	458	566	848	56	344	452	560
522	630	738	126	234	530	638	746	134	242	524	632	740	128	236
13515	13515	13515	13515	13515	13515	13515	13515	13515	13515	13515	13515	13515	13515	13515

16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	
(II)															13515
34	322	430	718	826	17	305	413	701	809	8	296	404	692	800	13515
610	898	106	214	502	593	881	89	197	485	584	872	80	188	476	13515
286	394	682	790	178	269	377	665	773	161	260	368	656	764	152	13515
862	70	358	466	574	845	53	341	449	557	836	44	332	440	548	13515
538	646	754	142	250	521	629	737	125	233	512	620	728	116	224	13515
14	302	410	698	806	21	309	417	705	813	5	293	401	689	797	13515
590	878	86	194	482	597	885	93	201	489	581	869	77	185	473	13515
266	374	662	770	158	273	381	669	777	165	257	365	653	761	149	13515
842	50	338	446	554	849	57	345	453	561	833	41	329	437	545	13515
518	626	734	122	230	525	633	741	129	237	509	617	725	113	221	13515
27	315	423	711	819	31	319	427	715	823	22	310	418	706	814	13515
603	891	99	207	495	607	895	103	211	499	598	886	94	202	490	13515
279	387	675	783	171	283	391	679	787	175	274	382	670	778	166	13515
855	63	351	459	567	859	67	355	463	571	850	58	346	454	562	13515
531	639	747	135	243	535	643	751	139	247	526	634	742	130	238	13515
24	312	420	708	816	10	298	406	694	802	25	313	421	709	817	13515
600	888	96	204	492	586	874	82	190	478	601	889	97	205	493	13515
276	384	672	780	168	262	370	658	766	154	277	385	673	781	169	13515
852	60	348	456	564	838	46	334	442	550	853	61	349	457	565	13515
528	636	744	132	240	514	622	730	118	226	529	637	745	133	241	13515
3	291	399	687	795	30	318	426	714	822	15	303	411	699	807	13515
579	867	75	183	471	606	894	102	210	498	591	879	87	195	483	13515
255	363	651	759	147	282	390	678	786	174	267	375	663	771	159	13515
831	39	327	435	543	858	66	354	462	570	843	51	339	447	555	13515
507	615	723	111	219	534	642	750	138	246	519	627	735	123	231	13515
9	297	405	693	801	2	290	398	686	794	36	324	432	720	828	13515
585	873	81	189	477	578	866	74	182	470	612	900	108	216	504	13515
261	369	657	765	153	254	362	650	758	146	288	396	684	792	180	13515
837	45	333	441	549	830	38	326	434	542	864	72	360	468	576	13515
513	621	729	117	225	506	614	722	110	218	540	648	756	144	252	13515
13515	13515	13515	13515	13515	13515	13515	13515	13515	13515	13515	13515	13515	13515	13515	13515

Combining Parts (I) and (II) we get the required result. In this case, the magic square sum is $S_{30\times 30} = 13515$. Each 5×5 block a **pan magic square** of order 5 with different magic square sums given in Example 22.

3.5.2 Second Approach

In the previous subsection, we gave a construction of magic square of order 30 using 36 blocks of magic squares of order 5 with different magic square sums. Even though, we can divide the total magic square sum of order 30 in 6 parts, i.e., $\frac{13515}{5} = 975$, still we don't have blocks of order 6 with equal magic square sums, resulting in magic square of order 30 [27]. Below is a construction of magic square of order 30, with different sub-block sums of magic square of order 6.

Distribution 7. *Let's consider the following distribution to construct* ***magic square*** *of order 30. To divide 900 numbers, from 1 to 900 in 25 blocks of equal difference with 36 elements in each block.*

	1	2	3	4	5	6	...	...	31	32	33	34	35	36	Total
A1	1	26	51	76	101	126	...	...	751	776	801	826	851	876	15786
A2	2	27	52	77	102	127	...	...	752	777	802	827	852	877	15822
A3	3	28	53	78	103	128	...	...	753	778	803	828	853	878	15858
A4	4	29	54	79	104	129	...	...	754	779	804	829	854	879	15894
A5	5	30	55	80	105	130	...	...	755	780	805	830	855	880	15930
A6	6	31	56	81	106	131	...	...	756	781	806	831	856	881	15966
A7	7	32	57	82	107	132	...	...	757	782	807	832	857	882	16002
A8	8	33	58	83	108	133	...	...	758	783	808	833	858	883	16038
A9	9	34	59	84	109	134	...	...	759	784	809	834	859	884	16074
A10	10	35	60	85	110	135	...	...	760	785	810	835	860	885	16110
A11	11	36	61	86	111	136	...	...	761	786	811	836	861	886	16146
A12	12	37	62	87	112	137	...	...	762	787	812	837	862	887	16182
A13	13	38	63	88	113	138	...	...	763	788	813	838	863	888	16218
A14	14	39	64	89	114	139	...	...	764	789	814	839	864	889	16254
A15	15	40	65	90	115	140	...	...	765	790	815	840	865	890	16290
A16	16	41	66	91	116	141	...	...	766	791	816	841	866	891	16326
A17	17	42	67	92	117	142	...	...	767	792	817	842	867	892	16362
A18	18	43	68	93	118	143	...	...	768	793	818	843	868	893	16398
A19	19	44	69	94	119	144	...	...	769	794	819	844	869	894	16434
A20	20	45	70	95	120	145	...	...	770	795	820	845	870	895	16470
A21	21	46	71	96	121	146	...	...	771	796	821	846	871	896	16506
A22	22	47	72	97	122	147	...	...	772	797	822	847	872	897	16542
A23	23	48	73	98	123	148	...	...	773	798	823	848	873	898	16578
A24	24	49	74	99	124	149	...	...	774	799	824	849	874	899	16614
A25	25	50	75	100	125	150	...	...	775	800	825	850	875	900	16650

Let's construct 25 blocks of magic squares of order 6 according to Example 4. In this case, all the 25 blocks are of different magic squares sums.

Construction 7. *Below are 25 magic squares of order 6 constructed according to Example 4 with the entries given in Distribution 7.*

A1						2631
1	551	676	826	401	176	2631
701	151	851	326	501	101	2631
276	126	301	651	751	526	2631
776	376	76	576	226	601	2631
451	801	251	51	726	351	2631
426	626	476	201	26	876	2631
2631	2631	2631	2631	2631	2631	2631

A2						2637
2	552	677	827	402	177	2637
702	152	852	327	502	102	2637
277	127	302	652	752	527	2637
777	377	77	577	227	602	2637
452	802	252	52	727	352	2637
427	627	477	202	27	877	2637
2637	2637	2637	2637	2637	2637	2637

A3						2643
3	553	678	828	403	178	2643
703	153	853	328	503	103	2643
278	128	303	653	753	528	2643
778	378	78	578	228	603	2643
453	803	253	53	728	353	2643
428	628	478	203	28	878	2643
2643	2643	2643	2643	2643	2643	2643

A4						2649
4	554	679	829	404	179	2649
704	154	854	329	504	104	2649
279	129	304	654	754	529	2649
779	379	79	579	229	604	2649
454	804	254	54	729	354	2649
429	629	479	204	29	879	2649
2649	2649	2649	2649	2649	2649	2649

A5						2655
5	555	680	830	405	180	2655
705	155	855	330	505	105	2655
280	130	305	655	755	530	2655
780	380	80	580	230	605	2655
455	805	255	55	730	355	2655
430	630	480	205	30	880	2655
2655	2655	2655	2655	2655	2655	2655

A6						2661
6	556	681	831	406	181	2661
706	156	856	331	506	106	2661
281	131	306	656	756	531	2661
781	381	81	581	231	606	2661
456	806	256	56	731	356	2661
431	631	481	206	31	881	2661
2661	2661	2661	2661	2661	2661	2661

A7						2667
7	557	682	832	407	182	2667
707	157	857	332	507	107	2667
282	132	307	657	757	532	2667
782	382	82	582	232	607	2667
457	807	257	57	732	357	2667
432	632	482	207	32	882	2667
2667	2667	2667	2667	2667	2667	2667

A8						2673
8	558	683	833	408	183	2673
708	158	858	333	508	108	2673
283	133	308	658	758	533	2673
783	383	83	583	233	608	2673
458	808	258	58	733	358	2673
433	633	483	208	33	883	2673
2673	2673	2673	2673	2673	2673	2673

A9						2679
9	559	684	834	409	184	2679
709	159	859	334	509	109	2679
284	134	309	659	759	534	2679
784	384	84	584	234	609	2679
459	809	259	59	734	359	2679
434	634	484	209	34	884	2679
2679	2679	2679	2679	2679	2679	2679

A10						2685
10	560	685	835	410	185	2685
710	160	860	335	510	110	2685
285	135	310	660	760	535	2685
785	385	85	585	235	610	2685
460	810	260	60	735	360	2685
435	635	485	210	35	885	2685
2685	2685	2685	2685	2685	2685	2685

A11						2691
11	561	686	836	411	186	2691
711	161	861	336	511	111	2691
286	136	311	661	761	536	2691
786	386	86	586	236	611	2691
461	811	261	61	736	361	2691
436	636	486	211	36	886	2691
2691	2691	2691	2691	2691	2691	2691

A12						2697
12	562	687	837	412	187	2697
712	162	862	337	512	112	2697
287	137	312	662	762	537	2697
787	387	87	587	237	612	2697
462	812	262	62	737	362	2697
437	637	487	212	37	887	2697
2697	2697	2697	2697	2697	2697	2697

A13						2703
13	563	688	838	413	188	2703
713	163	863	338	513	113	2703
288	138	313	663	763	538	2703
788	388	88	588	238	613	2703
463	813	263	63	738	363	2703
438	638	488	213	38	888	2703
2703	2703	2703	2703	2703	2703	2703

A14						2709
14	564	689	839	414	189	2709
714	164	864	339	514	114	2709
289	139	314	664	764	539	2709
789	389	89	589	239	614	2709
464	814	264	64	739	364	2709
439	639	489	214	39	889	2709
2709	2709	2709	2709	2709	2709	2709

A15						2715
15	565	690	840	415	190	2715
715	165	865	340	515	115	2715
290	140	315	665	765	540	2715
790	390	90	590	240	615	2715
465	815	265	65	740	365	2715
440	640	490	215	40	890	2715
2715	2715	2715	2715	2715	2715	2715

A16						2721
16	566	691	841	416	191	2721
716	166	866	341	516	116	2721
291	141	316	666	766	541	2721
791	391	91	591	241	616	2721
466	816	266	66	741	366	2721
441	641	491	216	41	891	2721
2721	2721	2721	2721	2721	2721	2721

A17						2727
17	567	692	842	417	192	2727
717	167	867	342	517	117	2727
292	142	317	667	767	542	2727
792	392	92	592	242	617	2727
467	817	267	67	742	367	2727
442	642	492	217	42	892	2727
2727	2727	2727	2727	2727	2727	2727

A18						2733
18	568	693	843	418	193	2733
718	168	868	343	518	118	2733
293	143	318	668	768	543	2733
793	393	93	593	243	618	2733
468	818	268	68	743	368	2733
443	643	493	218	43	893	2733
2733	2733	2733	2733	2733	2733	2733

A19						2739
19	569	694	844	419	194	2739
719	169	869	344	519	119	2739
294	144	319	669	769	544	2739
794	394	94	594	244	619	2739
469	819	269	69	744	369	2739
444	644	494	219	44	894	2739
2739	2739	2739	2739	2739	2739	2739

A20						2745
20	570	695	845	420	195	2745
720	170	870	345	520	120	2745
295	145	320	670	770	545	2745
795	395	95	595	245	620	2745
470	820	270	70	745	370	2745
445	645	495	220	45	895	2745
2745	2745	2745	2745	2745	2745	2745

A21						2751
21	571	696	846	421	196	2751
721	171	871	346	521	121	2751
296	146	321	671	771	546	2751
796	396	96	596	246	621	2751
471	821	271	71	746	371	2751
446	646	496	221	46	896	2751
2751	2751	2751	2751	2751	2751	2751

A22						2757
22	572	697	847	422	197	2757
722	172	872	347	522	122	2757
297	147	322	672	772	547	2757
797	397	97	597	247	622	2757
472	822	272	72	747	372	2757
447	647	497	222	47	897	2757
2757	2757	2757	2757	2757	2757	2757

A23						2763
23	573	698	848	423	198	2763
723	173	873	348	523	123	2763
298	148	323	673	773	548	2763
798	398	98	598	248	623	2763
473	823	273	73	748	373	2763
448	648	498	223	48	898	2763
2763	2763	2763	2763	2763	2763	2763

A24						2769
24	574	699	849	424	199	2769
724	174	874	349	524	124	2769
299	149	324	674	774	549	2769
799	399	99	599	249	624	2769
474	824	274	74	749	374	2769
449	649	499	224	49	899	2769
2769	2769	2769	2769	2769	2769	2769

A25						2775
25	575	700	850	425	200	2775
725	175	875	350	525	125	2775
300	150	325	675	775	550	2775
800	400	100	600	250	625	2775
475	825	275	75	750	375	2775
450	650	500	225	50	900	2775
2775	2775	2775	2775	2775	2775	2775

Structure 4. *Let's put 25 blocks of magic square given Construction 7 according to following Structure:*

A1	A9	A12	A20	A23
A17	A25	A3	A6	A14
A8	A11	A19	A22	A5
A24	A2	A10	A13	A16
A15	A18	A21	A4	A7

The above Structure 4 we have two magic squares. One is **pan magic square** of order 5 based on sums of each block given in Construction 7 and another a magic square of order 30.

Example 24. *According to Structure 4, and magic sums given in Construction 7 according to Distribution 7 give the following* ***pan magic square*** *of order 5:*

		13515	13515	13515	13515	13515
	2631	2679	2697	2745	2763	13515
13515	2727	2775	2643	2661	2709	13515
13515	2673	2691	2739	2757	2655	13515
13515	2769	2637	2685	2703	2721	13515
13515	2715	2733	2751	2649	2667	13515
	13515	13515	13515	13515	13515	13515

Example 25. *According to Distribution 7 and Structure 4 we have a* ***magic square*** *of order 30 given by*

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
(I)														
1	551	676	826	401	176	9	559	684	834	409	184	12	562	687
701	151	851	326	501	101	709	159	859	334	509	109	712	162	862
276	126	301	651	751	526	284	134	309	659	759	534	287	137	312
776	376	76	576	226	601	784	384	84	584	234	609	787	387	87
451	801	251	51	726	351	459	809	259	59	734	359	462	812	262
426	626	476	201	26	876	434	634	484	209	34	884	437	637	487
17	567	692	842	417	192	25	575	700	850	425	200	3	553	678
717	167	867	342	517	117	725	175	875	350	525	125	703	153	853
292	142	317	667	767	542	300	150	325	675	775	550	278	128	303
792	392	92	592	242	617	800	400	100	600	250	625	778	378	78
467	817	267	67	742	367	475	825	275	75	750	375	453	803	253
442	642	492	217	42	892	450	650	500	225	50	900	428	628	478
8	558	683	833	408	183	11	561	686	836	411	186	19	569	694
708	158	858	333	508	108	711	161	861	336	511	111	719	169	869
283	133	308	658	758	533	286	136	311	661	761	536	294	144	319
783	383	83	583	233	608	786	386	86	586	236	611	794	394	94
458	808	258	58	733	358	461	811	261	61	736	361	469	819	269
433	633	483	208	33	883	436	636	486	211	36	886	444	644	494
24	574	699	849	424	199	2	552	677	827	402	177	10	560	685
724	174	874	349	524	124	702	152	852	327	502	102	710	160	860
299	149	324	674	774	549	277	127	302	652	752	527	285	135	310
799	399	99	599	249	624	777	377	77	577	227	602	785	385	85
474	824	274	74	749	374	452	802	252	52	727	352	460	810	260
449	649	499	224	49	899	427	627	477	202	27	877	435	635	485
15	565	690	840	415	190	18	568	693	843	418	193	21	571	696
715	165	865	340	515	115	718	168	868	343	518	118	721	171	871
290	140	315	665	765	540	293	143	318	668	768	543	296	146	321
790	390	90	590	240	615	793	393	93	593	243	618	796	396	96
465	815	265	65	740	365	468	818	268	68	743	368	471	821	271
440	640	490	215	40	890	443	643	493	218	43	893	446	646	496
13515	**13515**	**13515**	**13515**	**13515**	**13515**	**13515**	**13515**	**13515**	**13515**	**13515**	**13515**	**13515**	**13515**	**13515**

16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	
(II)															13515
837	412	187	20	570	695	845	420	195	23	573	698	848	423	198	13515
337	512	112	720	170	870	345	520	120	723	173	873	348	523	123	13515
662	762	537	295	145	320	670	770	545	298	148	323	673	773	548	13515
587	237	612	795	395	95	595	245	620	798	398	98	598	248	623	13515
62	737	362	470	820	270	70	745	370	473	823	273	73	748	373	13515
212	37	887	445	645	495	220	45	895	448	648	498	223	48	898	13515
828	403	178	6	556	681	831	406	181	14	564	689	839	414	189	13515
328	503	103	706	156	856	331	506	106	714	164	864	339	514	114	13515
653	753	528	281	131	306	656	756	531	289	139	314	664	764	539	13515
578	228	603	781	381	81	581	231	606	789	389	89	589	239	614	13515
53	728	353	456	806	256	56	731	356	464	814	264	64	739	364	13515
203	28	878	431	631	481	206	31	881	439	639	489	214	39	889	13515
844	419	194	22	572	697	847	422	197	5	555	680	830	405	180	13515
344	519	119	722	172	872	347	522	122	705	155	855	330	505	105	13515
669	769	544	297	147	322	672	772	547	280	130	305	655	755	530	13515
594	244	619	797	397	97	597	247	622	780	380	80	580	230	605	13515
69	744	369	472	822	272	72	747	372	455	805	255	55	730	355	13515
219	44	894	447	647	497	222	47	897	430	630	480	205	30	880	13515
835	410	185	13	563	688	838	413	188	16	566	691	841	416	191	13515
335	510	110	713	163	863	338	513	113	716	166	866	341	516	116	13515
660	760	535	288	138	313	663	763	538	291	141	316	666	766	541	13515
585	235	610	788	388	88	588	238	613	791	391	91	591	241	616	13515
60	735	360	463	813	263	63	738	363	466	816	266	66	741	366	13515
210	35	885	438	638	488	213	38	888	441	641	491	216	41	891	13515
846	421	196	4	554	679	829	404	179	7	557	682	832	407	182	13515
346	521	121	704	154	854	329	504	104	707	157	857	332	507	107	13515
671	771	546	279	129	304	654	754	529	282	132	307	657	757	532	13515
596	246	621	779	379	79	579	229	604	782	382	82	582	232	607	13515
71	746	371	454	804	254	54	729	354	457	807	257	57	732	357	13515
221	46	896	429	629	479	204	29	879	432	632	482	207	32	882	13515
13515	13515	13515	13515	13515	13515	13515	13515	13515	13515	13515	13515	13515	13515	13515	13515

Combining Parts (I) and (II) we get the required result. In this case, the magic square sum is $S_{30\times 30} = 13515$. Each 5×5 block a pan magic square of order 5 with different magic square sums given in Construction 7.

3.6 Magic Square of Order 36

As in other cases, here also, it is not possible write a magic square of order 36 with each block of order 9 a magic square of with equal sums, because, the magic sum of order 36 divided by 4 give a fraction value, i.e., $\frac{23346}{4} = \frac{11673}{2} = 5836.5$. The only alternative is to have 16 blocks of order 9 with different magic sums.

Distribution 8. *Let's consider the following Distribution to construct* ***magic square*** *of order 36:*

	1	2	3	4	5	6	7	8	...	...	76	77	78	79	80	81	Total
A1	1	17	33	49	65	81	97	113	...	...	1201	1217	1233	1249	1265	1281	51921
A2	2	18	34	50	66	82	98	114	...	...	1202	1218	1234	1250	1266	1282	52002
A3	3	19	35	51	67	83	99	115	...	...	1203	1219	1235	1251	1267	1283	52083
A4	4	20	36	52	68	84	100	116	...	...	1204	1220	1236	1252	1268	1284	52164
A5	5	21	37	53	69	85	101	117	...	...	1205	1221	1237	1253	1269	1285	52245
A6	6	22	38	54	70	86	102	118	...	...	1206	1222	1238	1254	1270	1286	52326
A7	7	23	39	55	71	87	103	119	...	...	1207	1223	1239	1255	1271	1287	52407
A8	8	24	40	56	72	88	104	120	...	...	1208	1224	1240	1256	1272	1288	52488
A9	9	25	41	57	73	89	105	121	...	...	1209	1225	1241	1257	1273	1289	52569
A10	10	26	42	58	74	90	106	122	...	...	1210	1226	1242	1258	1274	1290	52650
A11	11	27	43	59	75	91	107	123	...	...	1211	1227	1243	1259	1275	1291	52731
A12	12	28	44	60	76	92	108	124	...	...	1212	1228	1244	1260	1276	1292	52812
A13	13	29	45	61	77	93	109	125	...	...	1213	1229	1245	1261	1277	1293	52893
A14	14	30	46	62	78	94	110	126	...	...	1214	1230	1246	1262	1278	1294	52974
A15	15	31	47	63	79	95	111	127	...	...	1215	1231	1247	1263	1279	1295	53055
A16	16	32	48	64	80	96	112	128	...	...	1216	1232	1248	1264	1280	1296	53136

There are total 16 blocks of 81 numbers each with equal difference in each row, i.e, 1, 17, 33,etc. Here the difference is of 16 numbers. Lets make 16 **pan magic squares** of order 9 according to Example 6 and put them according to **pan magic square** of order 4 given in Example 2:

Construction 8. *Below are 16 blocks of order 9 pan magic squares constructed according to Example 6 based on the values given in Distribution* **??**:

A1		5769	5769	5769	5769	5769	5769	5769	5769	5769
	113	769	1041	961	369	593	561	1217	145	5769
5769	577	993	353	177	545	1201	1025	97	801	5769
5769	1233	161	529	785	1009	129	337	609	977	5769
5769	209	433	1281	1057	33	833	657	881	385	5769
5769	817	1089	17	417	641	865	1265	193	465	5769
5769	897	401	625	449	1249	225	1	849	1073	5769
5769	305	673	945	1153	273	497	753	1121	49	5769
5769	481	1185	257	81	737	1105	929	289	705	5769
5769	1137	65	721	689	913	321	241	513	1169	5769
	5769	5769	5769	5769	5769	5769	5769	5769	5769	5769

A2		5778	5778	5778	5778	5778	5778	5778	5778	5778
	114	770	1042	962	370	594	562	1218	146	5778
5778	578	994	354	178	546	1202	1026	98	802	5778
5778	1234	162	530	786	1010	130	338	610	978	5778
5778	210	434	1282	1058	34	834	658	882	386	5778
5778	818	1090	18	418	642	866	1266	194	466	5778
5778	898	402	626	450	1250	226	2	850	1074	5778
5778	306	674	946	1154	274	498	754	1122	50	5778
5778	482	1186	258	82	738	1106	930	290	706	5778
5778	1138	66	722	690	914	322	242	514	1170	5778
	5778	5778	5778	5778	5778	5778	5778	5778	5778	5778

A3		5787	5787	5787	5787	5787	5787	5787	5787	5787
	115	771	1043	963	371	595	563	1219	147	5787
5787	579	995	355	179	547	1203	1027	99	803	5787
5787	1235	163	531	787	1011	131	339	611	979	5787
5787	211	435	1283	1059	35	835	659	883	387	5787
5787	819	1091	19	419	643	867	1267	195	467	5787
5787	899	403	627	451	1251	227	3	851	1075	5787
5787	307	675	947	1155	275	499	755	1123	51	5787
5787	483	1187	259	83	739	1107	931	291	707	5787
5787	1139	67	723	691	915	323	243	515	1171	5787
	5787	5787	5787	5787	5787	5787	5787	5787	5787	5787

A4		5796	5796	5796	5796	5796	5796	5796	5796	5796
	116	772	1044	964	372	596	564	1220	148	5796
5796	580	996	356	180	548	1204	1028	100	804	5796
5796	1236	164	532	788	1012	132	340	612	980	5796
5796	212	436	1284	1060	36	836	660	884	388	5796
5796	820	1092	20	420	644	868	1268	196	468	5796
5796	900	404	628	452	1252	228	4	852	1076	5796
5796	308	676	948	1156	276	500	756	1124	52	5796
5796	484	1188	260	84	740	1108	932	292	708	5796
5796	1140	68	724	692	916	324	244	516	1172	5796
	5796	5796	5796	5796	5796	5796	5796	5796	5796	5796

A5		5805	5805	5805	5805	5805	5805	5805	5805	5805
	117	773	1045	965	373	597	565	1221	149	5805
5805	581	997	357	181	549	1205	1029	101	805	5805
5805	1237	165	533	789	1013	133	341	613	981	5805
5805	213	437	1285	1061	37	837	661	885	389	5805
5805	821	1093	21	421	645	869	1269	197	469	5805
5805	901	405	629	453	1253	229	5	853	1077	5805
5805	309	677	949	1157	277	501	757	1125	53	5805
5805	485	1189	261	85	741	1109	933	293	709	5805
5805	1141	69	725	693	917	325	245	517	1173	5805
	5805	5805	5805	5805	5805	5805	5805	5805	5805	5805

A6		5814	5814	5814	5814	5814	5814	5814	5814	5814
	118	774	1046	966	374	598	566	1222	150	5814
5814	582	998	358	182	550	1206	1030	102	806	5814
5814	1238	166	534	790	1014	134	342	614	982	5814
5814	214	438	1286	1062	38	838	662	886	390	5814
5814	822	1094	22	422	646	870	1270	198	470	5814
5814	902	406	630	454	1254	230	6	854	1078	5814
5814	310	678	950	1158	278	502	758	1126	54	5814
5814	486	1190	262	86	742	1110	934	294	710	5814
5814	1142	70	726	694	918	326	246	518	1174	5814
	5814	5814	5814	5814	5814	5814	5814	5814	5814	5814

A7		5823	5823	5823	5823	5823	5823	5823	5823	5823
	119	775	1047	967	375	599	567	1223	151	5823
5823	583	999	359	183	551	1207	1031	103	807	5823
5823	1239	167	535	791	1015	135	343	615	983	5823
5823	215	439	1287	1063	39	839	663	887	391	5823
5823	823	1095	23	423	647	871	1271	199	471	5823
5823	903	407	631	455	1255	231	7	855	1079	5823
5823	311	679	951	1159	279	503	759	1127	55	5823
5823	487	1191	263	87	743	1111	935	295	711	5823
5823	1143	71	727	695	919	327	247	519	1175	5823
	5823	5823	5823	5823	5823	5823	5823	5823	5823	5823

A8		5832	5832	5832	5832	5832	5832	5832	5832	5832
	120	776	1048	968	376	600	568	1224	152	5832
5832	584	1000	360	184	552	1208	1032	104	808	5832
5832	1240	168	536	792	1016	136	344	616	984	5832
5832	216	440	1288	1064	40	840	664	888	392	5832
5832	824	1096	24	424	648	872	1272	200	472	5832
5832	904	408	632	456	1256	232	8	856	1080	5832
5832	312	680	952	1160	280	504	760	1128	56	5832
5832	488	1192	264	88	744	1112	936	296	712	5832
5832	1144	72	728	696	920	328	248	520	1176	5832
	5832	5832	5832	5832	5832	5832	5832	5832	5832	5832

A9		5841	5841	5841	5841	5841	5841	5841	5841	5841
	121	777	1049	969	377	601	569	1225	153	5841
5841	585	1001	361	185	553	1209	1033	105	809	5841
5841	1241	169	537	793	1017	137	345	617	985	5841
5841	217	441	1289	1065	41	841	665	889	393	5841
5841	825	1097	25	425	649	873	1273	201	473	5841
5841	905	409	633	457	1257	233	9	857	1081	5841
5841	313	681	953	1161	281	505	761	1129	57	5841
5841	489	1193	265	89	745	1113	937	297	713	5841
5841	1145	73	729	697	921	329	249	521	1177	5841
	5841	5841	5841	5841	5841	5841	5841	5841	5841	5841

A10		5850	5850	5850	5850	5850	5850	5850	5850	5850
	122	778	1050	970	378	602	570	1226	154	5850
5850	586	1002	362	186	554	1210	1034	106	810	5850
5850	1242	170	538	794	1018	138	346	618	986	5850
5850	218	442	1290	1066	42	842	666	890	394	5850
5850	826	1098	26	426	650	874	1274	202	474	5850
5850	906	410	634	458	1258	234	10	858	1082	5850
5850	314	682	954	1162	282	506	762	1130	58	5850
5850	490	1194	266	90	746	1114	938	298	714	5850
5850	1146	74	730	698	922	330	250	522	1178	5850
	5850	5850	5850	5850	5850	5850	5850	5850	5850	5850

A11		5859	5859	5859	5859	5859	5859	5859	5859	5859
	123	779	1051	971	379	603	571	1227	155	5859
5859	587	1003	363	187	555	1211	1035	107	811	5859
5859	1243	171	539	795	1019	139	347	619	987	5859
5859	219	443	1291	1067	43	843	667	891	395	5859
5859	827	1099	27	427	651	875	1275	203	475	5859
5859	907	411	635	459	1259	235	11	859	1083	5859
5859	315	683	955	1163	283	507	763	1131	59	5859
5859	491	1195	267	91	747	1115	939	299	715	5859
5859	1147	75	731	699	923	331	251	523	1179	5859
	5859	5859	5859	5859	5859	5859	5859	5859	5859	5859

A12		5868	5868	5868	5868	5868	5868	5868	5868	5868
	124	780	1052	972	380	604	572	1228	156	5868
5868	588	1004	364	188	556	1212	1036	108	812	5868
5868	1244	172	540	796	1020	140	348	620	988	5868
5868	220	444	1292	1068	44	844	668	892	396	5868
5868	828	1100	28	428	652	876	1276	204	476	5868
5868	908	412	636	460	1260	236	12	860	1084	5868
5868	316	684	956	1164	284	508	764	1132	60	5868
5868	492	1196	268	92	748	1116	940	300	716	5868
5868	1148	76	732	700	924	332	252	524	1180	5868
	5868	5868	5868	5868	5868	5868	5868	5868	5868	5868

A13		5877	5877	5877	5877	5877	5877	5877	5877	5877
	125	781	1053	973	381	605	573	1229	157	5877
5877	589	1005	365	189	557	1213	1037	109	813	5877
5877	1245	173	541	797	1021	141	349	621	989	5877
5877	221	445	1293	1069	45	845	669	893	397	5877
5877	829	1101	29	429	653	877	1277	205	477	5877
5877	909	413	637	461	1261	237	13	861	1085	5877
5877	317	685	957	1165	285	509	765	1133	61	5877
5877	493	1197	269	93	749	1117	941	301	717	5877
5877	1149	77	733	701	925	333	253	525	1181	5877
	5877	5877	5877	5877	5877	5877	5877	5877	5877	5877

A14		5884	5884	5884	5884	5884	5884	5884	5884	5884
	126	782	1054	974	382	606	574	1230	158	5884
5884	590	1006	366	190	558	1214	1038	110	814	5884
5884	1246	174	542	798	1022	142	350	622	990	5884
5884	222	446	1294	1070	46	846	670	894	398	5884
5884	830	1102	30	430	654	878	1278	206	478	5884
5884	910	414	638	462	1262	238	14	862	1086	5884
5884	318	686	958	1166	286	510	766	1134	62	5884
5884	494	1198	270	94	750	1118	942	302	718	5884
5884	1150	78	734	702	926	334	254	526	1182	5884
	5884	5884	5884	5884	5884	5884	5884	5884	5884	5884

A15		5895	5895	5895	5895	5895	5895	5895	5895	5895
	127	783	1055	975	383	607	575	1231	159	5895
5895	591	1007	367	191	559	1215	1039	111	815	5895
5895	1247	175	543	799	1023	143	351	623	991	5895
5895	223	447	1295	1071	47	847	671	895	399	5895
5895	831	1103	31	431	655	879	1279	207	479	5895
5895	911	415	639	463	1263	239	15	863	1087	5895
5895	319	687	959	1167	287	511	767	1135	63	5895
5895	495	1199	271	95	751	1119	943	303	719	5895
5895	1151	79	735	703	927	335	255	527	1183	5895
	5895	5895	5895	5895	5895	5895	5895	5895	5895	5895

A16		5904	5904	5904	5904	5904	5904	5904	5904	5904
	128	784	1056	976	384	608	576	1232	160	5904
5904	592	1008	368	192	560	1216	1040	112	816	5904
5904	1248	176	544	800	1024	144	352	624	992	5904
5904	224	448	1296	1072	48	848	672	896	400	5904
5904	832	1104	32	432	656	880	1280	208	480	5904
5904	912	416	640	464	1264	240	16	864	1088	5904
5904	320	688	960	1168	288	512	768	1136	64	5904
5904	496	1200	272	96	752	1120	944	304	720	5904
5904	1152	80	736	704	928	336	256	528	1184	5904
	5904	5904	5904	5904	5904	5904	5904	5904	5904	5904

Let's put above 16 pan magic squares of order 9 according to Structure 1. It results in two **pan magic squares** of orders 4 and 36. Order 4 is obtained using magic sums, and order 36 is obtained with all entries.

Example 26. *The **pan magic square** of order 4 based on the magic square sums given Construction **??** is given by*

		23346	23346	23346	23346
	5823	5868	5769	5886	23346
23346	5778	5877	5832	5859	23346
23346	5904	5787	5850	5805	23346
23346	5841	5814	5895	5796	23346
	23346	23346	23346	23346	23346

Example 27. *A **pan magic square** of order 36 with each sub block a **pan magic square** of order 9 is given by*

(I)	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18
		23346	23346	23346	23346	23346	23346	23346	23346	23346	23346	23346	23346	23346	23346	23346	23346	23346
23346	7	695	1239	23	711	1207	39	679	1223	12	700	1244	28	716	1212	44	684	1228
23346	647	1191	103	663	1159	119	631	1175	135	652	1196	108	668	1164	124	636	1180	140
23346	1287	55	599	1255	71	615	1271	87	583	1292	60	604	1260	76	620	1276	92	588
23346	775	887	279	791	903	247	807	871	263	780	892	284	796	908	252	812	876	268
23346	983	231	727	999	199	743	967	215	759	988	236	732	1004	204	748	972	220	764
23346	183	823	935	151	839	951	167	855	919	188	828	940	156	844	956	172	860	924
23346	1111	359	471	1127	375	439	1143	343	455	1116	364	476	1132	380	444	1148	348	460
23346	311	567	1063	327	535	1079	295	551	1095	316	572	1068	332	540	1084	300	556	1100
23346	519	1015	407	487	1031	423	503	1047	391	524	1020	412	492	1036	428	508	1052	396
23346	2	690	1234	18	706	1202	34	674	1218	13	701	1245	29	717	1213	45	685	1229
23346	642	1186	98	658	1154	114	626	1170	130	653	1197	109	669	1165	125	637	1181	141
23346	1282	50	594	1250	66	610	1266	82	578	1293	61	605	1261	77	621	1277	93	589
23346	770	882	274	786	898	242	802	866	258	781	893	285	797	909	253	813	877	269
23346	978	226	722	994	194	738	962	210	754	989	237	733	1005	205	749	973	221	765
23346	178	818	930	146	834	946	162	850	914	189	829	941	157	845	957	173	861	925
23346	1106	354	466	1122	370	434	1138	338	450	1117	365	477	1133	381	445	1149	349	461
23346	306	562	1058	322	530	1074	290	546	1090	317	573	1069	333	541	1085	301	557	1101
23346	514	1010	402	482	1026	418	498	1042	386	525	1021	413	493	1037	429	509	1053	397
23346	16	704	1248	32	720	1216	48	688	1232	3	691	1235	19	707	1203	35	675	1219
23346	656	1200	112	672	1168	128	640	1184	144	643	1187	99	659	1155	115	627	1171	131
23346	1296	64	608	1264	80	624	1280	96	592	1283	51	595	1251	67	611	1267	83	579
23346	784	896	288	800	912	256	816	880	272	771	883	275	787	899	243	803	867	259
23346	992	240	736	1008	208	752	976	224	768	979	227	723	995	195	739	963	211	755
23346	192	832	944	160	848	960	176	864	928	179	819	931	147	835	947	163	851	915
23346	1120	368	480	1136	384	448	1152	352	464	1107	355	467	1123	371	435	1139	339	451
23346	320	576	1072	336	544	1088	304	560	1104	307	563	1059	323	531	1075	291	547	1091
23346	528	1024	416	496	1040	432	512	1056	400	515	1011	403	483	1027	419	499	1043	387
23346	9	697	1241	25	713	1209	41	681	1225	6	694	1238	22	710	1206	38	678	1222
23346	649	1193	105	665	1161	121	633	1177	137	646	1190	102	662	1158	118	630	1174	134
23346	1289	57	601	1257	73	617	1273	89	585	1286	54	598	1254	70	614	1270	86	582
23346	777	889	281	793	905	249	809	873	265	774	886	278	790	902	246	806	870	262
23346	985	233	729	1001	201	745	969	217	761	982	230	726	998	198	742	966	214	758
23346	185	825	937	153	841	953	169	857	921	182	822	934	150	838	950	166	854	918
23346	1113	361	473	1129	377	441	1145	345	457	1110	358	470	1126	374	438	1142	342	454
23346	313	569	1065	329	537	1081	297	553	1097	310	566	1062	326	534	1078	294	550	1094
23346	521	1017	409	489	1033	425	505	1049	393	518	1014	406	486	1030	422	502	1046	390
	23346	23346	23346	23346	23346	23346	23346	23346	23346	23346	23346	23346	23346	23346	23346	23346	23346	23346

19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	(II)
23346	23346	23346	23346	23346	23346	23346	23346	23346	23346	23346	23346	23346	23346	23346	23346	23346	23346	23346
1	689	1233	17	705	1201	33	673	1217	14	702	1246	30	718	1214	46	686	1230	23346
641	1185	97	657	1153	113	625	1169	129	654	1198	110	670	1166	126	638	1182	142	23346
1281	49	593	1249	65	609	1265	81	577	1294	62	606	1262	78	622	1278	94	590	23346
769	881	273	785	897	241	801	865	257	782	894	286	798	910	254	814	878	270	23346
977	225	721	993	193	737	961	209	753	990	238	734	1006	206	750	974	222	766	23346
177	817	929	145	833	945	161	849	913	190	830	942	158	846	958	174	862	926	23346
1105	353	465	1121	369	433	1137	337	449	1118	366	478	1134	382	446	1150	350	462	23346
305	561	1057	321	529	1073	289	545	1089	318	574	1070	334	542	1086	302	558	1102	23346
513	1009	401	481	1025	417	497	1041	385	526	1022	414	494	1038	430	510	1054	398	23346
8	696	1240	24	712	1208	40	680	1224	11	699	1243	27	715	1211	43	683	1227	23346
648	1192	104	664	1160	120	632	1176	136	651	1195	107	667	1163	123	635	1179	139	23346
1288	56	600	1256	72	616	1272	88	584	1291	59	603	1259	75	619	1275	91	587	23346
776	888	280	792	904	248	808	872	264	779	891	283	795	907	251	811	875	267	23346
984	232	728	1000	200	744	968	216	760	987	235	731	1003	203	747	971	219	763	23346
184	824	936	152	840	952	168	856	920	187	827	939	155	843	955	171	859	923	23346
1112	360	472	1128	376	440	1144	344	456	1115	363	475	1131	379	443	1147	347	459	23346
312	568	1064	328	536	1080	296	552	1096	315	571	1067	331	539	1083	299	555	1099	23346
520	1016	408	488	1032	424	504	1048	392	523	1019	411	491	1035	427	507	1051	395	23346
10	698	1242	26	714	1210	42	682	1226	5	693	1237	21	709	1205	37	677	1221	23346
650	1194	106	666	1162	122	634	1178	138	645	1189	101	661	1157	117	629	1173	133	23346
1290	58	602	1258	74	618	1274	90	586	1285	53	597	1253	69	613	1269	85	581	23346
778	890	282	794	906	250	810	874	266	773	885	277	789	901	245	805	869	261	23346
986	234	730	1002	202	746	970	218	762	981	229	725	997	197	741	965	213	757	23346
186	826	938	154	842	954	170	858	922	181	821	933	149	837	949	165	853	917	23346
1114	362	474	1130	378	442	1146	346	458	1109	357	469	1125	373	437	1141	341	453	23346
314	570	1066	330	538	1082	298	554	1098	309	565	1061	325	533	1077	293	549	1093	23346
522	1018	410	490	1034	426	506	1050	394	517	1013	405	485	1029	421	501	1045	389	23346
15	703	1247	31	719	1215	47	687	1231	4	692	1236	20	708	1204	36	676	1220	23346
655	1199	111	671	1167	127	639	1183	143	644	1188	100	660	1156	116	628	1172	132	23346
1295	63	607	1263	79	623	1279	95	591	1284	52	596	1252	68	612	1268	84	580	23346
783	895	287	799	911	255	815	879	271	772	884	276	788	900	244	804	868	260	23346
991	239	735	1007	207	751	975	223	767	980	228	724	996	196	740	964	212	756	23346
191	831	943	159	847	959	175	863	927	180	820	932	148	836	948	164	852	916	23346
1119	367	479	1135	383	447	1151	351	463	1108	356	468	1124	372	436	1140	340	452	23346
319	575	1071	335	543	1087	303	559	1103	308	564	1060	324	532	1076	292	548	1092	23346
527	1023	415	495	1039	431	511	1055	399	516	1012	404	484	1028	420	500	1044	388	23346
23346	23346	23346	23346	23346	23346	23346	23346	23346	23346	23346	23346	23346	23346	23346	23346	23346	23346	23346

Combiningt Parts (I) and (II) we get the required result. In this case, the magic sum is $S_{36\times 36} = 23346$. Each 9×9 block is a **pan magic square** of order 9 with different magic sums given according to Example 26.

4 Final Comments

In previous works [26, 27], block-wise magic squares are constructed with equal sums blocks. The work is for the magic squares of order 4 to 36. These results are considered in two different situations. One when orders are multiple of 4 and another with orders multiples of 3. In case of orders of type $4k$, the magic squares obtained are **pan magic squares**, where blocks are equal sums **pan magic square** of order 4. The work done is for the magic squares of orders 8, 12, 16, 20, 24, 28 and 32. It is observed that for writing these magic squares just the knowledge of magic square of order 4 is sufficient. The magic square of order 4 used is Khajuraho's **pan magic square** of order 4. In case of order of type $3k$ (excluding multiples of orders $4k$) the results obtained are with equal sub-blocks sums, except the order 18. In this case, the sum of sub-blocks entries are of equal sums. There are situations fo magic squares, where it is impossible to find sub-blocks with equal sums magic squares. For example, in case of order 12, it is impossible to divide equal sums of magic squares of order 3. In case of order 20, it is impossible to divide equal sums sub-blocks of order 5, etc. This paper brings this types of block-wise unequal sums magic squares. In this paper, we brings block-wise constructions of magic squares, where sub-blocks are with different magic sums. This we have done for magic squares of orders, 12 (3×4), 18 (6×3), 20 (5×4), 30 (5×6) and 36 (9×4). The cases of magic squares of orders 12 and 30, two different ways are given to construct these magic squares. The two approaches of magic squares of order 12, one is normal magic square while second is **pan magic square**. Still two extra examples due to Dwane [**?**] are also given for the **pan magic square** of order 12. The Dwane's construction is little different what we applied in our work. The case of magic square of order 30, a second possibility of (6×5) is also considered. Some ideas of composite magic squares can be seen at [3, 4]. Some computerized examples up to order 20 can be done using Danielsson's [7] web-site.

- **Author's Contributions to Magic Squares**

The item-wise author's work on magic squares is as follows:

(i) **Digital numbers** magic squares - [11, 12, 13, 14, 15, 16];
(ii) **Block-wise construction** of **bimagic squares** - [17];
(iii) Connections with **genetic tables** and **Shannon's entropy** - [18];
(iv) **Selfie** and **palindromic-type** magic squares - [19];
(v) **Intervally distributed** and **block-wise** magic squares - [20, 21, 22];
(vi) **Multi-digits** magic squares - [23];
(vii) **Perfect square sum** magic squares with **uniformity** and **minimum Sum** - [24, 25];
(viii) **Pythagorean triples** to generate **perfect square sum** magic squares - [25];
(ix) **Block-wise** equal sums **pan magic squares of order** $4k$ - [26];
(x) **Block-wise** equal sums **magic squares of order** $3k$- [27].

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